Computer algebra independent integration tests

4-Trig-functions/4.5-Secant/4.5.2.1-a+b-sec-^m-c+d-sec-^n

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [241]. This is test number [121].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 99.59 (240)	% 0.41 (1)
Mathematica	% 93.36 (225)	% 6.64 (16)
Maple	% 89.63 (216)	% 10.37 (25)
Maxima	% 39.42 (95)	% 60.58 (146)
Fricas	% 60.17 (145)	% 39.83 (96)
Sympy	% 1.24 (3)	% 98.76 (238)
Giac	% 27.8 (67)	% 72.2 (174)

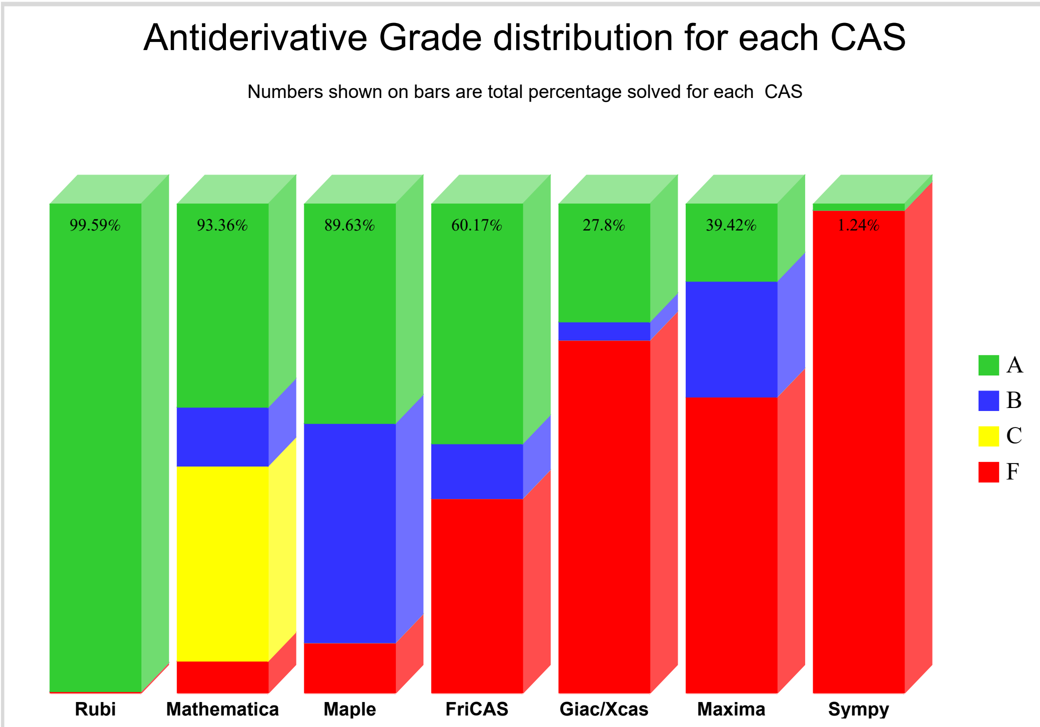
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

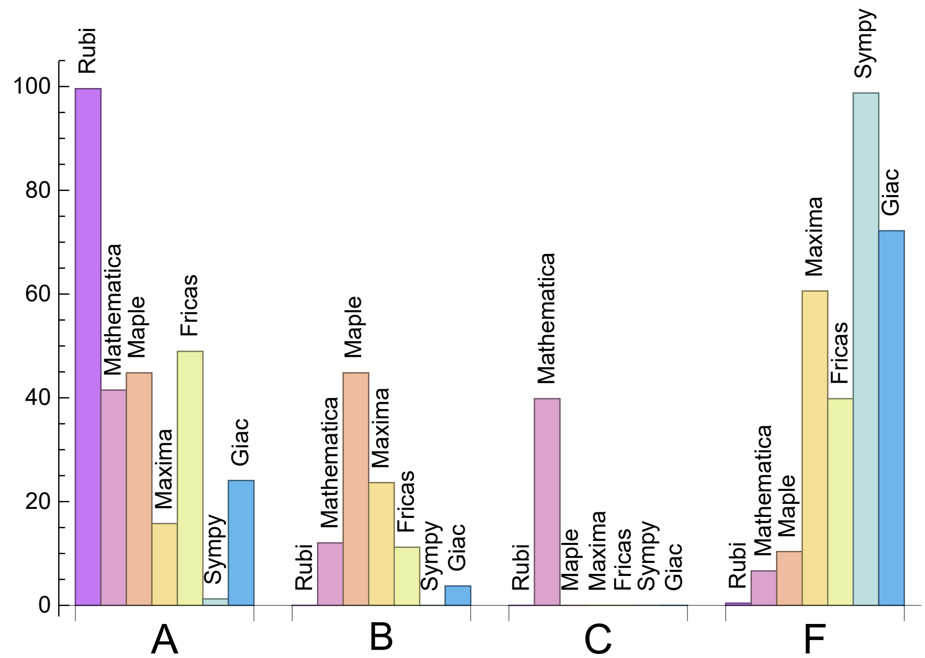
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	99.59	0.	0.	0.41
Mathematica	41.49	12.03	39.83	6.64
Maple	44.81	44.81	0.	10.37
Maxima	15.77	23.65	0.	60.58
Fricas	48.96	11.2	0.	39.83
Sympy	1.24	0.	0.	98.76
Giac	24.07	3.73	0.	72.2

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.35	211.5	0.96	153.	1.
Mathematica	9.02	25113.2	50.45	171.	1.16
Maple	0.87	14801.1	29.56	287.	1.8
Maxima	2.81	919.81	5.72	328.	3.
Fricas	10.24	1446.99	7.87	1006.	7.46
Sympy	0.	0.	0.	0.	0.
Giac	1.43	293.34	1.63	149.	1.47

1.4 list of integrals that has no closed form antiderivative

{221, 222, 223, 224, 225, 226, 227, 228, 229, 236}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {62, 63, 64, 70, 71, 77, 78, 85, 144, 147, 148, 149, 151, 152, 153, 158, 159, 163, 164, 165, 166, 167, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 200, 202, 206, 207, 209, 210, 211, 212, 213, 214, 215, 216, 217, 220, 230, 240, 241}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

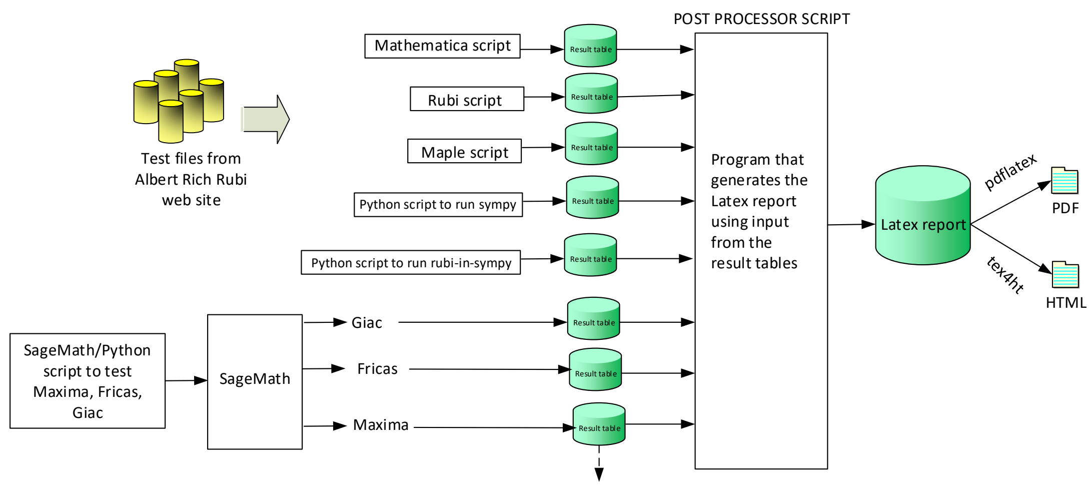
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241 }

B grade: { }

C grade: { }

F grade: { 217}

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 8, 9, 10, 11, 12, 13, 14, 15, 19, 20, 24, 25, 26, 29, 30, 32, 33, 34, 35, 36, 41, 42, 43, 44, 45, 46, 50, 51, 52, 53, 54, 57, 58, 59, 60, 61, 65, 66, 67, 68, 69, 72, 73, 74, 75, 76, 79, 80, 81, 82, 83, 84, 145, 146, 150, 154, 155, 156, 157, 160, 161, 162, 164, 168, 184, 185, 186, 187, 188, 189, 190, 191, 192, 194, 196, 199, 201, 203, 204, 208, 218, 221, 222, 223, 224, 225, 226, 227, 228, 229, 233, 236, 237, 238, 239 }

B grade: { 6, 16, 17, 21, 22, 23, 28, 31, 37, 39, 40, 144, 193, 195, 197, 200, 202, 209, 210, 211, 212, 213, 214, 215, 216, 220, 230, 240, 241 }

C grade: { 7, 18, 27, 38, 47, 48, 49, 55, 56, 62, 63, 64, 70, 71, 77, 78, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 143, 147, 148, 149, 151, 152, 153, 158, 159, 163, 165, 166, 167, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 198, 205, 206, 207, 217, 219 }

F grade: { 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 231, 232, 234, 235}

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 40, 41, 44, 45, 46, 51, 60, 68, 69, 86, 87, 88, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 108, 109, 110, 111, 112, 113, 115, 116, 117, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 143, 144, 145, 146, 149, 189, 201, 203, 204, 207, 208, 217, 218, 219, 221, 222, 223, 224, 225, 226, 227, 228, 229, 236 }

B grade: { 42, 43, 47, 48, 49, 50, 52, 53, 54, 55, 56, 57, 58, 59, 61, 62, 63, 64, 65, 66, 67, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 89, 90, 105, 106, 107, 114, 118, 147, 148, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 202, 205, 206, 209, 210, 211, 212, 213, 214, 215, 216, 220 }

C grade: { }

F grade: { 27, 38, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241 }

2.1.4 Maxima

A grade: { 1, 2, 4, 5, 13, 15, 26, 27, 28, 29, 30, 35, 36, 37, 38, 39, 40, 41, 89, 90, 97, 98, 106, 112, 113, 114, 119, 125, 221, 222, 223, 224, 225, 226, 227, 228, 229, 236

B grade: { 3, 6, 7, 8, 9, 10, 11, 12, 14, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 31, 32, 33, 34, 45, 52, 59, 86, 87, 88, 91, 92, 93, 94, 95, 96, 99, 100, 101, 102, 103, 107, 108, 115, 116, 117, 120, 121, 122, 123, 126, 127, 128, 129, 130, 150, 156, 162 }

C grade: { }

F grade: { 42, 43, 44, 46, 47, 48, 49, 50, 51, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 104, 105, 109, 110, 111, 118, 124, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 151, 152, 153, 154, 155, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 77, 78, 79, 80, 81, 82, 86, 87, 88, 94, 95, 96, 101, 102, 103, 130, 143, 147, 148, 149, 150, 151, 152, 154, 155, 156, 157, 158, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 178, 179, 184, 185, 187, 188, 189, 236 }

B grade: { 5, 22, 23, 61, 62, 63, 74, 75, 83, 89, 105, 114, 118, 122, 153, 159, 174, 180, 186, 190, 191, 192, 193, 194, 195, 196, 197 }

C grade: { }

F grade: { 76, 84, 85, 90, 91, 92, 93, 97, 98, 99, 100, 104, 106, 107, 108, 109, 110, 111, 112, 113, 115, 116, 117, 119, 120, 121, 123, 124, 125, 126, 127, 128, 129, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 144, 145, 146, 176, 177, 181, 182, 183, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241 }

2.1.6 Sympy

A grade: { 221, 224, 236 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 222, 223, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 40, 41, 113, 118, 119, 120, 125, 126, 127, 189, 190, 192, 221, 222, 223, 224, 225, 226, 227, 228, 229, 236 }

B grade: { 5, 27, 38, 191, 193, 194, 195, 196, 197 }

C grade: { }

F grade: { 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 114, 115, 116, 117, 121, 122, 123, 124, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	165	186	451	452	0	271
normalized size	1	1.	0.84	0.95	2.3	2.31	0.	1.38
time (sec)	N/A	0.302	2.003	0.033	1.032	1.171	0.	1.519

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	146	161	324	404	0	244
normalized size	1	1.	1.04	1.15	2.31	2.89	0.	1.74
time (sec)	N/A	0.199	1.152	0.028	1.038	1.135	0.	1.446

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	122	136	274	359	0	217
normalized size	1	1.	1.26	1.4	2.82	3.7	0.	2.24
time (sec)	N/A	0.118	0.738	0.025	1.029	1.164	0.	1.52

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	45	58	77	144	0	69
normalized size	1	1.	0.96	1.23	1.64	3.06	0.	1.47
time (sec)	N/A	0.065	0.034	0.021	1.014	1.077	0.	1.434

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	72	76	128	259	0	147
normalized size	1	1.	1.31	1.38	2.33	4.71	0.	2.67
time (sec)	N/A	0.063	0.291	0.02	1.07	1.076	0.	1.361

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	169	90	207	217	0	109
normalized size	1	1.	3.02	1.61	3.7	3.88	0.	1.95
time (sec)	N/A	0.164	0.286	0.079	1.599	1.096	0.	1.384

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	53	67	235	204	0	81
normalized size	1	1.	0.75	0.94	3.31	2.87	0.	1.14
time (sec)	N/A	0.236	0.057	0.091	1.633	1.034	0.	1.342

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	171	89	290	313	0	103
normalized size	1	1.	1.68	0.87	2.84	3.07	0.	1.01
time (sec)	N/A	0.329	0.62	0.1	1.78	1.027	0.	1.366

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	227	111	397	424	0	126
normalized size	1	1.	1.71	0.83	2.98	3.19	0.	0.95
time (sec)	N/A	0.427	0.616	0.109	1.602	1.054	0.	1.365

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	283	133	452	535	0	149
normalized size	1	1.	1.73	0.81	2.76	3.26	0.	0.91
time (sec)	N/A	0.543	0.886	0.119	1.635	1.05	0.	1.652

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	189	211	481	498	0	298
normalized size	1	1.	1.01	1.12	2.56	2.65	0.	1.59
time (sec)	N/A	0.237	2.219	0.035	1.01	1.219	0.	1.429

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	165	186	451	448	0	271
normalized size	1	1.	1.25	1.41	3.42	3.39	0.	2.05
time (sec)	N/A	0.151	1.834	0.028	1.02	1.188	0.	1.361

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	61	93	127	189	0	93
normalized size	1	1.	0.9	1.37	1.87	2.78	0.	1.37
time (sec)	N/A	0.074	0.04	0.023	1.035	1.066	0.	1.475

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	122	136	274	359	0	217
normalized size	1	1.	1.26	1.4	2.82	3.7	0.	2.24
time (sec)	N/A	0.114	0.791	0.025	1.019	1.147	0.	1.345

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	101	98	144	298	0	149
normalized size	1	1.	1.31	1.27	1.87	3.87	0.	1.94
time (sec)	N/A	0.147	0.44	0.023	1.018	1.109	0.	1.392

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	240	137	370	313	0	158
normalized size	1	1.	3.08	1.76	4.74	4.01	0.	2.03
time (sec)	N/A	0.209	2.369	0.083	1.553	1.097	0.	1.35

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	177	90	370	379	0	113
normalized size	1	1.	2.01	1.02	4.2	4.31	0.	1.28
time (sec)	N/A	0.361	1.17	0.087	1.533	1.125	0.	1.346

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	53	89	381	312	0	104
normalized size	1	1.	0.52	0.87	3.74	3.06	0.	1.02
time (sec)	N/A	0.452	0.085	0.104	1.587	1.073	0.	1.441

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	227	111	517	424	0	126
normalized size	1	1.	1.71	0.83	3.89	3.19	0.	0.95
time (sec)	N/A	0.579	0.611	0.114	1.563	1.046	0.	1.377

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	283	133	544	535	0	149
normalized size	1	1.	1.73	0.81	3.32	3.26	0.	0.91
time (sec)	N/A	0.733	0.88	0.128	1.664	1.115	0.	1.509

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	153	384	207	814	601	0	217
normalized size	1	1.12	2.82	1.52	5.99	4.42	0.	1.6
time (sec)	N/A	0.402	3.019	0.105	1.591	1.15	0.	1.466

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	448	159	558	541	0	190
normalized size	1	1.	4.39	1.56	5.47	5.3	0.	1.86
time (sec)	N/A	0.309	6.016	0.088	1.543	1.132	0.	1.328

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	216	90	362	425	0	113
normalized size	1	1.	2.54	1.06	4.26	5.	0.	1.33
time (sec)	N/A	0.331	1.092	0.086	1.581	1.076	0.	1.338

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	67	65	230	225	0	85
normalized size	1	1.	1.	0.97	3.43	3.36	0.	1.27
time (sec)	N/A	0.228	0.052	0.08	1.559	1.044	0.	1.236

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	113	59	161	212	0	76
normalized size	1	1.	1.85	0.97	2.64	3.48	0.	1.25
time (sec)	N/A	0.145	0.314	0.08	1.548	1.025	0.	1.424

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	135	87	138	180	0	115
normalized size	1	1.	1.96	1.26	2.	2.61	0.	1.67
time (sec)	N/A	0.114	0.543	0.056	1.516	1.042	0.	1.343

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	39	0	62	188	0	135
normalized size	1	1.	0.85	0.	1.35	4.09	0.	2.93
time (sec)	N/A	0.071	0.05	180.	1.525	1.058	0.	1.411

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	257	130	198	374	0	157
normalized size	1	1.	2.62	1.33	2.02	3.82	0.	1.6
time (sec)	N/A	0.145	1.314	0.065	1.546	1.034	0.	1.478

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	315	153	225	424	0	174
normalized size	1	1.	1.9	0.92	1.36	2.55	0.	1.05
time (sec)	N/A	0.211	1.257	0.071	1.529	1.085	0.	1.433

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	383	175	251	598	0	193
normalized size	1	1.	1.82	0.83	1.2	2.85	0.	0.92
time (sec)	N/A	0.286	1.235	0.074	1.555	1.104	0.	1.406

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	557	179	759	721	0	219
normalized size	1	1.	3.44	1.1	4.69	4.45	0.	1.35
time (sec)	N/A	0.443	5.623	0.099	1.586	1.19	0.	1.549

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	231	110	535	601	0	144
normalized size	1	1.	1.56	0.74	3.61	4.06	0.	0.97
time (sec)	N/A	0.606	1.175	0.103	1.596	1.132	0.	1.456

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	90	87	374	340	0	113
normalized size	1	1.	0.94	0.91	3.9	3.54	0.	1.18
time (sec)	N/A	0.421	0.079	0.102	1.646	1.024	0.	1.481

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	171	87	285	338	0	113
normalized size	1	1.	1.78	0.91	2.97	3.52	0.	1.18
time (sec)	N/A	0.304	0.461	0.091	1.572	1.002	0.	1.376

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	169	79	215	313	0	101
normalized size	1	1.	1.92	0.9	2.44	3.56	0.	1.15
time (sec)	N/A	0.202	0.457	0.086	1.574	0.995	0.	1.377

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	197	109	165	284	0	144
normalized size	1	1.	1.56	0.87	1.31	2.25	0.	1.14
time (sec)	N/A	0.193	0.929	0.061	1.553	1.039	0.	1.332

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	257	131	197	374	0	165
normalized size	1	1.	2.57	1.31	1.97	3.74	0.	1.65
time (sec)	N/A	0.143	0.917	0.063	1.546	1.08	0.	1.392

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	39	0	76	290	0	184
normalized size	1	1.	0.58	0.	1.13	4.33	0.	2.75
time (sec)	N/A	0.081	0.074	180.	1.575	1.119	0.	1.456

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	362	174	252	581	0	203
normalized size	1	1.	2.81	1.35	1.95	4.5	0.	1.57
time (sec)	N/A	0.173	1.371	0.069	1.564	1.062	0.	1.511

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	441	197	277	687	0	220
normalized size	1	1.	2.1	0.94	1.32	3.27	0.	1.05
time (sec)	N/A	0.236	1.729	0.075	1.549	1.163	0.	1.44

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	499	219	306	799	0	242
normalized size	1	1.	1.98	0.87	1.21	3.17	0.	0.96
time (sec)	N/A	0.3	2.216	0.076	1.606	1.16	0.	1.3

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	121	391	0	960	0	0
normalized size	1	1.	0.69	2.23	0.	5.49	0.	0.
time (sec)	N/A	0.176	1.012	0.319	0.	1.262	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	111	302	0	882	0	0
normalized size	1	1.	0.79	2.16	0.	6.3	0.	0.
time (sec)	N/A	0.168	1.206	0.291	0.	1.21	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	97	142	0	792	0	0
normalized size	1	1.	0.92	1.35	0.	7.54	0.	0.
time (sec)	N/A	0.158	0.772	0.263	0.	1.172	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	70	115	198	620	0	0
normalized size	1	1.	1.06	1.74	3.	9.39	0.	0.
time (sec)	N/A	0.109	0.32	0.212	1.713	1.146	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	90	116	0	680	0	0
normalized size	1	1.	1.3	1.68	0.	9.86	0.	0.
time (sec)	N/A	0.148	0.58	0.251	0.	1.465	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	78	214	0	872	0	0
normalized size	1	1.	0.75	2.06	0.	8.38	0.	0.
time (sec)	N/A	0.157	0.241	0.289	0.	1.449	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	78	311	0	1056	0	0
normalized size	1	1.	0.56	2.24	0.	7.6	0.	0.
time (sec)	N/A	0.167	0.26	0.329	0.	1.566	0.	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	78	402	0	1245	0	0
normalized size	1	1.	0.45	2.31	0.	7.16	0.	0.
time (sec)	N/A	0.177	0.25	0.37	0.	1.646	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	122	392	0	990	0	0
normalized size	1	1.	0.69	2.21	0.	5.59	0.	0.
time (sec)	N/A	0.184	0.959	0.272	0.	1.275	0.	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	112	232	0	900	0	0
normalized size	1	1.	0.79	1.63	0.	6.34	0.	0.
time (sec)	N/A	0.17	0.842	0.247	0.	1.196	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	96	212	1347	792	0	0
normalized size	1	1.	0.95	2.1	13.34	7.84	0.	0.
time (sec)	N/A	0.123	0.67	0.222	1.912	1.173	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	93	194	0	689	0	0
normalized size	1	1.	1.33	2.77	0.	9.84	0.	0.
time (sec)	N/A	0.156	0.577	0.191	0.	1.409	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	113	215	0	888	0	0
normalized size	1	1.	1.11	2.11	0.	8.71	0.	0.
time (sec)	N/A	0.162	0.653	0.255	0.	1.517	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	102	304	0	1083	0	0
normalized size	1	1.	0.74	2.22	0.	7.91	0.	0.
time (sec)	N/A	0.178	0.77	0.276	0.	1.577	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	102	401	0	1283	0	0
normalized size	1	1.	0.59	2.33	0.	7.46	0.	0.
time (sec)	N/A	0.187	1.327	0.314	0.	1.625	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	134	483	0	1098	0	0
normalized size	1	1.	0.63	2.28	0.	5.18	0.	0.
time (sec)	N/A	0.191	1.214	0.28	0.	1.307	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	124	323	0	1019	0	0
normalized size	1	1.	0.7	1.82	0.	5.76	0.	0.
time (sec)	N/A	0.174	0.921	0.26	0.	1.237	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	110	303	1885	887	0	0
normalized size	1	1.	0.83	2.3	14.28	6.72	0.	0.
time (sec)	N/A	0.141	0.803	0.229	2.032	1.245	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	96	120	0	724	0	0
normalized size	1	1.	0.93	1.17	0.	7.03	0.	0.
time (sec)	N/A	0.172	0.68	0.227	0.	1.471	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	102	351	0	844	0	0
normalized size	1	1.	1.38	4.74	0.	11.41	0.	0.
time (sec)	N/A	0.168	4.226	0.24	0.	1.491	0.	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	B	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	104	104	196	306	0	1106	0	0
normalized size	1	1.	1.88	2.94	0.	10.63	0.	0.
time (sec)	N/A	0.177	5.409	0.27	0.	1.55	0.	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	B	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	140	140	361	395	0	1310	0	0
normalized size	1	1.	2.58	2.82	0.	9.36	0.	0.
time (sec)	N/A	0.184	7.938	0.295	0.	1.611	0.	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	172	172	205	492	0	1512	0	0
normalized size	1	1.	1.19	2.86	0.	8.79	0.	0.
time (sec)	N/A	0.198	3.477	0.357	0.	1.67	0.	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	153	544	0	1420	0	0
normalized size	1	1.	0.83	2.94	0.	7.68	0.	0.
time (sec)	N/A	0.276	1.424	0.321	0.	5.261	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	166	372	0	1342	0	0
normalized size	1	1.	1.09	2.45	0.	8.83	0.	0.
time (sec)	N/A	0.228	1.316	0.299	0.	4.499	0.	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	124	330	0	1138	0	0
normalized size	1	1.	1.04	2.77	0.	9.56	0.	0.
time (sec)	N/A	0.188	0.475	0.277	0.	2.759	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	82	144	0	787	0	0
normalized size	1	1.	0.94	1.66	0.	9.05	0.	0.
time (sec)	N/A	0.138	0.276	0.219	0.	2.119	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	101	194	0	1143	0	0
normalized size	1	1.	0.83	1.6	0.	9.45	0.	0.
time (sec)	N/A	0.192	0.553	0.267	0.	2.359	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	161	161	5586	377	0	1395	0	0
normalized size	1	1.	34.7	2.34	0.	8.66	0.	0.
time (sec)	N/A	0.235	23.791	0.303	0.	2.321	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	196	196	5602	545	0	1627	0	0
normalized size	1	1.	28.58	2.78	0.	8.3	0.	0.
time (sec)	N/A	0.284	23.501	0.359	0.	2.623	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	196	552	0	1616	0	0
normalized size	1	1.	0.97	2.72	0.	7.96	0.	0.
time (sec)	N/A	0.286	1.53	0.28	0.	10.802	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	132	376	0	1405	0	0
normalized size	1	1.	0.78	2.22	0.	8.31	0.	0.
time (sec)	N/A	0.24	1.739	0.257	0.	5.021	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	134	128	369	0	1389	0	0
normalized size	1	1.13	1.08	3.1	0.	11.67	0.	0.
time (sec)	N/A	0.192	1.068	0.189	0.	3.802	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	130	130	371	0	1347	0	0
normalized size	1	1.15	1.15	3.28	0.	11.92	0.	0.
time (sec)	N/A	0.166	0.993	0.181	0.	4.327	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	154	377	0	0	0	0
normalized size	1	1.	0.87	2.13	0.	0.	0.	0.
time (sec)	N/A	0.244	1.174	0.254	0.	0.	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	214	214	5622	387	0	1478	0	0
normalized size	1	1.	26.27	1.81	0.	6.91	0.	0.
time (sec)	N/A	0.278	23.793	0.293	0.	2.008	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	249	249	5639	725	0	1867	0	0
normalized size	1	1.	22.65	2.91	0.	7.5	0.	0.
time (sec)	N/A	0.34	23.795	0.362	0.	2.18	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	260	180	726	0	1879	0	0
normalized size	1	1.	0.69	2.79	0.	7.23	0.	0.
time (sec)	N/A	0.341	3.485	0.308	0.	14.54	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	164	550	0	1673	0	0
normalized size	1	1.	0.72	2.4	0.	7.31	0.	0.
time (sec)	N/A	0.296	2.599	0.272	0.	11.877	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	136	553	0	1643	0	0
normalized size	1	1.	0.71	2.9	0.	8.6	0.	0.
time (sec)	N/A	0.252	1.537	0.252	0.	9.106	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	136	545	0	1655	0	0
normalized size	1	1.	0.72	2.88	0.	8.76	0.	0.
time (sec)	N/A	0.23	1.482	0.194	0.	6.558	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	181	134	543	0	1605	0	0
normalized size	1	1.22	0.91	3.67	0.	10.84	0.	0.
time (sec)	N/A	0.192	1.458	0.179	0.	3.964	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	158	545	0	0	0	0
normalized size	1	1.	0.69	2.37	0.	0.	0.	0.
time (sec)	N/A	0.306	1.443	0.265	0.	0.	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-2)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	269	269	5660	725	0	0	0	0
normalized size	1	1.	21.04	2.7	0.	0.	0.	0.
time (sec)	N/A	0.335	24.052	0.326	0.	0.	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	149	194	1740	1135	0	0
normalized size	1	1.	0.81	1.05	9.41	6.14	0.	0.
time (sec)	N/A	0.372	5.818	0.356	2.141	1.696	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	162	184	959	1052	0	0
normalized size	1	1.	1.17	1.32	6.9	7.57	0.	0.
time (sec)	N/A	0.273	2.16	0.309	1.882	1.69	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	99	164	328	887	0	0
normalized size	1	1.	1.06	1.76	3.53	9.54	0.	0.
time (sec)	N/A	0.178	1.226	0.297	1.796	1.657	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	102	127	53	506	0	0
normalized size	1	1.	2.12	2.65	1.1	10.54	0.	0.
time (sec)	N/A	0.084	0.573	0.308	1.838	1.62	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	86	97	88	0	0	0
normalized size	1	1.	1.69	1.9	1.73	0.	0.	0.
time (sec)	N/A	0.086	0.878	0.276	1.52	0.	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	107	164	539	0	0	0
normalized size	1	1.	1.11	1.71	5.61	0.	0.	0.
time (sec)	N/A	0.179	1.001	0.293	1.803	0.	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	152	226	1584	0	0	0
normalized size	1	1.	1.07	1.59	11.15	0.	0.	0.
time (sec)	N/A	0.271	1.231	0.303	2.415	0.	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	198	288	3299	0	0	0
normalized size	1	1.	1.05	1.53	17.55	0.	0.	0.
time (sec)	N/A	0.366	1.784	0.306	11.007	0.	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	157	187	1831	1154	0	0
normalized size	1	1.	0.83	0.98	9.64	6.07	0.	0.
time (sec)	N/A	0.364	1.253	0.29	2.173	1.697	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	159	171	644	873	0	0
normalized size	1	1.	1.54	1.66	6.25	8.48	0.	0.
time (sec)	N/A	0.11	1.371	0.276	1.824	1.688	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	128	151	328	884	0	0
normalized size	1	1.	1.38	1.62	3.53	9.51	0.	0.
time (sec)	N/A	0.169	0.727	0.309	1.781	1.66	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	105	149	81	0	0	0
normalized size	1	1.	1.01	1.43	0.78	0.	0.	0.
time (sec)	N/A	0.103	1.185	0.262	1.846	0.	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	115	163	128	0	0	0
normalized size	1	1.	1.15	1.63	1.28	0.	0.	0.
time (sec)	N/A	0.186	0.671	0.268	1.535	0.	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	153	227	2411	0	0	0
normalized size	1	1.	1.05	1.55	16.51	0.	0.	0.
time (sec)	N/A	0.283	1.21	0.265	2.512	0.	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	199	289	4698	0	0	0
normalized size	1	1.	1.02	1.47	23.97	0.	0.	0.
time (sec)	N/A	0.382	2.096	0.275	11.301	0.	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	164	191	2186	987	0	0
normalized size	1	1.	1.07	1.25	14.29	6.45	0.	0.
time (sec)	N/A	0.12	1.492	0.299	2.432	1.742	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	149	199	1831	1152	0	0
normalized size	1	1.	0.78	1.05	9.64	6.06	0.	0.
time (sec)	N/A	0.363	1.166	0.281	2.19	1.727	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	164	179	959	1049	0	0
normalized size	1	1.	1.18	1.29	6.9	7.55	0.	0.
time (sec)	N/A	0.263	1.259	0.307	1.843	1.641	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	292	183	0	0	0	0
normalized size	1	1.	1.92	1.2	0.	0.	0.	0.
time (sec)	N/A	0.114	6.767	0.319	0.	0.	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	111	237	0	1062	0	0
normalized size	1	1.	1.16	2.47	0.	11.06	0.	0.
time (sec)	N/A	0.179	1.22	0.27	0.	1.412	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	155	229	188	0	0	0
normalized size	1	1.	1.55	2.29	1.88	0.	0.	0.
time (sec)	N/A	0.181	1.37	0.266	1.514	0.	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	202	281	5046	0	0	0
normalized size	1	1.	1.36	1.9	34.09	0.	0.	0.
time (sec)	N/A	0.278	2.615	0.277	11.149	0.	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	285	353	8281	0	0	0
normalized size	1	1.	1.47	1.82	42.69	0.	0.	0.
time (sec)	N/A	0.376	5.435	0.282	76.433	0.	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	299	415	0	0	0	0
normalized size	1	1.	1.23	1.7	0.	0.	0.	0.
time (sec)	N/A	0.474	5.949	0.292	0.	0.	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	153	189	0	0	0	0
normalized size	1	1.	0.75	0.93	0.	0.	0.	0.
time (sec)	N/A	0.124	15.341	0.309	0.	0.	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	181	169	0	0	0	0
normalized size	1	1.	1.2	1.12	0.	0.	0.	0.
time (sec)	N/A	0.113	3.886	0.3	0.	0.	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	103	93	81	0	0	0
normalized size	1	1.	1.01	0.91	0.79	0.	0.	0.
time (sec)	N/A	0.107	9.154	0.287	1.811	0.	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	127	75	46	0	0	92
normalized size	1	1.	2.59	1.53	0.94	0.	0.	1.88
time (sec)	N/A	0.084	0.911	0.302	1.576	0.	0.	1.796

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	104	100	53	682	0	0
normalized size	1	1.	2.26	2.17	1.15	14.83	0.	0.
time (sec)	N/A	0.09	1.069	0.26	1.772	1.876	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	217	143	167	1104	0	0	0
normalized size	1	1.29	0.85	0.99	6.57	0.	0.	0.
time (sec)	N/A	0.137	8.145	0.294	1.941	0.	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	194	229	2978	0	0	0
normalized size	1	1.	0.71	0.84	10.87	0.	0.	0.
time (sec)	N/A	0.161	1.879	0.303	2.55	0.	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	204	276	3231	0	0	0
normalized size	1	1.	0.95	1.28	15.03	0.	0.	0.
time (sec)	N/A	0.145	2.082	0.276	2.719	0.	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	116	236	0	1098	0	185
normalized size	1	1.	1.21	2.46	0.	11.44	0.	1.93
time (sec)	N/A	0.193	0.731	0.27	0.	1.353	0.	3.058

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	114	106	95	0	0	170
normalized size	1	1.	1.16	1.08	0.97	0.	0.	1.73
time (sec)	N/A	0.196	1.113	0.256	1.493	0.	0.	2.348

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	106	119	533	0	0	158
normalized size	1	1.	1.13	1.27	5.67	0.	0.	1.68
time (sec)	N/A	0.189	0.561	0.291	1.789	0.	0.	1.9

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	141	161	1104	0	0	0
normalized size	1	1.	0.66	0.75	5.13	0.	0.	0.
time (sec)	N/A	0.146	1.384	0.3	1.916	0.	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	121	175	656	1242	0	0
normalized size	1	1.	1.2	1.73	6.5	12.3	0.	0.
time (sec)	N/A	0.121	1.478	0.264	1.837	2.438	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	347	347	275	291	5767	0	0	0
normalized size	1	1.	0.79	0.84	16.62	0.	0.	0.
time (sec)	N/A	0.184	2.553	0.273	11.272	0.	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	157	335	0	0	0	0
normalized size	1	1.	0.71	1.52	0.	0.	0.	0.
time (sec)	N/A	0.144	2.395	0.26	0.	0.	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	154	144	138	0	0	170
normalized size	1	1.	1.57	1.47	1.41	0.	0.	1.73
time (sec)	N/A	0.187	1.52	0.252	1.511	0.	0.	7.161

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	152	152	2411	0	0	316
normalized size	1	1.	1.06	1.06	16.74	0.	0.	2.19
time (sec)	N/A	0.288	0.839	0.253	2.546	0.	0.	4.428

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	151	152	1573	0	0	215
normalized size	1	1.	1.08	1.09	11.24	0.	0.	1.54
time (sec)	N/A	0.282	0.641	0.291	2.434	0.	0.	2.403

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	195	223	2978	0	0	0
normalized size	1	1.	0.72	0.83	11.03	0.	0.	0.
time (sec)	N/A	0.154	1.605	0.298	2.602	0.	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	345	345	275	286	5767	0	0	0
normalized size	1	1.	0.8	0.83	16.72	0.	0.	0.
time (sec)	N/A	0.181	2.263	0.268	11.514	0.	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	149	237	1871	1446	0	0
normalized size	1	1.	0.99	1.57	12.39	9.58	0.	0.
time (sec)	N/A	0.131	2.105	0.276	2.487	3.188	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	92	92	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.094	1.042	0.407	0.	0.	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	109	109	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.122	0.429	0.392	0.	0.	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.1	3.217	0.427	0.	0.	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.099	1.443	0.389	0.	0.	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	99	99	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.075	1.577	0.391	0.	0.	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	99	99	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.1	0.97	0.396	0.	0.	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.1	1.574	0.287	0.	0.	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	172	172	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.145	8.511	0.282	0.	0.	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	119	119	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.162	11.806	0.302	0.	0.	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	68	68	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.083	0.147	0.302	0.	0.	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	139	139	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.112	1.369	0.295	0.	0.	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	205	205	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.168	1.653	0.264	0.	0.	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	133	141	0	782	0	0
normalized size	1	1.	1.46	1.55	0.	8.59	0.	0.
time (sec)	N/A	0.082	0.706	0.227	0.	1.584	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	231	231	814	295	0	0	0	0
normalized size	1	1.	3.52	1.28	0.	0.	0.	0.
time (sec)	N/A	0.257	17.997	0.398	0.	0.	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	180	285	0	0	0	0
normalized size	1	1.	0.8	1.27	0.	0.	0.	0.
time (sec)	N/A	0.211	8.52	0.414	0.	0.	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	319	319	193	327	0	0	0	0
normalized size	1	1.	0.61	1.03	0.	0.	0.	0.
time (sec)	N/A	0.37	5.374	0.351	0.	0.	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	271	271	589	546	0	1181	0	0
normalized size	1	1.	2.17	2.01	0.	4.36	0.	0.
time (sec)	N/A	0.173	14.618	0.413	0.	1.699	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	205	205	519	389	0	976	0	0
normalized size	1	1.	2.53	1.9	0.	4.76	0.	0.
time (sec)	N/A	0.141	14.566	0.307	0.	1.592	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	144	144	444	248	0	819	0	0
normalized size	1	1.	3.08	1.72	0.	5.69	0.	0.
time (sec)	N/A	0.116	6.591	0.272	0.	1.301	0.	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	76	118	198	620	0	0
normalized size	1	1.	1.15	1.79	3.	9.39	0.	0.
time (sec)	N/A	0.087	0.316	0.214	1.678	1.155	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	105	105	2686	501	0	1704	0	0
normalized size	1	1.	25.58	4.77	0.	16.23	0.	0.
time (sec)	N/A	0.23	24.991	0.288	0.	2.954	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	219	219	2943	97143	0	3494	0	0
normalized size	1	1.	13.44	443.58	0.	15.95	0.	0.
time (sec)	N/A	0.222	28.425	1.534	0.	13.778	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	287	287	3106	330372	0	5542	0	0
normalized size	1	1.	10.82	1151.12	0.	19.31	0.	0.
time (sec)	N/A	0.307	24.137	14.995	0.	27.238	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	219	539	0	1230	0	0
normalized size	1	1.	0.91	2.24	0.	5.1	0.	0.
time (sec)	N/A	0.2	3.869	0.316	0.	1.345	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	145	382	0	1006	0	0
normalized size	1	1.	0.82	2.17	0.	5.72	0.	0.
time (sec)	N/A	0.144	1.336	0.271	0.	1.217	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	102	237	1347	819	0	0
normalized size	1	1.	0.97	2.26	12.83	7.8	0.	0.
time (sec)	N/A	0.15	0.562	0.229	1.903	1.204	0.	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	135	864	0	1804	0	0
normalized size	1	1.	1.23	7.85	0.	16.4	0.	0.
time (sec)	N/A	0.241	0.478	0.217	0.	8.844	0.	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	229	229	2886	62283	0	4466	0	0
normalized size	1	1.	12.6	271.98	0.	19.5	0.	0.
time (sec)	N/A	0.249	24.763	1.636	0.	30.339	0.	0.

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	310	310	3190	234091	0	6322	0	0
normalized size	1	1.	10.29	755.13	0.	20.39	0.	0.
time (sec)	N/A	0.342	24.581	10.07	0.	79.148	0.	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	336	286	677	0	1513	0	0
normalized size	1	1.	0.85	2.01	0.	4.5	0.	0.
time (sec)	N/A	0.209	6.189	0.343	0.	1.095	0.	0.

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	258	191	504	0	1233	0	0
normalized size	1	1.	0.74	1.95	0.	4.78	0.	0.
time (sec)	N/A	0.177	2.684	0.283	0.	1.103	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	128	341	1885	976	0	0
normalized size	1	1.	0.9	2.4	13.27	6.87	0.	0.
time (sec)	N/A	0.232	0.922	0.241	1.97	1.025	0.	0.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	203	203	343	1487	0	2773	0	0
normalized size	1	1.	1.69	7.33	0.	13.66	0.	0.
time (sec)	N/A	0.232	6.489	0.237	0.	19.611	0.	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	329	329	280	46082	0	4579	0	0
normalized size	1	1.	0.85	140.07	0.	13.92	0.	0.
time (sec)	N/A	0.337	3.725	0.848	0.	89.091	0.	0.

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	536	536	3368	209489	0	7337	0	0
normalized size	1	1.	6.28	390.84	0.	13.69	0.	0.
time (sec)	N/A	0.492	25.668	6.465	0.	172.515	0.	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	258	258	787	907	0	1553	0	0
normalized size	1	1.	3.05	3.52	0.	6.02	0.	0.
time (sec)	N/A	0.207	8.115	0.307	0.	72.664	0.	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	183	183	295	358	0	1246	0	0
normalized size	1	1.	1.61	1.96	0.	6.81	0.	0.
time (sec)	N/A	0.158	2.468	0.24	0.	16.946	0.	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	92	194	0	824	0	0
normalized size	1	1.	1.01	2.13	0.	9.05	0.	0.
time (sec)	N/A	0.11	0.287	0.217	0.	5.146	0.	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	166	166	431238	662	0	2695	0	0
normalized size	1	1.	2597.82	3.99	0.	16.23	0.	0.
time (sec)	N/A	0.372	38.708	0.249	0.	100.191	0.	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	416	416	472069	117715	0	6564	0	0
normalized size	1	1.	1134.78	282.97	0.	15.78	0.	0.
time (sec)	N/A	0.422	35.861	2.335	0.	96.091	0.	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	653	653	652560	402966	0	10966	0	0
normalized size	1	1.	999.33	617.1	0.	16.79	0.	0.
time (sec)	N/A	0.646	38.679	20.612	0.	132.514	0.	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	324	324	856	957	0	1740	0	0
normalized size	1	1.	2.64	2.95	0.	5.37	0.	0.
time (sec)	N/A	0.237	6.507	0.268	0.	66.132	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	290	290	16163	756	0	1578	0	0
normalized size	1	1.	55.73	2.61	0.	5.44	0.	0.
time (sec)	N/A	0.217	27.706	0.19	0.	29.978	0.	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	127	127	10115	554	0	1416	0	0
normalized size	1	1.	79.65	4.36	0.	11.15	0.	0.
time (sec)	N/A	0.183	26.611	0.174	0.	7.86	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	394	394	377837	2076	0	5720	0	0
normalized size	1	1.	958.98	5.27	0.	14.52	0.	0.
time (sec)	N/A	0.335	35.08	0.294	0.	54.434	0.	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	560	560	581056	164796	0	0	0	0
normalized size	1	1.	1037.6	294.28	0.	0.	0.	0.
time (sec)	N/A	0.511	37.741	3.779	0.	0.	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	802	802	774154	480553	0	0	0	0
normalized size	1	1.	965.28	599.19	0.	0.	0.	0.
time (sec)	N/A	0.797	40.873	28.445	0.	0.	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	480	480	21204	1444	0	2188	0	0
normalized size	1	1.	44.18	3.01	0.	4.56	0.	0.
time (sec)	N/A	0.316	29.311	0.278	0.	145.17	0.	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	468	468	16259	1133	0	1967	0	0
normalized size	1	1.	34.74	2.42	0.	4.2	0.	0.
time (sec)	N/A	0.295	27.949	0.208	0.	65.139	0.	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	164	164	10177	824	0	1754	0	0
normalized size	1	1.	62.05	5.02	0.	10.7	0.	0.
time (sec)	N/A	0.265	26.881	0.185	0.	16.705	0.	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	592	592	484869	3860	0	0	0	0
normalized size	1	1.	819.04	6.52	0.	0.	0.	0.
time (sec)	N/A	0.457	36.987	0.29	0.	0.	0.	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	756	756	686248	197500	0	0	0	0
normalized size	1	1.	907.74	261.24	0.	0.	0.	0.
time (sec)	N/A	0.634	39.996	5.676	0.	0.	0.	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	999	999	891356	556423	0	0	0	0
normalized size	1	1.	892.25	556.98	0.	0.	0.	0.
time (sec)	N/A	0.914	43.595	37.042	0.	0.	0.	0.

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	240	1563	0	2034	0	0
normalized size	1	1.	1.95	12.71	0.	16.54	0.	0.
time (sec)	N/A	0.339	18.159	0.42	0.	1.403	0.	0.

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	102	189	0	502	0	0
normalized size	1	1.	1.67	3.1	0.	8.23	0.	0.
time (sec)	N/A	0.097	0.214	0.318	0.	0.798	0.	0.

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	135	377	0	1262	0	0
normalized size	1	1.	1.22	3.4	0.	11.37	0.	0.
time (sec)	N/A	0.357	0.956	0.338	0.	0.92	0.	0.

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	184	494	0	2190	0	0
normalized size	1	1.	1.3	3.5	0.	15.53	0.	0.
time (sec)	N/A	0.369	14.504	0.33	0.	1.892	0.	0.

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	171	424	0	2290	0	0
normalized size	1	1.	1.21	3.01	0.	16.24	0.	0.
time (sec)	N/A	0.339	0.343	0.332	0.	3.868	0.	0.

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	68	113	0	540	0	142
normalized size	1	1.	1.01	1.69	0.	8.06	0.	2.12
time (sec)	N/A	0.126	0.151	0.066	0.	0.593	0.	1.477

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	155	328	0	1226	0	282
normalized size	1	1.	1.26	2.67	0.	9.97	0.	2.29
time (sec)	N/A	0.247	0.643	0.086	0.	0.602	0.	1.46

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	267	1063	0	2479	0	644
normalized size	1	1.	1.31	5.21	0.	12.15	0.	3.16
time (sec)	N/A	0.51	1.311	0.091	0.	0.747	0.	1.552

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	136	462	0	1426	0	332
normalized size	1	1.	1.02	3.47	0.	10.72	0.	2.5
time (sec)	N/A	0.285	0.643	0.085	0.	0.621	0.	1.46

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	493	1593	0	2946	0	926
normalized size	1	1.	2.08	6.72	0.	12.43	0.	3.91
time (sec)	N/A	0.796	2.019	0.098	0.	0.821	0.	1.579

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	377	377	438	3293	0	5010	0	1692
normalized size	1	1.	1.16	8.73	0.	13.29	0.	4.49
time (sec)	N/A	1.999	3.322	0.113	0.	1.109	0.	1.632

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	517	2031	0	3343	0	1150
normalized size	1	1.	2.04	8.	0.	13.16	0.	4.53
time (sec)	N/A	1.132	2.252	0.111	0.	0.928	0.	1.565

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	412	412	459	4330	0	5782	0	2213
normalized size	1	1.	1.11	10.51	0.	14.03	0.	5.37
time (sec)	N/A	1.062	3.805	0.167	0.	1.273	0.	1.79

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	622	622	1285	8573	0	9385	0	4470
normalized size	1	1.	2.07	13.78	0.	15.09	0.	7.19
time (sec)	N/A	1.769	6.842	0.159	0.	1.999	0.	2.036

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	320	913	1372	0	0	0	0
normalized size	1	1.	2.85	4.29	0.	0.	0.	0.
time (sec)	N/A	0.283	17.81	0.38	0.	0.	0.	0.

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	229	443	0	0	0	0
normalized size	1	1.	1.04	2.01	0.	0.	0.	0.
time (sec)	N/A	0.244	4.205	0.335	0.	0.	0.	0.

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	380	380	6093	2337	0	0	0	0
normalized size	1	1.	16.03	6.15	0.	0.	0.	0.
time (sec)	N/A	0.433	24.346	0.424	0.	0.	0.	0.

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	326	326	233	581	0	0	0	0
normalized size	1	1.	0.71	1.78	0.	0.	0.	0.
time (sec)	N/A	0.361	5.627	0.305	0.	0.	0.	0.

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	442	442	7168	3285	0	0	0	0
normalized size	1	1.	16.22	7.43	0.	0.	0.	0.
time (sec)	N/A	0.629	25.286	0.687	0.	0.	0.	0.

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	147	215	0	0	0	0
normalized size	1	1.	0.71	1.03	0.	0.	0.	0.
time (sec)	N/A	0.117	2.483	0.325	0.	0.	0.	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	254	318	0	0	0	0
normalized size	1	1.	1.18	1.47	0.	0.	0.	0.
time (sec)	N/A	0.238	9.955	0.31	0.	0.	0.	0.

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	376	376	1491	2009	0	0	0	0
normalized size	1	1.	3.97	5.34	0.	0.	0.	0.
time (sec)	N/A	0.428	14.561	0.327	0.	0.	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	495	495	2083	5712	0	0	0	0
normalized size	1	1.	4.21	11.54	0.	0.	0.	0.
time (sec)	N/A	0.778	17.104	0.368	0.	0.	0.	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	389	389	39925	543	0	0	0	0
normalized size	1	1.	102.63	1.4	0.	0.	0.	0.
time (sec)	N/A	0.447	32.517	0.434	0.	0.	0.	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	336	352	0	0	0	0
normalized size	1	1.	1.7	1.78	0.	0.	0.	0.
time (sec)	N/A	0.106	5.191	0.376	0.	0.	0.	0.

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	598	598	1678	2847	0	0	0	0
normalized size	1	1.	2.81	4.76	0.	0.	0.	0.
time (sec)	N/A	0.902	9.258	0.484	0.	0.	0.	0.

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	899	899	1960	15728	0	0	0	0
normalized size	1	1.	2.18	17.49	0.	0.	0.	0.
time (sec)	N/A	2.258	6.719	0.621	0.	0.	0.	0.

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	744	744	1720	4298	0	0	0	0
normalized size	1	1.	2.31	5.78	0.	0.	0.	0.
time (sec)	N/A	1.106	9.454	0.442	0.	0.	0.	0.

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	919	919	1930	13060	0	0	0	0
normalized size	1	1.	2.1	14.21	0.	0.	0.	0.
time (sec)	N/A	2.132	6.635	0.536	0.	0.	0.	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1122	1122	2355	39418	0	0	0	0
normalized size	1	1.	2.1	35.13	0.	0.	0.	0.
time (sec)	N/A	3.163	7.319	1.249	0.	0.	0.	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	891	891	1996	15912	0	0	0	0
normalized size	1	1.	2.24	17.86	0.	0.	0.	0.
time (sec)	N/A	2.009	6.695	0.607	0.	0.	0.	0.

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1150	1150	2314	32283	0	0	0	0
normalized size	1	1.	2.01	28.07	0.	0.	0.	0.
time (sec)	N/A	3.435	7.167	1.121	0.	0.	0.	0.

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1428	1428	2949	75468	0	0	0	0
normalized size	1	1.	2.07	52.85	0.	0.	0.	0.
time (sec)	N/A	5.438	8.174	2.485	0.	0.	0.	0.

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	C	A	F	F(-1)	F(-1)	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	652	0	49385	491	0	0	0	0
normalized size	1	0.	75.74	0.75	0.	0.	0.	0.
time (sec)	N/A	0.093	32.658	0.386	0.	0.	0.	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	325	352	0	0	0	0
normalized size	1	1.	1.64	1.78	0.	0.	0.	0.
time (sec)	N/A	0.111	5.25	0.372	0.	0.	0.	0.

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	398	398	249	292	0	0	0	0
normalized size	1	1.	0.63	0.73	0.	0.	0.	0.
time (sec)	N/A	0.437	1.983	0.372	0.	0.	0.	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	622	763	1731	3451	0	0	0	0
normalized size	1	1.23	2.78	5.55	0.	0.	0.	0.
time (sec)	N/A	1.316	9.535	0.463	0.	0.	0.	0.

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	88	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.191	2.16	0.257	0.	0.	0.	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.092	46.951	0.143	0.	0.	0.	0.

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.091	77.687	0.18	0.	0.	0.	0.

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	88	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.22	2.171	0.227	0.	0.	0.	0.

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.097	47.852	0.16	0.	0.	0.	0.

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.096	82.067	0.172	0.	0.	0.	0.

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	88	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.218	57.332	0.143	0.	0.	0.	0.

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.097	92.543	0.158	0.	0.	0.	0.

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.097	132.591	0.168	0.	0.	0.	0.

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	106	106	2425	0	0	0	0	0
normalized size	1	1.	22.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.189	14.44	0.349	0.	0.	0.	0.

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	275	275	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.437	2.398	0.173	0.	0.	0.	0.

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	205	205	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.249	1.	0.161	0.	0.	0.	0.

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	124	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.146	0.202	0.148	0.	0.	0.	0.

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	208	208	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.271	1.205	0.15	0.	0.	0.	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	248	248	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.451	1.797	0.161	0.	0.	0.	0.

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	55	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.116	2.421	0.251	0.	0.	0.	0.

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	296	278	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.515	1.052	0.163	0.	0.	0.	0.

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	200	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.242	0.493	0.15	0.	0.	0.	0.

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	125	0	0	0	0	0
normalized size	1	1.	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.144	0.222	0.133	0.	0.	0.	0.

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	206	206	5411	0	0	0	0	0
normalized size	1	1.	26.27	0.	0.	0.	0.	0.
time (sec)	N/A	0.402	25.592	0.147	0.	0.	0.	0.

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	322	322	10678	0	0	0	0	0
normalized size	1	1.	33.16	0.	0.	0.	0.	0.
time (sec)	N/A	0.559	32.853	0.154	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [31] had the largest ratio of [0.5769]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	15	9	1.	26	0.346
2	A	11	8	1.	26	0.308
3	A	5	3	1.	26	0.115
4	A	4	3	1.	26	0.115
5	A	4	3	1.	24	0.125
6	A	8	6	1.	26	0.231
7	A	9	6	1.	26	0.231
8	A	12	7	1.	26	0.269
9	A	15	7	1.	26	0.269
10	A	18	7	1.	26	0.269
11	A	13	8	1.	26	0.308
12	A	6	3	1.	26	0.115
13	A	5	3	1.	26	0.115
14	A	5	3	1.	26	0.115
15	A	9	8	1.	24	0.333
16	A	15	11	1.	26	0.423
17	A	13	9	1.	26	0.346
18	A	15	9	1.	26	0.346
19	A	19	9	1.	26	0.346
20	A	23	9	1.	26	0.346
21	A	26	14	1.12	26	0.538
22	A	21	13	1.	26	0.5
23	A	13	9	1.	26	0.346

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#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
24	A	9	6	1.	26	0.231
25	A	7	5	1.	24	0.208
26	A	4	3	1.	26	0.115
27	A	4	3	1.	26	0.115
28	A	5	3	1.	26	0.115
29	A	13	8	1.	26	0.308
30	A	17	9	1.	26	0.346
31	A	29	15	1.	26	0.577
32	A	20	13	1.	26	0.5
33	A	15	9	1.	26	0.346
34	A	12	7	1.	26	0.269
35	A	9	6	1.	24	0.25
36	A	12	8	1.	26	0.308
37	A	5	3	1.	26	0.115
38	A	5	3	1.	26	0.115
39	A	6	3	1.	26	0.115
40	A	14	8	1.	26	0.308
41	A	18	9	1.	26	0.346
42	A	5	4	1.	28	0.143
43	A	5	4	1.	28	0.143
44	A	5	4	1.	28	0.143
45	A	4	4	1.	26	0.154
46	A	4	4	1.	28	0.143
47	A	5	4	1.	28	0.143
48	A	6	4	1.	28	0.143
49	A	7	4	1.	28	0.143
50	A	6	5	1.	28	0.179
51	A	6	5	1.	28	0.179
52	A	5	5	1.	26	0.192
53	A	4	4	1.	28	0.143
54	A	5	5	1.	28	0.179
55	A	6	5	1.	28	0.179

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
56	A	7	5	1.	28	0.179
57	A	5	4	1.	28	0.143
58	A	5	4	1.	28	0.143
59	A	5	4	1.	26	0.154
60	A	5	4	1.	28	0.143
61	A	5	4	1.	28	0.143
62	A	5	4	1.	28	0.143
63	A	5	4	1.	28	0.143
64	A	5	4	1.	28	0.143
65	A	8	6	1.	28	0.214
66	A	7	6	1.	28	0.214
67	A	6	5	1.	28	0.179
68	A	5	4	1.	26	0.154
69	A	6	5	1.	28	0.179
70	A	7	6	1.	28	0.214
71	A	8	6	1.	28	0.214
72	A	8	6	1.	28	0.214
73	A	7	6	1.	28	0.214
74	A	7	6	1.13	28	0.214
75	A	6	5	1.15	26	0.192
76	A	7	6	1.	28	0.214
77	A	8	6	1.	28	0.214
78	A	9	6	1.	28	0.214
79	A	9	7	1.	28	0.25
80	A	8	7	1.	28	0.25
81	A	7	6	1.	28	0.214
82	A	7	6	1.	28	0.214
83	A	7	6	1.22	26	0.231
84	A	8	7	1.	28	0.25
85	A	9	7	1.	28	0.25
86	A	5	3	1.	30	0.1
87	A	4	3	1.	30	0.1

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
88	A	3	3	1.	30	0.1
89	A	2	2	1.	30	0.067
90	A	2	2	1.	30	0.067
91	A	3	3	1.	30	0.1
92	A	4	3	1.	30	0.1
93	A	5	3	1.	30	0.1
94	A	5	4	1.	30	0.133
95	A	3	3	1.	30	0.1
96	A	3	3	1.	30	0.1
97	A	3	2	1.	30	0.067
98	A	3	3	1.	30	0.1
99	A	4	4	1.	30	0.133
100	A	5	4	1.	30	0.133
101	A	4	3	1.	30	0.1
102	A	5	4	1.	30	0.133
103	A	4	3	1.	30	0.1
104	A	3	2	1.	30	0.067
105	A	3	3	1.	30	0.1
106	A	3	3	1.	30	0.1
107	A	4	4	1.	30	0.133
108	A	5	4	1.	30	0.133
109	A	6	4	1.	30	0.133
110	A	3	2	1.	30	0.067
111	A	3	2	1.	30	0.067
112	A	3	2	1.	30	0.067
113	A	2	2	1.	30	0.067
114	A	2	2	1.	30	0.067
115	A	3	2	1.29	30	0.067
116	A	3	2	1.	30	0.067
117	A	3	2	1.	30	0.067
118	A	3	3	1.	30	0.1
119	A	3	3	1.	30	0.1

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
120	A	3	3	1.	30	0.1
121	A	3	2	1.	30	0.067
122	A	3	3	1.	30	0.1
123	A	3	2	1.	30	0.067
124	A	3	2	1.	30	0.067
125	A	3	3	1.	30	0.1
126	A	4	4	1.	30	0.133
127	A	4	3	1.	30	0.1
128	A	3	2	1.	30	0.067
129	A	3	2	1.	30	0.067
130	A	4	3	1.	30	0.1
131	A	2	2	1.	24	0.083
132	A	3	3	1.	26	0.115
133	A	3	3	1.	26	0.115
134	A	3	3	1.	26	0.115
135	A	3	3	1.	24	0.125
136	A	3	3	1.	26	0.115
137	A	3	3	1.	26	0.115
138	A	4	3	1.	28	0.107
139	A	3	3	1.	28	0.107
140	A	2	2	1.	28	0.071
141	A	4	4	1.	28	0.143
142	A	5	5	1.	28	0.179
143	A	6	5	1.	27	0.185
144	A	3	3	1.	27	0.111
145	A	3	3	1.	27	0.111
146	A	5	5	1.	27	0.185
147	A	5	4	1.	27	0.148
148	A	5	4	1.	27	0.148
149	A	5	4	1.	27	0.148
150	A	4	4	1.	25	0.16
151	A	5	5	1.	27	0.185

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
152	A	7	6	1.	27	0.222
153	A	8	7	1.	27	0.259
154	A	6	5	1.	27	0.185
155	A	5	5	1.	27	0.185
156	A	5	5	1.	25	0.2
157	A	5	5	1.	27	0.185
158	A	7	6	1.	27	0.222
159	A	8	6	1.	27	0.222
160	A	5	4	1.	27	0.148
161	A	5	4	1.	27	0.148
162	A	6	5	1.	25	0.2
163	A	7	5	1.	27	0.185
164	A	10	6	1.	27	0.222
165	A	14	6	1.	27	0.222
166	A	9	5	1.	27	0.185
167	A	7	4	1.	27	0.148
168	A	5	4	1.	25	0.16
169	A	8	7	1.	27	0.259
170	A	12	6	1.	27	0.222
171	A	16	6	1.	27	0.222
172	A	10	5	1.	27	0.185
173	A	10	5	1.	27	0.185
174	A	6	5	1.	25	0.2
175	A	12	6	1.	27	0.222
176	A	15	6	1.	27	0.222
177	A	19	6	1.	27	0.222
178	A	14	5	1.	27	0.185
179	A	14	5	1.	27	0.185
180	A	7	5	1.	25	0.2
181	A	16	6	1.	27	0.222
182	A	19	6	1.	27	0.222
183	A	23	6	1.	27	0.222

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
184	A	5	5	1.	29	0.172
185	A	2	2	1.	29	0.069
186	A	5	5	1.	29	0.172
187	A	5	4	1.	29	0.138
188	A	5	4	1.	29	0.138
189	A	4	4	1.	23	0.174
190	A	5	5	1.	23	0.217
191	A	6	6	1.	23	0.261
192	A	5	5	1.	25	0.2
193	A	6	6	1.	25	0.24
194	A	7	7	1.	25	0.28
195	A	6	6	1.	25	0.24
196	A	7	7	1.	25	0.28
197	A	8	8	1.	25	0.32
198	A	5	5	1.	25	0.2
199	A	3	3	1.	27	0.111
200	A	6	6	1.	25	0.24
201	A	5	5	1.	27	0.185
202	A	7	7	1.	25	0.28
203	A	3	3	1.	25	0.12
204	A	3	3	1.	27	0.111
205	A	6	6	1.	25	0.24
206	A	7	7	1.	25	0.28
207	A	3	3	1.	29	0.103
208	A	1	1	1.	29	0.034
209	A	5	5	1.	29	0.172
210	A	7	7	1.	29	0.241
211	A	6	6	1.	29	0.207
212	A	7	7	1.	29	0.241
213	A	8	8	1.	29	0.276
214	A	7	7	1.	29	0.241
215	A	8	8	1.	29	0.276

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
216	A	9	8	1.	29	0.276
217	F	0	0	N/A	0	N/A
218	A	1	1	1.	29	0.034
219	A	3	3	1.	29	0.103
220	A	6	6	1.23	29	0.207
221	A	0	0	0.	0	0.
222	A	0	0	0.	0	0.
223	A	0	0	0.	0	0.
224	A	0	0	0.	0	0.
225	A	0	0	0.	0	0.
226	A	0	0	0.	0	0.
227	A	0	0	0.	0	0.
228	A	0	0	0.	0	0.
229	A	0	0	0.	0	0.
230	A	4	4	1.	27	0.148
231	A	8	6	1.	27	0.222
232	A	7	5	1.	27	0.185
233	A	6	4	1.	25	0.16
234	A	7	5	1.	27	0.185
235	A	8	6	1.	27	0.222
236	A	0	0	0.	0	0.
237	A	8	6	1.	27	0.222
238	A	7	5	1.	27	0.185
239	A	6	4	1.	25	0.16
240	A	7	5	1.	27	0.185
241	A	10	5	1.	27	0.185

Chapter 3

Listing of integrals

3.1 $\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^5\,dx$

Optimal. Leaf size=196

$$\frac{3a^2c^5\tan^5(e+fx)}{5f}+\frac{a^2c^5\tan^3(e+fx)}{3f}-\frac{a^2c^5\tan(e+fx)}{f}-\frac{19a^2c^5\tanh^{-1}(\sin(e+fx))}{16f}-\frac{a^2c^5\tan^3(e+fx)\sec^3(e+}{6f}$$

```
[Out] a^2*c^5*x - (19*a^2*c^5*ArcTanh[Sin[e + f*x]])/(16*f) - (a^2*c^5*Tan[e + f*
x])/f + (17*a^2*c^5*Sec[e + f*x]*Tan[e + f*x])/(16*f) + (a^2*c^5*Sec[e + f*
x]^3*Tan[e + f*x])/(8*f) + (a^2*c^5*Tan[e + f*x]^3)/(3*f) - (3*a^2*c^5*Sec[
e + f*x]*Tan[e + f*x]^3)/(4*f) - (a^2*c^5*Sec[e + f*x]^3*Tan[e + f*x]^3)/(6
*f) + (3*a^2*c^5*Tan[e + f*x]^5)/(5*f)
```

Rubi [A] time = 0.302005, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {3904, 3886, 3473, 8, 2611, 3770, 2607, 30, 3768}

$$\frac{3a^2c^5\tan^5(e+fx)}{5f}+\frac{a^2c^5\tan^3(e+fx)}{3f}-\frac{a^2c^5\tan(e+fx)}{f}-\frac{19a^2c^5\tanh^{-1}(\sin(e+fx))}{16f}-\frac{a^2c^5\tan^3(e+fx)\sec^3(e+}{6f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^5,x]
```

```
[Out] a^2*c^5*x - (19*a^2*c^5*ArcTanh[Sin[e + f*x]])/(16*f) - (a^2*c^5*Tan[e + f*
x])/f + (17*a^2*c^5*Sec[e + f*x]*Tan[e + f*x])/(16*f) + (a^2*c^5*Sec[e + f*
x]^3*Tan[e + f*x])/(8*f) + (a^2*c^5*Tan[e + f*x]^3)/(3*f) - (3*a^2*c^5*Sec[
```

```
e + f*x]*Tan[e + f*x]^3)/(4*f) - (a^2*c^5*Sec[e + f*x]^3*Tan[e + f*x]^3)/(6
*f) + (3*a^2*c^5*Tan[e + f*x]^5)/(5*f)
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^5\,dx = \left(a^2c^2\right)\int (c-c\sec(e+fx))^3\tan^4(e+fx)\,dx$$
$$= \left(a^2c^2\right)\int \left(c^3\tan^4(e+fx)-3c^3\sec(e+fx)\tan^4(e+fx)+3c^3\sec^2(e+\right.$$
$$= \left(a^2c^5\right)\int \tan^4(e+fx)\,dx-\left(a^2c^5\right)\int \sec^3(e+fx)\tan^4(e+fx)\,dx-\left(3a\right.$$
$$= \frac{a^2c^5\tan^3(e+fx)}{3f}-\frac{3a^2c^5\sec(e+fx)\tan^3(e+fx)}{4f}-\frac{a^2c^5\sec^3(e+fx)}{6f}$$
$$= -\frac{a^2c^5\tan(e+fx)}{f}+\frac{9a^2c^5\sec(e+fx)\tan(e+fx)}{8f}+\frac{a^2c^5\sec^3(e+fx)}{8f}$$
$$= a^2c^5x-\frac{9a^2c^5\tanh^{-1}(\sin(e+fx))}{8f}-\frac{a^2c^5\tan(e+fx)}{f}+\frac{17a^2c^5\sec(e+}{1}$$
$$= a^2c^5x-\frac{19a^2c^5\tanh^{-1}(\sin(e+fx))}{16f}-\frac{a^2c^5\tan(e+fx)}{f}+\frac{17a^2c^5\sec(e}{}$$

Mathematica [A] time = 2.00263, size = 165, normalized size = 0.84

$$\frac{a^2c^5\sec^6(e+fx)\left(-210\sin(e+fx)-120\sin(2(e+fx))+865\sin(3(e+fx))-768\sin(4(e+fx))+435\sin(5(e+fx))\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^5,x]
```

```
[Out] (a^2*c^5*Sec[e + f*x]^6*(1200*e + 1200*f*x - 4560*ArcTanh[Sin[e + f*x]]*Cos
[e + f*x]^6 + 1800*(e + f*x)*Cos[2*(e + f*x)] + 720*e*Cos[4*(e + f*x)] + 72
0*f*x*Cos[4*(e + f*x)] + 120*e*Cos[6*(e + f*x)] + 120*f*x*Cos[6*(e + f*x)]
- 210*Sin[e + f*x] - 120*Sin[2*(e + f*x)] + 865*Sin[3*(e + f*x)] - 768*Sin[
4*(e + f*x)] + 435*Sin[5*(e + f*x)] - 88*Sin[6*(e + f*x)]))/(3840*f)
```

Maple [A] time = 0.033, size = 186, normalized size = 1.

$$-\frac{11\,c^5a^2\left(\sec\left(fx+e\right)\right)^3\tan\left(fx+e\right)}{24\,f}+\frac{29\,c^5a^2\sec\left(fx+e\right)\tan\left(fx+e\right)}{16\,f}-\frac{19\,c^5a^2\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{16\,f}-\frac{11}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^5,x)
```

```
[Out] -11/24*a^2*c^5*sec(f*x+e)^3*tan(f*x+e)/f+29/16*a^2*c^5*sec(f*x+e)*tan(f*x+e
)/f-19/16/f*c^5*a^2*ln(sec(f*x+e)+tan(f*x+e))-11/15*a^2*c^5*tan(f*x+e)/f-13
/15/f*c^5*a^2*tan(f*x+e)*sec(f*x+e)^2+a^2*c^5*x+1/f*a^2*c^5*e+3/5/f*c^5*a^2
*tan(f*x+e)*sec(f*x+e)^4-1/6/f*c^5*a^2*tan(f*x+e)*sec(f*x+e)^5
```

Maxima [A] time = 1.03172, size = 451, normalized size = 2.3

$$\frac{96\left(3\,\tan\left(fx+e\right)^5+10\,\tan\left(fx+e\right)^3+15\,\tan\left(fx+e\right)\right)a^2c^5-800\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)a^2c^5+480\left(fx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^5,x, algorithm="maxima")
```

```
[Out] 1/480*(96*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^2*c^5
- 800*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c^5 + 480*(f*x + e)*a^2*c^5 + 5
*a^2*c^5*(2*(15*sin(f*x + e)^5 - 40*sin(f*x + e)^3 + 33*sin(f*x + e))/(sin(
f*x + e)^6 - 3*sin(f*x + e)^4 + 3*sin(f*x + e)^2 - 1) - 15*log(sin(f*x + e)
 + 1) + 15*log(sin(f*x + e) - 1)) + 30*a^2*c^5*(2*(3*sin(f*x + e)^3 - 5*sin
(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1
) + 3*log(sin(f*x + e) - 1)) - 600*a^2*c^5*(2*sin(f*x + e)/(sin(f*x + e)^2
- 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 1440*a^2*c^5*log(se
```

```
c(f*x + e) + tan(f*x + e)) + 480*a^2*c^5*tan(f*x + e))/f
```

Fricas [A] time = 1.17058, size = 452, normalized size = 2.31

$$\frac{480\,a^2c^5fx\cos\left(fx+e\right)^6-285\,a^2c^5\cos\left(fx+e\right)^6\log\left(\sin\left(fx+e\right)+1\right)+285\,a^2c^5\cos\left(fx+e\right)^6\log\left(-\sin\left(fx+e\right)+1\right)\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^5,x, algorithm="fricas")
```

```
[Out] 1/480*(480*a^2*c^5*f*x*cos(f*x + e)^6 - 285*a^2*c^5*cos(f*x + e)^6*log(sin(
f*x + e) + 1) + 285*a^2*c^5*cos(f*x + e)^6*log(-sin(f*x + e) + 1) - 2*(176*
a^2*c^5*cos(f*x + e)^5 - 435*a^2*c^5*cos(f*x + e)^4 + 208*a^2*c^5*cos(f*x +
 e)^3 + 110*a^2*c^5*cos(f*x + e)^2 - 144*a^2*c^5*cos(f*x + e) + 40*a^2*c^5)
*sin(f*x + e))/(f*cos(f*x + e)^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-a^2c^5\left(\int(-1)\,dx+\int 3\sec\left(e+fx\right)dx+\int-\sec^2\left(e+fx\right)dx+\int-5\sec^3\left(e+fx\right)dx+\int 5\sec^4\left(e+fx\right)dx+\int \mathrm{s}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**5,x)
```

```
[Out] -a**2*c**5*(Integral(-1, x) + Integral(3*sec(e + f*x), x) + Integral(-sec(e
 + f*x)**2, x) + Integral(-5*sec(e + f*x)**3, x) + Integral(5*sec(e + f*x)*
*4, x) + Integral(sec(e + f*x)**5, x) + Integral(-3*sec(e + f*x)**6, x) + I
ntegral(sec(e + f*x)**7, x))
```

Giac [A] time = 1.51932, size = 271, normalized size = 1.38

$$\frac{240\left(fx+e\right)a^2c^5-285\,a^2c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)+285\,a^2c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)+\frac{2\left(525\,a^2c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{1\cdots}\right.}{\cdots}}{240\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^5,x, algorithm="giac")
```

```
[Out] 1/240*(240*(f*x + e)*a^2*c^5 - 285*a^2*c^5*log(abs(tan(1/2*f*x + 1/2*e) + 1
)) + 285*a^2*c^5*log(abs(tan(1/2*f*x + 1/2*e) - 1)) + 2*(525*a^2*c^5*tan(1/
2*f*x + 1/2*e)^11 - 3135*a^2*c^5*tan(1/2*f*x + 1/2*e)^9 + 1746*a^2*c^5*tan(
1/2*f*x + 1/2*e)^7 - 366*a^2*c^5*tan(1/2*f*x + 1/2*e)^5 - 95*a^2*c^5*tan(1/
2*f*x + 1/2*e)^3 + 45*a^2*c^5*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2
 - 1)^6)/f
```

3.2 $\int (a + a\sec(e + fx))^2 (c - c\sec(e + fx))^4 \, dx$

Optimal. Leaf size=140

$$\frac{a^2c^4\tan^5(e+fx)}{5f} + \frac{a^2c^4\tan^3(e+fx)}{3f} - \frac{a^2c^4\tan(e+fx)}{f} - \frac{3a^2c^4\tanh^{-1}(\sin(e+fx))}{4f} - \frac{a^2c^4\tan^3(e+fx)\sec(e+fx)}{2f}$$

```
[Out] a^2*c^4*x - (3*a^2*c^4*ArcTanh[Sin[e + f*x]])/(4*f) - (a^2*c^4*Tan[e + f*x]
)/f + (3*a^2*c^4*Sec[e + f*x]*Tan[e + f*x])/(4*f) + (a^2*c^4*Tan[e + f*x]^3
)/(3*f) - (a^2*c^4*Sec[e + f*x]*Tan[e + f*x]^3)/(2*f) + (a^2*c^4*Tan[e + f*
x]^5)/(5*f)
```

Rubi [A] time = 0.198988, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3904, 3886, 3473, 8, 2611, 3770, 2607, 30}

$$\frac{a^2c^4\tan^5(e+fx)}{5f} + \frac{a^2c^4\tan^3(e+fx)}{3f} - \frac{a^2c^4\tan(e+fx)}{f} - \frac{3a^2c^4\tanh^{-1}(\sin(e+fx))}{4f} - \frac{a^2c^4\tan^3(e+fx)\sec(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^4,x]
```


```
[Out] a^2*c^4*x - (3*a^2*c^4*ArcTanh[Sin[e + f*x]])/(4*f) - (a^2*c^4*Tan[e + f*x]
)/f + (3*a^2*c^4*Sec[e + f*x]*Tan[e + f*x])/(4*f) + (a^2*c^4*Tan[e + f*x]^3
)/(3*f) - (a^2*c^4*Sec[e + f*x]*Tan[e + f*x]^3)/(2*f) + (a^2*c^4*Tan[e + f*
x]^5)/(5*f)
```


Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^4\,dx = \left(a^2c^2\right)\int (c-c\sec(e+fx))^2\tan^4(e+fx)\,dx$$
$$= \left(a^2c^2\right)\int \left(c^2\tan^4(e+fx)-2c^2\sec(e+fx)\tan^4(e+fx)+c^2\sec^2(e+f$$
$$= \left(a^2c^4\right)\int \tan^4(e+fx)\,dx+\left(a^2c^4\right)\int \sec^2(e+fx)\tan^4(e+fx)\,dx-\left(2a\right.$$
$$= \frac{a^2c^4\tan^3(e+fx)}{3f}-\frac{a^2c^4\sec(e+fx)\tan^3(e+fx)}{2f}-\left(a^2c^4\right)\int \tan^2(e+$$
$$= -\frac{a^2c^4\tan(e+fx)}{f}+\frac{3a^2c^4\sec(e+fx)\tan(e+fx)}{4f}+\frac{a^2c^4\tan^3(e+fx)}{3f}$$
$$= a^2c^4x-\frac{3a^2c^4\tanh^{-1}(\sin(e+fx))}{4f}-\frac{a^2c^4\tan(e+fx)}{f}+\frac{3a^2c^4\sec(e+}{4}$$

Mathematica [A] time = 1.15177, size = 146, normalized size = 1.04

$$\frac{a^2c^4\sec^5(e+fx)\left(40\sin(e+fx)+60\sin(2(e+fx))-220\sin(3(e+fx))+150\sin(4(e+fx))-68\sin(5(e+fx))+60\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (a^2*c^4*Sec[e + f*x]^5*(600*(e + f*x)*Cos[e + f*x] - 720*ArcTanh[Sin[e + f
*x]]*Cos[e + f*x]^5 + 300*e*Cos[3*(e + f*x)] + 300*f*x*Cos[3*(e + f*x)] + 6
0*e*Cos[5*(e + f*x)] + 60*f*x*Cos[5*(e + f*x)] + 40*Sin[e + f*x] + 60*Sin[2
*(e + f*x)] - 220*Sin[3*(e + f*x)] + 150*Sin[4*(e + f*x)] - 68*Sin[5*(e + f
*x)]))/(960*f)
```

Maple [A] time = 0.028, size = 161, normalized size = 1.2

$$-\frac{17\,c^4a^2\tan\left(fx+e\right)}{15\,f}-\frac{c^4a^2\tan\left(fx+e\right)\left(\sec\left(fx+e\right)\right)^2}{15\,f}+\frac{5\,c^4a^2\sec\left(fx+e\right)\tan\left(fx+e\right)}{4\,f}-\frac{3\,c^4a^2\ln\left(\sec\left(fx+e\right)+\right.}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^4,x)
```

```
[Out] -17/15*a^2*c^4*tan(f*x+e)/f-1/15/f*c^4*a^2*tan(f*x+e)*sec(f*x+e)^2+5/4*a^2*
c^4*sec(f*x+e)*tan(f*x+e)/f-3/4/f*c^4*a^2*ln(sec(f*x+e)+tan(f*x+e))+a^2*c^4
*x+1/f*a^2*c^4*e-1/2/f*c^4*a^2*tan(f*x+e)*sec(f*x+e)^3+1/5/f*c^4*a^2*tan(f*
x+e)*sec(f*x+e)^4
```

Maxima [A] time = 1.03812, size = 324, normalized size = 2.31

$$\frac{8\left(3\,\tan\left(fx+e\right)^5+10\,\tan\left(fx+e\right)^3+15\,\tan\left(fx+e\right)\right)a^2c^4-40\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)a^2c^4+120\left(fx+e\right)a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] 1/120*(8*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^2*c^4 -
 40*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c^4 + 120*(f*x + e)*a^2*c^4 + 15*
a^2*c^4*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x
+ e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) - 120*a^2*
c^4*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(
f*x + e) - 1)) - 240*a^2*c^4*log(sec(f*x + e) + tan(f*x + e)) - 120*a^2*c^4
*tan(f*x + e))/f
```

Fricas [A] time = 1.13531, size = 404, normalized size = 2.89

$$\frac{120\,a^2c^4fx\cos\left(fx+e\right)^5-45\,a^2c^4\cos\left(fx+e\right)^5\log\left(\sin\left(fx+e\right)+1\right)+45\,a^2c^4\cos\left(fx+e\right)^5\log\left(-\sin\left(fx+e\right)+1\right)-}{120\,f\cos\left(f\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] 1/120*(120*a^2*c^4*f*x*cos(f*x + e)^5 - 45*a^2*c^4*cos(f*x + e)^5*log(sin(f
*x + e) + 1) + 45*a^2*c^4*cos(f*x + e)^5*log(-sin(f*x + e) + 1) - 2*(68*a^2
*c^4*cos(f*x + e)^4 - 75*a^2*c^4*cos(f*x + e)^3 + 4*a^2*c^4*cos(f*x + e)^2
+ 30*a^2*c^4*cos(f*x + e) - 12*a^2*c^4)*sin(f*x + e))/(f*cos(f*x + e)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2c^4\left(\int 1\,dx+\int -2\sec\left(e+fx\right)dx+\int -\sec^2\left(e+fx\right)dx+\int 4\sec^3\left(e+fx\right)dx+\int -\sec^4\left(e+fx\right)dx+\int -2\sec$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**4,x)
```

```
[Out] a**2*c**4*(Integral(1, x) + Integral(-2*sec(e + f*x), x) + Integral(-sec(e
+ f*x)**2, x) + Integral(4*sec(e + f*x)**3, x) + Integral(-sec(e + f*x)**4,
 x) + Integral(-2*sec(e + f*x)**5, x) + Integral(sec(e + f*x)**6, x))
```

Giac [A] time = 1.44588, size = 244, normalized size = 1.74

$$\frac{60\left(fx+e\right)a^2c^4-45\,a^2c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)+45\,a^2c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)+\frac{2\left(105\,a^2c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^9-530}{}}{60\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^4,x, algorithm="giac")
```

```
[Out] 1/60*(60*(f*x + e)*a^2*c^4 - 45*a^2*c^4*log(abs(tan(1/2*f*x + 1/2*e) + 1))
+ 45*a^2*c^4*log(abs(tan(1/2*f*x + 1/2*e) - 1)) + 2*(105*a^2*c^4*tan(1/2*f*
x + 1/2*e)^9 - 530*a^2*c^4*tan(1/2*f*x + 1/2*e)^7 + 328*a^2*c^4*tan(1/2*f*x
 + 1/2*e)^5 - 110*a^2*c^4*tan(1/2*f*x + 1/2*e)^3 + 15*a^2*c^4*tan(1/2*f*x +
 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^5)/f
```

3.3 $\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^3\,dx$

Optimal. Leaf size=97

$$-\frac{3a^2c^3\tanh^{-1}(\sin(e+fx))}{8f}+\frac{a^2\tan^3(e+fx)\left(4c^3-3c^3\sec(e+fx)\right)}{12f}-\frac{a^2\tan(e+fx)\left(8c^3-3c^3\sec(e+fx)\right)}{8f}+a^2c^3x$$

```
[Out] a^2*c^3*x - (3*a^2*c^3*ArcTanh[Sin[e + f*x]])/(8*f) - (a^2*(8*c^3 - 3*c^3*S
ec[e + f*x])*Tan[e + f*x])/(8*f) + (a^2*(4*c^3 - 3*c^3*Sec[e + f*x])*Tan[e
+ f*x]^3)/(12*f)
```

Rubi [A] time = 0.117992, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3904, 3881, 3770}

$$-\frac{3a^2c^3\tanh^{-1}(\sin(e+fx))}{8f}+\frac{a^2\tan^3(e+fx)\left(4c^3-3c^3\sec(e+fx)\right)}{12f}-\frac{a^2\tan(e+fx)\left(8c^3-3c^3\sec(e+fx)\right)}{8f}+a^2c^3x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^3,x]
```

```
[Out] a^2*c^3*x - (3*a^2*c^3*ArcTanh[Sin[e + f*x]])/(8*f) - (a^2*(8*c^3 - 3*c^3*S
ec[e + f*x])*Tan[e + f*x])/(8*f) + (a^2*(4*c^3 - 3*c^3*Sec[e + f*x])*Tan[e
+ f*x]^3)/(12*f)
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3881

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(e*(e*Cot[c + d*x])^(m - 1)*(a*m + b*(m - 1)*Csc[c
 + d*x]))/(d*m*(m - 1)), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a*m
 + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1
]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^3\,dx = \left(a^2c^2\right)\int (c-c\sec(e+fx))\tan^4(e+fx)\,dx$$
$$= \frac{a^2\left(4c^3-3c^3\sec(e+fx)\right)\tan^3(e+fx)}{12f} - \frac{1}{4}\left(a^2c^2\right)\int (4c-3c\sec(e+fx$$
$$= -\frac{a^2\left(8c^3-3c^3\sec(e+fx)\right)\tan(e+fx)}{8f} + \frac{a^2\left(4c^3-3c^3\sec(e+fx)\right)\tan}{12f}$$
$$= a^2c^3x - \frac{a^2\left(8c^3-3c^3\sec(e+fx)\right)\tan(e+fx)}{8f} + \frac{a^2\left(4c^3-3c^3\sec(e+f}{12f}$$
$$= a^2c^3x - \frac{3a^2c^3\tanh^{-1}(\sin(e+fx))}{8f} - \frac{a^2\left(8c^3-3c^3\sec(e+fx)\right)\tan(e+}{8f}$$

Mathematica [A] time = 0.737631, size = 122, normalized size = 1.26

$$\frac{a^2c^3\sec^4(e+fx)\left(-18\sin(e+fx)-32\sin(2(e+fx))+30\sin(3(e+fx))-32\sin(4(e+fx))+96(e+fx)\cos(2(e+f}{192f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (a^2*c^3*Sec[e + f*x]^4*(72*e + 72*f*x - 72*ArcTanh[Sin[e + f*x]]*Cos[e + f
*x]^4 + 96*(e + f*x)*Cos[2*(e + f*x)] + 24*e*Cos[4*(e + f*x)] + 24*f*x*Cos[
4*(e + f*x)] - 18*Sin[e + f*x] - 32*Sin[2*(e + f*x)] + 30*Sin[3*(e + f*x)]
- 32*Sin[4*(e + f*x)]))/(192*f)
```

Maple [A] time = 0.025, size = 136, normalized size = 1.4

$$\frac{5c^3a^2\sec(fx+e)\tan(fx+e)}{8f} - \frac{3c^3a^2\ln(\sec(fx+e)+\tan(fx+e))}{8f} - \frac{4c^3a^2\tan(fx+e)}{3f} + a^2c^3x + \frac{a^2c^3e}{f} + \frac{c^3a^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^3,x)
```

```
[Out] 5/8/f*c^3*a^2*sec(f*x+e)*tan(f*x+e)-3/8/f*c^3*a^2*ln(sec(f*x+e)+tan(f*x+e))
-4/3/f*c^3*a^2*tan(f*x+e)+a^2*c^3*x+1/f*a^2*c^3*e+1/3/f*c^3*a^2*tan(f*x+e)*
sec(f*x+e)^2-1/4/f*c^3*a^2*tan(f*x+e)*sec(f*x+e)^3
```

Maxima [B] time = 1.0294, size = 274, normalized size = 2.82

$$\frac{16\left(\tan\left(fx+e\right)^3+3\tan\left(fx+e\right)\right)a^2c^3+48\left(fx+e\right)a^2c^3+3a^2c^3\left(\frac{2\left(3\sin\left(fx+e\right)^3-5\sin\left(fx+e\right)\right)}{\sin\left(fx+e\right)^4-2\sin\left(fx+e\right)^2+1}-3\log\left(\sin\left(fx+e\right)+1\right)+\cdots\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/48*(16*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c^3 + 48*(f*x + e)*a^2*c^3 +
 3*a^2*c^3*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f
*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) - 24*a^
2*c^3*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(si
n(f*x + e) - 1)) - 48*a^2*c^3*log(sec(f*x + e) + tan(f*x + e)) - 96*a^2*c^3
*tan(f*x + e))/f
```

Fricas [A] time = 1.16392, size = 359, normalized size = 3.7

$$\frac{48a^2c^3fx\cos\left(fx+e\right)^4-9a^2c^3\cos\left(fx+e\right)^4\log\left(\sin\left(fx+e\right)+1\right)+9a^2c^3\cos\left(fx+e\right)^4\log\left(-\sin\left(fx+e\right)+1\right)-2\left(3\cdots\right.}{48f\cos\left(fx+e\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/48*(48*a^2*c^3*f*x*cos(f*x + e)^4 - 9*a^2*c^3*cos(f*x + e)^4*log(sin(f*x
+ e) + 1) + 9*a^2*c^3*cos(f*x + e)^4*log(-sin(f*x + e) + 1) - 2*(32*a^2*c^3
*cos(f*x + e)^3 - 15*a^2*c^3*cos(f*x + e)^2 - 8*a^2*c^3*cos(f*x + e) + 6*a^
2*c^3)*sin(f*x + e))/(f*cos(f*x + e)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-a^2c^3\left(\int (-1)\,dx + \int \sec\left(e+fx\right)dx + \int 2\sec^2\left(e+fx\right)dx + \int -2\sec^3\left(e+fx\right)dx + \int -\sec^4\left(e+fx\right)dx + \int \sec$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**3,x)
```

```
[Out] -a**2*c**3*(Integral(-1, x) + Integral(sec(e + f*x), x) + Integral(2*sec(e
+ f*x)**2, x) + Integral(-2*sec(e + f*x)**3, x) + Integral(-sec(e + f*x)**4
, x) + Integral(sec(e + f*x)**5, x))
```

Giac [A] time = 1.51961, size = 217, normalized size = 2.24

$$\frac{24\left(fx+e\right)a^2c^3 - 9\,a^2c^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right) + 9\,a^2c^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right) + \frac{2\left(33\,a^2c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7 - 137\,a^2c^3}{}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/24*(24*(f*x + e)*a^2*c^3 - 9*a^2*c^3*log(abs(tan(1/2*f*x + 1/2*e) + 1)) +
 9*a^2*c^3*log(abs(tan(1/2*f*x + 1/2*e) - 1)) + 2*(33*a^2*c^3*tan(1/2*f*x +
 1/2*e)^7 - 137*a^2*c^3*tan(1/2*f*x + 1/2*e)^5 + 71*a^2*c^3*tan(1/2*f*x + 1
/2*e)^3 - 15*a^2*c^3*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^4)/
f
```

3.4 $\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^2\,dx$

Optimal. Leaf size=47

$$\frac{a^2c^2\tan^3(e+fx)}{3f}-\frac{a^2c^2\tan(e+fx)}{f}+a^2c^2x$$

```
[Out] a^2*c^2*x - (a^2*c^2*Tan[e + f*x])/f + (a^2*c^2*Tan[e + f*x]^3)/(3*f)
```

Rubi [A] time = 0.0646025, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3904, 3473, 8}

$$\frac{a^2c^2\tan^3(e+fx)}{3f}-\frac{a^2c^2\tan(e+fx)}{f}+a^2c^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] a^2*c^2*x - (a^2*c^2*Tan[e + f*x])/f + (a^2*c^2*Tan[e + f*x]^3)/(3*f)
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^2\,dx = \left(a^2c^2\right)\int \tan^4(e+fx)\,dx$$
$$= \frac{a^2c^2\tan^3(e+fx)}{3f} - \left(a^2c^2\right)\int \tan^2(e+fx)\,dx$$
$$= -\frac{a^2c^2\tan(e+fx)}{f} + \frac{a^2c^2\tan^3(e+fx)}{3f} + \left(a^2c^2\right)\int 1\,dx$$
$$= a^2c^2x - \frac{a^2c^2\tan(e+fx)}{f} + \frac{a^2c^2\tan^3(e+fx)}{3f}$$

Mathematica [A] time = 0.0339771, size = 45, normalized size = 0.96

$$a^2c^2\left(\frac{\tan^3(e+fx)}{3f} + \frac{\tan^{-1}(\tan(e+fx))}{f} - \frac{\tan(e+fx)}{f}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] a^2*c^2*(ArcTan[Tan[e + f*x]]/f - Tan[e + f*x]/f + Tan[e + f*x]^3/(3*f))
```

Maple [A] time = 0.021, size = 58, normalized size = 1.2

$$\frac{1}{f}\left(-2\,c^2a^2\tan\left(fx+e\right) + c^2a^2\left(fx+e\right) - c^2a^2\left(-\frac{2}{3} - \frac{\left(\sec\left(fx+e\right)\right)^2}{3}\right)\tan\left(fx+e\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^2,x)
```

```
[Out] 1/f*(-2*c^2*a^2*tan(f*x+e)+c^2*a^2*(f*x+e)-c^2*a^2*(-2/3-1/3*sec(f*x+e)^2)*
tan(f*x+e))
```

Maxima [A] time = 1.01444, size = 77, normalized size = 1.64

$$\frac{\left(\tan\left(fx+e\right)^3 + 3\,\tan\left(fx+e\right)\right)a^2c^2 + 3\left(fx+e\right)a^2c^2 - 6\,a^2c^2\tan\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/3*((tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c^2 + 3*(f*x + e)*a^2*c^2 - 6*a^
2*c^2*tan(f*x + e))/f
```

Fricas [A] time = 1.07713, size = 144, normalized size = 3.06

$$\frac{3\,a^2c^2fx\cos\left(fx+e\right)^3-\left(4\,a^2c^2\cos\left(fx+e\right)^2-a^2c^2\right)\sin\left(fx+e\right)}{3\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^2*c^2*f*x*cos(f*x + e)^3 - (4*a^2*c^2*cos(f*x + e)^2 - a^2*c^2)*si
n(f*x + e))/(f*cos(f*x + e)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2c^2\left(\int 1\,dx+\int -2\sec^2\left(e+fx\right)dx+\int \sec^4\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**2,x)
```

```
[Out] a**2*c**2*(Integral(1, x) + Integral(-2*sec(e + f*x)**2, x) + Integral(sec(
e + f*x)**4, x))
```

Giac [A] time = 1.43435, size = 69, normalized size = 1.47

$$\frac{a^2c^2\tan\left(fx+e\right)^3+3\left(fx+e\right)a^2c^2-3\,a^2c^2\tan\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/3*(a^2*c^2*tan(f*x + e)^3 + 3*(f*x + e)*a^2*c^2 - 3*a^2*c^2*tan(f*x + e))
/f
```

3.5 $\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx$

Optimal. Leaf size=55

$$\frac{a^2c\tanh^{-1}(\sin(e+fx))}{2f}-\frac{c\tan(e+fx)\left(a^2\sec(e+fx)+2a^2\right)}{2f}+a^2cx$$

```
[Out] a^2*c*x + (a^2*c*ArcTanh[Sin[e + f*x]])/(2*f) - (c*(2*a^2 + a^2*Sec[e + f*x
])*Tan[e + f*x])/(2*f)
```

Rubi [A] time = 0.0626252, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3904, 3881, 3770}

$$\frac{a^2c\tanh^{-1}(\sin(e+fx))}{2f}-\frac{c\tan(e+fx)\left(a^2\sec(e+fx)+2a^2\right)}{2f}+a^2cx$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x]),x]
```

```
[Out] a^2*c*x + (a^2*c*ArcTanh[Sin[e + f*x]])/(2*f) - (c*(2*a^2 + a^2*Sec[e + f*x
])*Tan[e + f*x])/(2*f)
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3881

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(e*(e*Cot[c + d*x])^(m - 1)*(a*m + b*(m - 1)*Csc[c
 + d*x]))/(d*m*(m - 1)), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a*m
 + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1
]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx &= -\left((ac)\int (a+a\sec(e+fx))\tan^2(e+fx)\,dx\right)\\ &= -\frac{c\left(2a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{2f}+\frac{1}{2}(ac)\int (2a+a\sec(e+fx))\,dx\\ &= a^2cx-\frac{c\left(2a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{2f}+\frac{1}{2}\left(a^2c\right)\int \sec(e+fx)\,dx\\ &= a^2cx+\frac{a^2c\tanh^{-1}(\sin(e+fx))}{2f}-\frac{c\left(2a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{2f}\end{aligned}$$

Mathematica [A] time = 0.291376, size = 72, normalized size = 1.31

$$\frac{a^2c\sec^2(e+fx)\left(-\sin(e+fx)-\sin(2(e+fx))+(e+fx)\cos(2(e+fx))+\cos^2(e+fx)\tanh^{-1}(\sin(e+fx))+e+fx\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x]),x]
```

```
[Out] (a^2*c*Sec[e + f*x]^2*(e + f*x + ArcTanh[Sin[e + f*x]]*Cos[e + f*x]^2 + (e
+ f*x)*Cos[2*(e + f*x)] - Sin[e + f*x] - Sin[2*(e + f*x)]))/(2*f)
```

Maple [A] time = 0.02, size = 76, normalized size = 1.4

$$\frac{a^2c\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{2f}+a^2cx+\frac{a^2ce}{f}-\frac{a^2c\tan\left(fx+e\right)}{f}-\frac{a^2c\sec\left(fx+e\right)\tan\left(fx+e\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x)
```

```
[Out] 1/2/f*a^2*c*ln(sec(f*x+e)+tan(f*x+e))+a^2*c*x+1/f*a^2*c*e-1/f*a^2*c*tan(f*x
+e)-1/2/f*a^2*c*sec(f*x+e)*tan(f*x+e)
```

Maxima [A] time = 1.07022, size = 128, normalized size = 2.33

$$\frac{4\left(fx+e\right)a^2c+a^2c\left(\frac{2\sin\left(fx+e\right)}{\sin\left(fx+e\right)^2-1}-\log\left(\sin\left(fx+e\right)+1\right)+\log\left(\sin\left(fx+e\right)-1\right)\right)+4\,a^2c\log\left(\sec\left(fx+e\right)+\tan\left(fx+e\right.\right.}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(f*x + e)*a^2*c + a^2*c*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(s
in(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 4*a^2*c*log(sec(f*x + e) + tan(
f*x + e)) - 4*a^2*c*tan(f*x + e))/f
```

Fricas [B] time = 1.07603, size = 259, normalized size = 4.71

$$\frac{4\,a^2cfx\cos\left(fx+e\right)^2+a^2c\cos\left(fx+e\right)^2\log\left(\sin\left(fx+e\right)+1\right)-a^2c\cos\left(fx+e\right)^2\log\left(-\sin\left(fx+e\right)+1\right)-2\left(2\,a^2c\cos\right.}{4\,f\cos\left(fx+e\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*a^2*c*f*x*cos(f*x + e)^2 + a^2*c*cos(f*x + e)^2*log(sin(f*x + e) + 1
) - a^2*c*cos(f*x + e)^2*log(-sin(f*x + e) + 1) - 2*(2*a^2*c*cos(f*x + e) +
 a^2*c)*sin(f*x + e))/(f*cos(f*x + e)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-a^2c\left(\int(-1)\,dx+\int-\sec\left(e+fx\right)dx+\int\sec^2\left(e+fx\right)dx+\int\sec^3\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**2*(c-c*sec(f*x+e)),x)
```

```
[Out] -a**2*c*(Integral(-1, x) + Integral(-sec(e + f*x), x) + Integral(sec(e + f*
x)**2, x) + Integral(sec(e + f*x)**3, x))
```

Giac [B] time = 1.36062, size = 147, normalized size = 2.67

$$\frac{2\left(fx+e\right)a^2c + a^2c\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right) - a^2c\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right) + \frac{2\left(a^2c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3 - 3a^2c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)^2}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*(2*(f*x + e)*a^2*c + a^2*c*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - a^2*c*l
og(abs(tan(1/2*f*x + 1/2*e) - 1)) + 2*(a^2*c*tan(1/2*f*x + 1/2*e)^3 - 3*a^2
*c*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^2)/f
```

3.6 $\int \frac{(a+a\sec(e+fx))^2}{c-c\sec(e+fx)}\,dx$

Optimal. Leaf size=56

$$-\frac{a^2\tanh^{-1}(\sin(e+fx))}{cf}-\frac{4a^2\tan(e+fx)}{cf(1-\sec(e+fx))}+\frac{a^2x}{c}$$

```
[Out] (a^2*x)/c - (a^2*ArcTanh[Sin[e + f*x]])/(c*f) - (4*a^2*Tan[e + f*x])/(c*f*(
1 - Sec[e + f*x]))
```

Rubi [A] time = 0.163921, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3903, 3777, 8, 3794, 3789, 3770}

$$-\frac{a^2\tanh^{-1}(\sin(e+fx))}{cf}-\frac{4a^2\tan(e+fx)}{cf(1-\sec(e+fx))}+\frac{a^2x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x]),x]
```

```
[Out] (a^2*x)/c - (a^2*ArcTanh[Sin[e + f*x]])/(c*f) - (4*a^2*Tan[e + f*x])/(c*f*(
1 - Sec[e + f*x]))
```

Rule 3903

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Dist[c^n, Int[ExpandTrig[(1 + (d*csc[e + f*
x])/c)^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
&& LtQ[m + n, 2]
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
rQ[2*n]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3789

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a
+ b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sec(e+fx))^2}{c-c\sec(e+fx)}\,dx &= \frac{\int \left(\frac{a^2}{1-\sec(e+fx)}+\frac{2a^2\sec(e+fx)}{1-\sec(e+fx)}+\frac{a^2\sec^2(e+fx)}{1-\sec(e+fx)}\right)dx}{c}\\
&= \frac{a^2\int \frac{1}{1-\sec(e+fx)}\,dx}{c}+\frac{a^2\int \frac{\sec^2(e+fx)}{1-\sec(e+fx)}\,dx}{c}+\frac{\left(2a^2\right)\int \frac{\sec(e+fx)}{1-\sec(e+fx)}\,dx}{c}\\
&= -\frac{3a^2\tan(e+fx)}{cf(1-\sec(e+fx))}-\frac{a^2\int -1\,dx}{c}-\frac{a^2\int \sec(e+fx)\,dx}{c}+\frac{a^2\int \frac{\sec(e+fx)}{1-\sec(e+fx)}\,dx}{c}\\
&= \frac{a^2x}{c}-\frac{a^2\tanh^{-1}(\sin(e+fx))}{cf}-\frac{4a^2\tan(e+fx)}{cf(1-\sec(e+fx))}
\end{aligned}$$

Mathematica [B] time = 0.285926, size = 169, normalized size = 3.02

$$\frac{a^2\csc\left(\frac{e}{2}\right)\sin\left(\frac{1}{2}(e+fx)\right)\left(-\cos\left(\frac{fx}{2}\right)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right.\right.\right.}{cf(\cos}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x]),x]
```

```
[Out] (a^2*Csc[e/2]*(-(Cos[(f*x)/2]*(f*x + Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2
]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])) + Cos[e + (f*x)/2]*(f*x + L
og[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f
*x)/2]]) + 8*Sin[(f*x)/2])*Sin[(e + f*x)/2])/(c*f*(-1 + Cos[e + f*x]))
```

Maple [A] time = 0.079, size = 90, normalized size = 1.6

$$2\,{\frac {{a}^{2}\arctan \left(\tan \left(1/2\,fx+e/2 \right) \right) }{fc}}-{\frac {{a}^{2}}{fc}\ln \left(\tan \left({\frac {fx}{2}}+{\frac {e}{2}} \right) +1 \right) }+{\frac {{a}^{2}}{fc}\ln \left(\tan \left({\frac {fx}{2}}+{\frac {e}{2}} \right) -1 \right) }+4\,{\frac {{a}^{2}}{fc\tan \left(1/2\,fx+e/2 \right) }}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x)
```

```
[Out] 2/f*a^2/c*arctan(tan(1/2*f*x+1/2*e))-1/f*a^2/c*ln(tan(1/2*f*x+1/2*e)+1)+1/f
*a^2/c*ln(tan(1/2*f*x+1/2*e)-1)+4/f*a^2/c/tan(1/2*f*x+1/2*e)
```

Maxima [B] time = 1.59887, size = 207, normalized size = 3.7

$$\frac{a^{2} {\left(\frac{2 \, \arctan\left(\frac{\sin\left(f x + e\right)}{\cos\left(f x + e\right) + 1}\right)}{c} + \frac{\cos\left(f x + e\right) + 1}{c \sin\left(f x + e\right)}\right)} - a^{2} {\left(\frac{\log\left(\frac{\sin\left(f x + e\right)}{\cos\left(f x + e\right) + 1} + 1\right)}{c} - \frac{\log\left(\frac{\sin\left(f x + e\right)}{\cos\left(f x + e\right) + 1} - 1\right)}{c} - \frac{\cos\left(f x + e\right) + 1}{c \sin\left(f x + e\right)}\right)} + \frac{2 \, a^{2} {\left(\cos\left(f x + e\right) + 1\right)}}{c \sin\left(f x + e\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] (a^2*(2*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c + (cos(f*x + e) + 1)/(c*s
in(f*x + e))) - a^2*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/c - log(sin(f
*x + e)/(cos(f*x + e) + 1) - 1)/c - (cos(f*x + e) + 1)/(c*sin(f*x + e))) +
2*a^2*(cos(f*x + e) + 1)/(c*sin(f*x + e)))/f
```

Fricas [A] time = 1.09638, size = 217, normalized size = 3.88

$$\frac{2\,a^{2}fx\sin\left(fx+e\right)-a^{2}\log\left(\sin\left(fx+e\right)+1\right)\sin\left(fx+e\right)+a^{2}\log\left(-\sin\left(fx+e\right)+1\right)\sin\left(fx+e\right)+8\,a^{2}\cos\left(fx+e\right)}{2\,cf\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^2*f*x*sin(f*x + e) - a^2*log(sin(f*x + e) + 1)*sin(f*x + e) + a^2*
log(-sin(f*x + e) + 1)*sin(f*x + e) + 8*a^2*cos(f*x + e) + 8*a^2)/(c*f*sin(
f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^{2}\left(\int\frac{2\sec\left(e+fx\right)}{\sec\left(e+fx\right)-1}\,dx+\int\frac{\sec^{2}\left(e+fx\right)}{\sec\left(e+fx\right)-1}\,dx+\int\frac{1}{\sec\left(e+fx\right)-1}\,dx\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**2/(c-c*sec(f*x+e)),x)
```

```
[Out] -a**2*(Integral(2*sec(e + f*x)/(sec(e + f*x) - 1), x) + Integral(sec(e + f*
x)**2/(sec(e + f*x) - 1), x) + Integral(1/(sec(e + f*x) - 1), x))/c
```

Giac [A] time = 1.38402, size = 109, normalized size = 1.95

$$\frac{\frac{\left(fx+e\right)a^{2}}{c}-\frac{a^{2}\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{c}+\frac{a^{2}\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{c}+\frac{4\,a^{2}}{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] ((f*x + e)*a^2/c - a^2*log(abs(tan(1/2*f*x + 1/2*e) + 1))/c + a^2*log(abs(t
an(1/2*f*x + 1/2*e) - 1))/c + 4*a^2/(c*tan(1/2*f*x + 1/2*e)))/f
```

3.7 $\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^2}\,dx$

Optimal. Leaf size=71

$$-\frac{4a^2\tan(e+fx)}{3c^2f(1-\sec(e+fx))}-\frac{4a^2\tan(e+fx)}{3c^2f(1-\sec(e+fx))^2}+\frac{a^2x}{c^2}$$

```
[Out] (a^2*x)/c^2 - (4*a^2*Tan[e + f*x])/(3*c^2*f*(1 - Sec[e + f*x])^2) - (4*a^2*
Tan[e + f*x])/(3*c^2*f*(1 - Sec[e + f*x]))
```

Rubi [A] time = 0.236315, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3903, 3777, 3919, 3794, 3796, 3797}

$$-\frac{4a^2\tan(e+fx)}{3c^2f(1-\sec(e+fx))}-\frac{4a^2\tan(e+fx)}{3c^2f(1-\sec(e+fx))^2}+\frac{a^2x}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (a^2*x)/c^2 - (4*a^2*Tan[e + f*x])/(3*c^2*f*(1 - Sec[e + f*x])^2) - (4*a^2*
Tan[e + f*x])/(3*c^2*f*(1 - Sec[e + f*x]))
```

Rule 3903

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Dist[c^n, Int[ExpandTrig[(1 + (d*csc[e + f*
x])/c)^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
&& LtQ[m + n, 2]
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
rQ[2*n]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3797

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(2*m + 1)), x]
+ Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x],
x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^2}\,dx &= \frac{\int \left(\frac{a^2}{(1-\sec(e+fx))^2}+\frac{2a^2\sec(e+fx)}{(1-\sec(e+fx))^2}+\frac{a^2\sec^2(e+fx)}{(1-\sec(e+fx))^2}\right)dx}{c^2}\\
&= \frac{a^2\int \frac{1}{(1-\sec(e+fx))^2}\,dx}{c^2}+\frac{a^2\int \frac{\sec^2(e+fx)}{(1-\sec(e+fx))^2}\,dx}{c^2}+\frac{\left(2a^2\right)\int \frac{\sec(e+fx)}{(1-\sec(e+fx))^2}\,dx}{c^2}\\
&= -\frac{4a^2\tan(e+fx)}{3c^2f(1-\sec(e+fx))^2}-\frac{a^2\int \frac{-3-\sec(e+fx)}{1-\sec(e+fx)}\,dx}{3c^2}\\
&= \frac{a^2x}{c^2}-\frac{4a^2\tan(e+fx)}{3c^2f(1-\sec(e+fx))^2}+\frac{\left(4a^2\right)\int \frac{\sec(e+fx)}{1-\sec(e+fx)}\,dx}{3c^2}\\
&= \frac{a^2x}{c^2}-\frac{4a^2\tan(e+fx)}{3c^2f(1-\sec(e+fx))^2}-\frac{4a^2\tan(e+fx)}{3c^2f(1-\sec(e+fx))}
\end{aligned}$$

Mathematica [C] time = 0.0571273, size = 53, normalized size = 0.75

$$-\frac{2a^2\cot^3\left(\frac{e}{2}+\frac{fx}{2}\right)\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{3c^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (-2*a^2*Cot[e/2 + (f*x)/2]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[e/2 + (f
*x)/2]^2])/(3*c^2*f)
```

Maple [A] time = 0.091, size = 67, normalized size = 0.9

$$2\,\frac{a^2\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)}{fc^2}-\frac{2\,a^2}{3\,fc^2}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-3}+2\,\frac{a^2}{fc^2\tan\left(1/2\,fx+e/2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x)
```

```
[Out] 2/f*a^2/c^2*arctan(tan(1/2*f*x+1/2*e))-2/3/f*a^2/c^2/tan(1/2*f*x+1/2*e)^3+2
/f*a^2/c^2/tan(1/2*f*x+1/2*e)
```

Maxima [B] time = 1.63322, size = 235, normalized size = 3.31

$$\frac{a^2\left(\frac{12\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^2}+\frac{\left(\frac{9\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-1\right)(\cos(fx+e)+1)^3}{c^2\sin(fx+e)^3}\right)-\frac{a^2\left(\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)(\cos(fx+e)+1)^3}{c^2\sin(fx+e)^3}+\frac{2a^2\left(\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-1\right)(\cos(fx+e)+1)^3}{c^2\sin(fx+e)^3}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(a^2*(12*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^2 + (9*sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 - 1)*(cos(f*x + e) + 1)^3/(c^2*sin(f*x + e)^3)) - a^
```

```
2*(3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)*(cos(f*x + e) + 1)^3/(c^2*sin
(f*x + e)^3) + 2*a^2*(3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1)*(cos(f*x +
 e) + 1)^3/(c^2*sin(f*x + e)^3))/f
```

Fricas [A] time = 1.03363, size = 204, normalized size = 2.87

$$\frac{8\,a^2\cos\left(fx+e\right)^2+4\,a^2\cos\left(fx+e\right)-4\,a^2+3\left(a^2fx\cos\left(fx+e\right)-a^2fx\right)\sin\left(fx+e\right)}{3\left(c^2f\cos\left(fx+e\right)-c^2f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(8*a^2*cos(f*x + e)^2 + 4*a^2*cos(f*x + e) - 4*a^2 + 3*(a^2*f*x*cos(f*x
 + e) - a^2*f*x)*sin(f*x + e))/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^2\left(\int\frac{2\sec\left(e+fx\right)}{\sec^2\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx+\int\frac{\sec^2\left(e+fx\right)}{\sec^2\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx+\int\frac{1}{\sec^2\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**2,x)
```

```
[Out] a**2*(Integral(2*sec(e + f*x)/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) +
Integral(sec(e + f*x)**2/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) + Integ
ral(1/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x))/c**2
```

Giac [A] time = 1.34247, size = 81, normalized size = 1.14

$$\frac{\frac{3\left(fx+e\right)a^2}{c^2}+\frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-a^2\right)}{c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*(f*x + e)*a^2/c^2 + 2*(3*a^2*tan(1/2*f*x + 1/2*e)^2 - a^2)/(c^2*tan(
1/2*f*x + 1/2*e)^3))/f
```

3.8 $\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}\,dx$

Optimal. Leaf size=102

$$-\frac{23a^2\tan(e+fx)}{15c^3f(1-\sec(e+fx))}-\frac{8a^2\tan(e+fx)}{15c^3f(1-\sec(e+fx))^2}-\frac{4a^2\tan(e+fx)}{5c^3f(1-\sec(e+fx))^3}+\frac{a^2x}{c^3}$$

```
[Out] (a^2*x)/c^3 - (4*a^2*Tan[e + f*x])/(5*c^3*f*(1 - Sec[e + f*x])^3) - (8*a^2*
Tan[e + f*x])/(15*c^3*f*(1 - Sec[e + f*x])^2) - (23*a^2*Tan[e + f*x])/(15*c
^3*f*(1 - Sec[e + f*x]))
```

Rubi [A] time = 0.329081, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {3903, 3777, 3922, 3919, 3794, 3796, 3797}

$$-\frac{23a^2\tan(e+fx)}{15c^3f(1-\sec(e+fx))}-\frac{8a^2\tan(e+fx)}{15c^3f(1-\sec(e+fx))^2}-\frac{4a^2\tan(e+fx)}{5c^3f(1-\sec(e+fx))^3}+\frac{a^2x}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (a^2*x)/c^3 - (4*a^2*Tan[e + f*x])/(5*c^3*f*(1 - Sec[e + f*x])^3) - (8*a^2*
Tan[e + f*x])/(15*c^3*f*(1 - Sec[e + f*x])^2) - (23*a^2*Tan[e + f*x])/(15*c
^3*f*(1 - Sec[e + f*x]))
```

Rule 3903

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Dist[c^n, Int[ExpandTrig[(1 + (d*csc[e + f*
x])/c)^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
&& LtQ[m + n, 2]
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
```

```
rQ[2*n]
```

Rule 3922

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*
x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x
])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ
[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3797

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(2*m + 1)), x]
+ Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x],
x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}\,dx = \frac{\int \left(\frac{a^2}{(1-\sec(e+fx))^3} + \frac{2a^2\sec(e+fx)}{(1-\sec(e+fx))^3} + \frac{a^2\sec^2(e+fx)}{(1-\sec(e+fx))^3}\right)dx}{c^3}$$

$$= \frac{a^2\int \frac{1}{(1-\sec(e+fx))^3}dx}{c^3} + \frac{a^2\int \frac{\sec^2(e+fx)}{(1-\sec(e+fx))^3}dx}{c^3} + \frac{(2a^2)\int \frac{\sec(e+fx)}{(1-\sec(e+fx))^3}dx}{c^3}$$

$$= -\frac{4a^2\tan(e+fx)}{5c^3f(1-\sec(e+fx))^3} - \frac{a^2\int \frac{-5-2\sec(e+fx)}{(1-\sec(e+fx))^2}dx}{5c^3} - \frac{(3a^2)\int \frac{\sec(e+fx)}{(1-\sec(e+fx))^2}dx}{5c^3} + \frac{(4a^2)\int \frac{\sec}{(1-\sec}}{5c^3}$$

$$= -\frac{4a^2\tan(e+fx)}{5c^3f(1-\sec(e+fx))^3} - \frac{8a^2\tan(e+fx)}{15c^3f(1-\sec(e+fx))^2} + \frac{a^2\int \frac{15+7\sec(e+fx)}{1-\sec(e+fx)}dx}{15c^3} - \frac{a^2\int \frac{\sec(e+f}{1-\sec(e+}}{5c^3}$$

$$= \frac{a^2x}{c^3} - \frac{4a^2\tan(e+fx)}{5c^3f(1-\sec(e+fx))^3} - \frac{8a^2\tan(e+fx)}{15c^3f(1-\sec(e+fx))^2} - \frac{a^2\tan(e+fx)}{15c^3f(1-\sec(e+fx))} + \frac{(22a}{}$$

$$= \frac{a^2x}{c^3} - \frac{4a^2\tan(e+fx)}{5c^3f(1-\sec(e+fx))^3} - \frac{8a^2\tan(e+fx)}{15c^3f(1-\sec(e+fx))^2} - \frac{23a^2\tan(e+fx)}{15c^3f(1-\sec(e+fx))}$$

Mathematica [A] time = 0.61974, size = 171, normalized size = 1.68

$$\frac{a^2\csc\left(\frac{e}{2}\right)\csc^5\left(\frac{1}{2}(e+fx)\right)\left(-360\sin\left(e+\frac{fx}{2}\right)+280\sin\left(e+\frac{3fx}{2}\right)+150\sin\left(2e+\frac{3fx}{2}\right)-86\sin\left(2e+\frac{5fx}{2}\right)-150fx\cos\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (a^2*Csc[e/2]*Csc[(e + f*x)/2]^5*(150*f*x*Cos[(f*x)/2] - 150*f*x*Cos[e + (f
*x)/2] - 75*f*x*Cos[e + (3*f*x)/2] + 75*f*x*Cos[2*e + (3*f*x)/2] + 15*f*x*C
os[2*e + (5*f*x)/2] - 15*f*x*Cos[3*e + (5*f*x)/2] - 500*Sin[(f*x)/2] - 360*
Sin[e + (f*x)/2] + 280*Sin[e + (3*f*x)/2] + 150*Sin[2*e + (3*f*x)/2] - 86*S
in[2*e + (5*f*x)/2]))/(480*c^3*f)
```

Maple [A] time = 0.1, size = 89, normalized size = 0.9

$$2\,\frac{a^2\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)}{fc^3}+\frac{a^2}{5\,fc^3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-5}-\frac{2\,a^2}{3\,fc^3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-3}+2\,\frac{a^2}{fc^3\tan\left(1/2\,fx+e/2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x)
```

```
[Out] 2/f*a^2/c^3*arctan(tan(1/2*f*x+1/2*e))+1/5/f*a^2/c^3/tan(1/2*f*x+1/2*e)^5-2
/3/f*a^2/c^3/tan(1/2*f*x+1/2*e)^3+2/f*a^2/c^3/tan(1/2*f*x+1/2*e)
```

Maxima [B] time = 1.78005, size = 290, normalized size = 2.84

$$\frac{a^2\left(\frac{120\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^3} - \frac{\left(\frac{20\sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{105\sin(fx+e)^4}{(\cos(fx+e)+1)^4} - 3\right)(\cos(fx+e)+1)^5}{c^3\sin(fx+e)^5}\right) - \frac{2a^2\left(\frac{10\sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4} - 3\right)(\cos(fx+e)+1)^5}{c^3\sin(fx+e)^5} - \frac{3a^2(\cdots}{}}{60f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/60*(a^2*(120*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^3 - (20*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 - 105*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3)*(c
os(f*x + e) + 1)^5/(c^3*sin(f*x + e)^5)) - 2*a^2*(10*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 - 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3)*(cos(f*x + e) +
 1)^5/(c^3*sin(f*x + e)^5) - 3*a^2*(5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 -
 1)*(cos(f*x + e) + 1)^5/(c^3*sin(f*x + e)^5))/f
```

Fricas [A] time = 1.02706, size = 313, normalized size = 3.07

$$\frac{43\,a^2\cos\left(fx+e\right)^3 - 11\,a^2\cos\left(fx+e\right)^2 - 31\,a^2\cos\left(fx+e\right) + 23\,a^2 + 15\left(a^2fx\cos\left(fx+e\right)^2 - 2\,a^2fx\cos\left(fx+e\right) + a^2 \cdots\right.}{15\left(c^3f\cos\left(fx+e\right)^2 - 2\,c^3f\cos\left(fx+e\right) + c^3f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(43*a^2*cos(f*x + e)^3 - 11*a^2*cos(f*x + e)^2 - 31*a^2*cos(f*x + e) +
 23*a^2 + 15*(a^2*f*x*cos(f*x + e)^2 - 2*a^2*f*x*cos(f*x + e) + a^2*f*x)*si
n(f*x + e))/((c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x
+ e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^2\left(\int \frac{2\sec\left(e+fx\right)}{\sec^3\left(e+fx\right)-3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)-1}\,dx+\int \frac{\sec^2\left(e+fx\right)}{\sec^3\left(e+fx\right)-3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)-1}\,dx+\int \frac{1}{\sec^3\left(e+fx\right)-3\sec^2\left(e+fx\right)+3\sec\left(e+\right.}\right.}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**3,x)
```

```
[Out] -a**2*(Integral(2*sec(e + f*x)/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*sec
(e + f*x) - 1), x) + Integral(sec(e + f*x)**2/(sec(e + f*x)**3 - 3*sec(e +
f*x)**2 + 3*sec(e + f*x) - 1), x) + Integral(1/(sec(e + f*x)**3 - 3*sec(e +
 f*x)**2 + 3*sec(e + f*x) - 1), x))/c**3
```

Giac [A] time = 1.36649, size = 103, normalized size = 1.01

$$\frac{\frac{15\left(fx+e\right)a^2}{c^3}+\frac{30\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-10\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+3\,a^2}{c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/15*(15*(f*x + e)*a^2/c^3 + (30*a^2*tan(1/2*f*x + 1/2*e)^4 - 10*a^2*tan(1/
2*f*x + 1/2*e)^2 + 3*a^2)/(c^3*tan(1/2*f*x + 1/2*e)^5))/f
```

3.9 $\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^4}\,dx$

Optimal. Leaf size=133

$$-\frac{164a^2\tan(e+fx)}{105c^4f(1-\sec(e+fx))}-\frac{59a^2\tan(e+fx)}{105c^4f(1-\sec(e+fx))^2}-\frac{12a^2\tan(e+fx)}{35c^4f(1-\sec(e+fx))^3}-\frac{4a^2\tan(e+fx)}{7c^4f(1-\sec(e+fx))^4}+\frac{a^2x}{c^4}$$

```
[Out] (a^2*x)/c^4 - (4*a^2*Tan[e + f*x])/(7*c^4*f*(1 - Sec[e + f*x])^4) - (12*a^2
*Tan[e + f*x])/(35*c^4*f*(1 - Sec[e + f*x])^3) - (59*a^2*Tan[e + f*x])/(105
*c^4*f*(1 - Sec[e + f*x])^2) - (164*a^2*Tan[e + f*x])/(105*c^4*f*(1 - Sec[e
 + f*x]))
```

Rubi [A] time = 0.42722, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {3903, 3777, 3922, 3919, 3794, 3796, 3797}

$$-\frac{164a^2\tan(e+fx)}{105c^4f(1-\sec(e+fx))}-\frac{59a^2\tan(e+fx)}{105c^4f(1-\sec(e+fx))^2}-\frac{12a^2\tan(e+fx)}{35c^4f(1-\sec(e+fx))^3}-\frac{4a^2\tan(e+fx)}{7c^4f(1-\sec(e+fx))^4}+\frac{a^2x}{c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (a^2*x)/c^4 - (4*a^2*Tan[e + f*x])/(7*c^4*f*(1 - Sec[e + f*x])^4) - (12*a^2
*Tan[e + f*x])/(35*c^4*f*(1 - Sec[e + f*x])^3) - (59*a^2*Tan[e + f*x])/(105
*c^4*f*(1 - Sec[e + f*x])^2) - (164*a^2*Tan[e + f*x])/(105*c^4*f*(1 - Sec[e
 + f*x]))
```

Rule 3903

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Dist[c^n, Int[ExpandTrig[(1 + (d*csc[e + f*
x])/c)^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
&& LtQ[m + n, 2]
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
```

```
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
rQ[2*n]
```

Rule 3922

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*
x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x
])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ
[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3797

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(2*m + 1)), x]
+ Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x],
x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^4}\,dx = \frac{\int\left(\frac{a^2}{(1-\sec(e+fx))^4}+\frac{2a^2\sec(e+fx)}{(1-\sec(e+fx))^4}+\frac{a^2\sec^2(e+fx)}{(1-\sec(e+fx))^4}\right)dx}{c^4}$$

$$= \frac{a^2\int\frac{1}{(1-\sec(e+fx))^4}dx}{c^4}+\frac{a^2\int\frac{\sec^2(e+fx)}{(1-\sec(e+fx))^4}dx}{c^4}+\frac{\left(2a^2\right)\int\frac{\sec(e+fx)}{(1-\sec(e+fx))^4}dx}{c^4}$$

$$= -\frac{4a^2\tan(e+fx)}{7c^4f(1-\sec(e+fx))^4}-\frac{a^2\int\frac{-7-3\sec(e+fx)}{(1-\sec(e+fx))^3}dx}{7c^4}-\frac{\left(4a^2\right)\int\frac{\sec(e+fx)}{(1-\sec(e+fx))^3}dx}{7c^4}+\frac{\left(6a^2\right)\int\frac{\sec(e\cdots}{(1-\sec(\cdots}}{7c^4}$$

$$= -\frac{4a^2\tan(e+fx)}{7c^4f(1-\sec(e+fx))^4}-\frac{12a^2\tan(e+fx)}{35c^4f(1-\sec(e+fx))^3}+\frac{a^2\int\frac{35+20\sec(e+fx)}{(1-\sec(e+fx))^2}dx}{35c^4}-\frac{\left(8a^2\right)\int\frac{\sec\cdots}{(1-\sec\cdots}}{35c\cdots}$$

$$= -\frac{4a^2\tan(e+fx)}{7c^4f(1-\sec(e+fx))^4}-\frac{12a^2\tan(e+fx)}{35c^4f(1-\sec(e+fx))^3}-\frac{59a^2\tan(e+fx)}{105c^4f(1-\sec(e+fx))^2}-\frac{a^2\int\frac{-105\cdots}{1-\cdots}}{1\cdots}$$

$$= \frac{a^2x}{c^4}-\frac{4a^2\tan(e+fx)}{7c^4f(1-\sec(e+fx))^4}-\frac{12a^2\tan(e+fx)}{35c^4f(1-\sec(e+fx))^3}-\frac{59a^2\tan(e+fx)}{105c^4f(1-\sec(e+fx))^2}-\frac{\cdots}{105c\cdots}$$

$$= \frac{a^2x}{c^4}-\frac{4a^2\tan(e+fx)}{7c^4f(1-\sec(e+fx))^4}-\frac{12a^2\tan(e+fx)}{35c^4f(1-\sec(e+fx))^3}-\frac{59a^2\tan(e+fx)}{105c^4f(1-\sec(e+fx))^2}-\frac{1\cdots}{105c\cdots}$$

Mathematica [A] time = 0.615846, size = 227, normalized size = 1.71

$$\frac{a^2\csc\left(\frac{e}{2}\right)\csc^7\left(\frac{1}{2}(e+fx)\right)\left(-10430\sin\left(e+\frac{fx}{2}\right)+8568\sin\left(e+\frac{3fx}{2}\right)+4830\sin\left(2e+\frac{3fx}{2}\right)-3206\sin\left(2e+\frac{5fx}{2}\right)-1260\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (a^2*Csc[e/2]*Csc[(e + f*x)/2]^7*(3675*f*x*Cos[(f*x)/2] - 3675*f*x*Cos[e +
(f*x)/2] - 2205*f*x*Cos[e + (3*f*x)/2] + 2205*f*x*Cos[2*e + (3*f*x)/2] + 73
5*f*x*Cos[2*e + (5*f*x)/2] - 735*f*x*Cos[3*e + (5*f*x)/2] - 105*f*x*Cos[3*e
 + (7*f*x)/2] + 105*f*x*Cos[4*e + (7*f*x)/2] - 11900*Sin[(f*x)/2] - 10430*S
in[e + (f*x)/2] + 8568*Sin[e + (3*f*x)/2] + 4830*Sin[2*e + (3*f*x)/2] - 320
6*Sin[2*e + (5*f*x)/2] - 1260*Sin[3*e + (5*f*x)/2] + 638*Sin[3*e + (7*f*x)/
2]))/(13440*c^4*f)
```

Maple [A] time = 0.109, size = 111, normalized size = 0.8

$$2\frac{a^2\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)}{fc^4}-\frac{a^2}{14\,fc^4}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-7}+\frac{3\,a^2}{10\,fc^4}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-5}-\frac{2\,a^2}{3\,fc^4}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x)
```

```
[Out] 2/f*a^2/c^4*arctan(tan(1/2*f*x+1/2*e))-1/14/f*a^2/c^4/tan(1/2*f*x+1/2*e)^7+
3/10/f*a^2/c^4/tan(1/2*f*x+1/2*e)^5-2/3/f*a^2/c^4/tan(1/2*f*x+1/2*e)^3+2/f*
a^2/c^4/tan(1/2*f*x+1/2*e)
```

Maxima [B] time = 1.6022, size = 397, normalized size = 2.98

$$\frac{5\,a^2\left(\frac{336\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^4}+\frac{\left(\frac{21\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{77\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{315\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-3\right)(\cos(fx+e)+1)^7}{c^4\sin(fx+e)^7}\right)+\frac{a^2\left(\frac{21\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{35\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{105\sin(fx}{(\cos(fx+e)}\right.}{c^4\sin(fx+e)^7}}{840\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] 1/840*(5*a^2*(336*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^4 + (21*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 - 77*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 315
*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 3)*(cos(f*x + e) + 1)^7/(c^4*sin(f*x
 + e)^7)) + a^2*(21*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 35*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 - 105*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 15)*(cos(
f*x + e) + 1)^7/(c^4*sin(f*x + e)^7) + 6*a^2*(21*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 35*sin(f*x + e)^6/(cos
(f*x + e) + 1)^6 - 5)*(cos(f*x + e) + 1)^7/(c^4*sin(f*x + e)^7))/f
```

Fricas [A] time = 1.05406, size = 424, normalized size = 3.19

$$\frac{319\,a^2\cos\left(fx+e\right)^4-327\,a^2\cos\left(fx+e\right)^3-95\,a^2\cos\left(fx+e\right)^2+387\,a^2\cos\left(fx+e\right)-164\,a^2+105\left(a^2fx\cos\left(fx+e\right.\right.}{105\left(c^4f\cos\left(fx+e\right)^3-3\,c^4f\cos\left(fx+e\right)^2+3\,c^4f\cos\left(fx+e\right)-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] 1/105*(319*a^2*cos(f*x + e)^4 - 327*a^2*cos(f*x + e)^3 - 95*a^2*cos(f*x + e
)^2 + 387*a^2*cos(f*x + e) - 164*a^2 + 105*(a^2*f*x*cos(f*x + e)^3 - 3*a^2*
f*x*cos(f*x + e)^2 + 3*a^2*f*x*cos(f*x + e) - a^2*f*x)*sin(f*x + e))/((c^4*
f*cos(f*x + e)^3 - 3*c^4*f*cos(f*x + e)^2 + 3*c^4*f*cos(f*x + e) - c^4*f)*s
in(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^2\left(\int \frac{2\sec\left(e+fx\right)}{\sec^4\left(e+fx\right)-4\sec^3\left(e+fx\right)+6\sec^2\left(e+fx\right)-4\sec\left(e+fx\right)+1}\,dx+\int \frac{\sec^2\left(e+fx\right)}{\sec^4\left(e+fx\right)-4\sec^3\left(e+fx\right)+6\sec^2\left(e+fx\right)-4\sec\left(e+fx\right)+1}\,dx+\int \frac{}{\sec^4\left(e+fx\right)}\right.}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**4,x)
```

```
[Out] a**2*(Integral(2*sec(e + f*x)/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(
e + f*x)**2 - 4*sec(e + f*x) + 1), x) + Integral(sec(e + f*x)**2/(sec(e + f
*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e + f*x)**2 - 4*sec(e + f*x) + 1), x) +
Integral(1/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e + f*x)**2 - 4*sec
(e + f*x) + 1), x))/c**4
```

Giac [A] time = 1.36476, size = 126, normalized size = 0.95

$$\frac{\frac{210\left(fx+e\right)a^2}{c^4}+\frac{420\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6-140\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+63\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-15\,a^2}{c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7}}{210\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="giac")
```

```
[Out] 1/210*(210*(f*x + e)*a^2/c^4 + (420*a^2*tan(1/2*f*x + 1/2*e)^6 - 140*a^2*ta
n(1/2*f*x + 1/2*e)^4 + 63*a^2*tan(1/2*f*x + 1/2*e)^2 - 15*a^2)/(c^4*tan(1/2
*f*x + 1/2*e)^7))/f
```

3.10 $\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5}\,dx$

Optimal. Leaf size=164

$$-\frac{494a^2\tan(e+fx)}{315c^5f(1-\sec(e+fx))}-\frac{179a^2\tan(e+fx)}{315c^5f(1-\sec(e+fx))^2}-\frac{37a^2\tan(e+fx)}{105c^5f(1-\sec(e+fx))^3}-\frac{16a^2\tan(e+fx)}{63c^5f(1-\sec(e+fx))^4}-\frac{4a^2}{9c^5f(1}$$

```
[Out] (a^2*x)/c^5 - (4*a^2*Tan[e + f*x])/(9*c^5*f*(1 - Sec[e + f*x])^5) - (16*a^2
*Tan[e + f*x])/(63*c^5*f*(1 - Sec[e + f*x])^4) - (37*a^2*Tan[e + f*x])/(105
*c^5*f*(1 - Sec[e + f*x])^3) - (179*a^2*Tan[e + f*x])/(315*c^5*f*(1 - Sec[e
 + f*x])^2) - (494*a^2*Tan[e + f*x])/(315*c^5*f*(1 - Sec[e + f*x]))
```

Rubi [A] time = 0.542792, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {3903, 3777, 3922, 3919, 3794, 3796, 3797}

$$-\frac{494a^2\tan(e+fx)}{315c^5f(1-\sec(e+fx))}-\frac{179a^2\tan(e+fx)}{315c^5f(1-\sec(e+fx))^2}-\frac{37a^2\tan(e+fx)}{105c^5f(1-\sec(e+fx))^3}-\frac{16a^2\tan(e+fx)}{63c^5f(1-\sec(e+fx))^4}-\frac{4a^2}{9c^5f(1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x])^5,x]
```


```
[Out] (a^2*x)/c^5 - (4*a^2*Tan[e + f*x])/(9*c^5*f*(1 - Sec[e + f*x])^5) - (16*a^2
*Tan[e + f*x])/(63*c^5*f*(1 - Sec[e + f*x])^4) - (37*a^2*Tan[e + f*x])/(105
*c^5*f*(1 - Sec[e + f*x])^3) - (179*a^2*Tan[e + f*x])/(315*c^5*f*(1 - Sec[e
 + f*x])^2) - (494*a^2*Tan[e + f*x])/(315*c^5*f*(1 - Sec[e + f*x]))
```


Rule 3903

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Dist[c^n, Int[ExpandTrig[(1 + (d*csc[e + f*
x])/c)^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
&& LtQ[m + n, 2]
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
```

```
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
rQ[2*n]
```

Rule 3922

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*
x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x
])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ
[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3797

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(2*m + 1)), x]
+ Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x],
x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5}\,dx = \frac{\int \left(\frac{a^2}{(1-\sec(e+fx))^5}+\frac{2a^2\sec(e+fx)}{(1-\sec(e+fx))^5}+\frac{a^2\sec^2(e+fx)}{(1-\sec(e+fx))^5}\right)dx}{c^5}$$

$$= \frac{a^2\int \frac{1}{(1-\sec(e+fx))^5}\,dx}{c^5}+\frac{a^2\int \frac{\sec^2(e+fx)}{(1-\sec(e+fx))^5}\,dx}{c^5}+\frac{\left(2a^2\right)\int \frac{\sec(e+fx)}{(1-\sec(e+fx))^5}\,dx}{c^5}$$

$$= -\frac{4a^2\tan(e+fx)}{9c^5f(1-\sec(e+fx))^5}-\frac{a^2\int \frac{-9-4\sec(e+fx)}{(1-\sec(e+fx))^4}\,dx}{9c^5}-\frac{\left(5a^2\right)\int \frac{\sec(e+fx)}{(1-\sec(e+fx))^4}\,dx}{9c^5}+\frac{\left(8a^2\right)\int \frac{\sec}{(1-\sec}}{9c^5}$$

$$= -\frac{4a^2\tan(e+fx)}{9c^5f(1-\sec(e+fx))^5}-\frac{16a^2\tan(e+fx)}{63c^5f(1-\sec(e+fx))^4}+\frac{a^2\int \frac{63+39\sec(e+fx)}{(1-\sec(e+fx))^3}\,dx}{63c^5}-\frac{\left(5a^2\right)\int \frac{\mathrm{se}}{(1-\mathrm{s}}}{21}$$

$$= -\frac{4a^2\tan(e+fx)}{9c^5f(1-\sec(e+fx))^5}-\frac{16a^2\tan(e+fx)}{63c^5f(1-\sec(e+fx))^4}-\frac{37a^2\tan(e+fx)}{105c^5f(1-\sec(e+fx))^3}-\frac{a^2\int \frac{-31}{(}}{}$$

$$= -\frac{4a^2\tan(e+fx)}{9c^5f(1-\sec(e+fx))^5}-\frac{16a^2\tan(e+fx)}{63c^5f(1-\sec(e+fx))^4}-\frac{37a^2\tan(e+fx)}{105c^5f(1-\sec(e+fx))^3}-\frac{179a}{315c^5f(}$$

$$= \frac{a^2x}{c^5}-\frac{4a^2\tan(e+fx)}{9c^5f(1-\sec(e+fx))^5}-\frac{16a^2\tan(e+fx)}{63c^5f(1-\sec(e+fx))^4}-\frac{37a^2\tan(e+fx)}{105c^5f(1-\sec(e+fx))^3}-\frac{}{31}$$

$$= \frac{a^2x}{c^5}-\frac{4a^2\tan(e+fx)}{9c^5f(1-\sec(e+fx))^5}-\frac{16a^2\tan(e+fx)}{63c^5f(1-\sec(e+fx))^4}-\frac{37a^2\tan(e+fx)}{105c^5f(1-\sec(e+fx))^3}-\frac{}{31}$$

Mathematica [A] time = 0.886377, size = 283, normalized size = 1.73

$$\frac{a^2\csc\left(\frac{e}{2}\right)\csc^9\left(\frac{1}{2}(e+fx)\right)\left(-117810\sin\left(e+\frac{fx}{2}\right)+100002\sin\left(e+\frac{3fx}{2}\right)+68670\sin\left(2e+\frac{3fx}{2}\right)-48978\sin\left(2e+\frac{5fx}{2}\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x])^5,x]
```

```
[Out] (a^2*Csc[e/2]*Csc[(e + f*x)/2]^9*(39690*f*x*Cos[(f*x)/2] - 39690*f*x*Cos[e
+ (f*x)/2] - 26460*f*x*Cos[e + (3*f*x)/2] + 26460*f*x*Cos[2*e + (3*f*x)/2]
+ 11340*f*x*Cos[2*e + (5*f*x)/2] - 11340*f*x*Cos[3*e + (5*f*x)/2] - 2835*f*
x*Cos[3*e + (7*f*x)/2] + 2835*f*x*Cos[4*e + (7*f*x)/2] + 315*f*x*Cos[4*e +
(9*f*x)/2] - 315*f*x*Cos[5*e + (9*f*x)/2] - 135198*Sin[(f*x)/2] - 117810*Si
n[e + (f*x)/2] + 100002*Sin[e + (3*f*x)/2] + 68670*Sin[2*e + (3*f*x)/2] - 4
8978*Sin[2*e + (5*f*x)/2] - 23310*Sin[3*e + (5*f*x)/2] + 13662*Sin[3*e + (7
*f*x)/2] + 4410*Sin[4*e + (7*f*x)/2] - 2008*Sin[4*e + (9*f*x)/2]))/(161280*
c^5*f)
```

Maple [A] time = 0.119, size = 133, normalized size = 0.8

$$2\,\frac{a^2\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)}{fc^5}+\frac{a^2}{36\,fc^5}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-9}-\frac{a^2}{7\,fc^5}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-7}+\frac{7\,a^2}{20\,fc^5}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-5}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x)
```

```
[Out] 2/f*a^2/c^5*arctan(tan(1/2*f*x+1/2*e))+1/36/f*a^2/c^5/tan(1/2*f*x+1/2*e)^9-
1/7/f*a^2/c^5/tan(1/2*f*x+1/2*e)^7+7/20/f*a^2/c^5/tan(1/2*f*x+1/2*e)^5-2/3/
f*a^2/c^5/tan(1/2*f*x+1/2*e)^3+2/f*a^2/c^5/tan(1/2*f*x+1/2*e)
```

Maxima [B] time = 1.63463, size = 452, normalized size = 2.76

$$\frac{a^2\left(\frac{10080\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^5}-\frac{\left(\frac{270\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{1008\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{2730\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{9765\sin(fx+e)^8}{(\cos(fx+e)+1)^8}-35\right)(\cos(fx+e)+1)^9}{c^5\sin(fx+e)^9}\right)-\frac{2\,a^2\left(\frac{180\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{378\sin(f}{(\cos(fx+}\right.}{}}{5040\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="maxima")
```

```
[Out] 1/5040*(a^2*(10080*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^5 - (270*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 - 1008*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 +
 2730*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 9765*sin(f*x + e)^8/(cos(f*x +
e) + 1)^8 - 35)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x + e)^9)) - 2*a^2*(180*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 - 378*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
+ 420*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 315*sin(f*x + e)^8/(cos(f*x + e
) + 1)^8 - 35)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x + e)^9) - 5*a^2*(18*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 - 42*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 63
*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 7)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x
 + e)^9))/f
```

Fricas [A] time = 1.04967, size = 535, normalized size = 3.26

$$\frac{1004\,a^2\cos\left(fx+e\right)^5-1811\,a^2\cos\left(fx+e\right)^4+797\,a^2\cos\left(fx+e\right)^3+1457\,a^2\cos\left(fx+e\right)^2-1661\,a^2\cos\left(fx+e\right)+49}{315\left(c^5f\cos\left(fx+e\right)^4-4\,c^5f\cos\left(fx+e\right)^3+6\,c^5f\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="fricas")
```

```
[Out] 1/315*(1004*a^2*cos(f*x + e)^5 - 1811*a^2*cos(f*x + e)^4 + 797*a^2*cos(f*x
+ e)^3 + 1457*a^2*cos(f*x + e)^2 - 1661*a^2*cos(f*x + e) + 494*a^2 + 315*(a
^2*f*x*cos(f*x + e)^4 - 4*a^2*f*x*cos(f*x + e)^3 + 6*a^2*f*x*cos(f*x + e)^2
 - 4*a^2*f*x*cos(f*x + e) + a^2*f*x)*sin(f*x + e))/((c^5*f*cos(f*x + e)^4 -
 4*c^5*f*cos(f*x + e)^3 + 6*c^5*f*cos(f*x + e)^2 - 4*c^5*f*cos(f*x + e) + c
^5*f)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^2\left(\int\frac{2\sec\left(e+fx\right)}{\sec^5\left(e+fx\right)-5\sec^4\left(e+fx\right)+10\sec^3\left(e+fx\right)-10\sec^2\left(e+fx\right)+5\sec\left(e+fx\right)-1}\,dx+\int\frac{\sec^2\left(e+fx\right)}{\sec^5\left(e+fx\right)-5\sec^4\left(e+fx\right)+10\sec^3\left(e+fx\right)-10\sec^2\left(e+fx\right)+}\right.}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**5,x)
```

```
[Out] -a**2*(Integral(2*sec(e + f*x)/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*se
c(e + f*x)**3 - 10*sec(e + f*x)**2 + 5*sec(e + f*x) - 1), x) + Integral(sec
(e + f*x)**2/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(e + f*x)**3 - 10
*sec(e + f*x)**2 + 5*sec(e + f*x) - 1), x) + Integral(1/(sec(e + f*x)**5 -
5*sec(e + f*x)**4 + 10*sec(e + f*x)**3 - 10*sec(e + f*x)**2 + 5*sec(e + f*x
) - 1), x))/c**5
```

Giac [A] time = 1.65229, size = 149, normalized size = 0.91

$$\frac{\frac{1260\left(fx+e\right)a^2}{c^5}+\frac{2520\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^8-840\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6+441\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-180\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+35\,a^2}{c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^9}}{1260\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="giac")
```

```
[Out] 1/1260*(1260*(f*x + e)*a^2/c^5 + (2520*a^2*tan(1/2*f*x + 1/2*e)^8 - 840*a^2
*tan(1/2*f*x + 1/2*e)^6 + 441*a^2*tan(1/2*f*x + 1/2*e)^4 - 180*a^2*tan(1/2*
f*x + 1/2*e)^2 + 35*a^2)/(c^5*tan(1/2*f*x + 1/2*e)^9))/f
```

3.11 $\int (a + a\sec(e + fx))^3(c - c\sec(e + fx))^5\,dx$

Optimal. Leaf size=188

$$-\frac{a^3c^5\tan^7(e+fx)}{7f}-\frac{a^3c^5\tan^5(e+fx)}{5f}+\frac{a^3c^5\tan^3(e+fx)}{3f}-\frac{a^3c^5\tan(e+fx)}{f}-\frac{5a^3c^5\tanh^{-1}(\sin(e+fx))}{8f}+\frac{a^3c^5\text{ta}}{}$$

```
[Out] a^3*c^5*x - (5*a^3*c^5*ArcTanh[Sin[e + f*x]])/(8*f) - (a^3*c^5*Tan[e + f*x]
)/f + (5*a^3*c^5*Sec[e + f*x]*Tan[e + f*x])/(8*f) + (a^3*c^5*Tan[e + f*x]^3
)/(3*f) - (5*a^3*c^5*Sec[e + f*x]*Tan[e + f*x]^3)/(12*f) - (a^3*c^5*Tan[e +
 f*x]^5)/(5*f) + (a^3*c^5*Sec[e + f*x]*Tan[e + f*x]^5)/(3*f) - (a^3*c^5*Tan
[e + f*x]^7)/(7*f)
```

Rubi [A] time = 0.23721, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3904, 3886, 3473, 8, 2611, 3770, 2607, 30}

$$-\frac{a^3c^5\tan^7(e+fx)}{7f}-\frac{a^3c^5\tan^5(e+fx)}{5f}+\frac{a^3c^5\tan^3(e+fx)}{3f}-\frac{a^3c^5\tan(e+fx)}{f}-\frac{5a^3c^5\tanh^{-1}(\sin(e+fx))}{8f}+\frac{a^3c^5\text{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^5,x]
```


```
[Out] a^3*c^5*x - (5*a^3*c^5*ArcTanh[Sin[e + f*x]])/(8*f) - (a^3*c^5*Tan[e + f*x]
)/f + (5*a^3*c^5*Sec[e + f*x]*Tan[e + f*x])/(8*f) + (a^3*c^5*Tan[e + f*x]^3
)/(3*f) - (5*a^3*c^5*Sec[e + f*x]*Tan[e + f*x]^3)/(12*f) - (a^3*c^5*Tan[e +
 f*x]^5)/(5*f) + (a^3*c^5*Sec[e + f*x]*Tan[e + f*x]^5)/(3*f) - (a^3*c^5*Tan
[e + f*x]^7)/(7*f)
```


Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
```

```
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^5\,dx = -\left(\left(a^3c^3\right)\int (c-c\sec(e+fx))^2\tan^6(e+fx)\,dx\right)$$
$$= -\left(\left(a^3c^3\right)\int \left(c^2\tan^6(e+fx)-2c^2\sec(e+fx)\tan^6(e+fx)+c^2\sec^2(e\right.\right.$$
$$= -\left(\left(a^3c^5\right)\int \tan^6(e+fx)\,dx\right)-\left(a^3c^5\right)\int \sec^2(e+fx)\tan^6(e+fx)\,dx+$$
$$= -\frac{a^3c^5\tan^5(e+fx)}{5f}+\frac{a^3c^5\sec(e+fx)\tan^5(e+fx)}{3f}+\left(a^3c^5\right)\int \tan^4(e$$
$$= \frac{a^3c^5\tan^3(e+fx)}{3f}-\frac{5a^3c^5\sec(e+fx)\tan^3(e+fx)}{12f}-\frac{a^3c^5\tan^5(e+fx)}{5f}$$
$$= -\frac{a^3c^5\tan(e+fx)}{f}+\frac{5a^3c^5\sec(e+fx)\tan(e+fx)}{8f}+\frac{a^3c^5\tan^3(e+fx)}{3f}$$
$$= a^3c^5x-\frac{5a^3c^5\tanh^{-1}(\sin(e+fx))}{8f}-\frac{a^3c^5\tan(e+fx)}{f}+\frac{5a^3c^5\sec(e+}{8}$$

Mathematica [A] time = 2.2194, size = 189, normalized size = 1.01

$$\frac{a^3c^5\sec^7(e+fx)\left(-4200\sin(e+fx)+2975\sin(2(e+fx))-2184\sin(3(e+fx))+980\sin(4(e+fx))-2408\sin(5(e+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^5,x]
```

```
[Out] (a^3*c^5*Sec[e + f*x]^7*(14700*(e + f*x)*Cos[e + f*x] - 16800*ArcTanh[Sin[e
 + f*x]]*Cos[e + f*x]^7 + 8820*e*Cos[3*(e + f*x)] + 8820*f*x*Cos[3*(e + f*x
)] + 2940*e*Cos[5*(e + f*x)] + 2940*f*x*Cos[5*(e + f*x)] + 420*e*Cos[7*(e +
 f*x)] + 420*f*x*Cos[7*(e + f*x)] - 4200*Sin[e + f*x] + 2975*Sin[2*(e + f*x
)] - 2184*Sin[3*(e + f*x)] + 980*Sin[4*(e + f*x)] - 2408*Sin[5*(e + f*x)] +
 1155*Sin[6*(e + f*x)] - 584*Sin[7*(e + f*x)]))/(26880*f)
```

Maple [A] time = 0.035, size = 211, normalized size = 1.1

$$-\frac{13\,c^5a^3\tan\left(fx+e\right)\left(\sec\left(fx+e\right)\right)^3}{12\,f}+\frac{11\,c^5a^3\sec\left(fx+e\right)\tan\left(fx+e\right)}{8\,f}-\frac{5\,c^5a^3\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{8\,f}-\frac{1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^5,x)
```

```
[Out] -13/12/f*c^5*a^3*tan(f*x+e)*sec(f*x+e)^3+11/8*a^3*c^5*sec(f*x+e)*tan(f*x+e)
/f-5/8/f*c^5*a^3*ln(sec(f*x+e)+tan(f*x+e))-146/105*a^3*c^5*tan(f*x+e)/f+a^3
*c^5*x+1/f*a^3*c^5*e+8/35/f*c^5*a^3*tan(f*x+e)*sec(f*x+e)^4+32/105/f*c^5*a^
3*tan(f*x+e)*sec(f*x+e)^2+1/3/f*c^5*a^3*tan(f*x+e)*sec(f*x+e)^5-1/7/f*c^5*a
^3*tan(f*x+e)*sec(f*x+e)^6
```

Maxima [B] time = 1.00956, size = 481, normalized size = 2.56

$$-\frac{48\left(5\tan\left(fx+e\right)^{7}+21\tan\left(fx+e\right)^{5}+35\tan\left(fx+e\right)^{3}+35\tan\left(fx+e\right)\right)a^{3}c^{5}-224\left(3\tan\left(fx+e\right)^{5}+10\tan\left(f\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^5,x, algorithm="maxima")
```

```
[Out] -1/1680*(48*(5*tan(f*x + e)^7 + 21*tan(f*x + e)^5 + 35*tan(f*x + e)^3 + 35*
tan(f*x + e))*a^3*c^5 - 224*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(
f*x + e))*a^3*c^5 - 1680*(f*x + e)*a^3*c^5 + 35*a^3*c^5*(2*(15*sin(f*x + e)
^5 - 40*sin(f*x + e)^3 + 33*sin(f*x + e))/(sin(f*x + e)^6 - 3*sin(f*x + e)^
4 + 3*sin(f*x + e)^2 - 1) - 15*log(sin(f*x + e) + 1) + 15*log(sin(f*x + e)
- 1)) - 630*a^3*c^5*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4
- 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)
) + 2520*a^3*c^5*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) +
1) + log(sin(f*x + e) - 1)) + 3360*a^3*c^5*log(sec(f*x + e) + tan(f*x + e))
 + 3360*a^3*c^5*tan(f*x + e))/f
```

Fricas [A] time = 1.21925, size = 498, normalized size = 2.65

$$\frac{1680\,a^{3}c^{5}fx\cos\left(fx+e\right)^{7}-525\,a^{3}c^{5}\cos\left(fx+e\right)^{7}\log\left(\sin\left(fx+e\right)+1\right)+525\,a^{3}c^{5}\cos\left(fx+e\right)^{7}\log\left(-\sin\left(fx+e\right)+1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^5,x, algorithm="fricas")
```

```
[Out] 1/1680*(1680*a^3*c^5*f*x*cos(f*x + e)^7 - 525*a^3*c^5*cos(f*x + e)^7*log(si
n(f*x + e) + 1) + 525*a^3*c^5*cos(f*x + e)^7*log(-sin(f*x + e) + 1) - 2*(11
68*a^3*c^5*cos(f*x + e)^6 - 1155*a^3*c^5*cos(f*x + e)^5 - 256*a^3*c^5*cos(f
*x + e)^4 + 910*a^3*c^5*cos(f*x + e)^3 - 192*a^3*c^5*cos(f*x + e)^2 - 280*a
^3*c^5*cos(f*x + e) + 120*a^3*c^5)*sin(f*x + e))/(f*cos(f*x + e)^7)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-a^3c^5\left(\int (-1)\,dx + \int 2\sec\left(e+fx\right)dx + \int 2\sec^2\left(e+fx\right)dx + \int -6\sec^3\left(e+fx\right)dx + \int 6\sec^5\left(e+fx\right)dx + \int -\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**5,x)
```

```
[Out] -a**3*c**5*(Integral(-1, x) + Integral(2*sec(e + f*x), x) + Integral(2*sec(
e + f*x)**2, x) + Integral(-6*sec(e + f*x)**3, x) + Integral(6*sec(e + f*x)
**5, x) + Integral(-2*sec(e + f*x)**6, x) + Integral(-2*sec(e + f*x)**7, x)
 + Integral(sec(e + f*x)**8, x))
```

Giac [A] time = 1.42914, size = 298, normalized size = 1.59

$$840\left(fx+e\right)a^3c^5 - 525\,a^3c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right) + 525\,a^3c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right) + \frac{2\left(1365\,a^3c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^5,x, algorithm="giac")
```

```
[Out] 1/840*(840*(f*x + e)*a^3*c^5 - 525*a^3*c^5*log(abs(tan(1/2*f*x + 1/2*e) + 1
)) + 525*a^3*c^5*log(abs(tan(1/2*f*x + 1/2*e) - 1)) + 2*(1365*a^3*c^5*tan(1
/2*f*x + 1/2*e)^13 - 9660*a^3*c^5*tan(1/2*f*x + 1/2*e)^11 + 29673*a^3*c^5*t
an(1/2*f*x + 1/2*e)^9 - 21216*a^3*c^5*tan(1/2*f*x + 1/2*e)^7 + 9863*a^3*c^5
*tan(1/2*f*x + 1/2*e)^5 - 2660*a^3*c^5*tan(1/2*f*x + 1/2*e)^3 + 315*a^3*c^5
*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^7)/f
```

3.12 $\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^4\,dx$

Optimal. Leaf size=132

$$-\frac{5a^3c^4\tanh^{-1}(\sin(e+fx))}{16f}-\frac{a^3\tan^5(e+fx)\left(6c^4-5c^4\sec(e+fx)\right)}{30f}+\frac{a^3\tan^3(e+fx)\left(8c^4-5c^4\sec(e+fx)\right)}{24f}-\frac{a^3\tan}{}$$

```
[Out] a^3*c^4*x - (5*a^3*c^4*ArcTanh[Sin[e + f*x]])/(16*f) - (a^3*(16*c^4 - 5*c^4
*Sec[e + f*x])*Tan[e + f*x])/(16*f) + (a^3*(8*c^4 - 5*c^4*Sec[e + f*x])*Tan
[e + f*x]^3)/(24*f) - (a^3*(6*c^4 - 5*c^4*Sec[e + f*x])*Tan[e + f*x]^5)/(30
*f)
```

Rubi [A] time = 0.151008, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3904, 3881, 3770}

$$-\frac{5a^3c^4\tanh^{-1}(\sin(e+fx))}{16f}-\frac{a^3\tan^5(e+fx)\left(6c^4-5c^4\sec(e+fx)\right)}{30f}+\frac{a^3\tan^3(e+fx)\left(8c^4-5c^4\sec(e+fx)\right)}{24f}-\frac{a^3\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^4,x]
```

```
[Out] a^3*c^4*x - (5*a^3*c^4*ArcTanh[Sin[e + f*x]])/(16*f) - (a^3*(16*c^4 - 5*c^4
*Sec[e + f*x])*Tan[e + f*x])/(16*f) + (a^3*(8*c^4 - 5*c^4*Sec[e + f*x])*Tan
[e + f*x]^3)/(24*f) - (a^3*(6*c^4 - 5*c^4*Sec[e + f*x])*Tan[e + f*x]^5)/(30
*f)
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3881

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(e*(e*Cot[c + d*x])^(m - 1)*(a*m + b*(m - 1)*Csc[c
 + d*x]))/(d*m*(m - 1)), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a*m
```

```
 + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1
]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a + a\sec(e+fx))^3(c - c\sec(e+fx))^4\,dx = -\left(\left(a^3c^3\right)\int (c - c\sec(e+fx))\tan^6(e+fx)\,dx\right)$$
$$= -\frac{a^3\left(6c^4 - 5c^4\sec(e+fx)\right)\tan^5(e+fx)}{30f} + \frac{1}{6}\left(a^3c^3\right)\int (6c - 5c\sec(e+$$
$$= \frac{a^3\left(8c^4 - 5c^4\sec(e+fx)\right)\tan^3(e+fx)}{24f} - \frac{a^3\left(6c^4 - 5c^4\sec(e+fx)\right)\tan}{30f}$$
$$= -\frac{a^3\left(16c^4 - 5c^4\sec(e+fx)\right)\tan(e+fx)}{16f} + \frac{a^3\left(8c^4 - 5c^4\sec(e+fx)\right)\text{ta}}{24f}$$
$$= a^3c^4x - \frac{a^3\left(16c^4 - 5c^4\sec(e+fx)\right)\tan(e+fx)}{16f} + \frac{a^3\left(8c^4 - 5c^4\sec(e+\right.}{24f}$$
$$= a^3c^4x - \frac{5a^3c^4\tanh^{-1}(\sin(e+fx))}{16f} - \frac{a^3\left(16c^4 - 5c^4\sec(e+fx)\right)\tan(e+}{16f}$$

Mathematica [A] time = 1.83372, size = 165, normalized size = 1.25

$$\frac{a^3c^4\sec^6(e+fx)\left(450\sin(e+fx) - 600\sin(2(e+fx)) - 25\sin(3(e+fx)) - 384\sin(4(e+fx)) + 165\sin(5(e+fx)) -\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (a^3*c^4*Sec[e + f*x]^6*(1200*e + 1200*f*x - 1200*ArcTanh[Sin[e + f*x]]*Cos
[e + f*x]^6 + 1800*(e + f*x)*Cos[2*(e + f*x)] + 720*e*Cos[4*(e + f*x)] + 72
0*f*x*Cos[4*(e + f*x)] + 120*e*Cos[6*(e + f*x)] + 120*f*x*Cos[6*(e + f*x)]
+ 450*Sin[e + f*x] - 600*Sin[2*(e + f*x)] - 25*Sin[3*(e + f*x)] - 384*Sin[4
*(e + f*x)] + 165*Sin[5*(e + f*x)] - 184*Sin[6*(e + f*x)]))/(3840*f)
```

Maple [A] time = 0.028, size = 186, normalized size = 1.4

$$-\frac{23\,c^4a^3\tan\left(fx+e\right)}{15\,f}+\frac{11\,c^4a^3\tan\left(fx+e\right)\left(\sec\left(fx+e\right)\right)^2}{15\,f}+\frac{11\,c^4a^3\sec\left(fx+e\right)\tan\left(fx+e\right)}{16\,f}-\frac{5\,c^4a^3\ln\left(\sec\left(fx+e\right.\right.}{16\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^4,x)
```

```
[Out] -23/15/f*c^4*a^3*tan(f*x+e)+11/15/f*c^4*a^3*tan(f*x+e)*sec(f*x+e)^2+11/16/f
*c^4*a^3*sec(f*x+e)*tan(f*x+e)-5/16/f*c^4*a^3*ln(sec(f*x+e)+tan(f*x+e))+a^3
*c^4*x+1/f*a^3*c^4*e-13/24/f*c^4*a^3*tan(f*x+e)*sec(f*x+e)^3-1/5/f*c^4*a^3*
tan(f*x+e)*sec(f*x+e)^4+1/6/f*c^4*a^3*tan(f*x+e)*sec(f*x+e)^5
```

Maxima [B] time = 1.01966, size = 451, normalized size = 3.42

$$-\frac{32\left(3\,\tan\left(fx+e\right)^5+10\,\tan\left(fx+e\right)^3+15\,\tan\left(fx+e\right)\right)a^3c^4-480\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)a^3c^4-480\left(fx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] -1/480*(32*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^3*c^4
 - 480*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c^4 - 480*(f*x + e)*a^3*c^4 +
5*a^3*c^4*(2*(15*sin(f*x + e)^5 - 40*sin(f*x + e)^3 + 33*sin(f*x + e))/(sin
(f*x + e)^6 - 3*sin(f*x + e)^4 + 3*sin(f*x + e)^2 - 1) - 15*log(sin(f*x + e
) + 1) + 15*log(sin(f*x + e) - 1)) - 90*a^3*c^4*(2*(3*sin(f*x + e)^3 - 5*si
n(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) +
1) + 3*log(sin(f*x + e) - 1)) + 360*a^3*c^4*(2*sin(f*x + e)/(sin(f*x + e)^2
 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 480*a^3*c^4*log(se
c(f*x + e) + tan(f*x + e)) + 1440*a^3*c^4*tan(f*x + e))/f
```

Fricas [A] time = 1.18828, size = 448, normalized size = 3.39

$$\frac{480\,a^3c^4fx\cos\left(fx+e\right)^6-75\,a^3c^4\cos\left(fx+e\right)^6\log\left(\sin\left(fx+e\right)+1\right)+75\,a^3c^4\cos\left(fx+e\right)^6\log\left(-\sin\left(fx+e\right)+1\right)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] 1/480*(480*a^3*c^4*f*x*cos(f*x + e)^6 - 75*a^3*c^4*cos(f*x + e)^6*log(sin(f
*x + e) + 1) + 75*a^3*c^4*cos(f*x + e)^6*log(-sin(f*x + e) + 1) - 2*(368*a^
3*c^4*cos(f*x + e)^5 - 165*a^3*c^4*cos(f*x + e)^4 - 176*a^3*c^4*cos(f*x + e
)^3 + 130*a^3*c^4*cos(f*x + e)^2 + 48*a^3*c^4*cos(f*x + e) - 40*a^3*c^4)*si
n(f*x + e))/(f*cos(f*x + e)^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3c^4\left(\int 1\,dx + \int -\sec\left(e+fx\right)dx + \int -3\sec^2\left(e+fx\right)dx + \int 3\sec^3\left(e+fx\right)dx + \int 3\sec^4\left(e+fx\right)dx + \int -3\sec$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**4,x)
```

```
[Out] a**3*c**4*(Integral(1, x) + Integral(-sec(e + f*x), x) + Integral(-3*sec(e
+ f*x)**2, x) + Integral(3*sec(e + f*x)**3, x) + Integral(3*sec(e + f*x)**4
, x) + Integral(-3*sec(e + f*x)**5, x) + Integral(-sec(e + f*x)**6, x) + In
tegral(sec(e + f*x)**7, x))
```

Giac [A] time = 1.36117, size = 271, normalized size = 2.05

$$\frac{240\left(fx+e\right)a^3c^4 - 75\,a^3c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right) + 75\,a^3c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right) + \frac{2\left(315\,a^3c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{11}-1}{}}{240\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^4,x, algorithm="giac")
```

```
[Out] 1/240*(240*(f*x + e)*a^3*c^4 - 75*a^3*c^4*log(abs(tan(1/2*f*x + 1/2*e) + 1)
) + 75*a^3*c^4*log(abs(tan(1/2*f*x + 1/2*e) - 1)) + 2*(315*a^3*c^4*tan(1/2*
f*x + 1/2*e)^11 - 1945*a^3*c^4*tan(1/2*f*x + 1/2*e)^9 + 5118*a^3*c^4*tan(1/
2*f*x + 1/2*e)^7 - 3138*a^3*c^4*tan(1/2*f*x + 1/2*e)^5 + 1095*a^3*c^4*tan(1
```

```
/2*f*x + 1/2*e)^3 - 165*a^3*c^4*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)
^2 - 1)^6)/f
```

3.13 $\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^3\,dx$

Optimal. Leaf size=68

$$-\frac{a^3c^3\tan^5(e+fx)}{5f}+\frac{a^3c^3\tan^3(e+fx)}{3f}-\frac{a^3c^3\tan(e+fx)}{f}+a^3c^3x$$

```
[Out] a^3*c^3*x - (a^3*c^3*Tan[e + f*x])/f + (a^3*c^3*Tan[e + f*x]^3)/(3*f) - (a^
3*c^3*Tan[e + f*x]^5)/(5*f)
```

Rubi [A] time = 0.07435, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3904, 3473, 8}

$$-\frac{a^3c^3\tan^5(e+fx)}{5f}+\frac{a^3c^3\tan^3(e+fx)}{3f}-\frac{a^3c^3\tan(e+fx)}{f}+a^3c^3x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^3,x]
```

```
[Out] a^3*c^3*x - (a^3*c^3*Tan[e + f*x])/f + (a^3*c^3*Tan[e + f*x]^3)/(3*f) - (a^
3*c^3*Tan[e + f*x]^5)/(5*f)
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^3\,dx &= -\left(\left(a^3c^3\right)\int \tan^6(e+fx)\,dx\right)\\ &= -\frac{a^3c^3\tan^5(e+fx)}{5f}+\left(a^3c^3\right)\int \tan^4(e+fx)\,dx\\ &= \frac{a^3c^3\tan^3(e+fx)}{3f}-\frac{a^3c^3\tan^5(e+fx)}{5f}-\left(a^3c^3\right)\int \tan^2(e+fx)\,dx\\ &= -\frac{a^3c^3\tan(e+fx)}{f}+\frac{a^3c^3\tan^3(e+fx)}{3f}-\frac{a^3c^3\tan^5(e+fx)}{5f}+\left(a^3c^3\right)\int 1\\ &= a^3c^3x-\frac{a^3c^3\tan(e+fx)}{f}+\frac{a^3c^3\tan^3(e+fx)}{3f}-\frac{a^3c^3\tan^5(e+fx)}{5f}\end{aligned}$$

Mathematica [A] time = 0.0401183, size = 61, normalized size = 0.9

$$-a^3c^3\left(\frac{\tan^5(e+fx)}{5f}-\frac{\tan^3(e+fx)}{3f}-\frac{\tan^{-1}(\tan(e+fx))}{f}+\frac{\tan(e+fx)}{f}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^3,x]
```

```
[Out] -(a^3*c^3*(-(ArcTan[Tan[e + f*x]]/f) + Tan[e + f*x]/f - Tan[e + f*x]^3/(3*f
) + Tan[e + f*x]^5/(5*f)))
```

Maple [A] time = 0.023, size = 93, normalized size = 1.4

$$\frac{1}{f}\left(-3\,c^3a^3\tan\left(fx+e\right)+c^3a^3\left(fx+e\right)-3\,c^3a^3\left(-2/3-1/3\,\left(\sec\left(fx+e\right)\right)^2\right)\tan\left(fx+e\right)+c^3a^3\left(-\frac{8}{15}-\frac{\left(\sec\left(fx+e\right)\right)^4}{5}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^3,x)
```

```
[Out] 1/f*(-3*c^3*a^3*tan(f*x+e)+c^3*a^3*(f*x+e)-3*c^3*a^3*(-2/3-1/3*sec(f*x+e)^2
)*tan(f*x+e)+c^3*a^3*(-8/15-1/5*sec(f*x+e)^4-4/15*sec(f*x+e)^2)*tan(f*x+e))
```

Maxima [A] time = 1.03518, size = 127, normalized size = 1.87

$$-\frac{\left(3\,\tan\left(fx+e\right)^{5}+10\,\tan\left(fx+e\right)^{3}+15\,\tan\left(fx+e\right)\right)a^{3}c^{3}-15\left(\tan\left(fx+e\right)^{3}+3\,\tan\left(fx+e\right)\right)a^{3}c^{3}-15\left(fx+e\right)a}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/15*((3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^3*c^3 - 1
5*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c^3 - 15*(f*x + e)*a^3*c^3 + 45*a^3
*c^3*tan(f*x + e))/f
```

Fricas [A] time = 1.06599, size = 189, normalized size = 2.78

$$\frac{15\,a^{3}c^{3}fx\cos\left(fx+e\right)^{5}-\left(23\,a^{3}c^{3}\cos\left(fx+e\right)^{4}-11\,a^{3}c^{3}\cos\left(fx+e\right)^{2}+3\,a^{3}c^{3}\right)\sin\left(fx+e\right)}{15\,f\cos\left(fx+e\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(15*a^3*c^3*f*x*cos(f*x + e)^5 - (23*a^3*c^3*cos(f*x + e)^4 - 11*a^3*c
^3*cos(f*x + e)^2 + 3*a^3*c^3)*sin(f*x + e))/(f*cos(f*x + e)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-a^{3}c^{3}\left(\int(-1)\,dx+\int 3\sec^{2}\left(e+fx\right)dx+\int -3\sec^{4}\left(e+fx\right)dx+\int\sec^{6}\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**3,x)
```

```
[Out] -a**3*c**3*(Integral(-1, x) + Integral(3*sec(e + f*x)**2, x) + Integral(-3*
sec(e + f*x)**4, x) + Integral(sec(e + f*x)**6, x))
```

Giac [A] time = 1.47474, size = 93, normalized size = 1.37

$$-\frac{3\,a^{3}c^{3}\tan\left(fx+e\right)^{5}-5\,a^{3}c^{3}\tan\left(fx+e\right)^{3}-15\left(fx+e\right)a^{3}c^{3}+15\,a^{3}c^{3}\tan\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/15*(3*a^3*c^3*tan(f*x + e)^5 - 5*a^3*c^3*tan(f*x + e)^3 - 15*(f*x + e)*a
^3*c^3 + 15*a^3*c^3*tan(f*x + e))/f
```

3.14 $\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^2\,dx$

Optimal. Leaf size=97

$$\frac{3a^3c^2\tanh^{-1}(\sin(e+fx))}{8f}+\frac{c^2\tan^3(e+fx)\left(3a^3\sec(e+fx)+4a^3\right)}{12f}-\frac{c^2\tan(e+fx)\left(3a^3\sec(e+fx)+8a^3\right)}{8f}+a^3c^2x$$

```
[Out] a^3*c^2*x + (3*a^3*c^2*ArcTanh[Sin[e + f*x]])/(8*f) - (c^2*(8*a^3 + 3*a^3*S
ec[e + f*x])*Tan[e + f*x])/(8*f) + (c^2*(4*a^3 + 3*a^3*Sec[e + f*x])*Tan[e
+ f*x]^3)/(12*f)
```

Rubi [A] time = 0.114357, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3904, 3881, 3770}

$$\frac{3a^3c^2\tanh^{-1}(\sin(e+fx))}{8f}+\frac{c^2\tan^3(e+fx)\left(3a^3\sec(e+fx)+4a^3\right)}{12f}-\frac{c^2\tan(e+fx)\left(3a^3\sec(e+fx)+8a^3\right)}{8f}+a^3c^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] a^3*c^2*x + (3*a^3*c^2*ArcTanh[Sin[e + f*x]])/(8*f) - (c^2*(8*a^3 + 3*a^3*S
ec[e + f*x])*Tan[e + f*x])/(8*f) + (c^2*(4*a^3 + 3*a^3*Sec[e + f*x])*Tan[e
+ f*x]^3)/(12*f)
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3881

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(e*(e*Cot[c + d*x])^(m - 1)*(a*m + b*(m - 1)*Csc[c
 + d*x]))/(d*m*(m - 1)), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a*m
 + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1
]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^2\,dx = \left(a^2c^2\right)\int (a+a\sec(e+fx))\tan^4(e+fx)\,dx$$
$$= \frac{c^2\left(4a^3+3a^3\sec(e+fx)\right)\tan^3(e+fx)}{12f} - \frac{1}{4}\left(a^2c^2\right)\int (4a+3a\sec(e+fx$$
$$= -\frac{c^2\left(8a^3+3a^3\sec(e+fx)\right)\tan(e+fx)}{8f} + \frac{c^2\left(4a^3+3a^3\sec(e+fx)\right)\tan}{12f}$$
$$= a^3c^2x - \frac{c^2\left(8a^3+3a^3\sec(e+fx)\right)\tan(e+fx)}{8f} + \frac{c^2\left(4a^3+3a^3\sec(e+fx\right.}{12f}$$
$$= a^3c^2x + \frac{3a^3c^2\tanh^{-1}(\sin(e+fx))}{8f} - \frac{c^2\left(8a^3+3a^3\sec(e+fx)\right)\tan(e+f}{8f}$$

Mathematica [A] time = 0.791387, size = 122, normalized size = 1.26

$$\frac{a^3c^2\sec^4(e+fx)\left(18\sin(e+fx)-32\sin(2(e+fx))-30\sin(3(e+fx))-32\sin(4(e+fx))+96(e+fx)\cos(2(e+fx))\right.}{192f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (a^3*c^2*Sec[e + f*x]^4*(72*e + 72*f*x + 72*ArcTanh[Sin[e + f*x]]*Cos[e + f
*x]^4 + 96*(e + f*x)*Cos[2*(e + f*x)] + 24*e*Cos[4*(e + f*x)] + 24*f*x*Cos[
4*(e + f*x)] + 18*Sin[e + f*x] - 32*Sin[2*(e + f*x)] - 30*Sin[3*(e + f*x)]
- 32*Sin[4*(e + f*x)]))/(192*f)
```

Maple [A] time = 0.025, size = 136, normalized size = 1.4

$$-\frac{4c^2a^3\tan(fx+e)}{3f} + \frac{3c^2a^3\ln\left(\sec(fx+e)+\tan(fx+e)\right)}{8f} + a^3c^2x + \frac{a^3c^2e}{f} - \frac{5c^2a^3\sec(fx+e)\tan(fx+e)}{8f} + \frac{c^2a^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^2,x)
```

```
[Out] -4/3/f*c^2*a^3*tan(f*x+e)+3/8/f*c^2*a^3*ln(sec(f*x+e)+tan(f*x+e))+a^3*c^2*x
+1/f*a^3*c^2*e-5/8/f*c^2*a^3*sec(f*x+e)*tan(f*x+e)+1/3/f*c^2*a^3*tan(f*x+e)
*sec(f*x+e)^2+1/4/f*c^2*a^3*tan(f*x+e)*sec(f*x+e)^3
```

Maxima [B] time = 1.01915, size = 274, normalized size = 2.82

$$\frac{16\left(\tan\left(fx+e\right)^{3}+3\,\tan\left(fx+e\right)\right)a^{3}c^{2}+48\left(fx+e\right)a^{3}c^{2}-3\,a^{3}c^{2}\left(\frac{2\left(3\,\sin\left(fx+e\right)^{3}-5\,\sin\left(fx+e\right)\right)}{\sin\left(fx+e\right)^{4}-2\,\sin\left(fx+e\right)^{2}+1}-3\,\log\left(\sin\left(fx+e\right)+1\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/48*(16*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c^2 + 48*(f*x + e)*a^3*c^2 -
 3*a^3*c^2*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f
*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) + 24*a^
3*c^2*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(si
n(f*x + e) - 1)) + 48*a^3*c^2*log(sec(f*x + e) + tan(f*x + e)) - 96*a^3*c^2
*tan(f*x + e))/f
```

Fricas [A] time = 1.14746, size = 359, normalized size = 3.7

$$\frac{48\,a^{3}c^{2}fx\cos\left(fx+e\right)^{4}+9\,a^{3}c^{2}\cos\left(fx+e\right)^{4}\log\left(\sin\left(fx+e\right)+1\right)-9\,a^{3}c^{2}\cos\left(fx+e\right)^{4}\log\left(-\sin\left(fx+e\right)+1\right)-2\,(}{48\,f\cos\left(fx+e\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/48*(48*a^3*c^2*f*x*cos(f*x + e)^4 + 9*a^3*c^2*cos(f*x + e)^4*log(sin(f*x
+ e) + 1) - 9*a^3*c^2*cos(f*x + e)^4*log(-sin(f*x + e) + 1) - 2*(32*a^3*c^2
*cos(f*x + e)^3 + 15*a^3*c^2*cos(f*x + e)^2 - 8*a^3*c^2*cos(f*x + e) - 6*a^
3*c^2)*sin(f*x + e))/(f*cos(f*x + e)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3c^2\left(\int 1\,dx + \int \sec\left(e+fx\right)dx + \int -2\sec^2\left(e+fx\right)dx + \int -2\sec^3\left(e+fx\right)dx + \int \sec^4\left(e+fx\right)dx + \int \sec^5\left(e+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**2,x)
```

```
[Out] a**3*c**2*(Integral(1, x) + Integral(sec(e + f*x), x) + Integral(-2*sec(e +
 f*x)**2, x) + Integral(-2*sec(e + f*x)**3, x) + Integral(sec(e + f*x)**4,
x) + Integral(sec(e + f*x)**5, x))
```

Giac [A] time = 1.34503, size = 217, normalized size = 2.24

$$\frac{24\left(fx+e\right)a^3c^2 + 9\,a^3c^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right) - 9\,a^3c^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right) + \frac{2\left(15\,a^3c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7 - 71\,a^3c^2\tan\cdots\right.}{\cdots}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/24*(24*(f*x + e)*a^3*c^2 + 9*a^3*c^2*log(abs(tan(1/2*f*x + 1/2*e) + 1)) -
 9*a^3*c^2*log(abs(tan(1/2*f*x + 1/2*e) - 1)) + 2*(15*a^3*c^2*tan(1/2*f*x +
 1/2*e)^7 - 71*a^3*c^2*tan(1/2*f*x + 1/2*e)^5 + 137*a^3*c^2*tan(1/2*f*x + 1
/2*e)^3 - 33*a^3*c^2*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^4)/
f
```

3.15 $\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx$

Optimal. Leaf size=77

$$-\frac{a^3c\tan^3(e+fx)}{3f}-\frac{a^3c\tan(e+fx)}{f}+\frac{a^3c\tanh^{-1}(\sin(e+fx))}{f}-\frac{a^3c\tan(e+fx)\sec(e+fx)}{f}+a^3cx$$

```
[Out] a^3*c*x + (a^3*c*ArcTanh[Sin[e + f*x]])/f - (a^3*c*Tan[e + f*x])/f - (a^3*c
*Sec[e + f*x]*Tan[e + f*x])/f - (a^3*c*Tan[e + f*x]^3)/(3*f)
```

Rubi [A] time = 0.147325, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3904, 3886, 3473, 8, 2611, 3770, 2607, 30}

$$-\frac{a^3c\tan^3(e+fx)}{3f}-\frac{a^3c\tan(e+fx)}{f}+\frac{a^3c\tanh^{-1}(\sin(e+fx))}{f}-\frac{a^3c\tan(e+fx)\sec(e+fx)}{f}+a^3cx$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x]),x]
```

```
[Out] a^3*c*x + (a^3*c*ArcTanh[Sin[e + f*x]])/f - (a^3*c*Tan[e + f*x])/f - (a^3*c
*Sec[e + f*x]*Tan[e + f*x])/f - (a^3*c*Tan[e + f*x]^3)/(3*f)
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int (a + a\sec(e + fx))^3(c - c\sec(e + fx))\,dx = -\left((ac)\int (a + a\sec(e + fx))^2\tan^2(e + fx)\,dx\right)$$
$$= -\left((ac)\int \left(a^2\tan^2(e + fx) + 2a^2\sec(e + fx)\tan^2(e + fx) + a^2\sec^2(e +$$
$$= -\left(\left(a^3c\right)\int \tan^2(e + fx)\,dx\right) - \left(a^3c\right)\int \sec^2(e + fx)\tan^2(e + fx)\,dx - (2$$
$$= -\frac{a^3c\tan(e + fx)}{f} - \frac{a^3c\sec(e + fx)\tan(e + fx)}{f} + \left(a^3c\right)\int 1\,dx + \left(a^3c\right)\int$$
$$= a^3cx + \frac{a^3c\tanh^{-1}(\sin(e + fx))}{f} - \frac{a^3c\tan(e + fx)}{f} - \frac{a^3c\sec(e + fx)\tan(}{f}$$

Mathematica [A] time = 0.440177, size = 101, normalized size = 1.31

$$\frac{a^3c\sec^3(e + fx)\left(-6\sin(e + fx) - 6\sin(2(e + fx)) - 2\sin(3(e + fx)) + 9(e + fx)\cos(e + fx) + 3e\cos(3(e + fx)) + 3fx\right.}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x]),x]
```

```
[Out] (a^3*c*Sec[e + f*x]^3*(9*(e + f*x)*Cos[e + f*x] + 12*ArcTanh[Sin[e + f*x]]*
Cos[e + f*x]^3 + 3*e*Cos[3*(e + f*x)] + 3*f*x*Cos[3*(e + f*x)] - 6*Sin[e +
f*x] - 6*Sin[2*(e + f*x)] - 2*Sin[3*(e + f*x)]))/(12*f)
```

Maple [A] time = 0.023, size = 98, normalized size = 1.3

$$\frac{a^3c\ln\left(\sec\left(fx + e\right) + \tan\left(fx + e\right)\right)}{f} + a^3cx + \frac{a^3ce}{f} - \frac{a^3c\sec\left(fx + e\right)\tan\left(fx + e\right)}{f} - \frac{2\,a^3c\tan\left(fx + e\right)}{3\,f} - \frac{a^3c\tan\left(fx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x)
```

```
[Out] 1/f*a^3*c*ln(sec(f*x+e)+tan(f*x+e))+a^3*c*x+1/f*a^3*c*e-a^3*c*sec(f*x+e)*ta
n(f*x+e)/f-2/3*a^3*c*tan(f*x+e)/f-1/3/f*a^3*c*tan(f*x+e)*sec(f*x+e)^2
```

Maxima [A] time = 1.01755, size = 144, normalized size = 1.87

$$-\frac{2\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)a^3c-6\left(fx+e\right)a^3c-3\,a^3c\left(\frac{2\,\sin\left(fx+e\right)}{\sin\left(fx+e\right)^2-1}-\log\left(\sin\left(fx+e\right)+1\right)+\log\left(\sin\left(fx+e\right)-\right.\right.}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/6*(2*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c - 6*(f*x + e)*a^3*c - 3*a^3
*c*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f
*x + e) - 1)) - 12*a^3*c*log(sec(f*x + e) + tan(f*x + e)))/f
```

Fricas [A] time = 1.10899, size = 298, normalized size = 3.87

$$\frac{6\,a^3cfx\cos\left(fx+e\right)^3+3\,a^3c\cos\left(fx+e\right)^3\log\left(\sin\left(fx+e\right)+1\right)-3\,a^3c\cos\left(fx+e\right)^3\log\left(-\sin\left(fx+e\right)+1\right)-2\left(2\,a^3c\right.}{6\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/6*(6*a^3*c*f*x*cos(f*x + e)^3 + 3*a^3*c*cos(f*x + e)^3*log(sin(f*x + e) +
 1) - 3*a^3*c*cos(f*x + e)^3*log(-sin(f*x + e) + 1) - 2*(2*a^3*c*cos(f*x +
e)^2 + 3*a^3*c*cos(f*x + e) + a^3*c)*sin(f*x + e))/(f*cos(f*x + e)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-a^3c\left(\int(-1)\,dx+\int-2\sec\left(e+fx\right)dx+\int2\sec^3\left(e+fx\right)dx+\int\sec^4\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**3*(c-c*sec(f*x+e)),x)
```

```
[Out] -a**3*c*(Integral(-1, x) + Integral(-2*sec(e + f*x), x) + Integral(2*sec(e
+ f*x)**3, x) + Integral(sec(e + f*x)**4, x))
```

Giac [A] time = 1.39224, size = 149, normalized size = 1.94

$$\frac{3\left(fx+e\right)a^3c+3\,a^3c\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-3\,a^3c\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{4\left(a^3c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-3\,a^3c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\right.}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/3*(3*(f*x + e)*a^3*c + 3*a^3*c*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 3*a^3
*c*log(abs(tan(1/2*f*x + 1/2*e) - 1)) - 4*(a^3*c*tan(1/2*f*x + 1/2*e)^3 - 3
*a^3*c*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^3)/f
```

3.16 $\int \frac{(a+a\sec(e+fx))^3}{c-c\sec(e+fx)}\,dx$

Optimal. Leaf size=78

$$-\frac{a^3\tan(e+fx)}{cf}+\frac{8a^3\cot(e+fx)}{cf}+\frac{8a^3\csc(e+fx)}{cf}-\frac{4a^3\tanh^{-1}(\sin(e+fx))}{cf}+\frac{a^3x}{c}$$

```
[Out] (a^3*x)/c - (4*a^3*ArcTanh[Sin[e + f*x]])/(c*f) + (8*a^3*Cot[e + f*x])/(c*f
) + (8*a^3*Csc[e + f*x])/(c*f) - (a^3*Tan[e + f*x])/(c*f)
```

Rubi [A] time = 0.208792, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {3904, 3886, 3473, 8, 2606, 3767, 2621, 321, 207, 2620, 14}

$$-\frac{a^3\tan(e+fx)}{cf}+\frac{8a^3\cot(e+fx)}{cf}+\frac{8a^3\csc(e+fx)}{cf}-\frac{4a^3\tanh^{-1}(\sin(e+fx))}{cf}+\frac{a^3x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x]),x]
```

```
[Out] (a^3*x)/c - (4*a^3*ArcTanh[Sin[e + f*x]])/(c*f) + (8*a^3*Cot[e + f*x])/(c*f
) + (8*a^3*Csc[e + f*x])/(c*f) - (a^3*Tan[e + f*x])/(c*f)
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^3}{c-c\sec(e+fx)}\,dx = -\frac{\int \cot^2(e+fx)(a+a\sec(e+fx))^4\,dx}{ac}$$
$$= -\frac{\int \left(a^4\cot^2(e+fx)+4a^4\cot(e+fx)\csc(e+fx)+6a^4\csc^2(e+fx)+4a^4\csc^2(e+fx)\sec\right.}{ac}$$
$$= -\frac{a^3\int \cot^2(e+fx)\,dx}{c} - \frac{a^3\int \csc^2(e+fx)\sec^2(e+fx)\,dx}{c} - \frac{\left(4a^3\right)\int \cot(e+fx)\csc(e+fx}{c}$$
$$= \frac{a^3\cot(e+fx)}{cf} + \frac{a^3\int 1\,dx}{c} - \frac{a^3\,\mathrm{Subst}\left(\int \frac{1+x^2}{x^2}\,dx, x, \tan(e+fx)\right)}{cf} + \frac{\left(4a^3\right)\mathrm{Subst}(\int 1\,dx, x,}{cf}$$
$$= \frac{a^3x}{c} + \frac{7a^3\cot(e+fx)}{cf} + \frac{8a^3\csc(e+fx)}{cf} - \frac{a^3\,\mathrm{Subst}\left(\int\left(1+\frac{1}{x^2}\right)dx, x, \tan(e+fx)\right)}{cf} + \frac{\left(4a\right.}{}$$
$$= \frac{a^3x}{c} - \frac{4a^3\tanh^{-1}(\sin(e+fx))}{cf} + \frac{8a^3\cot(e+fx)}{cf} + \frac{8a^3\csc(e+fx)}{cf} - \frac{a^3\tan(e+fx)}{cf}$$

Mathematica [B] time = 2.3685, size = 240, normalized size = 3.08

$$\frac{a^3\cos^2(e+fx)\tan\left(\frac{1}{2}(e+fx)\right)\sec^4\left(\frac{1}{2}(e+fx)\right)(\sec(e+fx)+1)^3\left(8\csc\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)\sec\left(\frac{1}{2}(e+fx)\right)+\tan\left(\frac{1}{2}(e+fx)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x]),x]
```

```
[Out] (a^3*Cos[e + f*x]^2*Sec[(e + f*x)/2]^4*(1 + Sec[e + f*x])^3*Tan[(e + f*x)/2
]*(8*Csc[e/2]*Sec[(e + f*x)/2]*Sin[(f*x)/2] + (-(f*x) - 4*Log[Cos[(e + f*x)
/2] - Sin[(e + f*x)/2]] + 4*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + Sin[
```

```
f*x]/((Cos[e/2] - Sin[e/2])*(Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(
e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])))*Tan[(e + f*x)/2]))/(4*
f*(c - c*Sec[e + f*x]))
```

Maple [A] time = 0.083, size = 137, normalized size = 1.8

$$2\,\frac{a^3\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)}{fc}+\frac{a^3}{fc}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{-1}-4\,\frac{a^3\ln\left(\tan\left(1/2\,fx+e/2\right)+1\right)}{fc}+\frac{a^3}{fc}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x)
```

```
[Out] 2/f*a^3/c*arctan(tan(1/2*f*x+1/2*e))+1/f*a^3/c/(tan(1/2*f*x+1/2*e)+1)-4/f*a
^3/c*ln(tan(1/2*f*x+1/2*e)+1)+1/f*a^3/c/(tan(1/2*f*x+1/2*e)-1)+4/f*a^3/c*ln
(tan(1/2*f*x+1/2*e)-1)+8/f*a^3/c/tan(1/2*f*x+1/2*e)
```

Maxima [B] time = 1.55327, size = 370, normalized size = 4.74

$$-\frac{a^3\left(\frac{\frac{3\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}-1}{\frac{c\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{c\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}}+\frac{\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)}{c}-\frac{\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)}{c}\right)-a^3\left(\frac{2\arctan\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)}{c}+\frac{\cos\left(fx+e\right)+1}{c\sin\left(fx+e\right)}\right)+3\,a^3\left(\frac{\log\left(\frac{\sin(\cdots}{\cos(\cdots}\right.}{\cdots}\right.\cdots}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] -(a^3*((3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1)/(c*sin(f*x + e)/(cos(f*x
 + e) + 1) - c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + log(sin(f*x + e)/(cos
(f*x + e) + 1) + 1)/c - log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c) - a^3*(
2*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c + (cos(f*x + e) + 1)/(c*sin(f*x
 + e))) + 3*a^3*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/c - log(sin(f*x +
 e)/(cos(f*x + e) + 1) - 1)/c - (cos(f*x + e) + 1)/(c*sin(f*x + e))) - 3*a^
3*(cos(f*x + e) + 1)/(c*sin(f*x + e)))/f
```

Fricas [A] time = 1.09735, size = 313, normalized size = 4.01

$$\frac{a^3fx\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a^3\cos\left(fx+e\right)\log\left(\sin\left(fx+e\right)+1\right)\sin\left(fx+e\right)+2\,a^3\cos\left(fx+e\right)\log\left(-\sin\left(fx+e\right.\right.}{cf\cos\left(fx+e\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] (a^3*f*x*cos(f*x + e)*sin(f*x + e) - 2*a^3*cos(f*x + e)*log(sin(f*x + e) +
1)*sin(f*x + e) + 2*a^3*cos(f*x + e)*log(-sin(f*x + e) + 1)*sin(f*x + e) +
9*a^3*cos(f*x + e)^2 + 8*a^3*cos(f*x + e) - a^3)/(c*f*cos(f*x + e)*sin(f*x
+ e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^3\left(\int\frac{3\sec\left(e+fx\right)}{\sec\left(e+fx\right)-1}\,dx+\int\frac{3\sec^2\left(e+fx\right)}{\sec\left(e+fx\right)-1}\,dx+\int\frac{\sec^3\left(e+fx\right)}{\sec\left(e+fx\right)-1}\,dx+\int\frac{1}{\sec\left(e+fx\right)-1}\,dx\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**3/(c-c*sec(f*x+e)),x)
```

```
[Out] -a**3*(Integral(3*sec(e + f*x)/(sec(e + f*x) - 1), x) + Integral(3*sec(e +
f*x)**2/(sec(e + f*x) - 1), x) + Integral(sec(e + f*x)**3/(sec(e + f*x) - 1
), x) + Integral(1/(sec(e + f*x) - 1), x))/c
```

Giac [A] time = 1.34976, size = 158, normalized size = 2.03

$$\frac{\frac{(fx+e)a^3}{c}-\frac{4\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{c}+\frac{4\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{c}+\frac{2\left(5\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-4\,a^3\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)c}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] ((f*x + e)*a^3/c - 4*a^3*log(abs(tan(1/2*f*x + 1/2*e) + 1))/c + 4*a^3*log(a
bs(tan(1/2*f*x + 1/2*e) - 1))/c + 2*(5*a^3*tan(1/2*f*x + 1/2*e)^2 - 4*a^3)/
((tan(1/2*f*x + 1/2*e)^3 - tan(1/2*f*x + 1/2*e))*c))/f
```

3.17 $\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^2} dx$

Optimal. Leaf size=88

$$\frac{a^3\tanh^{-1}(\sin(e+fx))}{c^2f}+\frac{4a^3\tan(e+fx)}{3c^2f(1-\sec(e+fx))}-\frac{8a^3\tan(e+fx)}{3c^2f(1-\sec(e+fx))^2}+\frac{a^3x}{c^2}$$

```
[Out] (a^3*x)/c^2 + (a^3*ArcTanh[Sin[e + f*x]])/(c^2*f) - (8*a^3*Tan[e + f*x])/(3
*c^2*f*(1 - Sec[e + f*x])^2) + (4*a^3*Tan[e + f*x])/(3*c^2*f*(1 - Sec[e + f
*x]))
```

Rubi [A] time = 0.361025, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {3903, 3777, 3919, 3794, 3796, 3797, 3799, 3998, 3770}

$$\frac{a^3\tanh^{-1}(\sin(e+fx))}{c^2f}+\frac{4a^3\tan(e+fx)}{3c^2f(1-\sec(e+fx))}-\frac{8a^3\tan(e+fx)}{3c^2f(1-\sec(e+fx))^2}+\frac{a^3x}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (a^3*x)/c^2 + (a^3*ArcTanh[Sin[e + f*x]])/(c^2*f) - (8*a^3*Tan[e + f*x])/(3
*c^2*f*(1 - Sec[e + f*x])^2) + (4*a^3*Tan[e + f*x])/(3*c^2*f*(1 - Sec[e + f
*x]))
```

Rule 3903

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Dist[c^n, Int[ExpandTrig[(1 + (d*csc[e + f*
x])/c)^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
&& LtQ[m + n, 2]
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
```

rQ[2*n]

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3797

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(2*m + 1)), x]
+ Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x],
x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3799

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^2}\,dx = \frac{\int \left(\frac{a^3}{(1-\sec(e+fx))^2}+\frac{3a^3\sec(e+fx)}{(1-\sec(e+fx))^2}+\frac{3a^3\sec^2(e+fx)}{(1-\sec(e+fx))^2}+\frac{a^3\sec^3(e+fx)}{(1-\sec(e+fx))^2}\right)dx}{c^2}$$
$$= \frac{a^3\int\frac{1}{(1-\sec(e+fx))^2}\,dx}{c^2}+\frac{a^3\int\frac{\sec^3(e+fx)}{(1-\sec(e+fx))^2}\,dx}{c^2}+\frac{(3a^3)\int\frac{\sec(e+fx)}{(1-\sec(e+fx))^2}\,dx}{c^2}+\frac{(3a^3)\int\frac{\sec^2(e+fx)}{(1-\sec(e+fx)}$$
$$= -\frac{8a^3\tan(e+fx)}{3c^2f(1-\sec(e+fx))^2}-\frac{a^3\int\frac{-3-\sec(e+fx)}{1-\sec(e+fx)}\,dx}{3c^2}+\frac{a^3\int\frac{(-2-3\sec(e+fx))\sec(e+fx)}{1-\sec(e+fx)}\,dx}{3c^2}+\frac{a^3\int\frac{\sec(}{1-\sec}}{c^2}$$
$$= \frac{a^3x}{c^2}-\frac{8a^3\tan(e+fx)}{3c^2f(1-\sec(e+fx))^2}+\frac{a^3\tan(e+fx)}{c^2f(1-\sec(e+fx))}+\frac{a^3\int\sec(e+fx)\,dx}{c^2}+\frac{(4a^3)\int\frac{\sec(e}{1-\sec}}{3c^2}$$
$$= \frac{a^3x}{c^2}+\frac{a^3\tanh^{-1}(\sin(e+fx))}{c^2f}-\frac{8a^3\tan(e+fx)}{3c^2f(1-\sec(e+fx))^2}+\frac{4a^3\tan(e+fx)}{3c^2f(1-\sec(e+fx))}$$

Mathematica [B] time = 1.17021, size = 177, normalized size = 2.01

$$\frac{a^3(\cos(e+fx)+1)^3\tan\left(\frac{1}{2}(e+fx)\right)\sec^2\left(\frac{1}{2}(e+fx)\right)\left(-4\cot\left(\frac{e}{2}\right)\tan\left(\frac{1}{2}(e+fx)\right)\sec^2\left(\frac{1}{2}(e+fx)\right)+4\csc\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)s}{6c^2f(co}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (a^3*(1 + Cos[e + f*x])^3*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]*(4*Csc[e/2]*S
ec[(e + f*x)/2]*Sin[(f*x)/2] - 4*Cot[e/2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/
2] + 3*(f*x - Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + Log[Cos[(e + f*x)/
2] + Sin[(e + f*x)/2]])*Tan[(e + f*x)/2]^3))/(6*c^2*f*(-1 + Cos[e + f*x])^2
)
```

Maple [A] time = 0.087, size = 90, normalized size = 1.

$$-\frac{4a^3}{3fc^2}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-3}+2\frac{a^3\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)}{fc^2}+\frac{a^3}{fc^2}\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)-\frac{a^3}{fc^2}\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x)
```

```
[Out] -4/3/f*a^3/c^2/tan(1/2*f*x+1/2*e)^3+2/f*a^3/c^2*arctan(tan(1/2*f*x+1/2*e))+
1/f*a^3/c^2*ln(tan(1/2*f*x+1/2*e)+1)-1/f*a^3/c^2*ln(tan(1/2*f*x+1/2*e)-1)
```

Maxima [B] time = 1.53255, size = 370, normalized size = 4.2

$$\frac{a^3\left(\frac{12\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^2}+\frac{\left(\frac{9\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-1\right)(\cos(fx+e)+1)^3}{c^2\sin(fx+e)^3}\right)+a^3\left(\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c^2}-\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c^2}-\frac{\left(\frac{9\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)(\cos(fx+e)+1)^3}{c^2\sin(fx+e)^3}\right.}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(a^3*(12*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^2 + (9*sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 - 1)*(cos(f*x + e) + 1)^3/(c^2*sin(f*x + e)^3)) + a^
3*(6*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/c^2 - 6*log(sin(f*x + e)/(cos
(f*x + e) + 1) - 1)/c^2 - (9*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)*(cos(
f*x + e) + 1)^3/(c^2*sin(f*x + e)^3)) - 3*a^3*(3*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 + 1)*(cos(f*x + e) + 1)^3/(c^2*sin(f*x + e)^3) + 3*a^3*(3*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 - 1)*(cos(f*x + e) + 1)^3/(c^2*sin(f*x + e)^3)
)/f
```

Fricas [A] time = 1.12544, size = 379, normalized size = 4.31

$$\frac{8a^3\cos(fx+e)^2+16a^3\cos(fx+e)+8a^3+3\left(a^3\cos(fx+e)-a^3\right)\log\left(\sin(fx+e)+1\right)\sin(fx+e)-3\left(a^3\cos(\right.}{6\left(c^2f\cos(fx+e)-c^2f\right)\sin(f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(8*a^3*cos(f*x + e)^2 + 16*a^3*cos(f*x + e) + 8*a^3 + 3*(a^3*cos(f*x +
e) - a^3)*log(sin(f*x + e) + 1)*sin(f*x + e) - 3*(a^3*cos(f*x + e) - a^3)*l
```

```
og(-sin(f*x + e) + 1)*sin(f*x + e) + 6*(a^3*f*x*cos(f*x + e) - a^3*f*x)*sin
(f*x + e))/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^3 \left(\int \frac{3\sec\left(e+fx\right)}{\sec^2\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx + \int \frac{3\sec^2\left(e+fx\right)}{\sec^2\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx + \int \frac{\sec^3\left(e+fx\right)}{\sec^2\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx + \int \frac{1}{\sec^2\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx$$

$$c^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**2,x)
```

```
[Out] a**3*(Integral(3*sec(e + f*x)/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) +
Integral(3*sec(e + f*x)**2/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) + Int
egral(sec(e + f*x)**3/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) + Integral
(1/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x))/c**2
```

Giac [A] time = 1.34594, size = 113, normalized size = 1.28

$$\frac{\frac{3\left(fx+e\right)a^3}{c^2} + \frac{3\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{c^2} - \frac{3\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{c^2} - \frac{4\,a^3}{c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*(f*x + e)*a^3/c^2 + 3*a^3*log(abs(tan(1/2*f*x + 1/2*e) + 1))/c^2 - 3
*a^3*log(abs(tan(1/2*f*x + 1/2*e) - 1))/c^2 - 4*a^3/(c^2*tan(1/2*f*x + 1/2*
e)^3))/f
```

3.18 $\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^3}\,dx$

Optimal. Leaf size=102

$$-\frac{26a^3\tan(e+fx)}{15c^3f(1-\sec(e+fx))}+\frac{4a^3\tan(e+fx)}{15c^3f(1-\sec(e+fx))^2}-\frac{8a^3\tan(e+fx)}{5c^3f(1-\sec(e+fx))^3}+\frac{a^3x}{c^3}$$

```
[Out] (a^3*x)/c^3 - (8*a^3*Tan[e + f*x])/(5*c^3*f*(1 - Sec[e + f*x])^3) + (4*a^3*
Tan[e + f*x])/(15*c^3*f*(1 - Sec[e + f*x])^2) - (26*a^3*Tan[e + f*x])/(15*c
^3*f*(1 - Sec[e + f*x]))
```

Rubi [A] time = 0.451617, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {3903, 3777, 3922, 3919, 3794, 3796, 3797, 3799, 4000}

$$-\frac{26a^3\tan(e+fx)}{15c^3f(1-\sec(e+fx))}+\frac{4a^3\tan(e+fx)}{15c^3f(1-\sec(e+fx))^2}-\frac{8a^3\tan(e+fx)}{5c^3f(1-\sec(e+fx))^3}+\frac{a^3x}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (a^3*x)/c^3 - (8*a^3*Tan[e + f*x])/(5*c^3*f*(1 - Sec[e + f*x])^3) + (4*a^3*
Tan[e + f*x])/(15*c^3*f*(1 - Sec[e + f*x])^2) - (26*a^3*Tan[e + f*x])/(15*c
^3*f*(1 - Sec[e + f*x]))
```

Rule 3903

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Dist[c^n, Int[ExpandTrig[(1 + (d*csc[e + f*
x])/c)^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
&& LtQ[m + n, 2]
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
```

rQ[2*n]

Rule 3922

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*
x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x
])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ
[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3797

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(2*m + 1)), x]
+ Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x],
x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3799

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
```

```
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4000

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[((A*b - a*B)*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^3}\,dx = \frac{\int \left(\frac{a^3}{(1-\sec(e+fx))^3}+\frac{3a^3\sec(e+fx)}{(1-\sec(e+fx))^3}+\frac{3a^3\sec^2(e+fx)}{(1-\sec(e+fx))^3}+\frac{a^3\sec^3(e+fx)}{(1-\sec(e+fx))^3}\right)dx}{c^3}$$

$$= \frac{a^3\int \frac{1}{(1-\sec(e+fx))^3}\,dx}{c^3}+\frac{a^3\int \frac{\sec^3(e+fx)}{(1-\sec(e+fx))^3}\,dx}{c^3}+\frac{\left(3a^3\right)\int \frac{\sec(e+fx)}{(1-\sec(e+fx))^3}\,dx}{c^3}+\frac{\left(3a^3\right)\int \frac{\sec^2(e+f\cdots}{(1-\sec(e+f\cdots}}{c^3}\cdots$$

$$= -\frac{8a^3\tan(e+fx)}{5c^3f(1-\sec(e+fx))^3}-\frac{a^3\int \frac{-5-2\sec(e+fx)}{(1-\sec(e+fx))^2}\,dx}{5c^3}+\frac{a^3\int \frac{(-3-5\sec(e+fx))\sec(e+fx)}{(1-\sec(e+fx))^2}\,dx}{5c^3}+\frac{\left(6a^3\right)\cdots}{\cdots}$$

$$= -\frac{8a^3\tan(e+fx)}{5c^3f(1-\sec(e+fx))^3}+\frac{4a^3\tan(e+fx)}{15c^3f(1-\sec(e+fx))^2}+\frac{a^3\int \frac{15+7\sec(e+fx)}{1-\sec(e+fx)}\,dx}{15c^3}+\frac{\left(2a^3\right)\int \frac{\sec\cdots}{1-\sec\cdots}}{5c^3}\cdots$$

$$= \frac{a^3x}{c^3}-\frac{8a^3\tan(e+fx)}{5c^3f(1-\sec(e+fx))^3}+\frac{4a^3\tan(e+fx)}{15c^3f(1-\sec(e+fx))^2}-\frac{4a^3\tan(e+fx)}{15c^3f(1-\sec(e+fx))}+\frac{\left(22a\cdots\right.}{\cdots}$$

$$= \frac{a^3x}{c^3}-\frac{8a^3\tan(e+fx)}{5c^3f(1-\sec(e+fx))^3}+\frac{4a^3\tan(e+fx)}{15c^3f(1-\sec(e+fx))^2}-\frac{26a^3\tan(e+fx)}{15c^3f(1-\sec(e+fx))}$$

Mathematica [C] time = 0.0847297, size = 53, normalized size = 0.52

$$\frac{2a^3\cot^5\left(\frac{e}{2}+\frac{fx}{2}\right)\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{5c^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (2*a^3*Cot[e/2 + (f*x)/2]^5*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[e/2 + (f*
x)/2]^2])/(5*c^3*f)
```

Maple [A] time = 0.104, size = 89, normalized size = 0.9

$$2\,{\frac{{a}^{3}\arctan \left(\tan \left(1/2\,fx+e/2 \right) \right) }{f{c}^{3}}}-{\frac{2\,{a}^{3}}{3\,f{c}^{3}} \left(\tan \left({\frac{fx}{2}}+{\frac{e}{2}} \right) \right) ^{-3}}+{\frac{2\,{a}^{3}}{5\,f{c}^{3}} \left(\tan \left({\frac{fx}{2}}+{\frac{e}{2}} \right) \right) ^{-5}}+2\,{\frac{{a}^{3}}{f{c}^{3}\tan \left(1/2\,fx+e/2 \right) }}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x)
```

```
[Out] 2/f*a^3/c^3*arctan(tan(1/2*f*x+1/2*e))-2/3/f*a^3/c^3/tan(1/2*f*x+1/2*e)^3+2
/5/f*a^3/c^3/tan(1/2*f*x+1/2*e)^5+2/f*a^3/c^3/tan(1/2*f*x+1/2*e)
```

Maxima [B] time = 1.58716, size = 381, normalized size = 3.74

$$\frac{a^{3}\left(\frac{120\,\arctan\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)}{c^{3}}-\frac{\left(\frac{20\,\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-\frac{105\,\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}-3\right)\left(\cos\left(fx+e\right)+1\right)^{5}}{c^{3}\sin\left(fx+e\right)^{5}}\right)+\frac{a^{3}\left(\frac{10\,\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{15\,\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}+3\right)\left(\cos\left(fx+e\right)+1\right)^{5}}{c^{3}\sin\left(fx+e\right)^{5}}-\frac{3\,a^{3}\left(10 \ldots\right.}{\ldots}}{60\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/60*(a^3*(120*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^3 - (20*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 - 105*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3)*(c
os(f*x + e) + 1)^5/(c^3*sin(f*x + e)^5)) + a^3*(10*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3)*(cos(f*x + e) + 1
)^5/(c^3*sin(f*x + e)^5) - 3*a^3*(10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 -
15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3)*(cos(f*x + e) + 1)^5/(c^3*sin(f
*x + e)^5) - 9*a^3*(5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1)*(cos(f*x + e
) + 1)^5/(c^3*sin(f*x + e)^5))/f
```

Fricas [A] time = 1.07334, size = 312, normalized size = 3.06

$$\frac{46\,a^{3}\cos\left(fx+e\right)^{3}-2\,a^{3}\cos\left(fx+e\right)^{2}-22\,a^{3}\cos\left(fx+e\right)+26\,a^{3}+15\left(a^{3}fx\cos\left(fx+e\right)^{2}-2\,a^{3}fx\cos\left(fx+e\right)+a^{3}f \ldots\right.}{15\left(c^{3}f\cos\left(fx+e\right)^{2}-2\,c^{3}f\cos\left(fx+e\right)+c^{3}f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(46*a^3*cos(f*x + e)^3 - 2*a^3*cos(f*x + e)^2 - 22*a^3*cos(f*x + e) +
26*a^3 + 15*(a^3*f*x*cos(f*x + e)^2 - 2*a^3*f*x*cos(f*x + e) + a^3*f*x)*sin
(f*x + e))/((c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x +
 e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^3\left(\int \frac{3\sec\left(e+fx\right)}{\sec^3\left(e+fx\right)-3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)-1}\,dx+\int \frac{3\sec^2\left(e+fx\right)}{\sec^3\left(e+fx\right)-3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)-1}\,dx+\int \frac{\sec^3\left(e+fx\right)}{\sec^3\left(e+fx\right)-3\sec^2\left(e+fx\right)+3\sec\left(e+\right.}}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**3,x)
```

```
[Out] -a**3*(Integral(3*sec(e + f*x)/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*sec
(e + f*x) - 1), x) + Integral(3*sec(e + f*x)**2/(sec(e + f*x)**3 - 3*sec(e
+ f*x)**2 + 3*sec(e + f*x) - 1), x) + Integral(sec(e + f*x)**3/(sec(e + f*x
)**3 - 3*sec(e + f*x)**2 + 3*sec(e + f*x) - 1), x) + Integral(1/(sec(e + f*
x)**3 - 3*sec(e + f*x)**2 + 3*sec(e + f*x) - 1), x))/c**3
```

Giac [A] time = 1.44092, size = 104, normalized size = 1.02

$$\frac{\frac{15\left(fx+e\right)a^3}{c^3}+\frac{2\left(15\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-5\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+3\,a^3\right)}{c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/15*(15*(f*x + e)*a^3/c^3 + 2*(15*a^3*tan(1/2*f*x + 1/2*e)^4 - 5*a^3*tan(1
/2*f*x + 1/2*e)^2 + 3*a^3)/(c^3*tan(1/2*f*x + 1/2*e)^5))/f
```

3.19 $\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^4} dx$

Optimal. Leaf size=133

$$-\frac{167a^3\tan(e+fx)}{105c^4f(1-\sec(e+fx))}-\frac{62a^3\tan(e+fx)}{105c^4f(1-\sec(e+fx))^2}+\frac{4a^3\tan(e+fx)}{35c^4f(1-\sec(e+fx))^3}-\frac{8a^3\tan(e+fx)}{7c^4f(1-\sec(e+fx))^4}+\frac{a^3x}{c^4}$$

```
[Out] (a^3*x)/c^4 - (8*a^3*Tan[e + f*x])/(7*c^4*f*(1 - Sec[e + f*x])^4) + (4*a^3*
Tan[e + f*x])/(35*c^4*f*(1 - Sec[e + f*x])^3) - (62*a^3*Tan[e + f*x])/(105*
c^4*f*(1 - Sec[e + f*x])^2) - (167*a^3*Tan[e + f*x])/(105*c^4*f*(1 - Sec[e
+ f*x]))
```

Rubi [A] time = 0.57925, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {3903, 3777, 3922, 3919, 3794, 3796, 3797, 3799, 4000}

$$-\frac{167a^3\tan(e+fx)}{105c^4f(1-\sec(e+fx))}-\frac{62a^3\tan(e+fx)}{105c^4f(1-\sec(e+fx))^2}+\frac{4a^3\tan(e+fx)}{35c^4f(1-\sec(e+fx))^3}-\frac{8a^3\tan(e+fx)}{7c^4f(1-\sec(e+fx))^4}+\frac{a^3x}{c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (a^3*x)/c^4 - (8*a^3*Tan[e + f*x])/(7*c^4*f*(1 - Sec[e + f*x])^4) + (4*a^3*
Tan[e + f*x])/(35*c^4*f*(1 - Sec[e + f*x])^3) - (62*a^3*Tan[e + f*x])/(105*
c^4*f*(1 - Sec[e + f*x])^2) - (167*a^3*Tan[e + f*x])/(105*c^4*f*(1 - Sec[e
+ f*x]))
```

Rule 3903

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Dist[c^n, Int[ExpandTrig[(1 + (d*csc[e + f*
x])/c)^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
&& LtQ[m + n, 2]
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
```

```
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
rQ[2*n]
```

Rule 3922

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*
x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x
])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ
[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3797

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(2*m + 1)), x]
+ Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x],
x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3799

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
```

```
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4000

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[((A*b - a*B)*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^4}\,dx = \frac{\int \left(\frac{a^3}{(1-\sec(e+fx))^4}+\frac{3a^3\sec(e+fx)}{(1-\sec(e+fx))^4}+\frac{3a^3\sec^2(e+fx)}{(1-\sec(e+fx))^4}+\frac{a^3\sec^3(e+fx)}{(1-\sec(e+fx))^4}\right)dx}{c^4}$$

$$= \frac{a^3\int \frac{1}{(1-\sec(e+fx))^4}\,dx}{c^4}+\frac{a^3\int \frac{\sec^3(e+fx)}{(1-\sec(e+fx))^4}\,dx}{c^4}+\frac{\left(3a^3\right)\int \frac{\sec(e+fx)}{(1-\sec(e+fx))^4}\,dx}{c^4}+\frac{\left(3a^3\right)\int \frac{\sec^2(e+fx)}{(1-\sec(e+fx)}}{c^4}$$

$$= -\frac{8a^3\tan(e+fx)}{7c^4f(1-\sec(e+fx))^4}-\frac{a^3\int \frac{-7-3\sec(e+fx)}{(1-\sec(e+fx))^3}\,dx}{7c^4}+\frac{a^3\int \frac{(-4-7\sec(e+fx))\sec(e+fx)}{(1-\sec(e+fx))^3}\,dx}{7c^4}+\frac{\left(9a^3\right)\int}{}$$

$$= -\frac{8a^3\tan(e+fx)}{7c^4f(1-\sec(e+fx))^4}+\frac{4a^3\tan(e+fx)}{35c^4f(1-\sec(e+fx))^3}+\frac{a^3\int \frac{35+20\sec(e+fx)}{(1-\sec(e+fx))^2}\,dx}{35c^4}+\frac{\left(13a^3\right)\int \frac{\text{se}}{(1-\text{s}}}{35c}$$

$$= -\frac{8a^3\tan(e+fx)}{7c^4f(1-\sec(e+fx))^4}+\frac{4a^3\tan(e+fx)}{35c^4f(1-\sec(e+fx))^3}-\frac{62a^3\tan(e+fx)}{105c^4f(1-\sec(e+fx))^2}-\frac{a^3\int \frac{-105}{1-}}{1}$$

$$= \frac{a^3x}{c^4}-\frac{8a^3\tan(e+fx)}{7c^4f(1-\sec(e+fx))^4}+\frac{4a^3\tan(e+fx)}{35c^4f(1-\sec(e+fx))^3}-\frac{62a^3\tan(e+fx)}{105c^4f(1-\sec(e+fx))^2}-\frac{}{15c^4}$$

$$= \frac{a^3x}{c^4}-\frac{8a^3\tan(e+fx)}{7c^4f(1-\sec(e+fx))^4}+\frac{4a^3\tan(e+fx)}{35c^4f(1-\sec(e+fx))^3}-\frac{62a^3\tan(e+fx)}{105c^4f(1-\sec(e+fx))^2}-\frac{1}{105c}$$

Mathematica [A] time = 0.611067, size = 227, normalized size = 1.71

$$\frac{a^3\csc\left(\frac{e}{2}\right)\csc^7\left(\frac{1}{2}(e+fx)\right)\left(-11270\sin\left(e+\frac{fx}{2}\right)+9114\sin\left(e+\frac{3fx}{2}\right)+5040\sin\left(2e+\frac{3fx}{2}\right)-3248\sin\left(2e+\frac{5fx}{2}\right)-1470\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (a^3*Csc[e/2]*Csc[(e + f*x)/2]^7*(3675*f*x*Cos[(f*x)/2] - 3675*f*x*Cos[e +
(f*x)/2] - 2205*f*x*Cos[e + (3*f*x)/2] + 2205*f*x*Cos[2*e + (3*f*x)/2] + 73
5*f*x*Cos[2*e + (5*f*x)/2] - 735*f*x*Cos[3*e + (5*f*x)/2] - 105*f*x*Cos[3*e
 + (7*f*x)/2] + 105*f*x*Cos[4*e + (7*f*x)/2] - 12320*Sin[(f*x)/2] - 11270*S
in[e + (f*x)/2] + 9114*Sin[e + (3*f*x)/2] + 5040*Sin[2*e + (3*f*x)/2] - 324
8*Sin[2*e + (5*f*x)/2] - 1470*Sin[3*e + (5*f*x)/2] + 674*Sin[3*e + (7*f*x)/
2]))/(13440*c^4*f)
```

Maple [A] time = 0.114, size = 111, normalized size = 0.8

$$2\,\frac{a^3\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)}{fc^4}-\frac{a^3}{7\,fc^4}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-7}+\frac{2\,a^3}{5\,fc^4}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-5}-\frac{2\,a^3}{3\,fc^4}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-3}+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x)
```

```
[Out] 2/f*a^3/c^4*arctan(tan(1/2*f*x+1/2*e))-1/7/f*a^3/c^4/tan(1/2*f*x+1/2*e)^7+2
/5/f*a^3/c^4/tan(1/2*f*x+1/2*e)^5-2/3/f*a^3/c^4/tan(1/2*f*x+1/2*e)^3+2/f*a^
3/c^4/tan(1/2*f*x+1/2*e)
```

Maxima [B] time = 1.56309, size = 517, normalized size = 3.89

$$5\,a^3\left(\frac{336\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^4}+\frac{\left(\frac{21\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{77\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{315\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-3\right)(\cos(fx+e)+1)^7}{c^4\sin(fx+e)^7}\right)+\frac{3\,a^3\left(\frac{21\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{35\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{105\sin(f}{(\cos(fx+}\right.}{c^4\sin(fx+e)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] 1/840*(5*a^3*(336*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^4 + (21*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 - 77*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 315
*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 3)*(cos(f*x + e) + 1)^7/(c^4*sin(f*x
```

```
 + e)^7)) + 3*a^3*(21*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 35*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4 - 105*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 15)*(co
s(f*x + e) + 1)^7/(c^4*sin(f*x + e)^7) + 9*a^3*(21*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 35*sin(f*x + e)^6/(c
os(f*x + e) + 1)^6 - 5)*(cos(f*x + e) + 1)^7/(c^4*sin(f*x + e)^7) - a^3*(21
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1)
^4 - 105*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 15)*(cos(f*x + e) + 1)^7/(c^
4*sin(f*x + e)^7))/f
```

Fricas [A] time = 1.04627, size = 424, normalized size = 3.19

$$\frac{337\,a^3\cos\left(fx+e\right)^4-276\,a^3\cos\left(fx+e\right)^3-50\,a^3\cos\left(fx+e\right)^2+396\,a^3\cos\left(fx+e\right)-167\,a^3+105\left(a^3fx\cos\left(fx+e\right)\right.}{105\left(c^4f\cos\left(fx+e\right)^3-3\,c^4f\cos\left(fx+e\right)^2+3\,c^4f\cos\left(fx+e\right)-c^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] 1/105*(337*a^3*cos(f*x + e)^4 - 276*a^3*cos(f*x + e)^3 - 50*a^3*cos(f*x + e
)^2 + 396*a^3*cos(f*x + e) - 167*a^3 + 105*(a^3*f*x*cos(f*x + e)^3 - 3*a^3*
f*x*cos(f*x + e)^2 + 3*a^3*f*x*cos(f*x + e) - a^3*f*x)*sin(f*x + e))/((c^4*
f*cos(f*x + e)^3 - 3*c^4*f*cos(f*x + e)^2 + 3*c^4*f*cos(f*x + e) - c^4*f)*s
in(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^3\left(\int\frac{3\sec\left(e+fx\right)}{\sec^4\left(e+fx\right)-4\sec^3\left(e+fx\right)+6\sec^2\left(e+fx\right)-4\sec\left(e+fx\right)+1}\,dx+\int\frac{3\sec^2\left(e+fx\right)}{\sec^4\left(e+fx\right)-4\sec^3\left(e+fx\right)+6\sec^2\left(e+fx\right)-4\sec\left(e+fx\right)+1}\,dx+\int\frac{}{\sec^4\left(e+fx\right)}\right.}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**4,x)
```

```
[Out] a**3*(Integral(3*sec(e + f*x)/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(
e + f*x)**2 - 4*sec(e + f*x) + 1), x) + Integral(3*sec(e + f*x)**2/(sec(e +
 f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e + f*x)**2 - 4*sec(e + f*x) + 1), x)
+ Integral(sec(e + f*x)**3/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e +
```

```
 f*x)**2 - 4*sec(e + f*x) + 1), x) + Integral(1/(sec(e + f*x)**4 - 4*sec(e
+ f*x)**3 + 6*sec(e + f*x)**2 - 4*sec(e + f*x) + 1), x))/c**4
```

Giac [A] time = 1.3774, size = 126, normalized size = 0.95

$$\frac{\frac{105\left(fx+e\right)a^3}{c^4}+\frac{210\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6-70\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+42\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-15\,a^3}{c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7}}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="giac")
```

```
[Out] 1/105*(105*(f*x + e)*a^3/c^4 + (210*a^3*tan(1/2*f*x + 1/2*e)^6 - 70*a^3*tan
(1/2*f*x + 1/2*e)^4 + 42*a^3*tan(1/2*f*x + 1/2*e)^2 - 15*a^3)/(c^4*tan(1/2*
f*x + 1/2*e)^7))/f
```

3.20 $\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^5}\,dx$

Optimal. Leaf size=164

$$-\frac{496a^3\tan(e+fx)}{315c^5f(1-\sec(e+fx))}-\frac{181a^3\tan(e+fx)}{315c^5f(1-\sec(e+fx))^2}-\frac{38a^3\tan(e+fx)}{105c^5f(1-\sec(e+fx))^3}+\frac{4a^3\tan(e+fx)}{63c^5f(1-\sec(e+fx))^4}-\frac{8a^3\text{ta}}{9c^5f(1-}$$

```
[Out] (a^3*x)/c^5 - (8*a^3*Tan[e + f*x])/(9*c^5*f*(1 - Sec[e + f*x])^5) + (4*a^3*
Tan[e + f*x])/(63*c^5*f*(1 - Sec[e + f*x])^4) - (38*a^3*Tan[e + f*x])/(105*
c^5*f*(1 - Sec[e + f*x])^3) - (181*a^3*Tan[e + f*x])/(315*c^5*f*(1 - Sec[e
+ f*x])^2) - (496*a^3*Tan[e + f*x])/(315*c^5*f*(1 - Sec[e + f*x]))
```

Rubi [A] time = 0.7334, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {3903, 3777, 3922, 3919, 3794, 3796, 3797, 3799, 4000}

$$-\frac{496a^3\tan(e+fx)}{315c^5f(1-\sec(e+fx))}-\frac{181a^3\tan(e+fx)}{315c^5f(1-\sec(e+fx))^2}-\frac{38a^3\tan(e+fx)}{105c^5f(1-\sec(e+fx))^3}+\frac{4a^3\tan(e+fx)}{63c^5f(1-\sec(e+fx))^4}-\frac{8a^3\text{ta}}{9c^5f(1-}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x])^5,x]
```

```
[Out] (a^3*x)/c^5 - (8*a^3*Tan[e + f*x])/(9*c^5*f*(1 - Sec[e + f*x])^5) + (4*a^3*
Tan[e + f*x])/(63*c^5*f*(1 - Sec[e + f*x])^4) - (38*a^3*Tan[e + f*x])/(105*
c^5*f*(1 - Sec[e + f*x])^3) - (181*a^3*Tan[e + f*x])/(315*c^5*f*(1 - Sec[e
+ f*x])^2) - (496*a^3*Tan[e + f*x])/(315*c^5*f*(1 - Sec[e + f*x]))
```

Rule 3903

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Dist[c^n, Int[ExpandTrig[(1 + (d*csc[e + f*
x])/c)^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
&& LtQ[m + n, 2]
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
```

```
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
rQ[2*n]
```

Rule 3922

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*
x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x
])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ
[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3797

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(2*m + 1)), x]
+ Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x],
x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3799

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
```

```
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4000

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[((A*b - a*B)*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^5}\,dx = \frac{\int \left(\frac{a^3}{(1-\sec(e+fx))^5}+\frac{3a^3\sec(e+fx)}{(1-\sec(e+fx))^5}+\frac{3a^3\sec^2(e+fx)}{(1-\sec(e+fx))^5}+\frac{a^3\sec^3(e+fx)}{(1-\sec(e+fx))^5}\right)dx}{c^5}$$

$$= \frac{a^3\int \frac{1}{(1-\sec(e+fx))^5}\,dx}{c^5}+\frac{a^3\int \frac{\sec^3(e+fx)}{(1-\sec(e+fx))^5}\,dx}{c^5}+\frac{\left(3a^3\right)\int \frac{\sec(e+fx)}{(1-\sec(e+fx))^5}\,dx}{c^5}+\frac{\left(3a^3\right)\int \frac{\sec^2(e+fx)}{(1-\sec(e+fx)}}{c^5}$$

$$= -\frac{8a^3\tan(e+fx)}{9c^5f(1-\sec(e+fx))^5}-\frac{a^3\int \frac{-9-4\sec(e+fx)}{(1-\sec(e+fx))^4}\,dx}{9c^5}+\frac{a^3\int \frac{(-5-9\sec(e+fx))\sec(e+fx)}{(1-\sec(e+fx))^4}\,dx}{9c^5}+\frac{\left(4a^3\right)\int}{}$$

$$= -\frac{8a^3\tan(e+fx)}{9c^5f(1-\sec(e+fx))^5}+\frac{4a^3\tan(e+fx)}{63c^5f(1-\sec(e+fx))^4}+\frac{a^3\int \frac{63+39\sec(e+fx)}{(1-\sec(e+fx))^3}\,dx}{63c^5}+\frac{a^3\int \frac{\sec(e+}{(1-\sec(e+}}{3c^5}$$

$$= -\frac{8a^3\tan(e+fx)}{9c^5f(1-\sec(e+fx))^5}+\frac{4a^3\tan(e+fx)}{63c^5f(1-\sec(e+fx))^4}-\frac{38a^3\tan(e+fx)}{105c^5f(1-\sec(e+fx))^3}-\frac{a^3\int \frac{-315}{(1-}}{}$$

$$= -\frac{8a^3\tan(e+fx)}{9c^5f(1-\sec(e+fx))^5}+\frac{4a^3\tan(e+fx)}{63c^5f(1-\sec(e+fx))^4}-\frac{38a^3\tan(e+fx)}{105c^5f(1-\sec(e+fx))^3}-\frac{181a^3}{315c^5f(1}$$

$$= \frac{a^3x}{c^5}-\frac{8a^3\tan(e+fx)}{9c^5f(1-\sec(e+fx))^5}+\frac{4a^3\tan(e+fx)}{63c^5f(1-\sec(e+fx))^4}-\frac{38a^3\tan(e+fx)}{105c^5f(1-\sec(e+fx))^3}-\frac{1}{315c}$$

$$= \frac{a^3x}{c^5}-\frac{8a^3\tan(e+fx)}{9c^5f(1-\sec(e+fx))^5}+\frac{4a^3\tan(e+fx)}{63c^5f(1-\sec(e+fx))^4}-\frac{38a^3\tan(e+fx)}{105c^5f(1-\sec(e+fx))^3}-\frac{1}{315c}$$

Mathematica [A] time = 0.879814, size = 283, normalized size = 1.73

$$a^3\csc\left(\frac{e}{2}\right)\csc^9\left(\frac{1}{2}(e+fx)\right)\left(-122850\sin\left(e+\frac{fx}{2}\right)+103278\sin\left(e+\frac{3fx}{2}\right)+73290\sin\left(2e+\frac{3fx}{2}\right)-51102\sin\left(2e+\frac{5fx}{2}\right)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x])^5,x]
```

```
[Out] (a^3*Csc[e/2]*Csc[(e + f*x)/2]^9*(39690*f*x*Cos[(f*x)/2] - 39690*f*x*Cos[e
+ (f*x)/2] - 26460*f*x*Cos[e + (3*f*x)/2] + 26460*f*x*Cos[2*e + (3*f*x)/2]
+ 11340*f*x*Cos[2*e + (5*f*x)/2] - 11340*f*x*Cos[3*e + (5*f*x)/2] - 2835*f*
x*Cos[3*e + (7*f*x)/2] + 2835*f*x*Cos[4*e + (7*f*x)/2] + 315*f*x*Cos[4*e +
(9*f*x)/2] - 315*f*x*Cos[5*e + (9*f*x)/2] - 142002*Sin[(f*x)/2] - 122850*Si
n[e + (f*x)/2] + 103278*Sin[e + (3*f*x)/2] + 73290*Sin[2*e + (3*f*x)/2] - 5
1102*Sin[2*e + (5*f*x)/2] - 24570*Sin[3*e + (5*f*x)/2] + 13878*Sin[3*e + (7
*f*x)/2] + 5040*Sin[4*e + (7*f*x)/2] - 2102*Sin[4*e + (9*f*x)/2]))/(161280*
c^5*f)
```

Maple [A] time = 0.128, size = 133, normalized size = 0.8

$$2\,\frac{a^3\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)}{fc^5}+\frac{a^3}{18\,fc^5}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-9}-\frac{3\,a^3}{14\,fc^5}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-7}+\frac{2\,a^3}{5\,fc^5}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-5}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x)
```

```
[Out] 2/f*a^3/c^5*arctan(tan(1/2*f*x+1/2*e))+1/18/f*a^3/c^5/tan(1/2*f*x+1/2*e)^9-
3/14/f*a^3/c^5/tan(1/2*f*x+1/2*e)^7+2/5/f*a^3/c^5/tan(1/2*f*x+1/2*e)^5-2/3/
f*a^3/c^5/tan(1/2*f*x+1/2*e)^3+2/f*a^3/c^5/tan(1/2*f*x+1/2*e)
```

Maxima [B] time = 1.66351, size = 544, normalized size = 3.32

$$a^3\left(\frac{10080\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^5}-\frac{\left(\frac{270\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{1008\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{2730\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{9765\sin(fx+e)^8}{(\cos(fx+e)+1)^8}-35\right)(\cos(fx+e)+1)^9}{c^5\sin(fx+e)^9}\right)-\frac{3\,a^3\left(\frac{180\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{378\sin}{(\cos(f}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="maxima")
```

```
[Out] 1/5040*(a^3*(10080*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^5 - (270*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 - 1008*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 +
 2730*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 9765*sin(f*x + e)^8/(cos(f*x +
e) + 1)^8 - 35)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x + e)^9)) - 3*a^3*(180*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 - 378*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
+ 420*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 315*sin(f*x + e)^8/(cos(f*x + e
) + 1)^8 - 35)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x + e)^9) - 15*a^3*(18*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 - 42*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 6
3*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 7)*(cos(f*x + e) + 1)^9/(c^5*sin(f*
x + e)^9) - 7*a^3*(18*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 45*sin(f*x + e)
^8/(cos(f*x + e) + 1)^8 - 5)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x + e)^9))/f
```

Fricas [A] time = 1.11534, size = 535, normalized size = 3.26

$$\frac{1051\,a^3\cos\left(fx+e\right)^5 - 1684\,a^3\cos\left(fx+e\right)^4 + 898\,a^3\cos\left(fx+e\right)^3 + 1468\,a^3\cos\left(fx+e\right)^2 - 1669\,a^3\cos\left(fx+e\right) + 496}{315\left(c^5 f\cos\left(fx+e\right)^4 - 4\,c^5 f\cos\left(fx+e\right)^3 + 6\,c^5 f\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="fricas")
```

```
[Out] 1/315*(1051*a^3*cos(f*x + e)^5 - 1684*a^3*cos(f*x + e)^4 + 898*a^3*cos(f*x
+ e)^3 + 1468*a^3*cos(f*x + e)^2 - 1669*a^3*cos(f*x + e) + 496*a^3 + 315*(a
^3*f*x*cos(f*x + e)^4 - 4*a^3*f*x*cos(f*x + e)^3 + 6*a^3*f*x*cos(f*x + e)^2
 - 4*a^3*f*x*cos(f*x + e) + a^3*f*x)*sin(f*x + e))/((c^5*f*cos(f*x + e)^4 -
 4*c^5*f*cos(f*x + e)^3 + 6*c^5*f*cos(f*x + e)^2 - 4*c^5*f*cos(f*x + e) + c
^5*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.50893, size = 149, normalized size = 0.91

$$\frac{\frac{630\,(fx+e)a^3}{c^5}+\frac{1260\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^8-420\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6+252\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-135\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+35\,a^3}{c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^9}}{630\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="giac")
```

```
[Out] 1/630*(630*(f*x + e)*a^3/c^5 + (1260*a^3*tan(1/2*f*x + 1/2*e)^8 - 420*a^3*t
an(1/2*f*x + 1/2*e)^6 + 252*a^3*tan(1/2*f*x + 1/2*e)^4 - 135*a^3*tan(1/2*f*
x + 1/2*e)^2 + 35*a^3)/(c^5*tan(1/2*f*x + 1/2*e)^9))/f
```

3.21 $\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=136

$$\frac{13c^5\tan(e+fx)}{2a^2f}-\frac{47c^5\tanh^{-1}(\sin(e+fx))}{2a^2f}+\frac{112c^5\tan(e+fx)}{3a^2f(\sec(e+fx)+1)}+\frac{\tan(e+fx)\left(c^5-c^5\sec(e+fx)\right)}{2a^2f}+\frac{c^5x}{a^2}-\frac{32}{3f(a}$$

```
[Out] (c^5*x)/a^2 - (47*c^5*ArcTanh[Sin[e + f*x]])/(2*a^2*f) + (13*c^5*Tan[e + f*
x])/(2*a^2*f) + (112*c^5*Tan[e + f*x])/(3*a^2*f*(1 + Sec[e + f*x])) - (32*c
^5*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2) + ((c^5 - c^5*Sec[e + f*x])*T
an[e + f*x])/(2*a^2*f)
```

Rubi [A] time = 0.402111, antiderivative size = 153, normalized size of antiderivative = 1.12, number of steps used = 26, number of rules used = 14, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3904, 3886, 3473, 8, 2606, 2607, 30, 3767, 2621, 302, 207, 2620, 270, 288}

$$\frac{7c^5\tan(e+fx)}{a^2f}-\frac{64c^5\cot^3(e+fx)}{3a^2f}-\frac{48c^5\cot(e+fx)}{a^2f}+\frac{131c^5\csc^3(e+fx)}{6a^2f}+\frac{33c^5\csc(e+fx)}{2a^2f}-\frac{47c^5\tanh^{-1}(\sin(e+}{2a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^5/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (c^5*x)/a^2 - (47*c^5*ArcTanh[Sin[e + f*x]])/(2*a^2*f) - (48*c^5*Cot[e + f*
x])/(a^2*f) - (64*c^5*Cot[e + f*x]^3)/(3*a^2*f) + (33*c^5*Csc[e + f*x])/(2*
a^2*f) + (131*c^5*Csc[e + f*x]^3)/(6*a^2*f) - (c^5*Csc[e + f*x]^3*Sec[e + f
*x]^2)/(2*a^2*f) + (7*c^5*Tan[e + f*x])/(a^2*f)
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
```

```
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx = \frac{\int \cot^4(e+fx)(c-c\sec(e+fx))^7\,dx}{a^2c^2}$$
$$= \frac{\int \left(c^7\cot^4(e+fx) - 7c^7\cot^3(e+fx)\csc(e+fx) + 21c^7\cot^2(e+fx)\csc^2(e+fx) - 35c^7\right.}{}$$
$$= \frac{c^5\int \cot^4(e+fx)\,dx}{a^2} - \frac{c^5\int \csc^4(e+fx)\sec^3(e+fx)\,dx}{a^2} - \frac{\left(7c^5\right)\int \cot^3(e+fx)\csc(e+f}{a^2}$$
$$= -\frac{c^5\cot^3(e+fx)}{3a^2f} - \frac{c^5\int \cot^2(e+fx)\,dx}{a^2} + \frac{c^5\,\mathrm{Subst}\left(\int \frac{x^6}{\left(-1+x^2\right)^2}\,dx, x, \csc(e+fx)\right)}{a^2f} + \frac{\left(7c^5\right.}{}$$
$$= -\frac{34c^5\cot(e+fx)}{a^2f} - \frac{19c^5\cot^3(e+fx)}{a^2f} - \frac{7c^5\csc(e+fx)}{a^2f} + \frac{14c^5\csc^3(e+fx)}{a^2f} - \frac{c^5\csc^3(e}{}$$
$$= \frac{c^5x}{a^2} - \frac{48c^5\cot(e+fx)}{a^2f} - \frac{64c^5\cot^3(e+fx)}{3a^2f} + \frac{14c^5\csc(e+fx)}{a^2f} + \frac{21c^5\csc^3(e+fx)}{a^2f} - \frac{c^5}{}$$
$$= \frac{c^5x}{a^2} - \frac{21c^5\tanh^{-1}(\sin(e+fx))}{a^2f} - \frac{48c^5\cot(e+fx)}{a^2f} - \frac{64c^5\cot^3(e+fx)}{3a^2f} + \frac{33c^5\csc(e+f}{2a^2f}$$
$$= \frac{c^5x}{a^2} - \frac{47c^5\tanh^{-1}(\sin(e+fx))}{2a^2f} - \frac{48c^5\cot(e+fx)}{a^2f} - \frac{64c^5\cot^3(e+fx)}{3a^2f} + \frac{33c^5\csc(e+f}{2a^2f}$$

Mathematica [B] time = 3.01918, size = 384, normalized size = 2.82

$$\frac{\cos^3(e+fx)\cot\left(\frac{1}{2}(e+fx)\right)\csc^6\left(\frac{1}{2}(e+fx)\right)(c-c\sec(e+fx))^5\left(-\frac{64\tan\left(\frac{e}{2}\right)\cot\left(\frac{1}{2}(e+fx)\right)\csc^2\left(\frac{1}{2}(e+fx)\right)}{f} - \frac{64\sec\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)\csc^3}{f}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^5/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (Cos[e + f*x]^3*Cot[(e + f*x)/2]*Csc[(e + f*x)/2]^6*(c - c*Sec[e + f*x])^5*
((-320*Cot[(e + f*x)/2]^2*Csc[(e + f*x)/2]*Sec[e/2]*Sin[(f*x)/2])/f - (64*C
sc[(e + f*x)/2]^3*Sec[e/2]*Sin[(f*x)/2])/f + 3*Cot[(e + f*x)/2]^3*(-4*x - (
94*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]])/f + (94*Log[Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2]])/f + 1/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2) - 1/(
f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2) - (28*Sin[f*x])/(f*(Cos[e/2] - S
in[e/2])*(Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(
e + f*x)/2] + Sin[(e + f*x)/2]))) - (64*Cot[(e + f*x)/2]*Csc[(e + f*x)/2]^2
```

```
*Tan[e/2])/f))/(96*a^2*(1 + Sec[e + f*x])^2)
```

Maple [A] time = 0.105, size = 207, normalized size = 1.5

$$\frac{16\,c^5}{3\,fa^2}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3+32\,\frac{c^5\tan\left(1/2\,fx+e/2\right)}{fa^2}+2\,\frac{c^5\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)}{fa^2}+\frac{c^5}{2\,fa^2}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{-2}-\frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x)
```

```
[Out] 16/3/f*c^5/a^2*tan(1/2*f*x+1/2*e)^3+32/f*c^5/a^2*tan(1/2*f*x+1/2*e)+2/f*c^5
/a^2*arctan(tan(1/2*f*x+1/2*e))+1/2/f*c^5/a^2/(tan(1/2*f*x+1/2*e)+1)^2-15/2
/f*c^5/a^2/(tan(1/2*f*x+1/2*e)+1)-47/2/f*c^5/a^2*ln(tan(1/2*f*x+1/2*e)+1)-1
/2/f*c^5/a^2/(tan(1/2*f*x+1/2*e)-1)^2-15/2/f*c^5/a^2/(tan(1/2*f*x+1/2*e)-1)
+47/2/f*c^5/a^2*ln(tan(1/2*f*x+1/2*e)-1)
```

Maxima [B] time = 1.59072, size = 814, normalized size = 5.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(c^5*(6*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3)/(a^2 - 2*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x
 + e)^4/(cos(f*x + e) + 1)^4) + (21*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f
*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 21*log(sin(f*x + e)/(cos(f*x + e) + 1
) + 1)/a^2 + 21*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2) + 5*c^5*((15*
sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2
- 12*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 12*log(sin(f*x + e)/(co
s(f*x + e) + 1) - 1)/a^2 + 12*sin(f*x + e)/((a^2 - a^2*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2)*(cos(f*x + e) + 1))) + 10*c^5*((9*sin(f*x + e)/(cos(f*x +
e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 6*log(sin(f*x + e)/(co
s(f*x + e) + 1) + 1)/a^2 + 6*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2)
- c^5*((9*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e) +
1)^3)/a^2 - 12*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + 10*c^5*(3*sin
(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 5
```

```
*c^5*(3*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e) + 1)
^3)/a^2)/f
```

Fricas [A] time = 1.15021, size = 601, normalized size = 4.42

$$\frac{12\,c^5 f x \cos\left(fx+e\right)^4 + 24\,c^5 f x \cos\left(fx+e\right)^3 + 12\,c^5 f x \cos\left(fx+e\right)^2 - 141\left(c^5 \cos\left(fx+e\right)^4 + 2\,c^5 \cos\left(fx+e\right)^3 + c^5 c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(12*c^5*f*x*cos(f*x + e)^4 + 24*c^5*f*x*cos(f*x + e)^3 + 12*c^5*f*x*co
s(f*x + e)^2 - 141*(c^5*cos(f*x + e)^4 + 2*c^5*cos(f*x + e)^3 + c^5*cos(f*x
 + e)^2)*log(sin(f*x + e) + 1) + 141*(c^5*cos(f*x + e)^4 + 2*c^5*cos(f*x +
e)^3 + c^5*cos(f*x + e)^2)*log(-sin(f*x + e) + 1) + 2*(202*c^5*cos(f*x + e)
^3 + 305*c^5*cos(f*x + e)^2 + 36*c^5*cos(f*x + e) - 3*c^5)*sin(f*x + e))/(a
^2*f*cos(f*x + e)^4 + 2*a^2*f*cos(f*x + e)^3 + a^2*f*cos(f*x + e)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{c^5\left(\int \frac{5\sec\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx + \int -\frac{10\sec^2\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx + \int \frac{10\sec^3\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx + \int -\frac{5\sec^4\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+f\right.}\right.}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**5/(a+a*sec(f*x+e))**2,x)
```

```
[Out] -c**5*(Integral(5*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) +
 Integral(-10*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) +
Integral(10*sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + In
tegral(-5*sec(e + f*x)**4/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Inte
gral(sec(e + f*x)**5/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Integral(
-1/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

Giac [A] time = 1.46604, size = 217, normalized size = 1.6

$$\frac{\frac{6\,(fx+e)c^5}{a^2} - \frac{141\,c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^2} + \frac{141\,c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^2} - \frac{6\left(15\,c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-13\,c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^2a^2} + \frac{32\left(a^4c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\right.\right.}{}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/6*(6*(f*x + e)*c^5/a^2 - 141*c^5*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^2 +
 141*c^5*log(abs(tan(1/2*f*x + 1/2*e) - 1))/a^2 - 6*(15*c^5*tan(1/2*f*x + 1
/2*e)^3 - 13*c^5*tan(1/2*f*x + 1/2*e))/((tan(1/2*f*x + 1/2*e)^2 - 1)^2*a^2)
 + 32*(a^4*c^5*tan(1/2*f*x + 1/2*e)^3 + 6*a^4*c^5*tan(1/2*f*x + 1/2*e))/a^6
)/f
```

3.22 $\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=102

$$\frac{c^4\tan(e+fx)}{a^2f}-\frac{32c^4\cot^3(e+fx)}{3a^2f}-\frac{16c^4\cot(e+fx)}{a^2f}+\frac{32c^4\csc^3(e+fx)}{3a^2f}-\frac{6c^4\tanh^{-1}(\sin(e+fx))}{a^2f}+\frac{c^4x}{a^2}$$

```
[Out] (c^4*x)/a^2 - (6*c^4*ArcTanh[Sin[e + f*x]])/(a^2*f) - (16*c^4*Cot[e + f*x])
/(a^2*f) - (32*c^4*Cot[e + f*x]^3)/(3*a^2*f) + (32*c^4*Csc[e + f*x]^3)/(3*a
^2*f) + (c^4*Tan[e + f*x])/(a^2*f)
```

Rubi [A] time = 0.308967, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 13, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3904, 3886, 3473, 8, 2606, 2607, 30, 3767, 2621, 302, 207, 2620, 270}

$$\frac{c^4\tan(e+fx)}{a^2f}-\frac{32c^4\cot^3(e+fx)}{3a^2f}-\frac{16c^4\cot(e+fx)}{a^2f}+\frac{32c^4\csc^3(e+fx)}{3a^2f}-\frac{6c^4\tanh^{-1}(\sin(e+fx))}{a^2f}+\frac{c^4x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^4/(a + a*Sec[e + f*x])^2,x]
```


```
[Out] (c^4*x)/a^2 - (6*c^4*ArcTanh[Sin[e + f*x]])/(a^2*f) - (16*c^4*Cot[e + f*x])
/(a^2*f) - (32*c^4*Cot[e + f*x]^3)/(3*a^2*f) + (32*c^4*Csc[e + f*x]^3)/(3*a
^2*f) + (c^4*Tan[e + f*x])/(a^2*f)
```


Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx = \frac{\int \cot^4(e+fx)(c-c\sec(e+fx))^6\,dx}{a^2c^2}$$

$$= \frac{\int \left(c^6\cot^4(e+fx) - 6c^6\cot^3(e+fx)\csc(e+fx) + 15c^6\cot^2(e+fx)\csc^2(e+fx) - 20c^6 \cdots\right.}{\cdots}$$

$$= \frac{c^4\int \cot^4(e+fx)\,dx}{a^2} + \frac{c^4\int \csc^4(e+fx)\sec^2(e+fx)\,dx}{a^2} - \frac{\left(6c^4\right)\int \cot^3(e+fx)\csc(e+f\cdots}{a^2}$$

$$= -\frac{c^4\cot^3(e+fx)}{3a^2f} - \frac{c^4\int \cot^2(e+fx)\,dx}{a^2} + \frac{c^4\,\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^4}\,dx, x, \tan(e+fx)\right)}{a^2f} + \frac{\left(6c^4\right)\cdots}{\cdots}$$

$$= -\frac{14c^4\cot(e+fx)}{a^2f} - \frac{31c^4\cot^3(e+fx)}{3a^2f} - \frac{6c^4\csc(e+fx)}{a^2f} + \frac{26c^4\csc^3(e+fx)}{3a^2f} + \frac{c^4\int 1\,dx}{a^2}\cdots$$

$$= \frac{c^4x}{a^2} - \frac{16c^4\cot(e+fx)}{a^2f} - \frac{32c^4\cot^3(e+fx)}{3a^2f} + \frac{32c^4\csc^3(e+fx)}{3a^2f} + \frac{c^4\tan(e+fx)}{a^2f} + \frac{\left(6c^4\right)\cdots}{\cdots}$$

$$= \frac{c^4x}{a^2} - \frac{6c^4\tanh^{-1}(\sin(e+fx))}{a^2f} - \frac{16c^4\cot(e+fx)}{a^2f} - \frac{32c^4\cot^3(e+fx)}{3a^2f} + \frac{32c^4\csc^3(e+f\cdots}{3a^2f}$$

Mathematica [B] time = 6.01563, size = 448, normalized size = 4.39

$$\frac{4c^4\sin^3\left(\frac{1}{2}(e+fx)\right)\cos\left(\frac{1}{2}(e+fx)\right)\left(-\frac{1}{16}\sec^3\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)(-64\cos(e+fx)-16\cos(2(e+fx))+90\cos(2e+fx)+27\cos\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^4/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (4*c^4*Cos[(e + f*x)/2]*Sin[(e + f*x)/2]^3*(-3*Cos[e]*Cot[(e + f*x)/2]^5*(f
*x + 6*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - 6*Log[Cos[(e + f*x)/2] +
Sin[(e + f*x)/2]])*Sec[e/2]^2 + 4*Cot[(e + f*x)/2]*Csc[(e + f*x)/2]^2*Sec[e
/2]^3*(-Sin[e/2] + Sin[(3*e)/2]) - 22*Cot[(e + f*x)/2]^4*Csc[(e + f*x)/2]*S
ec[e/2]*Sin[(f*x)/2] - ((-48 + 16*Cos[e] + 102*Cos[f*x] - 64*Cos[e + f*x] -
 16*Cos[2*(e + f*x)] + 90*Cos[2*e + f*x] + 27*Cos[e + 2*f*x] + 21*Cos[3*e +
 2*f*x])*Csc[(e + f*x)/2]^5*Sec[e/2]^3*Sin[(f*x)/2])/16 - Cot[(e + f*x)/2]^
3*(3*(f*x + 6*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - 6*Log[Cos[(e + f*x
)/2] + Sin[(e + f*x)/2]]) - 8*Csc[(e + f*x)/2]^2*Tan[e/2])*(-1 + Tan[e/2]^2
)))/(3*a^2*f*(1 + Cos[e + f*x])^2*(-1 + Cot[(e + f*x)/2])*(1 + Cot[(e + f*x
)/2])*(-1 + Tan[e/2])*(1 + Tan[e/2]))
```

Maple [A] time = 0.088, size = 159, normalized size = 1.6

$$\frac{8\,c^4}{3\,fa^2}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3+8\,\frac{c^4\tan\left(1/2\,fx+e/2\right)}{fa^2}+2\,\frac{c^4\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)}{fa^2}-\frac{c^4}{fa^2}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{-1}-6\,\frac{c^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x)
```

```
[Out] 8/3/f*c^4/a^2*tan(1/2*f*x+1/2*e)^3+8/f*c^4/a^2*tan(1/2*f*x+1/2*e)+2/f*c^4/a
^2*arctan(tan(1/2*f*x+1/2*e))-1/f*c^4/a^2/(tan(1/2*f*x+1/2*e)+1)-6/f*c^4/a^
2*ln(tan(1/2*f*x+1/2*e)+1)-1/f*c^4/a^2/(tan(1/2*f*x+1/2*e)-1)+6/f*c^4/a^2*l
n(tan(1/2*f*x+1/2*e)-1)
```

Maxima [B] time = 1.54283, size = 558, normalized size = 5.47

$$c^4\left(\frac{\frac{15\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}}{a^2}-\frac{12\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)}{a^2}+\frac{12\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)}{a^2}+\frac{12\sin\left(fx+e\right)}{\left(a^2-\frac{a^2\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}\right)\left(\cos\left(fx+e\right)+1\right)}\right)+4c^4\left(\frac{\frac{9\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\cdots}{a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(c^4*((15*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e
) + 1)^3)/a^2 - 12*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 12*log(si
n(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2 + 12*sin(f*x + e)/((a^2 - a^2*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1))) + 4*c^4*((9*sin(f*x + e
)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 6*log(sin
(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 6*log(sin(f*x + e)/(cos(f*x + e) +
1) - 1)/a^2) - c^4*((9*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(co
s(f*x + e) + 1)^3)/a^2 - 12*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) +
6*c^4*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1
)^3)/a^2 - 4*c^4*(3*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f
*x + e) + 1)^3)/a^2)/f
```

Fricas [B] time = 1.13238, size = 541, normalized size = 5.3

$$\frac{3\,c^4fx\cos\left(fx+e\right)^3+6\,c^4fx\cos\left(fx+e\right)^2+3\,c^4fx\cos\left(fx+e\right)-9\left(c^4\cos\left(fx+e\right)^3+2\,c^4\cos\left(fx+e\right)^2+c^4\cos\left(fx+e\right.\right.\cdots}{3\left(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*c^4*f*x*cos(f*x + e)^3 + 6*c^4*f*x*cos(f*x + e)^2 + 3*c^4*f*x*cos(f*
x + e) - 9*(c^4*cos(f*x + e)^3 + 2*c^4*cos(f*x + e)^2 + c^4*cos(f*x + e))*l
og(sin(f*x + e) + 1) + 9*(c^4*cos(f*x + e)^3 + 2*c^4*cos(f*x + e)^2 + c^4*c
os(f*x + e))*log(-sin(f*x + e) + 1) + (19*c^4*cos(f*x + e)^2 + 38*c^4*cos(f
*x + e) + 3*c^4)*sin(f*x + e))/(a^2*f*cos(f*x + e)^3 + 2*a^2*f*cos(f*x + e)
^2 + a^2*f*cos(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{c^4\left(\int -\frac{4\sec\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx+\int \frac{6\sec^2\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx+\int -\frac{4\sec^3\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx+\int \frac{\sec^4\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+}\right.}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**4/(a+a*sec(f*x+e))**2,x)
```

```
[Out] c**4*(Integral(-4*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) +
 Integral(6*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + In
tegral(-4*sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Inte
gral(sec(e + f*x)**4/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Integral(
1/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

Giac [A] time = 1.32759, size = 190, normalized size = 1.86

$$\frac{\frac{3\left(fx+e\right)c^4}{a^2}-\frac{18\,c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^2}+\frac{18\,c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^2}-\frac{6\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)a^2}+\frac{8\left(a^4c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+3\,a^4c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{a^6}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*(f*x + e)*c^4/a^2 - 18*c^4*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^2 +
18*c^4*log(abs(tan(1/2*f*x + 1/2*e) - 1))/a^2 - 6*c^4*tan(1/2*f*x + 1/2*e)/
((tan(1/2*f*x + 1/2*e)^2 - 1)*a^2) + 8*(a^4*c^4*tan(1/2*f*x + 1/2*e)^3 + 3*
a^4*c^4*tan(1/2*f*x + 1/2*e))/a^6)/f
```

3.23 $\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=85

$$-\frac{c^3\tanh^{-1}(\sin(e+fx))}{a^2f}+\frac{4c^3\tan(e+fx)}{3a^2f(\sec(e+fx)+1)}-\frac{8c^3\tan(e+fx)}{3a^2f(\sec(e+fx)+1)^2}+\frac{c^3x}{a^2}$$

```
[Out] (c^3*x)/a^2 - (c^3*ArcTanh[Sin[e + f*x]])/(a^2*f) - (8*c^3*Tan[e + f*x])/(3
*a^2*f*(1 + Sec[e + f*x])^2) + (4*c^3*Tan[e + f*x])/(3*a^2*f*(1 + Sec[e + f
*x]))
```

Rubi [A] time = 0.330986, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {3903, 3777, 3919, 3794, 3796, 3797, 3799, 3998, 3770}

$$-\frac{c^3\tanh^{-1}(\sin(e+fx))}{a^2f}+\frac{4c^3\tan(e+fx)}{3a^2f(\sec(e+fx)+1)}-\frac{8c^3\tan(e+fx)}{3a^2f(\sec(e+fx)+1)^2}+\frac{c^3x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (c^3*x)/a^2 - (c^3*ArcTanh[Sin[e + f*x]])/(a^2*f) - (8*c^3*Tan[e + f*x])/(3
*a^2*f*(1 + Sec[e + f*x])^2) + (4*c^3*Tan[e + f*x])/(3*a^2*f*(1 + Sec[e + f
*x]))
```

Rule 3903

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Dist[c^n, Int[ExpandTrig[(1 + (d*csc[e + f*
x])/c)^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
&& LtQ[m + n, 2]
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
```

rQ[2*n]

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3797

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(2*m + 1)), x]
+ Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x],
x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3799

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx = \frac{\int \left(\frac{c^3}{(1+\sec(e+fx))^2}-\frac{3c^3\sec(e+fx)}{(1+\sec(e+fx))^2}+\frac{3c^3\sec^2(e+fx)}{(1+\sec(e+fx))^2}-\frac{c^3\sec^3(e+fx)}{(1+\sec(e+fx))^2}\right)dx}{a^2}$$
$$= \frac{c^3\int \frac{1}{(1+\sec(e+fx))^2}\,dx}{a^2} - \frac{c^3\int \frac{\sec^3(e+fx)}{(1+\sec(e+fx))^2}\,dx}{a^2} - \frac{\left(3c^3\right)\int \frac{\sec(e+fx)}{(1+\sec(e+fx))^2}\,dx}{a^2} + \frac{\left(3c^3\right)\int \frac{\sec^2(e+fx}{(1+\sec(e+fx}}{a^2}$$
$$= -\frac{8c^3\tan(e+fx)}{3a^2f(1+\sec(e+fx))^2} - \frac{c^3\int \frac{-3+\sec(e+fx)}{1+\sec(e+fx)}\,dx}{3a^2} - \frac{c^3\int \frac{\sec(e+fx)(-2+3\sec(e+fx))}{1+\sec(e+fx)}\,dx}{3a^2} - \frac{c^3\int \frac{\sec}{1+se}}{}$$
$$= \frac{c^3x}{a^2} - \frac{8c^3\tan(e+fx)}{3a^2f(1+\sec(e+fx))^2} + \frac{c^3\tan(e+fx)}{a^2f(1+\sec(e+fx))} - \frac{c^3\int \sec(e+fx)\,dx}{a^2} - \frac{\left(4c^3\right)\int \frac{\sec}{1+se}}{3a}$$
$$= \frac{c^3x}{a^2} - \frac{c^3\tanh^{-1}(\sin(e+fx))}{a^2f} - \frac{8c^3\tan(e+fx)}{3a^2f(1+\sec(e+fx))^2} + \frac{4c^3\tan(e+fx)}{3a^2f(1+\sec(e+fx))}$$

Mathematica [B] time = 1.09175, size = 216, normalized size = 2.54

$$-\frac{c^3(\cos(e+fx)-1)^3\cot\left(\frac{1}{2}(e+fx)\right)\csc^2\left(\frac{1}{2}(e+fx)\right)\left(4\tan\left(\frac{e}{2}\right)\cot\left(\frac{1}{2}(e+fx)\right)\csc^2\left(\frac{1}{2}(e+fx)\right)+4\sec\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] -(c^3*(-1 + Cos[e + f*x])^3*Cot[(e + f*x)/2]*Csc[(e + f*x)/2]^2*(3*Cot[(e +
 f*x)/2]^3*(f*x + Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f
*x)/2] + Sin[(e + f*x)/2]]) - 4*Cot[(e + f*x)/2]^2*Csc[(e + f*x)/2]*Sec[e/2
]*Sin[(f*x)/2] + 4*Csc[(e + f*x)/2]^3*Sec[e/2]*Sin[(f*x)/2] + 4*Cot[(e + f*
x)/2]*Csc[(e + f*x)/2]^2*Tan[e/2]))/(6*a^2*f*(1 + Cos[e + f*x])^2)
```

Maple [A] time = 0.086, size = 90, normalized size = 1.1

$$\frac{4\,c^3}{3\,fa^2}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3+2\,\frac{c^3\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)}{fa^2}-\frac{c^3}{fa^2}\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)+\frac{c^3}{fa^2}\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x)
```

```
[Out] 4/3/f*c^3/a^2*tan(1/2*f*x+1/2*e)^3+2/f*c^3/a^2*arctan(tan(1/2*f*x+1/2*e))-1
/f*c^3/a^2*ln(tan(1/2*f*x+1/2*e)+1)+1/f*c^3/a^2*ln(tan(1/2*f*x+1/2*e)-1)
```

Maxima [B] time = 1.58067, size = 362, normalized size = 4.26

$$\frac{c^3\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2}+\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^2}\right)-c^3\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{12\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2}\right)+\cdots}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(c^3*((9*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3)/a^2 - 6*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 6*log(sin(f
*x + e)/(cos(f*x + e) + 1) - 1)/a^2) - c^3*((9*sin(f*x + e)/(cos(f*x + e) +
 1) - sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 12*arctan(sin(f*x + e)/(co
s(f*x + e) + 1))/a^2) + 3*c^3*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x
+ e)^3/(cos(f*x + e) + 1)^3)/a^2 - 3*c^3*(3*sin(f*x + e)/(cos(f*x + e) + 1)
 - sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2)/f
```

Fricas [B] time = 1.07578, size = 425, normalized size = 5.

$$\frac{6c^3fx\cos(fx+e)^2+12c^3fx\cos(fx+e)+6c^3fx-3\left(c^3\cos(fx+e)^2+2c^3\cos(fx+e)+c^3\right)\log(\sin(fx+e)+1)\cdots}{6\left(a^2f\cos(fx+e)^2+2a^2f\cos\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(6*c^3*f*x*cos(f*x + e)^2 + 12*c^3*f*x*cos(f*x + e) + 6*c^3*f*x - 3*(c^
3*cos(f*x + e)^2 + 2*c^3*cos(f*x + e) + c^3)*log(sin(f*x + e) + 1) + 3*(c^3
```

```
*cos(f*x + e)^2 + 2*c^3*cos(f*x + e) + c^3)*log(-sin(f*x + e) + 1) - 8*(c^3
*cos(f*x + e) - c^3)*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x
+ e) + a^2*f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{c^3\left(\int \frac{3\sec(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx+\int -\frac{3\sec^2(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx+\int \frac{\sec^3(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx+\int -\frac{1}{\sec^2(e+fx)+2\sec(e+f}\right.}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**3/(a+a*sec(f*x+e))**2,x)
```

```
[Out] -c**3*(Integral(3*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) +
 Integral(-3*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + I
ntegral(sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Integr
al(-1/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

Giac [A] time = 1.33767, size = 113, normalized size = 1.33

$$\frac{\frac{4c^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3}{a^2}+\frac{3(fx+e)c^3}{a^2}-\frac{3c^3\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{a^2}+\frac{3c^3\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{a^2}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/3*(4*c^3*tan(1/2*f*x + 1/2*e)^3/a^2 + 3*(f*x + e)*c^3/a^2 - 3*c^3*log(abs
(tan(1/2*f*x + 1/2*e) + 1))/a^2 + 3*c^3*log(abs(tan(1/2*f*x + 1/2*e) - 1))/
a^2)/f
```

3.24 $\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=67

$$-\frac{4c^2\tan(e+fx)}{3a^2f(\sec(e+fx)+1)}-\frac{4c^2\tan(e+fx)}{3a^2f(\sec(e+fx)+1)^2}+\frac{c^2x}{a^2}$$

```
[Out] (c^2*x)/a^2 - (4*c^2*Tan[e + f*x])/(3*a^2*f*(1 + Sec[e + f*x])^2) - (4*c^2*
Tan[e + f*x])/(3*a^2*f*(1 + Sec[e + f*x]))
```

Rubi [A] time = 0.228489, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3903, 3777, 3919, 3794, 3796, 3797}

$$-\frac{4c^2\tan(e+fx)}{3a^2f(\sec(e+fx)+1)}-\frac{4c^2\tan(e+fx)}{3a^2f(\sec(e+fx)+1)^2}+\frac{c^2x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (c^2*x)/a^2 - (4*c^2*Tan[e + f*x])/(3*a^2*f*(1 + Sec[e + f*x])^2) - (4*c^2*
Tan[e + f*x])/(3*a^2*f*(1 + Sec[e + f*x]))
```

Rule 3903

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Dist[c^n, Int[ExpandTrig[(1 + (d*csc[e + f*
x])/c)^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
&& LtQ[m + n, 2]
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
rQ[2*n]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3797

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(2*m + 1)), x]
+ Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x],
x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx &= \frac{\int \left(\frac{c^2}{(1+\sec(e+fx))^2}-\frac{2c^2\sec(e+fx)}{(1+\sec(e+fx))^2}+\frac{c^2\sec^2(e+fx)}{(1+\sec(e+fx))^2}\right)dx}{a^2}\\
&= \frac{c^2\int \frac{1}{(1+\sec(e+fx))^2}\,dx}{a^2}+\frac{c^2\int \frac{\sec^2(e+fx)}{(1+\sec(e+fx))^2}\,dx}{a^2}-\frac{\left(2c^2\right)\int \frac{\sec(e+fx)}{(1+\sec(e+fx))^2}\,dx}{a^2}\\
&= -\frac{4c^2\tan(e+fx)}{3a^2f(1+\sec(e+fx))^2}-\frac{c^2\int \frac{-3+\sec(e+fx)}{1+\sec(e+fx)}\,dx}{3a^2}\\
&= \frac{c^2x}{a^2}-\frac{4c^2\tan(e+fx)}{3a^2f(1+\sec(e+fx))^2}-\frac{\left(4c^2\right)\int \frac{\sec(e+fx)}{1+\sec(e+fx)}\,dx}{3a^2}\\
&= \frac{c^2x}{a^2}-\frac{4c^2\tan(e+fx)}{3a^2f(1+\sec(e+fx))^2}-\frac{4c^2\tan(e+fx)}{3a^2f(1+\sec(e+fx))}
\end{aligned}
$$

Mathematica [A] time = 0.0521523, size = 67, normalized size = 1.

$$\frac{c^2\left(\frac{2\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f}+\frac{2\tan^{-1}\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f}-\frac{2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (c^2*((2*ArcTan[Tan[e/2 + (f*x)/2]])/f - (2*Tan[e/2 + (f*x)/2])/f + (2*Tan[
e/2 + (f*x)/2]^3)/(3*f)))/a^2
```

Maple [A] time = 0.08, size = 65, normalized size = 1.

$$\frac{2c^2}{3fa^2}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3-2\frac{c^2\tan\left(1/2\,fx+e/2\right)}{fa^2}+2\frac{c^2\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)}{fa^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x)
```

```
[Out] 2/3/f*c^2/a^2*tan(1/2*f*x+1/2*e)^3-2/f*c^2/a^2*tan(1/2*f*x+1/2*e)+2/f*c^2/a
^2*arctan(tan(1/2*f*x+1/2*e))
```

Maxima [B] time = 1.55873, size = 230, normalized size = 3.43

$$-\frac{c^2\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{12\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2}\right)-\frac{c^2\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2}+\frac{2c^2\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -1/6*(c^2*((9*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e
) + 1)^3)/a^2 - 12*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - c^2*(3*si
```

```
n(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 +
2*c^2*(3*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e) + 1
)^3)/a^2)/f
```

Fricas [A] time = 1.04437, size = 225, normalized size = 3.36

$$\frac{3\,c^2 fx\cos\left(fx+e\right)^2+6\,c^2 fx\cos\left(fx+e\right)+3\,c^2 fx-4\left(2\,c^2\cos\left(fx+e\right)+c^2\right)\sin\left(fx+e\right)}{3\left(a^2 f\cos\left(fx+e\right)^2+2\,a^2 f\cos\left(fx+e\right)+a^2 f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*c^2*f*x*cos(f*x + e)^2 + 6*c^2*f*x*cos(f*x + e) + 3*c^2*f*x - 4*(2*c
^2*cos(f*x + e) + c^2)*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*
x + e) + a^2*f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{c^2\left(\int -\frac{2\sec\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx+\int \frac{\sec^2\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx+\int \frac{1}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**2/(a+a*sec(f*x+e))**2,x)
```

```
[Out] c**2*(Integral(-2*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) +
 Integral(sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Inte
gral(1/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

Giac [A] time = 1.23605, size = 85, normalized size = 1.27

$$\frac{\frac{3\left(fx+e\right)c^2}{a^2}+\frac{2\left(a^4c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-3\,a^4c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{a^6}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*(f*x + e)*c^2/a^2 + 2*(a^4*c^2*tan(1/2*f*x + 1/2*e)^3 - 3*a^4*c^2*ta
n(1/2*f*x + 1/2*e))/a^6)/f
```

3.25 $\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=61

$$-\frac{5c\tan(e+fx)}{3a^2f(\sec(e+fx)+1)}-\frac{2c\tan(e+fx)}{3a^2f(\sec(e+fx)+1)^2}+\frac{cx}{a^2}$$

```
[Out] (c*x)/a^2 - (2*c*Tan[e + f*x])/(3*a^2*f*(1 + Sec[e + f*x])^2) - (5*c*Tan[e
+ f*x])/(3*a^2*f*(1 + Sec[e + f*x]))
```

Rubi [A] time = 0.145132, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3903, 3777, 3919, 3794, 3796}

$$-\frac{5c\tan(e+fx)}{3a^2f(\sec(e+fx)+1)}-\frac{2c\tan(e+fx)}{3a^2f(\sec(e+fx)+1)^2}+\frac{cx}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (c*x)/a^2 - (2*c*Tan[e + f*x])/(3*a^2*f*(1 + Sec[e + f*x])^2) - (5*c*Tan[e
+ f*x])/(3*a^2*f*(1 + Sec[e + f*x]))
```

Rule 3903

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Dist[c^n, Int[ExpandTrig[(1 + (d*csc[e + f*
x])/c)^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
&& LtQ[m + n, 2]
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
rQ[2*n]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^2}\,dx &= \frac{\int \left(\frac{c}{(1+\sec(e+fx))^2}-\frac{c\sec(e+fx)}{(1+\sec(e+fx))^2}\right)dx}{a^2}\\
&= \frac{c\int \frac{1}{(1+\sec(e+fx))^2}\,dx}{a^2}-\frac{c\int \frac{\sec(e+fx)}{(1+\sec(e+fx))^2}\,dx}{a^2}\\
&= -\frac{2c\tan(e+fx)}{3a^2f(1+\sec(e+fx))^2}-\frac{c\int \frac{-3+\sec(e+fx)}{1+\sec(e+fx)}\,dx}{3a^2}-\frac{c\int \frac{\sec(e+fx)}{1+\sec(e+fx)}\,dx}{3a^2}\\
&= \frac{cx}{a^2}-\frac{2c\tan(e+fx)}{3a^2f(1+\sec(e+fx))^2}-\frac{c\tan(e+fx)}{3a^2f(1+\sec(e+fx))}-\frac{(4c)\int \frac{\sec(e+fx)}{1+\sec(e+fx)}\,dx}{3a^2}\\
&= \frac{cx}{a^2}-\frac{2c\tan(e+fx)}{3a^2f(1+\sec(e+fx))^2}-\frac{5c\tan(e+fx)}{3a^2f(1+\sec(e+fx))}
\end{aligned}
$$

Mathematica [A] time = 0.313902, size = 113, normalized size = 1.85

$$
\frac{c\sec\left(\frac{e}{2}\right)\sec^3\left(\frac{1}{2}(e+fx)\right)\left(18\sin\left(e+\frac{fx}{2}\right)-14\sin\left(e+\frac{3fx}{2}\right)+9fx\cos\left(e+\frac{fx}{2}\right)+3fx\cos\left(e+\frac{3fx}{2}\right)+3fx\cos\left(2e+\frac{3fx}{2}\right.\right.}{24a^2f}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (c*Sec[e/2]*Sec[(e + f*x)/2]^3*(9*f*x*Cos[(f*x)/2] + 9*f*x*Cos[e + (f*x)/2]
 + 3*f*x*Cos[e + (3*f*x)/2] + 3*f*x*Cos[2*e + (3*f*x)/2] - 24*Sin[(f*x)/2]
+ 18*Sin[e + (f*x)/2] - 14*Sin[e + (3*f*x)/2]))/(24*a^2*f)
```

Maple [A] time = 0.08, size = 59, normalized size = 1.

$$\frac{c}{3\,f a^2}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3-2\,\frac{c\tan\left(1/2\,fx+e/2\right)}{fa^2}+2\,\frac{c\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)}{fa^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x)
```

```
[Out] 1/3/f*c/a^2*tan(1/2*f*x+1/2*e)^3-2/f*c/a^2*tan(1/2*f*x+1/2*e)+2/f*c/a^2*arc
tan(tan(1/2*f*x+1/2*e))
```

Maxima [B] time = 1.54762, size = 161, normalized size = 2.64

$$-\frac{c\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{12\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2}\right)+\frac{c\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -1/6*(c*((9*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3)/a^2 - 12*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + c*(3*sin(f*
x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2)/f
```

Fricas [A] time = 1.02543, size = 212, normalized size = 3.48

$$\frac{3\,cfx\cos\left(fx+e\right)^2+6\,cfx\cos\left(fx+e\right)+3\,cfx-\left(7\,c\cos\left(fx+e\right)+5\,c\right)\sin\left(fx+e\right)}{3\left(a^2f\cos\left(fx+e\right)^2+2\,a^2f\cos\left(fx+e\right)+a^2f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*c*f*x*cos(f*x + e)^2 + 6*c*f*x*cos(f*x + e) + 3*c*f*x - (7*c*cos(f*x
 + e) + 5*c)*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a
^2*f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{c\left(\int\frac{\sec\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx+\int-\frac{1}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))**2,x)
```

```
[Out] -c*(Integral(sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Inte
gral(-1/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

Giac [A] time = 1.42365, size = 76, normalized size = 1.25

$$\frac{\frac{3\left(fx+e\right)c}{a^2}+\frac{a^4c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-6\,a^4c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{a^6}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*(f*x + e)*c/a^2 + (a^4*c*tan(1/2*f*x + 1/2*e)^3 - 6*a^4*c*tan(1/2*f*
x + 1/2*e))/a^6)/f
```

3.26 $\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))}\,dx$

Optimal. Leaf size=69

$$-\frac{\cot^3(e+fx)(1-\sec(e+fx))}{3a^2cf}+\frac{\cot(e+fx)(3-2\sec(e+fx))}{3a^2cf}+\frac{x}{a^2c}$$

```
[Out] x/(a^2*c) + (Cot[e + f*x]*(3 - 2*Sec[e + f*x]))/(3*a^2*c*f) - (Cot[e + f*x]
^3*(1 - Sec[e + f*x]))/(3*a^2*c*f)
```

Rubi [A] time = 0.113739, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3904, 3882, 8}

$$-\frac{\cot^3(e+fx)(1-\sec(e+fx))}{3a^2cf}+\frac{\cot(e+fx)(3-2\sec(e+fx))}{3a^2cf}+\frac{x}{a^2c}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])),x]
```

```
[Out] x/(a^2*c) + (Cot[e + f*x]*(3 - 2*Sec[e + f*x]))/(3*a^2*c*f) - (Cot[e + f*x]
^3*(1 - Sec[e + f*x]))/(3*a^2*c*f)
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))}\,dx = \frac{\int \cot^4(e+fx)(c-c\sec(e+fx))\,dx}{a^2c^2}$$

$$= -\frac{\cot^3(e+fx)(1-\sec(e+fx))}{3a^2cf} + \frac{\int \cot^2(e+fx)(-3c+2c\sec(e+fx))\,dx}{3a^2c^2}$$

$$= \frac{\cot(e+fx)(3-2\sec(e+fx))}{3a^2cf} - \frac{\cot^3(e+fx)(1-\sec(e+fx))}{3a^2cf} + \frac{\int 3c\,dx}{3a^2c^2}$$

$$= \frac{x}{a^2c} + \frac{\cot(e+fx)(3-2\sec(e+fx))}{3a^2cf} - \frac{\cot^3(e+fx)(1-\sec(e+fx))}{3a^2cf}$$

Mathematica [A] time = 0.543341, size = 135, normalized size = 1.96

$$\frac{\csc\left(\frac{e}{2}\right)\sec\left(\frac{e}{2}\right)\csc\left(\frac{1}{2}(e+fx)\right)\sec^3\left(\frac{1}{2}(e+fx)\right)(10\sin(e+fx)+5\sin(2(e+fx))-6\sin(2e+fx)-8\sin(e+2fx)-6f}{96a^2cf}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])),x]
```

```
[Out] (Csc[e/2]*Csc[(e + f*x)/2]*Sec[e/2]*Sec[(e + f*x)/2]^3*(6*f*x*Cos[f*x] - 6*
f*x*Cos[2*e + f*x] + 3*f*x*Cos[e + 2*f*x] - 3*f*x*Cos[3*e + 2*f*x] - 10*Sin
[f*x] + 10*Sin[e + f*x] + 5*Sin[2*(e + f*x)] - 6*Sin[2*e + f*x] - 8*Sin[e +
 2*f*x]))/(96*a^2*c*f)
```

Maple [A] time = 0.056, size = 87, normalized size = 1.3

$$\frac{1}{12\,fa^2c}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3 - \frac{1}{fa^2c}\tan\left(\frac{fx}{2}+\frac{e}{2}\right) + 2\,\frac{\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)}{fa^2c} + \frac{1}{4\,fa^2c}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x)
```

```
[Out] 1/12/f/a^2/c*tan(1/2*f*x+1/2*e)^3-1/f/a^2/c*tan(1/2*f*x+1/2*e)+2/f/a^2/c*ar
ctan(tan(1/2*f*x+1/2*e))+1/4/f/a^2/c/tan(1/2*f*x+1/2*e)
```

Maxima [A] time = 1.51585, size = 138, normalized size = 2.

$$-\frac{\frac{\frac{12\,\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}}{a^2c}-\frac{24\,\arctan\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)}{a^2c}-\frac{3\left(\cos\left(fx+e\right)+1\right)}{a^2c\sin\left(fx+e\right)}}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/12*((12*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e) +
 1)^3)/(a^2*c) - 24*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/(a^2*c) - 3*(co
s(f*x + e) + 1)/(a^2*c*sin(f*x + e)))/f
```

Fricas [A] time = 1.04229, size = 180, normalized size = 2.61

$$\frac{4\,\cos\left(fx+e\right)^2+3\left(fx\cos\left(fx+e\right)+fx\right)\sin\left(fx+e\right)+\cos\left(fx+e\right)-2}{3\left(a^2cf\cos\left(fx+e\right)+a^2cf\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/3*(4*cos(f*x + e)^2 + 3*(f*x*cos(f*x + e) + f*x)*sin(f*x + e) + cos(f*x +
 e) - 2)/((a^2*c*f*cos(f*x + e) + a^2*c*f)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int\frac{1}{\sec^3\left(e+fx\right)+\sec^2\left(e+fx\right)-\sec\left(e+fx\right)-1}\,dx}{a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e)),x)
```

```
[Out] -Integral(1/(sec(e + f*x)**3 + sec(e + f*x)**2 - sec(e + f*x) - 1), x)/(a**
2*c)
```

Giac [A] time = 1.34305, size = 115, normalized size = 1.67

$$\frac{\frac{12\left(fx+e\right)}{a^2c}+\frac{3}{a^2c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}+\frac{a^4c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-12a^4c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a^6c^3}}{12f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/12*(12*(f*x + e)/(a^2*c) + 3/(a^2*c*tan(1/2*f*x + 1/2*e)) + (a^4*c^2*tan(
1/2*f*x + 1/2*e)^3 - 12*a^4*c^2*tan(1/2*f*x + 1/2*e))/(a^6*c^3))/f
```

3.27 $\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2}\,dx$

Optimal. Leaf size=46

$$-\frac{\cot^3(e+fx)}{3a^2c^2f}+\frac{\cot(e+fx)}{a^2c^2f}+\frac{x}{a^2c^2}$$

```
[Out] x/(a^2*c^2) + Cot[e + f*x]/(a^2*c^2*f) - Cot[e + f*x]^3/(3*a^2*c^2*f)
```

Rubi [A] time = 0.0713255, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3904, 3473, 8}

$$-\frac{\cot^3(e+fx)}{3a^2c^2f}+\frac{\cot(e+fx)}{a^2c^2f}+\frac{x}{a^2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^2),x]
```

```
[Out] x/(a^2*c^2) + Cot[e + f*x]/(a^2*c^2*f) - Cot[e + f*x]^3/(3*a^2*c^2*f)
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2}\,dx = \frac{\int \cot^4(e+fx)\,dx}{a^2c^2}$$
$$= -\frac{\cot^3(e+fx)}{3a^2c^2f} - \frac{\int \cot^2(e+fx)\,dx}{a^2c^2}$$
$$= \frac{\cot(e+fx)}{a^2c^2f} - \frac{\cot^3(e+fx)}{3a^2c^2f} + \frac{\int 1\,dx}{a^2c^2}$$
$$= \frac{x}{a^2c^2} + \frac{\cot(e+fx)}{a^2c^2f} - \frac{\cot^3(e+fx)}{3a^2c^2f}$$

Mathematica [C] time = 0.0504444, size = 39, normalized size = 0.85

$$-\frac{\cot^3(e+fx)\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2(e+fx)\right)}{3a^2c^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^2),x]
```

```
[Out] -(Cot[e + f*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[e + f*x]^2])/(3*a^2*
c^2*f)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+a\sec\left(fx+e\right)\right)^2\left(c-c\sec\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x)
```

```
[Out] int(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x)
```

Maxima [A] time = 1.52462, size = 62, normalized size = 1.35

$$\frac{\frac{3\,(fx+e)}{a^2c^2} + \frac{3\,\tan(fx+e)^2-1}{a^2c^2\,\tan(fx+e)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(3*(f*x + e)/(a^2*c^2) + (3*tan(f*x + e)^2 - 1)/(a^2*c^2*tan(f*x + e)^3
))/f
```

Fricas [A] time = 1.05791, size = 188, normalized size = 4.09

$$\frac{4\,\cos\left(fx+e\right)^3 + 3\left(fx\cos\left(fx+e\right)^2 - fx\right)\sin\left(fx+e\right) - 3\,\cos\left(fx+e\right)}{3\left(a^2c^2f\,\cos\left(fx+e\right)^2 - a^2c^2f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(4*cos(f*x + e)^3 + 3*(f*x*cos(f*x + e)^2 - f*x)*sin(f*x + e) - 3*cos(f
*x + e))/((a^2*c^2*f*cos(f*x + e)^2 - a^2*c^2*f)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{\sec^4\left(e+fx\right)-2\sec^2\left(e+fx\right)+1}\,dx}{a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**2,x)
```

```
[Out] Integral(1/(sec(e + f*x)**4 - 2*sec(e + f*x)**2 + 1), x)/(a**2*c**2)
```

Giac [B] time = 1.41077, size = 135, normalized size = 2.93

$$\frac{\frac{24\left(fx+e\right)}{a^2c^2}+\frac{15\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1}{a^2c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3}+\frac{a^4c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-15a^4c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a^6c^6}}{24f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/24*(24*(f*x + e)/(a^2*c^2) + (15*tan(1/2*f*x + 1/2*e)^2 - 1)/(a^2*c^2*tan
(1/2*f*x + 1/2*e)^3) + (a^4*c^4*tan(1/2*f*x + 1/2*e)^3 - 15*a^4*c^4*tan(1/2
*f*x + 1/2*e))/(a^6*c^6))/f
```

3.28 $\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3}\,dx$

Optimal. Leaf size=98

$$\frac{\cot^5(e+fx)(\sec(e+fx)+1)}{5a^2c^3f}-\frac{\cot^3(e+fx)(4\sec(e+fx)+5)}{15a^2c^3f}+\frac{\cot(e+fx)(8\sec(e+fx)+15)}{15a^2c^3f}+\frac{x}{a^2c^3}$$

```
[Out] x/(a^2*c^3) + (Cot[e + f*x]^5*(1 + Sec[e + f*x]))/(5*a^2*c^3*f) - (Cot[e +
f*x]^3*(5 + 4*Sec[e + f*x]))/(15*a^2*c^3*f) + (Cot[e + f*x]*(15 + 8*Sec[e +
 f*x]))/(15*a^2*c^3*f)
```

Rubi [A] time = 0.144786, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3904, 3882, 8}

$$\frac{\cot^5(e+fx)(\sec(e+fx)+1)}{5a^2c^3f}-\frac{\cot^3(e+fx)(4\sec(e+fx)+5)}{15a^2c^3f}+\frac{\cot(e+fx)(8\sec(e+fx)+15)}{15a^2c^3f}+\frac{x}{a^2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^3),x]
```

```
[Out] x/(a^2*c^3) + (Cot[e + f*x]^5*(1 + Sec[e + f*x]))/(5*a^2*c^3*f) - (Cot[e +
f*x]^3*(5 + 4*Sec[e + f*x]))/(15*a^2*c^3*f) + (Cot[e + f*x]*(15 + 8*Sec[e +
 f*x]))/(15*a^2*c^3*f)
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3}\,dx = -\frac{\int \cot^6(e+fx)(a+a\sec(e+fx))\,dx}{a^3c^3}$$
$$= \frac{\cot^5(e+fx)(1+\sec(e+fx))}{5a^2c^3f} - \frac{\int \cot^4(e+fx)(-5a-4a\sec(e+fx))\,dx}{5a^3c^3}$$
$$= \frac{\cot^5(e+fx)(1+\sec(e+fx))}{5a^2c^3f} - \frac{\cot^3(e+fx)(5+4\sec(e+fx))}{15a^2c^3f} - \int \mathrm{cot}$$
$$= \frac{\cot^5(e+fx)(1+\sec(e+fx))}{5a^2c^3f} - \frac{\cot^3(e+fx)(5+4\sec(e+fx))}{15a^2c^3f} + \cot(e$$
$$= \frac{x}{a^2c^3} + \frac{\cot^5(e+fx)(1+\sec(e+fx))}{5a^2c^3f} - \frac{\cot^3(e+fx)(5+4\sec(e+fx))}{15a^2c^3f}$$

Mathematica [B] time = 1.31395, size = 257, normalized size = 2.62

$$\frac{\csc\left(\frac{e}{2}\right)\sec\left(\frac{e}{2}\right)\csc^5\left(\frac{1}{2}(e+fx)\right)\sec^3\left(\frac{1}{2}(e+fx)\right)(-534\sin(e+fx)+178\sin(2(e+fx))+178\sin(3(e+fx))-89\sin(4(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^3),x]
```

```
[Out] (Csc[e/2]*Csc[(e + f*x)/2]^5*Sec[e/2]*Sec[(e + f*x)/2]^3*(360*f*x*Cos[f*x]
- 360*f*x*Cos[2*e + f*x] - 120*f*x*Cos[e + 2*f*x] + 120*f*x*Cos[3*e + 2*f*x
] - 120*f*x*Cos[2*e + 3*f*x] + 120*f*x*Cos[4*e + 3*f*x] + 60*f*x*Cos[3*e +
4*f*x] - 60*f*x*Cos[5*e + 4*f*x] + 200*Sin[e] - 584*Sin[f*x] - 534*Sin[e +
f*x] + 178*Sin[2*(e + f*x)] + 178*Sin[3*(e + f*x)] - 89*Sin[4*(e + f*x)] -
520*Sin[2*e + f*x] + 248*Sin[e + 2*f*x] + 120*Sin[3*e + 2*f*x] + 248*Sin[2*
e + 3*f*x] + 120*Sin[4*e + 3*f*x] - 184*Sin[3*e + 4*f*x]))/(30720*a^2*c^3*f
)
```

Maple [A] time = 0.065, size = 130, normalized size = 1.3

$$\frac{1}{48fa^2c^3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3 - \frac{3}{8fa^2c^3}\tan\left(\frac{fx}{2}+\frac{e}{2}\right) + 2\frac{\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)}{fa^2c^3} + \frac{1}{80fa^2c^3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-5} - \frac{}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x)
```

```
[Out] 1/48/f/a^2/c^3*tan(1/2*f*x+1/2*e)^3-3/8/f/a^2/c^3*tan(1/2*f*x+1/2*e)+2/f/a^
2/c^3*arctan(tan(1/2*f*x+1/2*e))+1/80/f/a^2/c^3/tan(1/2*f*x+1/2*e)^5-1/8/f/
a^2/c^3/tan(1/2*f*x+1/2*e)^3+1/f/a^2/c^3/tan(1/2*f*x+1/2*e)
```

Maxima [A] time = 1.54595, size = 198, normalized size = 2.02

$$-\frac{\frac{5\left(\frac{18\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}\right)}{a^2c^3}-\frac{480\arctan\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)}{a^2c^3}+\frac{3\left(\frac{10\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}-\frac{80\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}-1\right)\left(\cos\left(fx+e\right)+1\right)^5}{a^2c^3\sin\left(fx+e\right)^5}}{240\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/240*(5*(18*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e
) + 1)^3)/(a^2*c^3) - 480*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/(a^2*c^3)
 + 3*(10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 80*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4 - 1)*(cos(f*x + e) + 1)^5/(a^2*c^3*sin(f*x + e)^5))/f
```

Fricas [A] time = 1.0342, size = 374, normalized size = 3.82

$$\frac{23\cos\left(fx+e\right)^4-8\cos\left(fx+e\right)^3-27\cos\left(fx+e\right)^2+15\left(fx\cos\left(fx+e\right)^3-fx\cos\left(fx+e\right)^2-fx\cos\left(fx+e\right)+fx\right.}{15\left(a^2c^3f\cos\left(fx+e\right)^3-a^2c^3f\cos\left(fx+e\right)^2-a^2c^3f\cos\left(fx+e\right)+a^2c^3f\right)\sin\left(fx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(23*cos(f*x + e)^4 - 8*cos(f*x + e)^3 - 27*cos(f*x + e)^2 + 15*(f*x*co
s(f*x + e)^3 - f*x*cos(f*x + e)^2 - f*x*cos(f*x + e) + f*x)*sin(f*x + e) +
7*cos(f*x + e) + 8)/((a^2*c^3*f*cos(f*x + e)^3 - a^2*c^3*f*cos(f*x + e)^2 -
 a^2*c^3*f*cos(f*x + e) + a^2*c^3*f)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{1}{\sec^{5}\left(e+fx\right)-\sec^{4}\left(e+fx\right)-2\sec^{3}\left(e+fx\right)+2\sec^{2}\left(e+fx\right)+\sec\left(e+fx\right)-1}\,dx}{a^{2}c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**3,x)
```

```
[Out] -Integral(1/(sec(e + f*x)**5 - sec(e + f*x)**4 - 2*sec(e + f*x)**3 + 2*sec(
e + f*x)**2 + sec(e + f*x) - 1), x)/(a**2*c**3)
```

Giac [A] time = 1.4785, size = 157, normalized size = 1.6

$$\frac{\frac{240\left(fx+e\right)}{a^{2}c^{3}}+\frac{3\left(80\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-10\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+1\right)}{a^{2}c^{3}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{5}}+\frac{5\left(a^{4}c^{6}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}-18a^{4}c^{6}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{a^{6}c^{9}}}{240f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/240*(240*(f*x + e)/(a^2*c^3) + 3*(80*tan(1/2*f*x + 1/2*e)^4 - 10*tan(1/2*
f*x + 1/2*e)^2 + 1)/(a^2*c^3*tan(1/2*f*x + 1/2*e)^5) + 5*(a^4*c^6*tan(1/2*f
*x + 1/2*e)^3 - 18*a^4*c^6*tan(1/2*f*x + 1/2*e))/(a^6*c^9))/f
```

3.29 $\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4}\,dx$

Optimal. Leaf size=166

$$-\frac{2\cot^7(e+fx)}{7a^2c^4f}+\frac{\cot^5(e+fx)}{5a^2c^4f}-\frac{\cot^3(e+fx)}{3a^2c^4f}+\frac{\cot(e+fx)}{a^2c^4f}-\frac{2\csc^7(e+fx)}{7a^2c^4f}+\frac{6\csc^5(e+fx)}{5a^2c^4f}-\frac{2\csc^3(e+fx)}{a^2c^4f}+2$$

```
[Out] x/(a^2*c^4) + Cot[e + f*x]/(a^2*c^4*f) - Cot[e + f*x]^3/(3*a^2*c^4*f) + Cot
[e + f*x]^5/(5*a^2*c^4*f) - (2*Cot[e + f*x]^7)/(7*a^2*c^4*f) + (2*Csc[e + f
*x])/(a^2*c^4*f) - (2*Csc[e + f*x]^3)/(a^2*c^4*f) + (6*Csc[e + f*x]^5)/(5*a
^2*c^4*f) - (2*Csc[e + f*x]^7)/(7*a^2*c^4*f)
```

Rubi [A] time = 0.211247, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3904, 3886, 3473, 8, 2606, 194, 2607, 30}

$$-\frac{2\cot^7(e+fx)}{7a^2c^4f}+\frac{\cot^5(e+fx)}{5a^2c^4f}-\frac{\cot^3(e+fx)}{3a^2c^4f}+\frac{\cot(e+fx)}{a^2c^4f}-\frac{2\csc^7(e+fx)}{7a^2c^4f}+\frac{6\csc^5(e+fx)}{5a^2c^4f}-\frac{2\csc^3(e+fx)}{a^2c^4f}+2$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^4),x]
```

```
[Out] x/(a^2*c^4) + Cot[e + f*x]/(a^2*c^4*f) - Cot[e + f*x]^3/(3*a^2*c^4*f) + Cot
[e + f*x]^5/(5*a^2*c^4*f) - (2*Cot[e + f*x]^7)/(7*a^2*c^4*f) + (2*Csc[e + f
*x])/(a^2*c^4*f) - (2*Csc[e + f*x]^3)/(a^2*c^4*f) + (6*Csc[e + f*x]^5)/(5*a
^2*c^4*f) - (2*Csc[e + f*x]^7)/(7*a^2*c^4*f)
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4}\,dx = \frac{\int \cot^8(e+fx)(a+a\sec(e+fx))^2\,dx}{a^4c^4}$$

$$= \frac{\int \left(a^2\cot^8(e+fx)+2a^2\cot^7(e+fx)\csc(e+fx)+a^2\cot^6(e+fx)\csc\right.}{a^4c^4}$$

$$= \frac{\int \cot^8(e+fx)\,dx}{a^2c^4} + \frac{\int \cot^6(e+fx)\csc^2(e+fx)\,dx}{a^2c^4} + \frac{2\int \cot^7(e+fx)}{a^2}$$

$$= -\frac{\cot^7(e+fx)}{7a^2c^4f} - \frac{\int \cot^6(e+fx)\,dx}{a^2c^4} + \frac{\text{Subst}\left(\int x^6\,dx,x,-\cot(e+fx)\right)}{a^2c^4f}$$

$$= \frac{\cot^5(e+fx)}{5a^2c^4f} - \frac{2\cot^7(e+fx)}{7a^2c^4f} + \frac{\int \cot^4(e+fx)\,dx}{a^2c^4} - \frac{2\,\text{Subst}\left(\int \left(-1+\right.\right.}{}$$

$$= -\frac{\cot^3(e+fx)}{3a^2c^4f} + \frac{\cot^5(e+fx)}{5a^2c^4f} - \frac{2\cot^7(e+fx)}{7a^2c^4f} + \frac{2\csc(e+fx)}{a^2c^4f} - \frac{2\,\text{cs}}{}$$

$$= \frac{\cot(e+fx)}{a^2c^4f} - \frac{\cot^3(e+fx)}{3a^2c^4f} + \frac{\cot^5(e+fx)}{5a^2c^4f} - \frac{2\cot^7(e+fx)}{7a^2c^4f} + \frac{2\csc(e}{a^2c}$$

$$= \frac{x}{a^2c^4} + \frac{\cot(e+fx)}{a^2c^4f} - \frac{\cot^3(e+fx)}{3a^2c^4f} + \frac{\cot^5(e+fx)}{5a^2c^4f} - \frac{2\cot^7(e+fx)}{7a^2c^4f} +$$

Mathematica [A] time = 1.2566, size = 315, normalized size = 1.9

$$\frac{\csc\left(\frac{e}{2}\right)\sec\left(\frac{e}{2}\right)\csc^7\left(\frac{1}{2}(e+fx)\right)\sec^3\left(\frac{1}{2}(e+fx)\right)(-16002\sin(e+fx)+9144\sin(2(e+fx))+3429\sin(3(e+fx))-45}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^4),x]
```

```
[Out] (Csc[e/2]*Csc[(e + f*x)/2]^7*Sec[e/2]*Sec[(e + f*x)/2]^3*(5880*f*x*Cos[f*x]
 - 5880*f*x*Cos[2*e + f*x] - 3360*f*x*Cos[e + 2*f*x] + 3360*f*x*Cos[3*e + 2
*f*x] - 1260*f*x*Cos[2*e + 3*f*x] + 1260*f*x*Cos[4*e + 3*f*x] + 1680*f*x*Co
s[3*e + 4*f*x] - 1680*f*x*Cos[5*e + 4*f*x] - 420*f*x*Cos[4*e + 5*f*x] + 420
*f*x*Cos[6*e + 5*f*x] + 4032*Sin[e] - 9632*Sin[f*x] - 16002*Sin[e + f*x] +
9144*Sin[2*(e + f*x)] + 3429*Sin[3*(e + f*x)] - 4572*Sin[4*(e + f*x)] + 114
3*Sin[5*(e + f*x)] - 11760*Sin[2*e + f*x] + 8864*Sin[e + 2*f*x] + 3360*Sin[
3*e + 2*f*x] + 2064*Sin[2*e + 3*f*x] + 2520*Sin[4*e + 3*f*x] - 4432*Sin[3*e
 + 4*f*x] - 1680*Sin[5*e + 4*f*x] + 1528*Sin[4*e + 5*f*x]))/(860160*a^2*c^4
*f)
```

Maple [A] time = 0.071, size = 153, normalized size = 0.9

$$\frac{1}{96\,fa^2c^4}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3-\frac{7}{32\,fa^2c^4}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2\,\frac{\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)}{fa^2c^4}-\frac{1}{224\,fa^2c^4}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-7}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x)
```

```
[Out] 1/96/f/a^2/c^4*tan(1/2*f*x+1/2*e)^3-7/32/f/a^2/c^4*tan(1/2*f*x+1/2*e)+2/f/a
^2/c^4*arctan(tan(1/2*f*x+1/2*e))-1/224/f/a^2/c^4/tan(1/2*f*x+1/2*e)^7+7/16
0/f/a^2/c^4/tan(1/2*f*x+1/2*e)^5-11/48/f/a^2/c^4/tan(1/2*f*x+1/2*e)^3+21/16
/f/a^2/c^4/tan(1/2*f*x+1/2*e)
```

Maxima [A] time = 1.52891, size = 225, normalized size = 1.36

$$-\frac{\frac{35\left(\frac{21\,\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}\right)}{a^2c^4}-\frac{6720\,\arctan\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)}{a^2c^4}-\frac{\left(\frac{147\,\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}-\frac{770\,\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}+\frac{4410\,\sin\left(fx+e\right)^6}{\left(\cos\left(fx+e\right)+1\right)^6}-15\right)\left(\cos\left(fx+e\right)+1\right)^7}{a^2c^4\sin\left(fx+e\right)^7}}{3360\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] -1/3360*(35*(21*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3)/(a^2*c^4) - 6720*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/(a^2*c
^4) - (147*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 770*sin(f*x + e)^4/(cos(f*
x + e) + 1)^4 + 4410*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 15)*(cos(f*x + e
) + 1)^7/(a^2*c^4*sin(f*x + e)^7))/f
```

Fricas [A] time = 1.08481, size = 424, normalized size = 2.55

$$\frac{191\,\cos\left(fx+e\right)^5-172\,\cos\left(fx+e\right)^4-253\,\cos\left(fx+e\right)^3+258\,\cos\left(fx+e\right)^2+105\left(fx\cos\left(fx+e\right)^4-2\,fx\cos\left(fx+\right.\right.}{105\left(a^2c^4f\cos\left(fx+e\right)^4-2\,a^2c^4f\cos\left(fx+e\right)^3+2\,a^2c^4f\cos\left(fx+e\right)-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] 1/105*(191*cos(f*x + e)^5 - 172*cos(f*x + e)^4 - 253*cos(f*x + e)^3 + 258*c
os(f*x + e)^2 + 105*(f*x*cos(f*x + e)^4 - 2*f*x*cos(f*x + e)^3 + 2*f*x*cos(
f*x + e) - f*x)*sin(f*x + e) + 87*cos(f*x + e) - 96)/((a^2*c^4*f*cos(f*x +
e)^4 - 2*a^2*c^4*f*cos(f*x + e)^3 + 2*a^2*c^4*f*cos(f*x + e) - a^2*c^4*f)*s
in(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{\sec^6(e+fx)-2\sec^5(e+fx)-\sec^4(e+fx)+4\sec^3(e+fx)-\sec^2(e+fx)-2\sec(e+fx)+1}\,dx}{a^2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**4,x)
```

```
[Out] Integral(1/(sec(e + f*x)**6 - 2*sec(e + f*x)**5 - sec(e + f*x)**4 + 4*sec(e
 + f*x)**3 - sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x)/(a**2*c**4)
```

Giac [A] time = 1.43285, size = 174, normalized size = 1.05

$$\frac{\frac{3360\,(fx+e)}{a^2c^4}+\frac{4410\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6-770\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+147\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-15}{a^2c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7}+\frac{35\left(a^4c^8\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-21\,a^4c^8\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{a^6c^{12}}}{3360\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="giac")
```

```
[Out] 1/3360*(3360*(f*x + e)/(a^2*c^4) + (4410*tan(1/2*f*x + 1/2*e)^6 - 770*tan(1
/2*f*x + 1/2*e)^4 + 147*tan(1/2*f*x + 1/2*e)^2 - 15)/(a^2*c^4*tan(1/2*f*x +
 1/2*e)^7) + 35*(a^4*c^8*tan(1/2*f*x + 1/2*e)^3 - 21*a^4*c^8*tan(1/2*f*x +
1/2*e))/(a^6*c^12))/f
```

3.30 $\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5}\,dx$

Optimal. Leaf size=210

$$\frac{4\cot^9(e+fx)}{9a^2c^5f}-\frac{\cot^7(e+fx)}{7a^2c^5f}+\frac{\cot^5(e+fx)}{5a^2c^5f}-\frac{\cot^3(e+fx)}{3a^2c^5f}+\frac{\cot(e+fx)}{a^2c^5f}+\frac{4\csc^9(e+fx)}{9a^2c^5f}-\frac{15\csc^7(e+fx)}{7a^2c^5f}+\frac{21\,\mathrm{cs}}{5}$$

```
[Out] x/(a^2*c^5) + Cot[e + f*x]/(a^2*c^5*f) - Cot[e + f*x]^3/(3*a^2*c^5*f) + Cot
[e + f*x]^5/(5*a^2*c^5*f) - Cot[e + f*x]^7/(7*a^2*c^5*f) + (4*Cot[e + f*x]^
9)/(9*a^2*c^5*f) + (3*Csc[e + f*x])/(a^2*c^5*f) - (13*Csc[e + f*x]^3)/(3*a^
2*c^5*f) + (21*Csc[e + f*x]^5)/(5*a^2*c^5*f) - (15*Csc[e + f*x]^7)/(7*a^2*c
^5*f) + (4*Csc[e + f*x]^9)/(9*a^2*c^5*f)
```

Rubi [A] time = 0.286163, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {3904, 3886, 3473, 8, 2606, 194, 2607, 30, 270}

$$\frac{4\cot^9(e+fx)}{9a^2c^5f}-\frac{\cot^7(e+fx)}{7a^2c^5f}+\frac{\cot^5(e+fx)}{5a^2c^5f}-\frac{\cot^3(e+fx)}{3a^2c^5f}+\frac{\cot(e+fx)}{a^2c^5f}+\frac{4\csc^9(e+fx)}{9a^2c^5f}-\frac{15\csc^7(e+fx)}{7a^2c^5f}+\frac{21\,\mathrm{cs}}{5}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^5),x]
```

```
[Out] x/(a^2*c^5) + Cot[e + f*x]/(a^2*c^5*f) - Cot[e + f*x]^3/(3*a^2*c^5*f) + Cot
[e + f*x]^5/(5*a^2*c^5*f) - Cot[e + f*x]^7/(7*a^2*c^5*f) + (4*Cot[e + f*x]^
9)/(9*a^2*c^5*f) + (3*Csc[e + f*x])/(a^2*c^5*f) - (13*Csc[e + f*x]^3)/(3*a^
2*c^5*f) + (21*Csc[e + f*x]^5)/(5*a^2*c^5*f) - (15*Csc[e + f*x]^7)/(7*a^2*c
^5*f) + (4*Csc[e + f*x]^9)/(9*a^2*c^5*f)
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
```

```
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5}\,dx = -\frac{\int \cot^{10}(e+fx)(a+a\sec(e+fx))^3\,dx}{a^5c^5}$$
$$= -\frac{\int \left(a^3\cot^{10}(e+fx)+3a^3\cot^9(e+fx)\csc(e+fx)+3a^3\cot^8(e+fx)\,c\right.}{a^5c^5}$$
$$= -\frac{\int \cot^{10}(e+fx)\,dx}{a^2c^5} - \frac{\int \cot^7(e+fx)\csc^3(e+fx)\,dx}{a^2c^5} - \frac{3\int \cot^9(e+fx}{a^2}$$
$$= \frac{\cot^9(e+fx)}{9a^2c^5f} + \frac{\int \cot^8(e+fx)\,dx}{a^2c^5} + \frac{\text{Subst}\left(\int x^2\left(-1+x^2\right)^3\,dx,x,\csc(e\right.}{a^2c^5f}$$
$$= -\frac{\cot^7(e+fx)}{7a^2c^5f} + \frac{4\cot^9(e+fx)}{9a^2c^5f} - \frac{\int \cot^6(e+fx)\,dx}{a^2c^5} + \frac{\text{Subst}\left(\int\left(-x^2+\right.\right.}{}$$
$$= \frac{\cot^5(e+fx)}{5a^2c^5f} - \frac{\cot^7(e+fx)}{7a^2c^5f} + \frac{4\cot^9(e+fx)}{9a^2c^5f} + \frac{3\csc(e+fx)}{a^2c^5f} - \frac{13\csc^3}{3a^2}$$
$$= -\frac{\cot^3(e+fx)}{3a^2c^5f} + \frac{\cot^5(e+fx)}{5a^2c^5f} - \frac{\cot^7(e+fx)}{7a^2c^5f} + \frac{4\cot^9(e+fx)}{9a^2c^5f} + \frac{3\csc(}{a^2}$$
$$= \frac{\cot(e+fx)}{a^2c^5f} - \frac{\cot^3(e+fx)}{3a^2c^5f} + \frac{\cot^5(e+fx)}{5a^2c^5f} - \frac{\cot^7(e+fx)}{7a^2c^5f} + \frac{4\cot^9(e+}{9a^2c^5f}$$
$$= \frac{x}{a^2c^5} + \frac{\cot(e+fx)}{a^2c^5f} - \frac{\cot^3(e+fx)}{3a^2c^5f} + \frac{\cot^5(e+fx)}{5a^2c^5f} - \frac{\cot^7(e+fx)}{7a^2c^5f} + \frac{4\,\text{c}}{}$$

<u>Mathematica</u> [A] time = 1.23536, size = 383, normalized size = 1.82

$$\frac{\csc\left(\frac{e}{2}\right)\sec\left(\frac{e}{2}\right)\tan(e+fx)\sec^6(e+fx)(-675036\sin(e+fx)+506277\sin(2(e+fx))+37502\sin(3(e+fx))-225012\,\text{s}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^5),x]
```

```
[Out] (Csc[e/2]*Sec[e/2]*Sec[e + f*x]^6*(181440*f*x*Cos[f*x] - 181440*f*x*Cos[2*e
 + f*x] - 136080*f*x*Cos[e + 2*f*x] + 136080*f*x*Cos[3*e + 2*f*x] - 10080*f
*x*Cos[2*e + 3*f*x] + 10080*f*x*Cos[4*e + 3*f*x] + 60480*f*x*Cos[3*e + 4*f*
x] - 60480*f*x*Cos[5*e + 4*f*x] - 30240*f*x*Cos[4*e + 5*f*x] + 30240*f*x*Co
s[6*e + 5*f*x] + 5040*f*x*Cos[5*e + 6*f*x] - 5040*f*x*Cos[7*e + 6*f*x] + 16
9344*Sin[e] - 338112*Sin[f*x] - 675036*Sin[e + f*x] + 506277*Sin[2*(e + f*x
)] + 37502*Sin[3*(e + f*x)] - 225012*Sin[4*(e + f*x)] + 112506*Sin[5*(e + f
*x)] - 18751*Sin[6*(e + f*x)] - 431424*Sin[2*e + f*x] + 375552*Sin[e + 2*f*
x] + 201600*Sin[3*e + 2*f*x] - 41248*Sin[2*e + 3*f*x] + 84000*Sin[4*e + 3*f
```

```
*x] - 155712*Sin[3*e + 4*f*x] - 100800*Sin[5*e + 4*f*x] + 98016*Sin[4*e + 5
*f*x] + 30240*Sin[6*e + 5*f*x] - 21376*Sin[5*e + 6*f*x])*Tan[e + f*x])/(645
120*a^2*c^5*f*(-1 + Sec[e + f*x])^5*(1 + Sec[e + f*x])^2)
```

Maple [A] time = 0.074, size = 175, normalized size = 0.8

$$\frac{1}{192\,fa^2c^5}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3-\frac{1}{8\,fa^2c^5}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2\,\frac{\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)}{fa^2c^5}+\frac{1}{576\,fa^2c^5}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-9}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x)
```

```
[Out] 1/192/f/a^2/c^5*tan(1/2*f*x+1/2*e)^3-1/8/f/a^2/c^5*tan(1/2*f*x+1/2*e)+2/f/a
^2/c^5*arctan(tan(1/2*f*x+1/2*e))+1/576/f/a^2/c^5/tan(1/2*f*x+1/2*e)^9-1/56
/f/a^2/c^5/tan(1/2*f*x+1/2*e)^7+29/320/f/a^2/c^5/tan(1/2*f*x+1/2*e)^5-1/3/f
/a^2/c^5/tan(1/2*f*x+1/2*e)^3+99/64/f/a^2/c^5/tan(1/2*f*x+1/2*e)
```

Maxima [A] time = 1.55535, size = 251, normalized size = 1.2

$$-\frac{\frac{105\left(\frac{24\,\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}\right)}{a^2c^5}-\frac{40320\,\arctan\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)}{a^2c^5}+\frac{\left(\frac{360\,\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}-\frac{1827\,\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}+\frac{6720\,\sin\left(fx+e\right)^6}{\left(\cos\left(fx+e\right)+1\right)^6}-\frac{31185\,\sin\left(fx+e\right)^8}{\left(\cos\left(fx+e\right)+1\right)^8}-35\right)\left(\cos\left(fx+e\right)\right.}{a^2c^5\sin\left(fx+e\right)^9}}{20160\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="maxima")
```

```
[Out] -1/20160*(105*(24*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3)/(a^2*c^5) - 40320*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/(a^
2*c^5) + (360*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1827*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 + 6720*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 31185*sin(f*
x + e)^8/(cos(f*x + e) + 1)^8 - 35)*(cos(f*x + e) + 1)^9/(a^2*c^5*sin(f*x +
 e)^9))/f
```

Fricas [A] time = 1.1041, size = 598, normalized size = 2.85

$$\frac{668 \cos\left(fx+e\right)^{6} - 1059 \cos\left(fx+e\right)^{5} - 573 \cos\left(fx+e\right)^{4} + 1813 \cos\left(fx+e\right)^{3} - 393 \cos\left(fx+e\right)^{2} + 315\left(fx\cos\left(fx\right.\right.}{315\left(a^{2}c^{5}f\cos\left(fx+e\right)^{5} - 3\,a^{2}c^{5}f\cos\left(fx+e\right)^{4} + 2\,a^{2}c^{5}f\cos\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="fricas")
```

```
[Out] 1/315*(668*cos(f*x + e)^6 - 1059*cos(f*x + e)^5 - 573*cos(f*x + e)^4 + 1813
*cos(f*x + e)^3 - 393*cos(f*x + e)^2 + 315*(f*x*cos(f*x + e)^5 - 3*f*x*cos(
f*x + e)^4 + 2*f*x*cos(f*x + e)^3 + 2*f*x*cos(f*x + e)^2 - 3*f*x*cos(f*x +
e) + f*x)*sin(f*x + e) - 789*cos(f*x + e) + 368)/((a^2*c^5*f*cos(f*x + e)^5
 - 3*a^2*c^5*f*cos(f*x + e)^4 + 2*a^2*c^5*f*cos(f*x + e)^3 + 2*a^2*c^5*f*co
s(f*x + e)^2 - 3*a^2*c^5*f*cos(f*x + e) + a^2*c^5*f)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{1}{\sec^{7}\left(e+fx\right)-3\sec^{6}\left(e+fx\right)+\sec^{5}\left(e+fx\right)+5\sec^{4}\left(e+fx\right)-5\sec^{3}\left(e+fx\right)-\sec^{2}\left(e+fx\right)+3\sec\left(e+fx\right)-1}\,dx}{a^{2}c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**5,x)
```

```
[Out] -Integral(1/(sec(e + f*x)**7 - 3*sec(e + f*x)**6 + sec(e + f*x)**5 + 5*sec(
e + f*x)**4 - 5*sec(e + f*x)**3 - sec(e + f*x)**2 + 3*sec(e + f*x) - 1), x)
/(a**2*c**5)
```

Giac [A] time = 1.40587, size = 193, normalized size = 0.92

$$\frac{\frac{20160\left(fx+e\right)}{a^{2}c^{5}} + \frac{31185\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{8} - 6720\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{6} + 1827\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4} - 360\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2} + 35}{a^{2}c^{5}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{9}} + \frac{105\left(a^{4}c^{10}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3} - 24\,a^{4}c^{10}\tan\right.}{a^{6}c^{15}}}{20160\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="giac")
```

```
[Out] 1/20160*(20160*(f*x + e)/(a^2*c^5) + (31185*tan(1/2*f*x + 1/2*e)^8 - 6720*t
an(1/2*f*x + 1/2*e)^6 + 1827*tan(1/2*f*x + 1/2*e)^4 - 360*tan(1/2*f*x + 1/2
*e)^2 + 35)/(a^2*c^5*tan(1/2*f*x + 1/2*e)^9) + 105*(a^4*c^10*tan(1/2*f*x +
1/2*e)^3 - 24*a^4*c^10*tan(1/2*f*x + 1/2*e))/(a^6*c^15))/f
```

3.31 $\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=162

$$-\frac{c^5\tan(e+fx)}{a^3f}+\frac{128c^5\cot^5(e+fx)}{5a^3f}+\frac{128c^5\cot^3(e+fx)}{3a^3f}+\frac{32c^5\cot(e+fx)}{a^3f}-\frac{128c^5\csc^5(e+fx)}{5a^3f}+\frac{64c^5\csc^3(e+fx}{3a^3f}$$

```
[Out] (c^5*x)/a^3 + (8*c^5*ArcTanh[Sin[e + f*x]])/(a^3*f) + (32*c^5*Cot[e + f*x])
/(a^3*f) + (128*c^5*Cot[e + f*x]^3)/(3*a^3*f) + (128*c^5*Cot[e + f*x]^5)/(5
*a^3*f) - (16*c^5*Csc[e + f*x])/(a^3*f) + (64*c^5*Csc[e + f*x]^3)/(3*a^3*f)
 - (128*c^5*Csc[e + f*x]^5)/(5*a^3*f) - (c^5*Tan[e + f*x])/(a^3*f)
```

Rubi [A] time = 0.443483, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 29, number of rules used = 15, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.577, Rules used = {3904, 3886, 3473, 8, 2606, 194, 2607, 30, 14, 3767, 2621, 302, 207, 2620, 270}

$$-\frac{c^5\tan(e+fx)}{a^3f}+\frac{128c^5\cot^5(e+fx)}{5a^3f}+\frac{128c^5\cot^3(e+fx)}{3a^3f}+\frac{32c^5\cot(e+fx)}{a^3f}-\frac{128c^5\csc^5(e+fx)}{5a^3f}+\frac{64c^5\csc^3(e+fx}{3a^3f}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^5/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (c^5*x)/a^3 + (8*c^5*ArcTanh[Sin[e + f*x]])/(a^3*f) + (32*c^5*Cot[e + f*x])
/(a^3*f) + (128*c^5*Cot[e + f*x]^3)/(3*a^3*f) + (128*c^5*Cot[e + f*x]^5)/(5
*a^3*f) - (16*c^5*Csc[e + f*x])/(a^3*f) + (64*c^5*Csc[e + f*x]^3)/(3*a^3*f)
 - (128*c^5*Csc[e + f*x]^5)/(5*a^3*f) - (c^5*Tan[e + f*x])/(a^3*f)
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
```

```
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx = -\frac{\int \cot^6(e+fx)(c-c\sec(e+fx))^8\,dx}{a^3c^3}$$

$$= -\frac{\int \left(c^8\cot^6(e+fx) - 8c^8\cot^5(e+fx)\csc(e+fx) + 28c^8\cot^4(e+fx)\csc^2(e+fx) - 56c$$

$$= -\frac{c^5\int \cot^6(e+fx)\,dx}{a^3} - \frac{c^5\int \csc^6(e+fx)\sec^2(e+fx)\,dx}{a^3} + \frac{(8c^5)\int \cot^5(e+fx)\csc(e+$$

$$= \frac{c^5\cot^5(e+fx)}{5a^3f} + \frac{c^5\int \cot^4(e+fx)\,dx}{a^3} - \frac{c^5\,\text{Subst}\left(\int \frac{(1+x^2)^3}{x^6}\,dx, x, \tan(e+fx)\right)}{a^3f} - \frac{(8c^5)\,\text{S}$$

$$= \frac{28c^5\cot(e+fx)}{a^3f} + \frac{55c^5\cot^3(e+fx)}{3a^3f} + \frac{57c^5\cot^5(e+fx)}{5a^3f} - \frac{56c^5\csc^5(e+fx)}{5a^3f} - \frac{c^5\int \cot$$

$$= \frac{32c^5\cot(e+fx)}{a^3f} + \frac{128c^5\cot^3(e+fx)}{3a^3f} + \frac{128c^5\cot^5(e+fx)}{5a^3f} - \frac{16c^5\csc(e+fx)}{a^3f} + \frac{64c^5\,\text{cs}$$

$$= \frac{c^5x}{a^3} + \frac{8c^5\tanh^{-1}(\sin(e+fx))}{a^3f} + \frac{32c^5\cot(e+fx)}{a^3f} + \frac{128c^5\cot^3(e+fx)}{3a^3f} + \frac{128c^5\cot^5(e+$$

Mathematica [B] time = 5.6226, size = 557, normalized size = 3.44

$$-\frac{c^5\sec\left(\frac{e}{2}\right)(\cos(e+fx)-1)^5\cot\left(\frac{1}{2}(e+fx)\right)\csc^4\left(\frac{1}{2}(e+fx)\right)\left(1016\sin\left(\frac{fx}{2}\right)\cot^6\left(\frac{1}{2}(e+fx)\right)\csc\left(\frac{1}{2}(e+fx)\right)+\sec^2\left(\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^5/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] -(c^5*(-1 + Cos[e + f*x])^5*Cot[(e + f*x)/2]*Csc[(e + f*x)/2]^4*Sec[e/2]*(-
60*Cos[e]*Cot[(e + f*x)/2]^7*(f*x - 8*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/
2]] + 8*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])*Sec[e/2] + 48*Cot[(e + f*
x)/2]*Csc[(e + f*x)/2]^4*Sec[e/2]^2*(Sin[e/2] - Sin[(3*e)/2]) + 8*(-7 + Cos
[e + f*x])*Cot[(e + f*x)/2]^3*Csc[(e + f*x)/2]^4*Sec[e/2]^2*(Sin[e/2] - Sin
[(3*e)/2]) + 1016*Cot[(e + f*x)/2]^6*Csc[(e + f*x)/2]*Sin[(f*x)/2] + (-140
+ 76*Cos[e] + 131*Cos[f*x] - 210*Cos[e + f*x] - 84*Cos[2*(e + f*x)] - 14*Co
s[3*(e + f*x)] + 131*Cos[2*e + f*x] + 66*Cos[e + 2*f*x] + 66*Cos[3*e + 2*f*
x] + 21*Cos[2*e + 3*f*x] + 21*Cos[4*e + 3*f*x])*Csc[(e + f*x)/2]^7*Sec[e/2]
^2*Sin[(f*x)/2] + 2*Cot[(e + f*x)/2]^5*Sec[e/2]*(30*Cos[e]*(f*x - 8*Log[Cos
[(e + f*x)/2] - Sin[(e + f*x)/2]] + 8*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/
```

```
2]]) - (Cos[e] + 15*(-1 + Cos[f*x] + Cos[e + f*x]))*Csc[(e + f*x)/2]^2*Tan[
e/2])))/(240*a^3*f*(1 + Cos[e + f*x])^3*(-1 + Cot[(e + f*x)/2])*(1 + Cot[(e
 + f*x)/2])*(-1 + Tan[e/2])*(1 + Tan[e/2]))
```

Maple [A] time = 0.099, size = 179, normalized size = 1.1

$$-\frac{8\,c^5}{5\,f a^3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^5-\frac{8\,c^5}{3\,f a^3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3-16\,\frac{c^5\tan\left(1/2\,fx+e/2\right)}{f a^3}+2\,\frac{c^5\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)}{f a^3}+8\,\frac{c^5\,l}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x)
```

```
[Out] -8/5/f*c^5/a^3*tan(1/2*f*x+1/2*e)^5-8/3/f*c^5/a^3*tan(1/2*f*x+1/2*e)^3-16/f
*c^5/a^3*tan(1/2*f*x+1/2*e)+2/f*c^5/a^3*arctan(tan(1/2*f*x+1/2*e))+8/f*c^5/
a^3*ln(tan(1/2*f*x+1/2*e)+1)+1/f*c^5/a^3/(tan(1/2*f*x+1/2*e)+1)+1/f*c^5/a^3
/(tan(1/2*f*x+1/2*e)-1)-8/f*c^5/a^3*ln(tan(1/2*f*x+1/2*e)-1)
```

Maxima [B] time = 1.58554, size = 759, normalized size = 4.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/60*(3*c^5*(40*sin(f*x + e)/((a^3 - a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2)*(cos(f*x + e) + 1)) + (85*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x
+ e)^3/(cos(f*x + e) + 1)^3 + sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 60
*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x + e)/(cos(f*
x + e) + 1) - 1)/a^3) + 5*c^5*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 20*si
n(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/
a^3 - 60*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x + e)
/(cos(f*x + e) + 1) - 1)/a^3) + c^5*((105*sin(f*x + e)/(cos(f*x + e) + 1) -
 20*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) +
1)^5)/a^3 - 120*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3) + 10*c^5*(15*s
in(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3
*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 + 5*c^5*(15*sin(f*x + e)/(cos(f*x
 + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos
```

```
(f*x + e) + 1)^5)/a^3 - 30*c^5*(5*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x
 + e)^5/(cos(f*x + e) + 1)^5)/a^3)/f
```

Fricas [A] time = 1.18968, size = 721, normalized size = 4.45

$$\frac{15\,c^5 fx\cos\left(fx+e\right)^4 + 45\,c^5 fx\cos\left(fx+e\right)^3 + 45\,c^5 fx\cos\left(fx+e\right)^2 + 15\,c^5 fx\cos\left(fx+e\right) + 60\left(c^5\cos\left(fx+e\right)^4 + 3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(15*c^5*f*x*cos(f*x + e)^4 + 45*c^5*f*x*cos(f*x + e)^3 + 45*c^5*f*x*co
s(f*x + e)^2 + 15*c^5*f*x*cos(f*x + e) + 60*(c^5*cos(f*x + e)^4 + 3*c^5*cos
(f*x + e)^3 + 3*c^5*cos(f*x + e)^2 + c^5*cos(f*x + e))*log(sin(f*x + e) + 1
) - 60*(c^5*cos(f*x + e)^4 + 3*c^5*cos(f*x + e)^3 + 3*c^5*cos(f*x + e)^2 +
c^5*cos(f*x + e))*log(-sin(f*x + e) + 1) - (239*c^5*cos(f*x + e)^3 + 477*c^
5*cos(f*x + e)^2 + 349*c^5*cos(f*x + e) + 15*c^5)*sin(f*x + e))/(a^3*f*cos(
f*x + e)^4 + 3*a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + a^3*f*cos(f*
x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{c^5\left(\int \frac{5\sec\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx + \int -\frac{10\sec^2\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx + \int \frac{10\sec^3\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**5/(a+a*sec(f*x+e))**3,x)
```

```
[Out] -c**5*(Integral(5*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec
(e + f*x) + 1), x) + Integral(-10*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(
e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(10*sec(e + f*x)**3/(sec(e
+ f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(-5*sec(e
 + f*x)**4/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) +
 Integral(sec(e + f*x)**5/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e +
f*x) + 1), x) + Integral(-1/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e
```

```
+ f*x) + 1), x))/a**3
```

Giac [A] time = 1.54874, size = 219, normalized size = 1.35

$$\frac{\frac{15\left(fx+e\right)c^5}{a^3}+\frac{120\,c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^3}-\frac{120\,c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^3}+\frac{30\,c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)a^3}-\frac{8\left(3\,a^{12}c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+5\,a^{12}c^5\tan\left(\frac{1}{2}\,fx\right.\right.}{a^{15}}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/15*(15*(f*x + e)*c^5/a^3 + 120*c^5*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^3
 - 120*c^5*log(abs(tan(1/2*f*x + 1/2*e) - 1))/a^3 + 30*c^5*tan(1/2*f*x + 1/
2*e)/((tan(1/2*f*x + 1/2*e)^2 - 1)*a^3) - 8*(3*a^12*c^5*tan(1/2*f*x + 1/2*e
)^5 + 5*a^12*c^5*tan(1/2*f*x + 1/2*e)^3 + 30*a^12*c^5*tan(1/2*f*x + 1/2*e))
/a^15)/f
```

3.32 $\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=148

$$\frac{c^4\tanh^{-1}(\sin(e+fx))}{a^3f}-\frac{c^4\tan(e+fx)\sec^2(e+fx)}{5a^3f(\sec(e+fx)+1)^3}-\frac{23c^4\tan(e+fx)}{5a^3f(\sec(e+fx)+1)}+\frac{14c^4\tan(e+fx)}{5a^3f(\sec(e+fx)+1)^2}-\frac{3c^4\tan(e}{a^3f(\sec(e+}$$

```
[Out] (c^4*x)/a^3 + (c^4*ArcTanh[Sin[e + f*x]])/(a^3*f) - (3*c^4*Tan[e + f*x])/(a
^3*f*(1 + Sec[e + f*x])^3) - (c^4*Sec[e + f*x]^2*Tan[e + f*x])/(5*a^3*f*(1
+ Sec[e + f*x])^3) + (14*c^4*Tan[e + f*x])/(5*a^3*f*(1 + Sec[e + f*x])^2) -
 (23*c^4*Tan[e + f*x])/(5*a^3*f*(1 + Sec[e + f*x]))
```

Rubi [A] time = 0.606057, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 13, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3903, 3777, 3922, 3919, 3794, 3796, 3797, 3799, 4000, 3816, 4008, 3998, 3770}

$$\frac{c^4\tanh^{-1}(\sin(e+fx))}{a^3f}-\frac{c^4\tan(e+fx)\sec^2(e+fx)}{5a^3f(\sec(e+fx)+1)^3}-\frac{23c^4\tan(e+fx)}{5a^3f(\sec(e+fx)+1)}+\frac{14c^4\tan(e+fx)}{5a^3f(\sec(e+fx)+1)^2}-\frac{3c^4\tan(e}{a^3f(\sec(e+}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^4/(a + a*Sec[e + f*x])^3,x]
```


```
[Out] (c^4*x)/a^3 + (c^4*ArcTanh[Sin[e + f*x]])/(a^3*f) - (3*c^4*Tan[e + f*x])/(a
^3*f*(1 + Sec[e + f*x])^3) - (c^4*Sec[e + f*x]^2*Tan[e + f*x])/(5*a^3*f*(1
+ Sec[e + f*x])^3) + (14*c^4*Tan[e + f*x])/(5*a^3*f*(1 + Sec[e + f*x])^2) -
 (23*c^4*Tan[e + f*x])/(5*a^3*f*(1 + Sec[e + f*x]))
```


Rule 3903

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Dist[c^n, Int[ExpandTrig[(1 + (d*csc[e + f*
x])/c)^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
&& LtQ[m + n, 2]
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
```

```
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
rQ[2*n]
```

Rule 3922

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*
x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x
])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ
[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3797

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(2*m + 1)), x]
+ Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x],
x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3799

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)),
```

```
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4000

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[((A*b - a*B)*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx = \frac{\int \left(\frac{c^4}{(1+\sec(e+fx))^3} - \frac{4c^4\sec(e+fx)}{(1+\sec(e+fx))^3} + \frac{6c^4\sec^2(e+fx)}{(1+\sec(e+fx))^3} - \frac{4c^4\sec^3(e+fx)}{(1+\sec(e+fx))^3} + \frac{c^4\sec^4(e+fx)}{(1+\sec(e+fx))^3}\right)dx}{a^3}$$

$$= \frac{c^4\int \frac{1}{(1+\sec(e+fx))^3}\,dx}{a^3} + \frac{c^4\int \frac{\sec^4(e+fx)}{(1+\sec(e+fx))^3}\,dx}{a^3} - \frac{(4c^4)\int \frac{\sec(e+fx)}{(1+\sec(e+fx))^3}\,dx}{a^3} - \frac{(4c^4)\int \frac{\sec^3(e+fx)}{(1+\sec(e+fx))}}{a^3}$$

$$= -\frac{3c^4\tan(e+fx)}{a^3f(1+\sec(e+fx))^3} - \frac{c^4\sec^2(e+fx)\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3} - \frac{c^4\int \frac{(2-5\sec(e+fx))\sec^2(e+fx)}{(1+\sec(e+fx))^2}\,dx}{5a^3} - \frac{c^4}{}$$

$$= -\frac{3c^4\tan(e+fx)}{a^3f(1+\sec(e+fx))^3} - \frac{c^4\sec^2(e+fx)\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3} + \frac{14c^4\tan(e+fx)}{5a^3f(1+\sec(e+fx))^2} + \frac{c^4\int \frac{15-7}{1+s}}{1}$$

$$= \frac{c^4x}{a^3} - \frac{3c^4\tan(e+fx)}{a^3f(1+\sec(e+fx))^3} - \frac{c^4\sec^2(e+fx)\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3} + \frac{14c^4\tan(e+fx)}{5a^3f(1+\sec(e+fx))^2} - \frac{6}{5a^3}$$

$$= \frac{c^4x}{a^3} + \frac{c^4\tanh^{-1}(\sin(e+fx))}{a^3f} - \frac{3c^4\tan(e+fx)}{a^3f(1+\sec(e+fx))^3} - \frac{c^4\sec^2(e+fx)\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3} + \frac{1}{5a^3}$$

Mathematica [A] time = 1.17546, size = 231, normalized size = 1.56

$$\frac{c^4(\cos(e+fx)-1)^4\cot\left(\frac{1}{2}(e+fx)\right)\csc^2\left(\frac{1}{2}(e+fx)\right)\left(8\tan\left(\frac{e}{2}\right)\cot^3\left(\frac{1}{2}(e+fx)\right)\csc^2\left(\frac{1}{2}(e+fx)\right)-4\tan\left(\frac{e}{2}\right)\cot\left(\frac{1}{2}(e+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^4/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (c^4*(-1 + Cos[e + f*x])^4*Cot[(e + f*x)/2]*Csc[(e + f*x)/2]^2*(5*Cot[(e +
f*x)/2]^5*(f*x - Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + Log[Cos[(e + f*
x)/2] + Sin[(e + f*x)/2]]) - (9 + 8*Cos[e + f*x] + 3*Cos[2*(e + f*x)])*Csc[
(e + f*x)/2]^5*Sec[e/2]*Sin[(f*x)/2] + 8*Cot[(e + f*x)/2]^3*Csc[(e + f*x)/2
]^2*Tan[e/2] - 4*Cot[(e + f*x)/2]*Csc[(e + f*x)/2]^4*Tan[e/2]))/(10*a^3*f*(
1 + Cos[e + f*x])^3)
```

Maple [A] time = 0.103, size = 110, normalized size = 0.7

$$-\frac{4c^4}{5fa^3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^5 - 4\frac{c^4\tan(1/2fx+e/2)}{fa^3} + 2\frac{c^4\arctan(\tan(1/2fx+e/2))}{fa^3} + \frac{c^4}{fa^3}\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right) - \frac{c^4}{fa}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x)
```

```
[Out] -4/5/f*c^4/a^3*tan(1/2*f*x+1/2*e)^5-4/f*c^4/a^3*tan(1/2*f*x+1/2*e)+2/f*c^4/
a^3*arctan(tan(1/2*f*x+1/2*e))+1/f*c^4/a^3*ln(tan(1/2*f*x+1/2*e)+1)-1/f*c^4
/a^3*ln(tan(1/2*f*x+1/2*e)-1)
```

Maxima [B] time = 1.59596, size = 535, normalized size = 3.61

$$-\frac{c^4\left(\frac{\frac{105\sin(fx+e)}{\cos(fx+e)+1}+\frac{20\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3}-\frac{60\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^3}+\frac{60\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^3}\right)+c^4\left(\frac{\frac{105\sin(fx+e)}{\cos(fx+e)+1}-\frac{20\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx}{(\cos(fx+e}}{a^3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/60*(c^4*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 20*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 60*log(sin(f*
x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x + e)/(cos(f*x + e) + 1)
 - 1)/a^3) + c^4*((105*sin(f*x + e)/(cos(f*x + e) + 1) - 20*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 120*arc
tan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3) + 4*c^4*(15*sin(f*x + e)/(cos(f*x
 + e) + 1) + 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos
(f*x + e) + 1)^5)/a^3 + 4*c^4*(15*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(
f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^
3 - 18*c^4*(5*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^5/(cos(f*x + e
) + 1)^5)/a^3)/f
```

Fricas [A] time = 1.1315, size = 601, normalized size = 4.06

$$\frac{10\,c^4 f x \cos\left(fx+e\right)^3+30\,c^4 f x\cos\left(fx+e\right)^2+30\,c^4 f x\cos\left(fx+e\right)+10\,c^4 f x+5\left(c^4\cos\left(fx+e\right)^3+3\,c^4\cos\left(fx+e\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/10*(10*c^4*f*x*cos(f*x + e)^3 + 30*c^4*f*x*cos(f*x + e)^2 + 30*c^4*f*x*co
s(f*x + e) + 10*c^4*f*x + 5*(c^4*cos(f*x + e)^3 + 3*c^4*cos(f*x + e)^2 + 3*
c^4*cos(f*x + e) + c^4)*log(sin(f*x + e) + 1) - 5*(c^4*cos(f*x + e)^3 + 3*c
^4*cos(f*x + e)^2 + 3*c^4*cos(f*x + e) + c^4)*log(-sin(f*x + e) + 1) - 16*(
3*c^4*cos(f*x + e)^2 + 4*c^4*cos(f*x + e) + 3*c^4)*sin(f*x + e))/(a^3*f*cos
(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{c^4\left(\int -\frac{4\sec\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int\frac{6\sec^2\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int -\frac{4\sec^3\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+\right.}\right.}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**4/(a+a*sec(f*x+e))**3,x)
```

```
[Out] c**4*(Integral(-4*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec
(e + f*x) + 1), x) + Integral(6*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e
+ f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(-4*sec(e + f*x)**3/(sec(e +
f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(sec(e + f*
x)**4/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Inte
gral(1/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x))/a**3
```

Giac [A] time = 1.45611, size = 144, normalized size = 0.97

$$\frac{\frac{5\left(fx+e\right)c^4}{a^3}+\frac{5\,c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^3}-\frac{5\,c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^3}-\frac{4\left(a^{12}c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+5\,a^{12}c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{a^{15}}}{5\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/5*(5*(f*x + e)*c^4/a^3 + 5*c^4*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^3 - 5
*c^4*log(abs(tan(1/2*f*x + 1/2*e) - 1))/a^3 - 4*(a^12*c^4*tan(1/2*f*x + 1/2
*e)^5 + 5*a^12*c^4*tan(1/2*f*x + 1/2*e))/a^15)/f
```

3.33 $\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=96

$$-\frac{26c^3\tan(e+fx)}{15a^3f(\sec(e+fx)+1)}+\frac{4c^3\tan(e+fx)}{15a^3f(\sec(e+fx)+1)^2}-\frac{8c^3\tan(e+fx)}{5a^3f(\sec(e+fx)+1)^3}+\frac{c^3x}{a^3}$$

```
[Out] (c^3*x)/a^3 - (8*c^3*Tan[e + f*x])/(5*a^3*f*(1 + Sec[e + f*x])^3) + (4*c^3*
Tan[e + f*x])/(15*a^3*f*(1 + Sec[e + f*x])^2) - (26*c^3*Tan[e + f*x])/(15*a
^3*f*(1 + Sec[e + f*x]))
```

Rubi [A] time = 0.420562, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {3903, 3777, 3922, 3919, 3794, 3796, 3797, 3799, 4000}

$$-\frac{26c^3\tan(e+fx)}{15a^3f(\sec(e+fx)+1)}+\frac{4c^3\tan(e+fx)}{15a^3f(\sec(e+fx)+1)^2}-\frac{8c^3\tan(e+fx)}{5a^3f(\sec(e+fx)+1)^3}+\frac{c^3x}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (c^3*x)/a^3 - (8*c^3*Tan[e + f*x])/(5*a^3*f*(1 + Sec[e + f*x])^3) + (4*c^3*
Tan[e + f*x])/(15*a^3*f*(1 + Sec[e + f*x])^2) - (26*c^3*Tan[e + f*x])/(15*a
^3*f*(1 + Sec[e + f*x]))
```

Rule 3903

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Dist[c^n, Int[ExpandTrig[(1 + (d*csc[e + f*
x])/c)^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
&& LtQ[m + n, 2]
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
```

rQ[2*n]

Rule 3922

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*
x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x
])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ
[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3797

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(2*m + 1)), x]
+ Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x],
x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3799

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
```

```
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4000

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[((A*b - a*B)*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^3}dx = \frac{\int \left(\frac{c^3}{(1+\sec(e+fx))^3}-\frac{3c^3\sec(e+fx)}{(1+\sec(e+fx))^3}+\frac{3c^3\sec^2(e+fx)}{(1+\sec(e+fx))^3}-\frac{c^3\sec^3(e+fx)}{(1+\sec(e+fx))^3}\right)dx}{a^3}$$

$$= \frac{c^3\int \frac{1}{(1+\sec(e+fx))^3}dx}{a^3}-\frac{c^3\int \frac{\sec^3(e+fx)}{(1+\sec(e+fx))^3}dx}{a^3}-\frac{(3c^3)\int \frac{\sec(e+fx)}{(1+\sec(e+fx))^3}dx}{a^3}+\frac{(3c^3)\int \frac{\sec^2(e+fx}{(1+\sec(e+fx}}{a^3}$$

$$= -\frac{8c^3\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3}-\frac{c^3\int \frac{-5+2\sec(e+fx)}{(1+\sec(e+fx))^2}dx}{5a^3}-\frac{c^3\int \frac{\sec(e+fx)(-3+5\sec(e+fx))}{(1+\sec(e+fx))^2}dx}{5a^3}-\frac{(6c^3)\int}{}$$

$$= -\frac{8c^3\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3}+\frac{4c^3\tan(e+fx)}{15a^3f(1+\sec(e+fx))^2}+\frac{c^3\int \frac{15-7\sec(e+fx)}{1+\sec(e+fx)}dx}{15a^3}-\frac{(2c^3)\int \frac{\sec}{1+\sec}}{5a^3}$$

$$= \frac{c^3x}{a^3}-\frac{8c^3\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3}+\frac{4c^3\tan(e+fx)}{15a^3f(1+\sec(e+fx))^2}-\frac{4c^3\tan(e+fx)}{15a^3f(1+\sec(e+fx))}-\frac{(22c}{}$$

$$= \frac{c^3x}{a^3}-\frac{8c^3\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3}+\frac{4c^3\tan(e+fx)}{15a^3f(1+\sec(e+fx))^2}-\frac{26c^3\tan(e+fx)}{15a^3f(1+\sec(e+fx))}$$

Mathematica [A] time = 0.0785299, size = 90, normalized size = 0.94

$$-\frac{c^3\left(\frac{2\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)}{5f}-\frac{2\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f}-\frac{2\tan^{-1}\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f}+\frac{2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] -((c^3*((-2*ArcTan[Tan[e/2 + (f*x)/2]])/f + (2*Tan[e/2 + (f*x)/2])/f - (2*T
an[e/2 + (f*x)/2]^3)/(3*f) + (2*Tan[e/2 + (f*x)/2]^5)/(5*f)))/a^3)
```

Maple [A] time = 0.102, size = 87, normalized size = 0.9

$$-\frac{2\,c^3}{5\,fa^3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^5+\frac{2\,c^3}{3\,fa^3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3-2\,\frac{c^3\tan\left(1/2\,fx+e/2\right)}{fa^3}+2\,\frac{c^3\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)}{fa^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x)
```

```
[Out] -2/5/f*c^3/a^3*tan(1/2*f*x+1/2*e)^5+2/3/f*c^3/a^3*tan(1/2*f*x+1/2*e)^3-2/f*
c^3/a^3*tan(1/2*f*x+1/2*e)+2/f*c^3/a^3*arctan(tan(1/2*f*x+1/2*e))
```

Maxima [B] time = 1.64579, size = 374, normalized size = 3.9

$$-\frac{c^3\left(\frac{\frac{105\,\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{20\,\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}+\frac{3\,\sin\left(fx+e\right)^5}{\left(\cos\left(fx+e\right)+1\right)^5}}{a^3}-\frac{120\,\arctan\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)}{a^3}\right)+\frac{c^3\left(\frac{15\,\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{10\,\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}+\frac{3\,\sin\left(fx+e\right)^5}{\left(\cos\left(fx+e\right)+1\right)^5}\right)}{a^3}+\frac{3\,c^3\left(\frac{15\,\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\cdots\right.}{}}{60\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/60*(c^3*((105*sin(f*x + e)/(cos(f*x + e) + 1) - 20*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 120*arctan(si
n(f*x + e)/(cos(f*x + e) + 1))/a^3) + c^3*(15*sin(f*x + e)/(cos(f*x + e) +
1) + 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e
) + 1)^5)/a^3 + 3*c^3*(15*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)
^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 9*c^
3*(5*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^5)
/a^3)/f
```

Fricas [A] time = 1.02357, size = 340, normalized size = 3.54

$$\frac{15\,c^3 f x \cos\left(f x+e\right)^3+45\,c^3 f x \cos\left(f x+e\right)^2+45\,c^3 f x \cos\left(f x+e\right)+15\,c^3 f x-2\left(23\,c^3 \cos\left(f x+e\right)^2+24\,c^3 \cos\left(f x\right.\right.}{15\left(a^3 f \cos\left(f x+e\right)^3+3\,a^3 f \cos\left(f x+e\right)^2+3\,a^3 f \cos\left(f x+e\right)+a^3 f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(15*c^3*f*x*cos(f*x + e)^3 + 45*c^3*f*x*cos(f*x + e)^2 + 45*c^3*f*x*co
s(f*x + e) + 15*c^3*f*x - 2*(23*c^3*cos(f*x + e)^2 + 24*c^3*cos(f*x + e) +
13*c^3)*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^
3*f*cos(f*x + e) + a^3*f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{c^3\left(\int \frac{3\sec\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int -\frac{3\sec^2\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int \frac{\sec^3\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(\right.}\right.}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**3/(a+a*sec(f*x+e))**3,x)
```

```
[Out] -c**3*(Integral(3*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec
(e + f*x) + 1), x) + Integral(-3*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e
 + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(sec(e + f*x)**3/(sec(e + f*
x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(-1/(sec(e +
f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x))/a**3
```

Giac [A] time = 1.48101, size = 113, normalized size = 1.18

$$\frac{\frac{15\left(fx+e\right)c^3}{a^3}-\frac{2\left(3\,a^{12}c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-5\,a^{12}c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+15\,a^{12}c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{a^{15}}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/15*(15*(f*x + e)*c^3/a^3 - 2*(3*a^12*c^3*tan(1/2*f*x + 1/2*e)^5 - 5*a^12*
c^3*tan(1/2*f*x + 1/2*e)^3 + 15*a^12*c^3*tan(1/2*f*x + 1/2*e))/a^15)/f
```

3.34 $\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=96

$$-\frac{23c^2\tan(e+fx)}{15a^3f(\sec(e+fx)+1)}-\frac{8c^2\tan(e+fx)}{15a^3f(\sec(e+fx)+1)^2}-\frac{4c^2\tan(e+fx)}{5a^3f(\sec(e+fx)+1)^3}+\frac{c^2x}{a^3}$$

```
[Out] (c^2*x)/a^3 - (4*c^2*Tan[e + f*x])/(5*a^3*f*(1 + Sec[e + f*x])^3) - (8*c^2*
Tan[e + f*x])/(15*a^3*f*(1 + Sec[e + f*x])^2) - (23*c^2*Tan[e + f*x])/(15*a
^3*f*(1 + Sec[e + f*x]))
```

Rubi [A] time = 0.303866, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {3903, 3777, 3922, 3919, 3794, 3796, 3797}

$$-\frac{23c^2\tan(e+fx)}{15a^3f(\sec(e+fx)+1)}-\frac{8c^2\tan(e+fx)}{15a^3f(\sec(e+fx)+1)^2}-\frac{4c^2\tan(e+fx)}{5a^3f(\sec(e+fx)+1)^3}+\frac{c^2x}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (c^2*x)/a^3 - (4*c^2*Tan[e + f*x])/(5*a^3*f*(1 + Sec[e + f*x])^3) - (8*c^2*
Tan[e + f*x])/(15*a^3*f*(1 + Sec[e + f*x])^2) - (23*c^2*Tan[e + f*x])/(15*a
^3*f*(1 + Sec[e + f*x]))
```

Rule 3903

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Dist[c^n, Int[ExpandTrig[(1 + (d*csc[e + f*
x])/c)^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
&& LtQ[m + n, 2]
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
```

```
rQ[2*n]
```

Rule 3922

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*
x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x
])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ
[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3797

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(2*m + 1)), x]
+ Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x],
x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx = \frac{\int \left(\frac{c^2}{(1+\sec(e+fx))^3} - \frac{2c^2\sec(e+fx)}{(1+\sec(e+fx))^3} + \frac{c^2\sec^2(e+fx)}{(1+\sec(e+fx))^3}\right)dx}{a^3}$$

$$= \frac{c^2\int \frac{1}{(1+\sec(e+fx))^3}\,dx}{a^3} + \frac{c^2\int \frac{\sec^2(e+fx)}{(1+\sec(e+fx))^3}\,dx}{a^3} - \frac{(2c^2)\int \frac{\sec(e+fx)}{(1+\sec(e+fx))^3}\,dx}{a^3}$$

$$= -\frac{4c^2\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3} - \frac{c^2\int \frac{-5+2\sec(e+fx)}{(1+\sec(e+fx))^2}\,dx}{5a^3} + \frac{(3c^2)\int \frac{\sec(e+fx)}{(1+\sec(e+fx))^2}\,dx}{5a^3} - \frac{(4c^2)\int \frac{\sec}{(1+\sec}}{5a^3}$$

$$= -\frac{4c^2\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3} - \frac{8c^2\tan(e+fx)}{15a^3f(1+\sec(e+fx))^2} + \frac{c^2\int \frac{15-7\sec(e+fx)}{1+\sec(e+fx)}\,dx}{15a^3} + \frac{c^2\int \frac{\sec(e+f}{1+\sec(e+}}{5a^3}$$

$$= \frac{c^2x}{a^3} - \frac{4c^2\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3} - \frac{8c^2\tan(e+fx)}{15a^3f(1+\sec(e+fx))^2} - \frac{c^2\tan(e+fx)}{15a^3f(1+\sec(e+fx))} - \frac{(22c}{}$$

$$= \frac{c^2x}{a^3} - \frac{4c^2\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3} - \frac{8c^2\tan(e+fx)}{15a^3f(1+\sec(e+fx))^2} - \frac{23c^2\tan(e+fx)}{15a^3f(1+\sec(e+fx))}$$

Mathematica [A] time = 0.46073, size = 171, normalized size = 1.78

$$\frac{c^2\sec\left(\frac{e}{2}\right)\sec^5\left(\frac{1}{2}(e+fx)\right)\left(360\sin\left(e+\frac{fx}{2}\right) - 280\sin\left(e+\frac{3fx}{2}\right) + 150\sin\left(2e+\frac{3fx}{2}\right) - 86\sin\left(2e+\frac{5fx}{2}\right) + 150fx\cos\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (c^2*Sec[e/2]*Sec[(e + f*x)/2]^5*(150*f*x*Cos[(f*x)/2] + 150*f*x*Cos[e + (f
*x)/2] + 75*f*x*Cos[e + (3*f*x)/2] + 75*f*x*Cos[2*e + (3*f*x)/2] + 15*f*x*C
os[2*e + (5*f*x)/2] + 15*f*x*Cos[3*e + (5*f*x)/2] - 500*Sin[(f*x)/2] + 360*
Sin[e + (f*x)/2] - 280*Sin[e + (3*f*x)/2] + 150*Sin[2*e + (3*f*x)/2] - 86*S
in[2*e + (5*f*x)/2]))/(480*a^3*f)
```

Maple [A] time = 0.091, size = 87, normalized size = 0.9

$$-\frac{c^2}{5fa^3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^5 + \frac{2c^2}{3fa^3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3 - 2\frac{c^2\tan\left(1/2\,fx+e/2\right)}{fa^3} + 2\frac{c^2\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)}{fa^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x)
```

```
[Out] -1/5/f*c^2/a^3*tan(1/2*f*x+1/2*e)^5+2/3/f*c^2/a^3*tan(1/2*f*x+1/2*e)^3-2/f*
c^2/a^3*tan(1/2*f*x+1/2*e)+2/f*c^2/a^3*arctan(tan(1/2*f*x+1/2*e))
```

Maxima [B] time = 1.57184, size = 285, normalized size = 2.97

$$-\frac{c^2\left(\frac{\frac{105\sin(fx+e)}{\cos(fx+e)+1}-\frac{20\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3}-\frac{120\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^3}\right)+\frac{2c^2\left(\frac{15\sin(fx+e)}{\cos(fx+e)+1}-\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}-\frac{3c^2\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}\right.}{a}}{60f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/60*(c^2*((105*sin(f*x + e)/(cos(f*x + e) + 1) - 20*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 120*arctan(si
n(f*x + e)/(cos(f*x + e) + 1))/a^3) + 2*c^2*(15*sin(f*x + e)/(cos(f*x + e)
+ 1) - 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x +
 e) + 1)^5)/a^3 - 3*c^2*(5*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^5
/(cos(f*x + e) + 1)^5)/a^3)/f
```

Fricas [A] time = 1.00222, size = 338, normalized size = 3.52

$$\frac{15\,c^2fx\cos\left(fx+e\right)^3+45\,c^2fx\cos\left(fx+e\right)^2+45\,c^2fx\cos\left(fx+e\right)+15\,c^2fx-\left(43\,c^2\cos\left(fx+e\right)^2+54\,c^2\cos\left(fx+e\right.\right.}{15\left(a^3f\cos\left(fx+e\right)^3+3\,a^3f\cos\left(fx+e\right)^2+3\,a^3f\cos\left(fx+e\right)+a^3f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(15*c^2*f*x*cos(f*x + e)^3 + 45*c^2*f*x*cos(f*x + e)^2 + 45*c^2*f*x*co
s(f*x + e) + 15*c^2*f*x - (43*c^2*cos(f*x + e)^2 + 54*c^2*cos(f*x + e) + 23
*c^2)*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*
f*cos(f*x + e) + a^3*f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{c^2\left(\int -\frac{2\sec\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int \frac{\sec^2\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int \frac{1}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+\right.}\right.}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**2/(a+a*sec(f*x+e))**3,x)
```

```
[Out] c**2*(Integral(-2*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec
(e + f*x) + 1), x) + Integral(sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e +
f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(1/(sec(e + f*x)**3 + 3*sec(e +
 f*x)**2 + 3*sec(e + f*x) + 1), x))/a**3
```

Giac [A] time = 1.3757, size = 113, normalized size = 1.18

$$\frac{\frac{15\left(fx+e\right)c^2}{a^3}-\frac{3\,a^{12}c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-10\,a^{12}c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+30\,a^{12}c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{a^{15}}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/15*(15*(f*x + e)*c^2/a^3 - (3*a^12*c^2*tan(1/2*f*x + 1/2*e)^5 - 10*a^12*c
^2*tan(1/2*f*x + 1/2*e)^3 + 30*a^12*c^2*tan(1/2*f*x + 1/2*e))/a^15)/f
```

3.35 $\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^3} dx$

Optimal. Leaf size=88

$$-\frac{8c\tan(e+fx)}{5a^3f(\sec(e+fx)+1)}-\frac{3c\tan(e+fx)}{5a^3f(\sec(e+fx)+1)^2}-\frac{2c\tan(e+fx)}{5a^3f(\sec(e+fx)+1)^3}+\frac{cx}{a^3}$$

```
[Out] (c*x)/a^3 - (2*c*Tan[e + f*x])/(5*a^3*f*(1 + Sec[e + f*x])^3) - (3*c*Tan[e
+ f*x])/(5*a^3*f*(1 + Sec[e + f*x])^2) - (8*c*Tan[e + f*x])/(5*a^3*f*(1 + S
ec[e + f*x]))
```

Rubi [A] time = 0.202123, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3903, 3777, 3922, 3919, 3794, 3796}

$$-\frac{8c\tan(e+fx)}{5a^3f(\sec(e+fx)+1)}-\frac{3c\tan(e+fx)}{5a^3f(\sec(e+fx)+1)^2}-\frac{2c\tan(e+fx)}{5a^3f(\sec(e+fx)+1)^3}+\frac{cx}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (c*x)/a^3 - (2*c*Tan[e + f*x])/(5*a^3*f*(1 + Sec[e + f*x])^3) - (3*c*Tan[e
+ f*x])/(5*a^3*f*(1 + Sec[e + f*x])^2) - (8*c*Tan[e + f*x])/(5*a^3*f*(1 + S
ec[e + f*x]))
```

Rule 3903

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Dist[c^n, Int[ExpandTrig[(1 + (d*csc[e + f*
x])/c)^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
&& LtQ[m + n, 2]
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
rQ[2*n]
```

Rule 3922

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*
x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x
])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ
[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^3}\,dx = \frac{\int \left(\frac{c}{(1+\sec(e+fx))^3}-\frac{c\sec(e+fx)}{(1+\sec(e+fx))^3}\right)dx}{a^3}$$
$$= \frac{c\int \frac{1}{(1+\sec(e+fx))^3}\,dx}{a^3}-\frac{c\int \frac{\sec(e+fx)}{(1+\sec(e+fx))^3}\,dx}{a^3}$$
$$= -\frac{2c\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3}-\frac{c\int \frac{-5+2\sec(e+fx)}{(1+\sec(e+fx))^2}\,dx}{5a^3}-\frac{(2c)\int \frac{\sec(e+fx)}{(1+\sec(e+fx))^2}\,dx}{5a^3}$$
$$= -\frac{2c\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3}-\frac{3c\tan(e+fx)}{5a^3f(1+\sec(e+fx))^2}+\frac{c\int \frac{15-7\sec(e+fx)}{1+\sec(e+fx)}\,dx}{15a^3}-\frac{(2c)\int \frac{\sec(e+fx)}{1+\sec(e+fx}}{15a^3}$$
$$= \frac{cx}{a^3}-\frac{2c\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3}-\frac{3c\tan(e+fx)}{5a^3f(1+\sec(e+fx))^2}-\frac{2c\tan(e+fx)}{15a^3f(1+\sec(e+fx))}-\frac{(22c)\int}{}$$
$$= \frac{cx}{a^3}-\frac{2c\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3}-\frac{3c\tan(e+fx)}{5a^3f(1+\sec(e+fx))^2}-\frac{8c\tan(e+fx)}{5a^3f(1+\sec(e+fx))}$$

Mathematica [A] time = 0.456782, size = 169, normalized size = 1.92

$$\frac{c\sec\left(\frac{e}{2}\right)\sec^5\left(\frac{1}{2}(e+fx)\right)\left(110\sin\left(e+\frac{fx}{2}\right)-90\sin\left(e+\frac{3fx}{2}\right)+40\sin\left(2e+\frac{3fx}{2}\right)-26\sin\left(2e+\frac{5fx}{2}\right)+50fx\cos\left(e+\frac{fx}{2}\right.\right.}{160}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (c*Sec[e/2]*Sec[(e + f*x)/2]^5*(50*f*x*Cos[(f*x)/2] + 50*f*x*Cos[e + (f*x)/
2] + 25*f*x*Cos[e + (3*f*x)/2] + 25*f*x*Cos[2*e + (3*f*x)/2] + 5*f*x*Cos[2*
e + (5*f*x)/2] + 5*f*x*Cos[3*e + (5*f*x)/2] - 150*Sin[(f*x)/2] + 110*Sin[e
+ (f*x)/2] - 90*Sin[e + (3*f*x)/2] + 40*Sin[2*e + (3*f*x)/2] - 26*Sin[2*e +
 (5*f*x)/2]))/(160*a^3*f)
```

Maple [A] time = 0.086, size = 79, normalized size = 0.9

$$-\frac{c}{10\,fa^3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^5+\frac{c}{2\,fa^3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3-2\,\frac{c\tan\left(1/2\,fx+e/2\right)}{fa^3}+2\,\frac{c\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)}{fa^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x)
```

```
[Out] -1/10/f*c/a^3*tan(1/2*f*x+1/2*e)^5+1/2/f*c/a^3*tan(1/2*f*x+1/2*e)^3-2/f*c/a
^3*tan(1/2*f*x+1/2*e)+2/f*c/a^3*arctan(tan(1/2*f*x+1/2*e))
```

Maxima [A] time = 1.5744, size = 215, normalized size = 2.44

$$-\frac{c\left(\frac{\frac{105\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{20\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}+\frac{3\sin\left(fx+e\right)^5}{\left(\cos\left(fx+e\right)+1\right)^5}}{a^3}-\frac{120\arctan\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)}{a^3}\right)+\frac{c\left(\frac{15\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{10\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}+\frac{3\sin\left(fx+e\right)^5}{\left(\cos\left(fx+e\right)+1\right)^5}\right)}{a^3}}{60\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/60*(c*((105*sin(f*x + e)/(cos(f*x + e) + 1) - 20*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 120*arctan(sin(
f*x + e)/(cos(f*x + e) + 1))/a^3) + c*(15*sin(f*x + e)/(cos(f*x + e) + 1) -
 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) +
1)^5)/a^3)/f
```

Fricas [A] time = 0.99497, size = 313, normalized size = 3.56

$$\frac{5\,cfx\cos\left(fx+e\right)^3+15\,cfx\cos\left(fx+e\right)^2+15\,cfx\cos\left(fx+e\right)+5\,cfx-\left(13\,c\cos\left(fx+e\right)^2+19\,c\cos\left(fx+e\right)+8\,c\right)}{5\left(a^3f\cos\left(fx+e\right)^3+3\,a^3f\cos\left(fx+e\right)^2+3\,a^3f\cos\left(fx+e\right)+a^3f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/5*(5*c*f*x*cos(f*x + e)^3 + 15*c*f*x*cos(f*x + e)^2 + 15*c*f*x*cos(f*x +
e) + 5*c*f*x - (13*c*cos(f*x + e)^2 + 19*c*cos(f*x + e) + 8*c)*sin(f*x + e)
)/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a
^3*f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{c\left(\int \frac{\sec\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int -\frac{1}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))**3,x)
```

```
[Out] -c*(Integral(sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e +
f*x) + 1), x) + Integral(-1/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e
+ f*x) + 1), x))/a**3
```

Giac [A] time = 1.37685, size = 101, normalized size = 1.15

$$\frac{\frac{10\left(fx+e\right)c}{a^3}-\frac{a^{12}c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-5\,a^{12}c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+20\,a^{12}c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{a^{15}}}{10\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/10*(10*(f*x + e)*c/a^3 - (a^12*c*tan(1/2*f*x + 1/2*e)^5 - 5*a^12*c*tan(1/
2*f*x + 1/2*e)^3 + 20*a^12*c*tan(1/2*f*x + 1/2*e))/a^15)/f
```

3.36 $\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))}\,dx$

Optimal. Leaf size=126

$$\frac{2\cot^5(e+fx)}{5a^3cf}-\frac{\cot^3(e+fx)}{3a^3cf}+\frac{\cot(e+fx)}{a^3cf}-\frac{2\csc^5(e+fx)}{5a^3cf}+\frac{4\csc^3(e+fx)}{3a^3cf}-\frac{2\csc(e+fx)}{a^3cf}+\frac{x}{a^3c}$$

```
[Out] x/(a^3*c) + Cot[e + f*x]/(a^3*c*f) - Cot[e + f*x]^3/(3*a^3*c*f) + (2*Cot[e
+ f*x]^5)/(5*a^3*c*f) - (2*Csc[e + f*x])/(a^3*c*f) + (4*Csc[e + f*x]^3)/(3*
a^3*c*f) - (2*Csc[e + f*x]^5)/(5*a^3*c*f)
```

Rubi [A] time = 0.193262, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3904, 3886, 3473, 8, 2606, 194, 2607, 30}

$$\frac{2\cot^5(e+fx)}{5a^3cf}-\frac{\cot^3(e+fx)}{3a^3cf}+\frac{\cot(e+fx)}{a^3cf}-\frac{2\csc^5(e+fx)}{5a^3cf}+\frac{4\csc^3(e+fx)}{3a^3cf}-\frac{2\csc(e+fx)}{a^3cf}+\frac{x}{a^3c}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])),x]
```

```
[Out] x/(a^3*c) + Cot[e + f*x]/(a^3*c*f) - Cot[e + f*x]^3/(3*a^3*c*f) + (2*Cot[e
+ f*x]^5)/(5*a^3*c*f) - (2*Csc[e + f*x])/(a^3*c*f) + (4*Csc[e + f*x]^3)/(3*
a^3*c*f) - (2*Csc[e + f*x]^5)/(5*a^3*c*f)
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))}\,dx = -\frac{\int \cot^6(e+fx)(c-c\sec(e+fx))^2\,dx}{a^3c^3}$$

$$= -\frac{\int \left(c^2\cot^6(e+fx) - 2c^2\cot^5(e+fx)\csc(e+fx) + c^2\cot^4(e+fx)\csc\right.}{a^3c^3}$$

$$= -\frac{\int \cot^6(e+fx)\,dx}{a^3c} - \frac{\int \cot^4(e+fx)\csc^2(e+fx)\,dx}{a^3c} + \frac{2\int \cot^5(e+fx}{a^3}$$

$$= \frac{\cot^5(e+fx)}{5a^3cf} + \frac{\int \cot^4(e+fx)\,dx}{a^3c} - \frac{\text{Subst}\left(\int x^4\,dx, x, -\cot(e+fx)\right)}{a^3cf} -$$

$$= -\frac{\cot^3(e+fx)}{3a^3cf} + \frac{2\cot^5(e+fx)}{5a^3cf} - \frac{\int \cot^2(e+fx)\,dx}{a^3c} - \frac{2\,\text{Subst}\left(\int (1-\right.}{}$$

$$= \frac{\cot(e+fx)}{a^3cf} - \frac{\cot^3(e+fx)}{3a^3cf} + \frac{2\cot^5(e+fx)}{5a^3cf} - \frac{2\csc(e+fx)}{a^3cf} + \frac{4\csc^3(e}{3a^3}$$

$$= \frac{x}{a^3c} + \frac{\cot(e+fx)}{a^3cf} - \frac{\cot^3(e+fx)}{3a^3cf} + \frac{2\cot^5(e+fx)}{5a^3cf} - \frac{2\csc(e+fx)}{a^3cf} + \frac{4}{}$$

Mathematica [A] time = 0.929026, size = 197, normalized size = 1.56

$$-\frac{\csc\left(\frac{e}{2}\right)\sec\left(\frac{e}{2}\right)\csc\left(\frac{1}{2}(e+fx)\right)\sec^5\left(\frac{1}{2}(e+fx)\right)(-445\sin(e+fx) - 356\sin(2(e+fx)) - 89\sin(3(e+fx)) + 240\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])),x]
```

```
[Out] -(Csc[e/2]*Csc[(e + f*x)/2]*Sec[e/2]*Sec[(e + f*x)/2]^5*(-150*f*x*Cos[f*x]
+ 150*f*x*Cos[2*e + f*x] - 120*f*x*Cos[e + 2*f*x] + 120*f*x*Cos[3*e + 2*f*x
] - 30*f*x*Cos[2*e + 3*f*x] + 30*f*x*Cos[4*e + 3*f*x] + 80*Sin[e] + 280*Sin
[f*x] - 445*Sin[e + f*x] - 356*Sin[2*(e + f*x)] - 89*Sin[3*(e + f*x)] + 240
*Sin[2*e + f*x] + 296*Sin[e + 2*f*x] + 120*Sin[3*e + 2*f*x] + 104*Sin[2*e +
 3*f*x]))/(3840*a^3*c*f)
```

Maple [A] time = 0.061, size = 109, normalized size = 0.9

$$-\frac{1}{40fa^3c}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^5 + \frac{5}{24fa^3c}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3 - \frac{11}{8fa^3c}\tan\left(\frac{fx}{2}+\frac{e}{2}\right) + 2\frac{\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)}{fa^3c} + \frac{1}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x)
```

```
[Out] -1/40/f/a^3/c*tan(1/2*f*x+1/2*e)^5+5/24/f/a^3/c*tan(1/2*f*x+1/2*e)^3-11/8/f
/a^3/c*tan(1/2*f*x+1/2*e)+2/f/a^3/c*arctan(tan(1/2*f*x+1/2*e))+1/8/f/a^3/c/
tan(1/2*f*x+1/2*e)
```

Maxima [A] time = 1.55291, size = 165, normalized size = 1.31

$$-\frac{\frac{\frac{165 \sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{25 \sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{3 \sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}}{a^{3}c}-\frac{240 \arctan\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)}{a^{3}c}-\frac{15 \left(\cos\left(fx+e\right)+1\right)}{a^{3}c \sin\left(fx+e\right)}}{120 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/120*((165*sin(f*x + e)/(cos(f*x + e) + 1) - 25*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3*c) - 240*arctan(si
n(f*x + e)/(cos(f*x + e) + 1))/(a^3*c) - 15*(cos(f*x + e) + 1)/(a^3*c*sin(f
*x + e)))/f
```

Fricas [A] time = 1.03937, size = 284, normalized size = 2.25

$$\frac{26 \cos\left(fx+e\right)^{3}+22 \cos\left(fx+e\right)^{2}+15\left(fx\cos\left(fx+e\right)^{2}+2\,fx\cos\left(fx+e\right)+fx\right)\sin\left(fx+e\right)-17 \cos\left(fx+e\right)-16}{15\left(a^{3}cf\cos\left(fx+e\right)^{2}+2\,a^{3}cf\cos\left(fx+e\right)+a^{3}cf\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/15*(26*cos(f*x + e)^3 + 22*cos(f*x + e)^2 + 15*(f*x*cos(f*x + e)^2 + 2*f*
x*cos(f*x + e) + f*x)*sin(f*x + e) - 17*cos(f*x + e) - 16)/((a^3*c*f*cos(f*
x + e)^2 + 2*a^3*c*f*cos(f*x + e) + a^3*c*f)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{1}{\sec^4(e+fx)+2\sec^3(e+fx)-2\sec(e+fx)-1}\,dx}{a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e)),x)
```

```
[Out] -Integral(1/(sec(e + f*x)**4 + 2*sec(e + f*x)**3 - 2*sec(e + f*x) - 1), x)/
(a**3*c)
```

Giac [A] time = 1.33161, size = 144, normalized size = 1.14

$$\frac{\frac{120\,(fx+e)}{a^3c}+\frac{15}{a^3c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}-\frac{3\,a^{12}c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-25\,a^{12}c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+165\,a^{12}c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{a^{15}c^5}}{120\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/120*(120*(f*x + e)/(a^3*c) + 15/(a^3*c*tan(1/2*f*x + 1/2*e)) - (3*a^12*c^
4*tan(1/2*f*x + 1/2*e)^5 - 25*a^12*c^4*tan(1/2*f*x + 1/2*e)^3 + 165*a^12*c^
4*tan(1/2*f*x + 1/2*e))/(a^15*c^5))/f
```

3.37 $\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx$

Optimal. Leaf size=100

$$\frac{\cot^5(e+fx)(1-\sec(e+fx))}{5a^3c^2f}-\frac{\cot^3(e+fx)(5-4\sec(e+fx))}{15a^3c^2f}+\frac{\cot(e+fx)(15-8\sec(e+fx))}{15a^3c^2f}+\frac{x}{a^3c^2}$$

```
[Out] x/(a^3*c^2) + (Cot[e + f*x]*(15 - 8*Sec[e + f*x]))/(15*a^3*c^2*f) - (Cot[e
+ f*x]^3*(5 - 4*Sec[e + f*x]))/(15*a^3*c^2*f) + (Cot[e + f*x]^5*(1 - Sec[e
+ f*x]))/(5*a^3*c^2*f)
```

Rubi [A] time = 0.142863, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3904, 3882, 8}

$$\frac{\cot^5(e+fx)(1-\sec(e+fx))}{5a^3c^2f}-\frac{\cot^3(e+fx)(5-4\sec(e+fx))}{15a^3c^2f}+\frac{\cot(e+fx)(15-8\sec(e+fx))}{15a^3c^2f}+\frac{x}{a^3c^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^2),x]
```

```
[Out] x/(a^3*c^2) + (Cot[e + f*x]*(15 - 8*Sec[e + f*x]))/(15*a^3*c^2*f) - (Cot[e
+ f*x]^3*(5 - 4*Sec[e + f*x]))/(15*a^3*c^2*f) + (Cot[e + f*x]^5*(1 - Sec[e
+ f*x]))/(5*a^3*c^2*f)
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx = -\frac{\int \cot^6(e+fx)(c-c\sec(e+fx))\,dx}{a^3c^3}$$
$$= \frac{\cot^5(e+fx)(1-\sec(e+fx))}{5a^3c^2f} - \frac{\int \cot^4(e+fx)(-5c+4c\sec(e+fx))\,d}{5a^3c^3}$$
$$= -\frac{\cot^3(e+fx)(5-4\sec(e+fx))}{15a^3c^2f} + \frac{\cot^5(e+fx)(1-\sec(e+fx))}{5a^3c^2f} - \int$$
$$= \frac{\cot(e+fx)(15-8\sec(e+fx))}{15a^3c^2f} - \frac{\cot^3(e+fx)(5-4\sec(e+fx))}{15a^3c^2f} + \frac{\mathrm{co}}{}$$
$$= \frac{x}{a^3c^2} + \frac{\cot(e+fx)(15-8\sec(e+fx))}{15a^3c^2f} - \frac{\cot^3(e+fx)(5-4\sec(e+f}{15a^3c^2f}$$

Mathematica [B] time = 0.91673, size = 257, normalized size = 2.57

$$\frac{\csc\left(\frac{e}{2}\right)\sec\left(\frac{e}{2}\right)\csc^3\left(\frac{1}{2}(e+fx)\right)\sec^5\left(\frac{1}{2}(e+fx)\right)(534\sin(e+fx)+178\sin(2(e+fx))-178\sin(3(e+fx))-89\sin(4(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^2),x]
```

```
[Out] (Csc[e/2]*Csc[(e + f*x)/2]^3*Sec[e/2]*Sec[(e + f*x)/2]^5*(360*f*x*Cos[f*x]
- 360*f*x*Cos[2*e + f*x] + 120*f*x*Cos[e + 2*f*x] - 120*f*x*Cos[3*e + 2*f*x
] - 120*f*x*Cos[2*e + 3*f*x] + 120*f*x*Cos[4*e + 3*f*x] - 60*f*x*Cos[3*e +
4*f*x] + 60*f*x*Cos[5*e + 4*f*x] - 200*Sin[e] - 584*Sin[f*x] + 534*Sin[e +
f*x] + 178*Sin[2*(e + f*x)] - 178*Sin[3*(e + f*x)] - 89*Sin[4*(e + f*x)] -
520*Sin[2*e + f*x] - 248*Sin[e + 2*f*x] - 120*Sin[3*e + 2*f*x] + 248*Sin[2*
e + 3*f*x] + 120*Sin[4*e + 3*f*x] + 184*Sin[3*e + 4*f*x]))/(30720*a^3*c^2*f
)
```

Maple [A] time = 0.063, size = 131, normalized size = 1.3

$$-\frac{1}{80\,fa^3c^2}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^5 + \frac{1}{8\,fa^3c^2}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3 - \frac{1}{fa^3c^2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right) + 2\,\frac{\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)}{fa^3c^2} - \frac{}{48\,}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x)
```

```
[Out] -1/80/f/a^3/c^2*tan(1/2*f*x+1/2*e)^5+1/8/f/a^3/c^2*tan(1/2*f*x+1/2*e)^3-1/f
/a^3/c^2*tan(1/2*f*x+1/2*e)+2/f/a^3/c^2*arctan(tan(1/2*f*x+1/2*e))-1/48/f/a
^3/c^2/tan(1/2*f*x+1/2*e)^3+3/8/f/a^3/c^2/tan(1/2*f*x+1/2*e)
```

Maxima [A] time = 1.54584, size = 197, normalized size = 1.97

$$-\frac{\frac{3\left(\frac{80\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{10\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{\sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}\right)}{a^{3}c^{2}}-\frac{480\arctan\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)}{a^{3}c^{2}}-\frac{5\left(\frac{18\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-1\right)\left(\cos\left(fx+e\right)+1\right)^{3}}{a^{3}c^{2}\sin\left(fx+e\right)^{3}}}{240\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -1/240*(3*(80*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3 + sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3*c^2) - 480*arctan(s
in(f*x + e)/(cos(f*x + e) + 1))/(a^3*c^2) - 5*(18*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 - 1)*(cos(f*x + e) + 1)^3/(a^3*c^2*sin(f*x + e)^3))/f
```

Fricas [A] time = 1.07982, size = 374, normalized size = 3.74

$$\frac{23\cos\left(fx+e\right)^{4}+8\cos\left(fx+e\right)^{3}-27\cos\left(fx+e\right)^{2}+15\left(fx\cos\left(fx+e\right)^{3}+fx\cos\left(fx+e\right)^{2}-fx\cos\left(fx+e\right)-fx\right)}{15\left(a^{3}c^{2}f\cos\left(fx+e\right)^{3}+a^{3}c^{2}f\cos\left(fx+e\right)^{2}-a^{3}c^{2}f\cos\left(fx+e\right)-a^{3}c^{2}f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/15*(23*cos(f*x + e)^4 + 8*cos(f*x + e)^3 - 27*cos(f*x + e)^2 + 15*(f*x*co
s(f*x + e)^3 + f*x*cos(f*x + e)^2 - f*x*cos(f*x + e) - f*x)*sin(f*x + e) -
7*cos(f*x + e) + 8)/((a^3*c^2*f*cos(f*x + e)^3 + a^3*c^2*f*cos(f*x + e)^2 -
 a^3*c^2*f*cos(f*x + e) - a^3*c^2*f)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{\sec^5(e+fx)+\sec^4(e+fx)-2\sec^3(e+fx)-2\sec^2(e+fx)+\sec(e+fx)+1}\,dx}{a^3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**2,x)
```

```
[Out] Integral(1/(sec(e + f*x)**5 + sec(e + f*x)**4 - 2*sec(e + f*x)**3 - 2*sec(e
 + f*x)**2 + sec(e + f*x) + 1), x)/(a**3*c**2)
```

Giac [A] time = 1.39182, size = 165, normalized size = 1.65

$$\frac{\frac{240\,(fx+e)}{a^3c^2}+\frac{5\left(18\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)}{a^3c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3}-\frac{3\left(a^{12}c^8\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-10\,a^{12}c^8\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+80\,a^{12}c^8\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{a^{15}c^{10}}}{240\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/240*(240*(f*x + e)/(a^3*c^2) + 5*(18*tan(1/2*f*x + 1/2*e)^2 - 1)/(a^3*c^2
*tan(1/2*f*x + 1/2*e)^3) - 3*(a^12*c^8*tan(1/2*f*x + 1/2*e)^5 - 10*a^12*c^8
*tan(1/2*f*x + 1/2*e)^3 + 80*a^12*c^8*tan(1/2*f*x + 1/2*e))/(a^15*c^10))/f
```

3.38 $\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3}\,dx$

Optimal. Leaf size=67

$$\frac{\cot^5(e+fx)}{5a^3c^3f}-\frac{\cot^3(e+fx)}{3a^3c^3f}+\frac{\cot(e+fx)}{a^3c^3f}+\frac{x}{a^3c^3}$$

```
[Out] x/(a^3*c^3) + Cot[e + f*x]/(a^3*c^3*f) - Cot[e + f*x]^3/(3*a^3*c^3*f) + Cot
[e + f*x]^5/(5*a^3*c^3*f)
```

Rubi [A] time = 0.0805624, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3904, 3473, 8}

$$\frac{\cot^5(e+fx)}{5a^3c^3f}-\frac{\cot^3(e+fx)}{3a^3c^3f}+\frac{\cot(e+fx)}{a^3c^3f}+\frac{x}{a^3c^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^3),x]
```

```
[Out] x/(a^3*c^3) + Cot[e + f*x]/(a^3*c^3*f) - Cot[e + f*x]^3/(3*a^3*c^3*f) + Cot
[e + f*x]^5/(5*a^3*c^3*f)
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3}\,dx = -\frac{\int \cot^6(e+fx)\,dx}{a^3c^3}$$
$$= \frac{\cot^5(e+fx)}{5a^3c^3f}+\frac{\int \cot^4(e+fx)\,dx}{a^3c^3}$$
$$= -\frac{\cot^3(e+fx)}{3a^3c^3f}+\frac{\cot^5(e+fx)}{5a^3c^3f}-\frac{\int \cot^2(e+fx)\,dx}{a^3c^3}$$
$$= \frac{\cot(e+fx)}{a^3c^3f}-\frac{\cot^3(e+fx)}{3a^3c^3f}+\frac{\cot^5(e+fx)}{5a^3c^3f}+\frac{\int 1\,dx}{a^3c^3}$$
$$= \frac{x}{a^3c^3}+\frac{\cot(e+fx)}{a^3c^3f}-\frac{\cot^3(e+fx)}{3a^3c^3f}+\frac{\cot^5(e+fx)}{5a^3c^3f}$$

Mathematica [C] time = 0.0737104, size = 39, normalized size = 0.58

$$\frac{\cot^5(e+fx)\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-\tan^2(e+fx)\right)}{5a^3c^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^3),x]
```

```
[Out] (Cot[e + f*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[e + f*x]^2])/(5*a^3*c
^3*f)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+a\sec\left(fx+e\right)\right)^3\left(c-c\sec\left(fx+e\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x)
```

```
[Out] int(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x)
```

Maxima [A] time = 1.57528, size = 76, normalized size = 1.13

$$\frac{\frac{15\,(fx+e)}{a^3c^3} + \frac{15\,\tan(fx+e)^4 - 5\,\tan(fx+e)^2 + 3}{a^3c^3\tan(fx+e)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/15*(15*(f*x + e)/(a^3*c^3) + (15*tan(f*x + e)^4 - 5*tan(f*x + e)^2 + 3)/(
a^3*c^3*tan(f*x + e)^5))/f
```

Fricas [A] time = 1.11877, size = 290, normalized size = 4.33

$$\frac{23\,\cos\left(fx+e\right)^5 - 35\,\cos\left(fx+e\right)^3 + 15\left(fx\cos\left(fx+e\right)^4 - 2\,fx\cos\left(fx+e\right)^2 + fx\right)\sin\left(fx+e\right) + 15\,\cos\left(fx+e\right)}{15\left(a^3c^3f\cos\left(fx+e\right)^4 - 2\,a^3c^3f\cos\left(fx+e\right)^2 + a^3c^3f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(23*cos(f*x + e)^5 - 35*cos(f*x + e)^3 + 15*(f*x*cos(f*x + e)^4 - 2*f*
x*cos(f*x + e)^2 + f*x)*sin(f*x + e) + 15*cos(f*x + e))/((a^3*c^3*f*cos(f*x
 + e)^4 - 2*a^3*c^3*f*cos(f*x + e)^2 + a^3*c^3*f)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{1}{\sec^6\left(e+fx\right) - 3\sec^4\left(e+fx\right) + 3\sec^2\left(e+fx\right) - 1}\,dx}{a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**3,x)
```

```
[Out] -Integral(1/(sec(e + f*x)**6 - 3*sec(e + f*x)**4 + 3*sec(e + f*x)**2 - 1),
x)/(a**3*c**3)
```

Giac [B] time = 1.45574, size = 184, normalized size = 2.75

$$\frac{\frac{480\left(fx+e\right)}{a^3c^3}+\frac{330\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-35\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+3}{a^3c^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5}-\frac{3a^{12}c^{12}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-35a^{12}c^{12}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+330a^{12}c^{12}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a^{15}c^{15}}}{480f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/480*(480*(f*x + e)/(a^3*c^3) + (330*tan(1/2*f*x + 1/2*e)^4 - 35*tan(1/2*f
*x + 1/2*e)^2 + 3)/(a^3*c^3*tan(1/2*f*x + 1/2*e)^5) - (3*a^12*c^12*tan(1/2*
f*x + 1/2*e)^5 - 35*a^12*c^12*tan(1/2*f*x + 1/2*e)^3 + 330*a^12*c^12*tan(1/
2*f*x + 1/2*e))/(a^15*c^15))/f
```

3.39 $\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4}\,dx$

Optimal. Leaf size=129

$$-\frac{\cot^7(e+fx)(\sec(e+fx)+1)}{7a^3c^4f}+\frac{\cot^5(e+fx)(6\sec(e+fx)+7)}{35a^3c^4f}-\frac{\cot^3(e+fx)(24\sec(e+fx)+35)}{105a^3c^4f}+\frac{\cot(e+fx)(16}{35}$$

```
[Out] x/(a^3*c^4) - (Cot[e + f*x]^7*(1 + Sec[e + f*x]))/(7*a^3*c^4*f) + (Cot[e +
f*x]^5*(7 + 6*Sec[e + f*x]))/(35*a^3*c^4*f) + (Cot[e + f*x]*(35 + 16*Sec[e
+ f*x]))/(35*a^3*c^4*f) - (Cot[e + f*x]^3*(35 + 24*Sec[e + f*x]))/(105*a^3*
c^4*f)
```

Rubi [A] time = 0.172945, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3904, 3882, 8}

$$-\frac{\cot^7(e+fx)(\sec(e+fx)+1)}{7a^3c^4f}+\frac{\cot^5(e+fx)(6\sec(e+fx)+7)}{35a^3c^4f}-\frac{\cot^3(e+fx)(24\sec(e+fx)+35)}{105a^3c^4f}+\frac{\cot(e+fx)(16}{35}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^4),x]
```

```
[Out] x/(a^3*c^4) - (Cot[e + f*x]^7*(1 + Sec[e + f*x]))/(7*a^3*c^4*f) + (Cot[e +
f*x]^5*(7 + 6*Sec[e + f*x]))/(35*a^3*c^4*f) + (Cot[e + f*x]*(35 + 16*Sec[e
+ f*x]))/(35*a^3*c^4*f) - (Cot[e + f*x]^3*(35 + 24*Sec[e + f*x]))/(105*a^3*
c^4*f)
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
```

```
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4}\,dx = \frac{\int \cot^8(e+fx)(a+a\sec(e+fx))\,dx}{a^4c^4}$$

$$= -\frac{\cot^7(e+fx)(1+\sec(e+fx))}{7a^3c^4f} + \frac{\int \cot^6(e+fx)(-7a-6a\sec(e+fx)}{7a^4c^4}$$

$$= -\frac{\cot^7(e+fx)(1+\sec(e+fx))}{7a^3c^4f} + \frac{\cot^5(e+fx)(7+6\sec(e+fx))}{35a^3c^4f} + \int$$

$$= -\frac{\cot^7(e+fx)(1+\sec(e+fx))}{7a^3c^4f} + \frac{\cot^5(e+fx)(7+6\sec(e+fx))}{35a^3c^4f} - \mathrm{co}$$

$$= -\frac{\cot^7(e+fx)(1+\sec(e+fx))}{7a^3c^4f} + \frac{\cot^5(e+fx)(7+6\sec(e+fx))}{35a^3c^4f} + \mathrm{co}$$

$$= \frac{x}{a^3c^4} - \frac{\cot^7(e+fx)(1+\sec(e+fx))}{7a^3c^4f} + \frac{\cot^5(e+fx)(7+6\sec(e+fx)}{35a^3c^4f}$$

Mathematica [B] time = 1.37081, size = 362, normalized size = 2.81

$$\frac{\csc\left(\frac{e}{2}\right)\sec\left(\frac{e}{2}\right)\csc^7\left(\frac{1}{2}(e+fx)\right)\sec^5\left(\frac{1}{2}(e+fx)\right)(-22860\sin(e+fx)+5715\sin(2(e+fx))+11430\sin(3(e+fx))-4}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^4),x]
```

```
[Out] (Csc[e/2]*Csc[(e + f*x)/2]^7*Sec[e/2]*Sec[(e + f*x)/2]^5*(16800*f*x*Cos[f*x
] - 16800*f*x*Cos[2*e + f*x] - 4200*f*x*Cos[e + 2*f*x] + 4200*f*x*Cos[3*e +
 2*f*x] - 8400*f*x*Cos[2*e + 3*f*x] + 8400*f*x*Cos[4*e + 3*f*x] + 3360*f*x*
Cos[3*e + 4*f*x] - 3360*f*x*Cos[5*e + 4*f*x] + 1680*f*x*Cos[4*e + 5*f*x] -
1680*f*x*Cos[6*e + 5*f*x] - 840*f*x*Cos[5*e + 6*f*x] + 840*f*x*Cos[7*e + 6*
f*x] + 3136*Sin[e] - 30112*Sin[f*x] - 22860*Sin[e + f*x] + 5715*Sin[2*(e +
f*x)] + 11430*Sin[3*(e + f*x)] - 4572*Sin[4*(e + f*x)] - 2286*Sin[5*(e + f*
x)] + 1143*Sin[6*(e + f*x)] - 26208*Sin[2*e + f*x] + 14080*Sin[e + 2*f*x] +
```

```
16400*Sin[2*e + 3*f*x] + 11760*Sin[4*e + 3*f*x] - 7904*Sin[3*e + 4*f*x] -
3360*Sin[5*e + 4*f*x] - 3952*Sin[4*e + 5*f*x] - 1680*Sin[6*e + 5*f*x] + 281
6*Sin[5*e + 6*f*x]))/(6881280*a^3*c^4*f)
```

Maple [A] time = 0.069, size = 174, normalized size = 1.4

$$-\frac{1}{320\,fa^3c^4}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^5+\frac{1}{24\,fa^3c^4}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3-\frac{29}{64\,fa^3c^4}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2\,\frac{\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)}{fa^3c^4}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x)
```

```
[Out] -1/320/f/a^3/c^4*tan(1/2*f*x+1/2*e)^5+1/24/f/a^3/c^4*tan(1/2*f*x+1/2*e)^3-2
9/64/f/a^3/c^4*tan(1/2*f*x+1/2*e)+2/f/a^3/c^4*arctan(tan(1/2*f*x+1/2*e))-1/
448/f/a^3/c^4/tan(1/2*f*x+1/2*e)^7+1/40/f/a^3/c^4/tan(1/2*f*x+1/2*e)^5-29/1
92/f/a^3/c^4/tan(1/2*f*x+1/2*e)^3+1/f/a^3/c^4/tan(1/2*f*x+1/2*e)
```

Maxima [A] time = 1.56434, size = 252, normalized size = 1.95

$$-\frac{\frac{7\left(\frac{435\,\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{40\,\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}+\frac{3\,\sin\left(fx+e\right)^5}{\left(\cos\left(fx+e\right)+1\right)^5}\right)}{a^3c^4}-\frac{13440\,\arctan\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)}{a^3c^4}-\frac{\left(\frac{168\,\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}-\frac{1015\,\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}+\frac{6720\,\sin\left(fx+e\right)^6}{\left(\cos\left(fx+e\right)+1\right)^6}-15\right)\left(\cos\left(fx+e\right)+1\right)^7}{a^3c^4\sin\left(fx+e\right)^7}}{6720\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] -1/6720*(7*(435*sin(f*x + e)/(cos(f*x + e) + 1) - 40*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3*c^4) - 13440*ar
ctan(sin(f*x + e)/(cos(f*x + e) + 1))/(a^3*c^4) - (168*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 - 1015*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 6720*sin(f*x +
 e)^6/(cos(f*x + e) + 1)^6 - 15)*(cos(f*x + e) + 1)^7/(a^3*c^4*sin(f*x + e)
^7))/f
```

Fricas [A] time = 1.06171, size = 581, normalized size = 4.5

$$\frac{176\cos\left(fx+e\right)^{6}-71\cos\left(fx+e\right)^{5}-335\cos\left(fx+e\right)^{4}+125\cos\left(fx+e\right)^{3}+225\cos\left(fx+e\right)^{2}+105\left(fx\cos\left(fx+\right.\right.}{105\left(a^{3}c^{4}f\cos\left(fx+e\right)^{5}-a^{3}c^{4}f\cos\left(fx+e\right)^{4}-2\,a^{3}c^{4}f\cos\left(fx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] 1/105*(176*cos(f*x + e)^6 - 71*cos(f*x + e)^5 - 335*cos(f*x + e)^4 + 125*co
s(f*x + e)^3 + 225*cos(f*x + e)^2 + 105*(f*x*cos(f*x + e)^5 - f*x*cos(f*x +
 e)^4 - 2*f*x*cos(f*x + e)^3 + 2*f*x*cos(f*x + e)^2 + f*x*cos(f*x + e) - f*
x)*sin(f*x + e) - 57*cos(f*x + e) - 48)/((a^3*c^4*f*cos(f*x + e)^5 - a^3*c^
4*f*cos(f*x + e)^4 - 2*a^3*c^4*f*cos(f*x + e)^3 + 2*a^3*c^4*f*cos(f*x + e)^
2 + a^3*c^4*f*cos(f*x + e) - a^3*c^4*f)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{\sec^{7}\left(e+fx\right)-\sec^{6}\left(e+fx\right)-3\sec^{5}\left(e+fx\right)+3\sec^{4}\left(e+fx\right)+3\sec^{3}\left(e+fx\right)-3\sec^{2}\left(e+fx\right)-\sec\left(e+fx\right)+1}\,dx}{a^{3}c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**4,x)
```

```
[Out] Integral(1/(sec(e + f*x)**7 - sec(e + f*x)**6 - 3*sec(e + f*x)**5 + 3*sec(e
 + f*x)**4 + 3*sec(e + f*x)**3 - 3*sec(e + f*x)**2 - sec(e + f*x) + 1), x)/
(a**3*c**4)
```

Giac [A] time = 1.51067, size = 203, normalized size = 1.57

$$\frac{\frac{6720\left(fx+e\right)}{a^{3}c^{4}}+\frac{6720\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{6}-1015\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+168\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-15}{a^{3}c^{4}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{7}}-\frac{7\left(3\,a^{12}c^{16}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{5}-40\,a^{12}c^{16}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}+435\,a^{12}c^{\cdots}\right.}{a^{15}c^{20}}}{6720\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="giac")
```

```
[Out] 1/6720*(6720*(f*x + e)/(a^3*c^4) + (6720*tan(1/2*f*x + 1/2*e)^6 - 1015*tan(
1/2*f*x + 1/2*e)^4 + 168*tan(1/2*f*x + 1/2*e)^2 - 15)/(a^3*c^4*tan(1/2*f*x
+ 1/2*e)^7) - 7*(3*a^12*c^16*tan(1/2*f*x + 1/2*e)^5 - 40*a^12*c^16*tan(1/2*
f*x + 1/2*e)^3 + 435*a^12*c^16*tan(1/2*f*x + 1/2*e))/(a^15*c^20))/f
```

3.40 $\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5}\,dx$

Optimal. Leaf size=210

$$\frac{2\cot^9(e+fx)}{9a^3c^5f}-\frac{\cot^7(e+fx)}{7a^3c^5f}+\frac{\cot^5(e+fx)}{5a^3c^5f}-\frac{\cot^3(e+fx)}{3a^3c^5f}+\frac{\cot(e+fx)}{a^3c^5f}+\frac{2\csc^9(e+fx)}{9a^3c^5f}-\frac{8\csc^7(e+fx)}{7a^3c^5f}+\frac{12\,\mathrm{cs}}{5}$$

```
[Out] x/(a^3*c^5) + Cot[e + f*x]/(a^3*c^5*f) - Cot[e + f*x]^3/(3*a^3*c^5*f) + Cot
[e + f*x]^5/(5*a^3*c^5*f) - Cot[e + f*x]^7/(7*a^3*c^5*f) + (2*Cot[e + f*x]^
9)/(9*a^3*c^5*f) + (2*Csc[e + f*x])/(a^3*c^5*f) - (8*Csc[e + f*x]^3)/(3*a^3
*c^5*f) + (12*Csc[e + f*x]^5)/(5*a^3*c^5*f) - (8*Csc[e + f*x]^7)/(7*a^3*c^5
*f) + (2*Csc[e + f*x]^9)/(9*a^3*c^5*f)
```

Rubi [A] time = 0.236406, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3904, 3886, 3473, 8, 2606, 194, 2607, 30}

$$\frac{2\cot^9(e+fx)}{9a^3c^5f}-\frac{\cot^7(e+fx)}{7a^3c^5f}+\frac{\cot^5(e+fx)}{5a^3c^5f}-\frac{\cot^3(e+fx)}{3a^3c^5f}+\frac{\cot(e+fx)}{a^3c^5f}+\frac{2\csc^9(e+fx)}{9a^3c^5f}-\frac{8\csc^7(e+fx)}{7a^3c^5f}+\frac{12\,\mathrm{cs}}{5}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^5),x]
```

```
[Out] x/(a^3*c^5) + Cot[e + f*x]/(a^3*c^5*f) - Cot[e + f*x]^3/(3*a^3*c^5*f) + Cot
[e + f*x]^5/(5*a^3*c^5*f) - Cot[e + f*x]^7/(7*a^3*c^5*f) + (2*Cot[e + f*x]^
9)/(9*a^3*c^5*f) + (2*Csc[e + f*x])/(a^3*c^5*f) - (8*Csc[e + f*x]^3)/(3*a^3
*c^5*f) + (12*Csc[e + f*x]^5)/(5*a^3*c^5*f) - (8*Csc[e + f*x]^7)/(7*a^3*c^5
*f) + (2*Csc[e + f*x]^9)/(9*a^3*c^5*f)
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
```

```
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5}\,dx = -\frac{\int \cot^{10}(e+fx)(a+a\sec(e+fx))^2\,dx}{a^5c^5}$$
$$= -\frac{\int \left(a^2\cot^{10}(e+fx)+2a^2\cot^9(e+fx)\csc(e+fx)+a^2\cot^8(e+fx)\right.}{a^5c^5}$$
$$= -\frac{\int \cot^{10}(e+fx)\,dx}{a^3c^5} - \frac{\int \cot^8(e+fx)\csc^2(e+fx)\,dx}{a^3c^5} - \frac{2\int \cot^9(e+f}{}$$
$$= \frac{\cot^9(e+fx)}{9a^3c^5f} + \frac{\int \cot^8(e+fx)\,dx}{a^3c^5} - \frac{\text{Subst}\left(\int x^8\,dx, x, -\cot(e+fx)\right)}{a^3c^5f}$$
$$= -\frac{\cot^7(e+fx)}{7a^3c^5f} + \frac{2\cot^9(e+fx)}{9a^3c^5f} - \frac{\int \cot^6(e+fx)\,dx}{a^3c^5} + \frac{2\,\text{Subst}\left(\int (1\right.}{}$$
$$= \frac{\cot^5(e+fx)}{5a^3c^5f} - \frac{\cot^7(e+fx)}{7a^3c^5f} + \frac{2\cot^9(e+fx)}{9a^3c^5f} + \frac{2\csc(e+fx)}{a^3c^5f} - \frac{8\csc}{3a}$$
$$= -\frac{\cot^3(e+fx)}{3a^3c^5f} + \frac{\cot^5(e+fx)}{5a^3c^5f} - \frac{\cot^7(e+fx)}{7a^3c^5f} + \frac{2\cot^9(e+fx)}{9a^3c^5f} + \frac{2\csc}{}$$
$$= \frac{\cot(e+fx)}{a^3c^5f} - \frac{\cot^3(e+fx)}{3a^3c^5f} + \frac{\cot^5(e+fx)}{5a^3c^5f} - \frac{\cot^7(e+fx)}{7a^3c^5f} + \frac{2\cot^9(e}{9a^3c}$$
$$= \frac{x}{a^3c^5} + \frac{\cot(e+fx)}{a^3c^5f} - \frac{\cot^3(e+fx)}{3a^3c^5f} + \frac{\cot^5(e+fx)}{5a^3c^5f} - \frac{\cot^7(e+fx)}{7a^3c^5f} + \frac{2}{}$$

Mathematica [B] time = 1.72925, size = 441, normalized size = 2.1

$$\frac{\csc\left(\frac{e}{2}\right)\sec\left(\frac{e}{2}\right)\tan(e+fx)\sec^7(e+fx)(-1152405\sin(e+fx)+512180\sin(2(e+fx))+486571\sin(3(e+fx))-4097}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^5),x]
```

```
[Out] (Csc[e/2]*Sec[e/2]*Sec[e + f*x]^7*(453600*f*x*Cos[f*x] - 453600*f*x*Cos[2*e
 + f*x] - 201600*f*x*Cos[e + 2*f*x] + 201600*f*x*Cos[3*e + 2*f*x] - 191520*
f*x*Cos[2*e + 3*f*x] + 191520*f*x*Cos[4*e + 3*f*x] + 161280*f*x*Cos[3*e + 4
*f*x] - 161280*f*x*Cos[5*e + 4*f*x] + 10080*f*x*Cos[4*e + 5*f*x] - 10080*f*
x*Cos[6*e + 5*f*x] - 40320*f*x*Cos[5*e + 6*f*x] + 40320*f*x*Cos[7*e + 6*f*x
] + 10080*f*x*Cos[6*e + 7*f*x] - 10080*f*x*Cos[8*e + 7*f*x] + 259584*Sin[e]
 - 897024*Sin[f*x] - 1152405*Sin[e + f*x] + 512180*Sin[2*(e + f*x)] + 48657
1*Sin[3*(e + f*x)] - 409744*Sin[4*(e + f*x)] - 25609*Sin[5*(e + f*x)] + 102
436*Sin[6*(e + f*x)] - 25609*Sin[7*(e + f*x)] - 825216*Sin[2*e + f*x] + 622
```

```
976*Sin[e + 2*f*x] + 142464*Sin[3*e + 2*f*x] + 297088*Sin[2*e + 3*f*x] + 43
0080*Sin[4*e + 3*f*x] - 424192*Sin[3*e + 4*f*x] - 188160*Sin[5*e + 4*f*x] +
 2048*Sin[4*e + 5*f*x] - 40320*Sin[6*e + 5*f*x] + 112768*Sin[5*e + 6*f*x] +
 40320*Sin[7*e + 6*f*x] - 38272*Sin[6*e + 7*f*x])*Tan[e + f*x])/(2580480*a^
3*c^5*f*(-1 + Sec[e + f*x])^5*(1 + Sec[e + f*x])^3)
```

Maple [A] time = 0.075, size = 197, normalized size = 0.9

$$-\frac{1}{640\,fa^3c^5}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^5+\frac{3}{128\,fa^3c^5}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3-\frac{37}{128\,fa^3c^5}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2\,\frac{\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)}{fa^3c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x)
```

```
[Out] -1/640/f/a^3/c^5*tan(1/2*f*x+1/2*e)^5+3/128/f/a^3/c^5*tan(1/2*f*x+1/2*e)^3-
37/128/f/a^3/c^5*tan(1/2*f*x+1/2*e)+2/f/a^3/c^5*arctan(tan(1/2*f*x+1/2*e))+
1/1152/f/a^3/c^5/tan(1/2*f*x+1/2*e)^9-9/896/f/a^3/c^5/tan(1/2*f*x+1/2*e)^7+
37/640/f/a^3/c^5/tan(1/2*f*x+1/2*e)^5-31/128/f/a^3/c^5/tan(1/2*f*x+1/2*e)^3
+163/128/f/a^3/c^5/tan(1/2*f*x+1/2*e)
```

Maxima [A] time = 1.54884, size = 277, normalized size = 1.32

$$-\frac{\frac{63\left(\frac{185\sin(fx+e)}{\cos(fx+e)+1}-\frac{15\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3c^5}-\frac{80640\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^3c^5}+\frac{\left(\frac{405\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{2331\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{9765\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{51345\sin(fx+e)^8}{(\cos(fx+e)+1)^8}-35\right.}{a^3c^5\sin(fx+e)^9}}{40320\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="maxima")
```

```
[Out] -1/40320*(63*(185*sin(f*x + e)/(cos(f*x + e) + 1) - 15*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 + sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3*c^5) - 80640*ar
ctan(sin(f*x + e)/(cos(f*x + e) + 1))/(a^3*c^5) + (405*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 - 2331*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 9765*sin(f*x +
 e)^6/(cos(f*x + e) + 1)^6 - 51345*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 35
)*(cos(f*x + e) + 1)^9/(a^3*c^5*sin(f*x + e)^9))/f
```

Fricas [A] time = 1.16328, size = 687, normalized size = 3.27

$$\frac{598\cos\left(fx+e\right)^{7}-566\cos\left(fx+e\right)^{6}-1212\cos\left(fx+e\right)^{5}+1310\cos\left(fx+e\right)^{4}+860\cos\left(fx+e\right)^{3}-1014\cos\left(fx+e\right)^{2}+\cdots}{315\left(a^{3}c^{5}f\cos\left(fx+e\right)^{6}-2\,a^{3}c^{5}f\cos\left(fx+e\right)^{5}-a^{3}\cdots\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="fricas")
```

```
[Out] 1/315*(598*cos(f*x + e)^7 - 566*cos(f*x + e)^6 - 1212*cos(f*x + e)^5 + 1310
*cos(f*x + e)^4 + 860*cos(f*x + e)^3 - 1014*cos(f*x + e)^2 + 315*(f*x*cos(f
*x + e)^6 - 2*f*x*cos(f*x + e)^5 - f*x*cos(f*x + e)^4 + 4*f*x*cos(f*x + e)^
3 - f*x*cos(f*x + e)^2 - 2*f*x*cos(f*x + e) + f*x)*sin(f*x + e) - 197*cos(f
*x + e) + 256)/((a^3*c^5*f*cos(f*x + e)^6 - 2*a^3*c^5*f*cos(f*x + e)^5 - a^
3*c^5*f*cos(f*x + e)^4 + 4*a^3*c^5*f*cos(f*x + e)^3 - a^3*c^5*f*cos(f*x + e
)^2 - 2*a^3*c^5*f*cos(f*x + e) + a^3*c^5*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.43953, size = 220, normalized size = 1.05

$$\frac{\frac{40320\left(fx+e\right)}{a^{3}c^{5}}+\frac{51345\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{8}-9765\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{6}+2331\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-405\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+35}{a^{3}c^{5}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{9}}-\frac{63\left(a^{12}c^{20}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{5}-15\,a^{12}c^{20}\,\mathrm{t}\cdots\right)}{\cdots}}{40320\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="giac")
```

```
[Out] 1/40320*(40320*(f*x + e)/(a^3*c^5) + (51345*tan(1/2*f*x + 1/2*e)^8 - 9765*t
an(1/2*f*x + 1/2*e)^6 + 2331*tan(1/2*f*x + 1/2*e)^4 - 405*tan(1/2*f*x + 1/2
*e)^2 + 35)/(a^3*c^5*tan(1/2*f*x + 1/2*e)^9) - 63*(a^12*c^20*tan(1/2*f*x +
1/2*e)^5 - 15*a^12*c^20*tan(1/2*f*x + 1/2*e)^3 + 185*a^12*c^20*tan(1/2*f*x
+ 1/2*e))/(a^15*c^25))/f
```

3.41 $\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6}\,dx$

Optimal. Leaf size=252

$$-\frac{4\cot^{11}(e+fx)}{11a^3c^6f}+\frac{\cot^9(e+fx)}{9a^3c^6f}-\frac{\cot^7(e+fx)}{7a^3c^6f}+\frac{\cot^5(e+fx)}{5a^3c^6f}-\frac{\cot^3(e+fx)}{3a^3c^6f}+\frac{\cot(e+fx)}{a^3c^6f}-\frac{4\csc^{11}(e+fx)}{11a^3c^6f}+\frac{19}{}$$

```
[Out] x/(a^3*c^6) + Cot[e + f*x]/(a^3*c^6*f) - Cot[e + f*x]^3/(3*a^3*c^6*f) + Cot
[e + f*x]^5/(5*a^3*c^6*f) - Cot[e + f*x]^7/(7*a^3*c^6*f) + Cot[e + f*x]^9/(
9*a^3*c^6*f) - (4*Cot[e + f*x]^11)/(11*a^3*c^6*f) + (3*Csc[e + f*x])/(a^3*c
^6*f) - (16*Csc[e + f*x]^3)/(3*a^3*c^6*f) + (34*Csc[e + f*x]^5)/(5*a^3*c^6*
f) - (36*Csc[e + f*x]^7)/(7*a^3*c^6*f) + (19*Csc[e + f*x]^9)/(9*a^3*c^6*f)
- (4*Csc[e + f*x]^11)/(11*a^3*c^6*f)
```

Rubi [A] time = 0.299534, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {3904, 3886, 3473, 8, 2606, 194, 2607, 30, 270}

$$-\frac{4\cot^{11}(e+fx)}{11a^3c^6f}+\frac{\cot^9(e+fx)}{9a^3c^6f}-\frac{\cot^7(e+fx)}{7a^3c^6f}+\frac{\cot^5(e+fx)}{5a^3c^6f}-\frac{\cot^3(e+fx)}{3a^3c^6f}+\frac{\cot(e+fx)}{a^3c^6f}-\frac{4\csc^{11}(e+fx)}{11a^3c^6f}+\frac{19}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^6),x]
```


```
[Out] x/(a^3*c^6) + Cot[e + f*x]/(a^3*c^6*f) - Cot[e + f*x]^3/(3*a^3*c^6*f) + Cot
[e + f*x]^5/(5*a^3*c^6*f) - Cot[e + f*x]^7/(7*a^3*c^6*f) + Cot[e + f*x]^9/(
9*a^3*c^6*f) - (4*Cot[e + f*x]^11)/(11*a^3*c^6*f) + (3*Csc[e + f*x])/(a^3*c
^6*f) - (16*Csc[e + f*x]^3)/(3*a^3*c^6*f) + (34*Csc[e + f*x]^5)/(5*a^3*c^6*
f) - (36*Csc[e + f*x]^7)/(7*a^3*c^6*f) + (19*Csc[e + f*x]^9)/(9*a^3*c^6*f)
- (4*Csc[e + f*x]^11)/(11*a^3*c^6*f)
```


Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6}\,dx = \frac{\int \cot^{12}(e+fx)(a+a\sec(e+fx))^3\,dx}{a^6c^6}$$
$$= \frac{\int \left(a^3\cot^{12}(e+fx)+3a^3\cot^{11}(e+fx)\csc(e+fx)+3a^3\cot^{10}(e+fx\right.}{a^6c^6}$$
$$= \frac{\int \cot^{12}(e+fx)\,dx}{a^3c^6}+\frac{\int \cot^9(e+fx)\csc^3(e+fx)\,dx}{a^3c^6}+\frac{3\int \cot^{11}(e+f}{a}$$
$$= -\frac{\cot^{11}(e+fx)}{11a^3c^6f}-\frac{\int \cot^{10}(e+fx)\,dx}{a^3c^6}-\frac{\text{Subst}\left(\int x^2\left(-1+x^2\right)^4\,dx,x,c\right.}{a^3c^6f}$$
$$= \frac{\cot^9(e+fx)}{9a^3c^6f}-\frac{4\cot^{11}(e+fx)}{11a^3c^6f}+\frac{\int \cot^8(e+fx)\,dx}{a^3c^6}-\frac{\text{Subst}\left(\int\left(x^2-\right.\right.}{}$$
$$= -\frac{\cot^7(e+fx)}{7a^3c^6f}+\frac{\cot^9(e+fx)}{9a^3c^6f}-\frac{4\cot^{11}(e+fx)}{11a^3c^6f}+\frac{3\csc(e+fx)}{a^3c^6f}-\frac{16}{}$$
$$= \frac{\cot^5(e+fx)}{5a^3c^6f}-\frac{\cot^7(e+fx)}{7a^3c^6f}+\frac{\cot^9(e+fx)}{9a^3c^6f}-\frac{4\cot^{11}(e+fx)}{11a^3c^6f}+\frac{3\csc}{a}$$
$$= -\frac{\cot^3(e+fx)}{3a^3c^6f}+\frac{\cot^5(e+fx)}{5a^3c^6f}-\frac{\cot^7(e+fx)}{7a^3c^6f}+\frac{\cot^9(e+fx)}{9a^3c^6f}-\frac{4\cot^1}{11}$$
$$= \frac{\cot(e+fx)}{a^3c^6f}-\frac{\cot^3(e+fx)}{3a^3c^6f}+\frac{\cot^5(e+fx)}{5a^3c^6f}-\frac{\cot^7(e+fx)}{7a^3c^6f}+\frac{\cot^9(e+}{9a^3c^6}$$
$$= \frac{x}{a^3c^6}+\frac{\cot(e+fx)}{a^3c^6f}-\frac{\cot^3(e+fx)}{3a^3c^6f}+\frac{\cot^5(e+fx)}{5a^3c^6f}-\frac{\cot^7(e+fx)}{7a^3c^6f}+\frac{c}{}$$

Mathematica [A] time = 2.21587, size = 499, normalized size = 1.98

$$\frac{\csc\left(\frac{e}{2}\right)\sec\left(\frac{e}{2}\right)\tan(e+fx)\sec^8(e+fx)(-86058610\sin(e+fx)+51635166\sin(2(e+fx))+26599934\sin(3(e+fx))}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^6),x]
```

```
[Out] (Csc[e/2]*Sec[e/2]*Sec[e + f*x]^8*(24393600*f*x*Cos[f*x] - 24393600*f*x*Cos
[2*e + f*x] - 14636160*f*x*Cos[e + 2*f*x] + 14636160*f*x*Cos[3*e + 2*f*x] -
 7539840*f*x*Cos[2*e + 3*f*x] + 7539840*f*x*Cos[4*e + 3*f*x] + 11088000*f*x
*Cos[3*e + 4*f*x] - 11088000*f*x*Cos[5*e + 4*f*x] - 2217600*f*x*Cos[4*e + 5
*f*x] + 2217600*f*x*Cos[6*e + 5*f*x] - 2217600*f*x*Cos[5*e + 6*f*x] + 22176
00*f*x*Cos[7*e + 6*f*x] + 1330560*f*x*Cos[6*e + 7*f*x] - 1330560*f*x*Cos[8*
```

```
e + 7*f*x] - 221760*f*x*Cos[7*e + 8*f*x] + 221760*f*x*Cos[9*e + 8*f*x] + 17
677440*Sin[e] - 49287040*Sin[f*x] - 86058610*Sin[e + f*x] + 51635166*Sin[2*
(e + f*x)] + 26599934*Sin[3*(e + f*x)] - 39117550*Sin[4*(e + f*x)] + 782351
0*Sin[5*(e + f*x)] + 7823510*Sin[6*(e + f*x)] - 4694106*Sin[7*(e + f*x)] +
782351*Sin[8*(e + f*x)] - 55651200*Sin[2*e + f*x] + 47971968*Sin[e + 2*f*x]
 + 14990976*Sin[3*e + 2*f*x] + 8100992*Sin[2*e + 3*f*x] + 24334464*Sin[4*e
+ 3*f*x] - 28627840*Sin[3*e + 4*f*x] - 19071360*Sin[5*e + 4*f*x] + 9687680*
Sin[4*e + 5*f*x] - 147840*Sin[6*e + 5*f*x] + 5548160*Sin[5*e + 6*f*x] + 399
1680*Sin[7*e + 6*f*x] - 4393344*Sin[6*e + 7*f*x] - 1330560*Sin[8*e + 7*f*x]
 + 953984*Sin[7*e + 8*f*x])*Tan[e + f*x])/(113541120*a^3*c^6*f*(-1 + Sec[e
+ f*x])^6*(1 + Sec[e + f*x])^3)
```

Maple [A] time = 0.076, size = 219, normalized size = 0.9

$$-\frac{1}{1280\,f a^3 c^6}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^5+\frac{5}{384\,f a^3 c^6}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3-\frac{23}{128\,f a^3 c^6}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2\,\frac{\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)}{f a^3 c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x)
```

```
[Out] -1/1280/f/a^3/c^6*tan(1/2*f*x+1/2*e)^5+5/384/f/a^3/c^6*tan(1/2*f*x+1/2*e)^3
-23/128/f/a^3/c^6*tan(1/2*f*x+1/2*e)+2/f/a^3/c^6*arctan(tan(1/2*f*x+1/2*e))
-1/2816/f/a^3/c^6/tan(1/2*f*x+1/2*e)^11+5/1152/f/a^3/c^6/tan(1/2*f*x+1/2*e)
^9-23/896/f/a^3/c^6/tan(1/2*f*x+1/2*e)^7+13/128/f/a^3/c^6/tan(1/2*f*x+1/2*e
)^5-1/3/f/a^3/c^6/tan(1/2*f*x+1/2*e)^3+191/128/f/a^3/c^6/tan(1/2*f*x+1/2*e)
```

Maxima [A] time = 1.60557, size = 306, normalized size = 1.21

$$-\frac{\frac{231\left(\frac{690\,\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{50\,\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}+\frac{3\,\sin\left(fx+e\right)^5}{\left(\cos\left(fx+e\right)+1\right)^5}\right)}{a^3c^6}-\frac{1774080\,\arctan\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)}{a^3c^6}-\frac{5\left(\frac{770\,\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}-\frac{4554\,\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}+\frac{18018\,\sin\left(fx+e\right)^6}{\left(\cos\left(fx+e\right)+1\right)^6}-\frac{59136\,\sin\left(fx+e\right.}{\left(\cos\left(fx+e\right)+1\right.}\right.}{a^3c^6\sin\left(fx+e\right)^{11}}}{887040\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x, algorithm="maxima")
```

```
[Out] -1/887040*(231*(690*sin(f*x + e)/(cos(f*x + e) + 1) - 50*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3*c^6) - 1774
```

```
080*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/(a^3*c^6) - 5*(770*sin(f*x + e)
^2/(cos(f*x + e) + 1)^2 - 4554*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 18018*
sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 59136*sin(f*x + e)^8/(cos(f*x + e) +
1)^8 + 264726*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 63)*(cos(f*x + e) + 1
)^11/(a^3*c^6*sin(f*x + e)^11))/f
```

Fricas [A] time = 1.16003, size = 799, normalized size = 3.17

$$\frac{7453\cos\left(fx+e\right)^{8}-11964\cos\left(fx+e\right)^{7}-11866\cos\left(fx+e\right)^{6}+30542\cos\left(fx+e\right)^{5}+90\cos\left(fx+e\right)^{4}-26438\,\mathrm{co}}{3465\left(a^{3}c^{6}f\cos\left(fx+e\right)^{7}-3\,a^{3}c^{6}f\,\mathrm{cos}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x, algorithm="fricas")
```

```
[Out] 1/3465*(7453*cos(f*x + e)^8 - 11964*cos(f*x + e)^7 - 11866*cos(f*x + e)^6 +
 30542*cos(f*x + e)^5 + 90*cos(f*x + e)^4 - 26438*cos(f*x + e)^3 + 8539*cos
(f*x + e)^2 + 3465*(f*x*cos(f*x + e)^7 - 3*f*x*cos(f*x + e)^6 + f*x*cos(f*x
 + e)^5 + 5*f*x*cos(f*x + e)^4 - 5*f*x*cos(f*x + e)^3 - f*x*cos(f*x + e)^2
+ 3*f*x*cos(f*x + e) - f*x)*sin(f*x + e) + 7671*cos(f*x + e) - 3712)/((a^3*
c^6*f*cos(f*x + e)^7 - 3*a^3*c^6*f*cos(f*x + e)^6 + a^3*c^6*f*cos(f*x + e)^
5 + 5*a^3*c^6*f*cos(f*x + e)^4 - 5*a^3*c^6*f*cos(f*x + e)^3 - a^3*c^6*f*cos
(f*x + e)^2 + 3*a^3*c^6*f*cos(f*x + e) - a^3*c^6*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**6,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30013, size = 242, normalized size = 0.96

$$\frac{\frac{887040\,(fx+e)}{a^3c^6}+\frac{5\left(264726\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{10}-59136\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}+18018\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}-4554\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+770\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-63\right)}{a^3c^6\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{11}}-\frac{231\left(3\,a\right.}{}}{887040\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x, algorithm="giac")
```

```
[Out] 1/887040*(887040*(f*x + e)/(a^3*c^6) + 5*(264726*tan(1/2*f*x + 1/2*e)^10 -
59136*tan(1/2*f*x + 1/2*e)^8 + 18018*tan(1/2*f*x + 1/2*e)^6 - 4554*tan(1/2*
f*x + 1/2*e)^4 + 770*tan(1/2*f*x + 1/2*e)^2 - 63)/(a^3*c^6*tan(1/2*f*x + 1/
2*e)^11) - 231*(3*a^12*c^24*tan(1/2*f*x + 1/2*e)^5 - 50*a^12*c^24*tan(1/2*f
*x + 1/2*e)^3 + 690*a^12*c^24*tan(1/2*f*x + 1/2*e))/(a^15*c^30))/f
```

3.42 $\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^4\,dx$

Optimal. Leaf size=175

$$\frac{2a^4c^4\tan^7(e+fx)}{7f(a\sec(e+fx)+a)^{7/2}}-\frac{2a^3c^4\tan^5(e+fx)}{5f(a\sec(e+fx)+a)^{5/2}}+\frac{2a^2c^4\tan^3(e+fx)}{3f(a\sec(e+fx)+a)^{3/2}}+\frac{2\sqrt{a}c^4\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}-\frac{2ac^4}{f\sqrt{a\,\mathrm{se}}}$$

```
[Out] (2*Sqrt[a]*c^4*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f -
 (2*a*c^4*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) + (2*a^2*c^4*Tan[e + f
*x]^3)/(3*f*(a + a*Sec[e + f*x])^(3/2)) - (2*a^3*c^4*Tan[e + f*x]^5)/(5*f*(
a + a*Sec[e + f*x])^(5/2)) + (2*a^4*c^4*Tan[e + f*x]^7)/(7*f*(a + a*Sec[e +
 f*x])^(7/2))
```

Rubi [A] time = 0.176185, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3904, 3887, 302, 203}

$$\frac{2a^4c^4\tan^7(e+fx)}{7f(a\sec(e+fx)+a)^{7/2}}-\frac{2a^3c^4\tan^5(e+fx)}{5f(a\sec(e+fx)+a)^{5/2}}+\frac{2a^2c^4\tan^3(e+fx)}{3f(a\sec(e+fx)+a)^{3/2}}+\frac{2\sqrt{a}c^4\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}-\frac{2ac^4}{f\sqrt{a\,\mathrm{se}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (2*Sqrt[a]*c^4*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f -
 (2*a*c^4*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) + (2*a^2*c^4*Tan[e + f
*x]^3)/(3*f*(a + a*Sec[e + f*x])^(3/2)) - (2*a^3*c^4*Tan[e + f*x]^5)/(5*f*(
a + a*Sec[e + f*x])^(5/2)) + (2*a^4*c^4*Tan[e + f*x]^7)/(7*f*(a + a*Sec[e +
 f*x])^(7/2))
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^4\,dx = \left(a^4c^4\right)\int \frac{\tan^8(e+fx)}{(a+a\sec(e+fx))^{7/2}}\,dx$$

$$= -\frac{\left(2a^5c^4\right)\text{Subst}\left(\int \frac{x^8}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= -\frac{\left(2a^5c^4\right)\text{Subst}\left(\int \left(-\frac{1}{a^4}+\frac{x^2}{a^3}-\frac{x^4}{a^2}+\frac{x^6}{a}+\frac{1}{a^4\left(1+ax^2\right)}\right)dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right.}{f}$$

$$= -\frac{2ac^4\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}+\frac{2a^2c^4\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}}-\frac{2a^3c^4\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5}}$$

$$= \frac{2\sqrt{a}c^4\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}-\frac{2ac^4\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}+\frac{2a^2c^4\tan^3(e+f}{3f(a+a\sec(e+f}$$

Mathematica [A] time = 1.01161, size = 121, normalized size = 0.69

$$\frac{2c^4\tan\left(\frac{1}{2}(e+fx)\right)\sec^3(e+fx)\sqrt{a(\sec(e+fx)+1)}\left((-198\cos(e+fx)+61\cos(2(e+fx))-44\cos(3(e+fx))+76)\sqrt{s}\right.}{105f\sqrt{\sec(e+fx)-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (2*c^4*(105*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Cos[e + f*x]^3 + (76 - 198*Cos[
e + f*x] + 61*Cos[2*(e + f*x)] - 44*Cos[3*(e + f*x)])*Sqrt[-1 + Sec[e + f*x
]])*Sec[e + f*x]^3*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(105*f*Sqrt
[-1 + Sec[e + f*x]])
```

Maple [B] time = 0.319, size = 391, normalized size = 2.2

$$\frac{c^4}{840 f \sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^3}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\left(105\sqrt{2}\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^3 \mathrm{Artanh}\left(1/2\,\frac{\sqrt{2}\sin\left(fx\right.}{\cos\left(fx+\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^4*(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/840*c^4/f*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(105*2^(1/2)*sin(f*x+e)*c
os(f*x+e)^3*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*
x+e)/cos(f*x+e))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(7/2)+315*2^(1/2)*sin(f*x+e
)*cos(f*x+e)^2*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin
(f*x+e)/cos(f*x+e))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(7/2)+315*2^(1/2)*sin(f*
x+e)*cos(f*x+e)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*si
n(f*x+e)/cos(f*x+e))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(7/2)+105*2^(1/2)*arcta
nh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*
(-2*cos(f*x+e)/(1+cos(f*x+e)))^(7/2)*sin(f*x+e)+2816*cos(f*x+e)^4-4768*cos(
f*x+e)^3+3008*cos(f*x+e)^2-1296*cos(f*x+e)+240)/sin(f*x+e)/cos(f*x+e)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^4*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.26172, size = 960, normalized size = 5.49

$$\left[\frac{105\left(c^4\cos\left(fx+e\right)^4+c^4\cos\left(fx+e\right)^3\right)\sqrt{-a}\log\left(\frac{2\,a\cos\left(fx+e\right)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)\sin\left(fx+e\right)+a\cos\left(fx+e\right)-a}{\cos\left(fx+e\right)+1}\right)-2\left(176\,c\right.}{105\left(f\cos\left(fx+e\right)^4+f\cos\left(fx\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^4*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/105*(105*(c^4*cos(f*x + e)^4 + c^4*cos(f*x + e)^3)*sqrt(-a)*log((2*a*cos
(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x +
e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(176*c^4*cos(
f*x + e)^3 - 122*c^4*cos(f*x + e)^2 + 66*c^4*cos(f*x + e) - 15*c^4)*sqrt((a
*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^4 + f*cos(f*
x + e)^3), -2/105*(105*(c^4*cos(f*x + e)^4 + c^4*cos(f*x + e)^3)*sqrt(a)*ar
ctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x
+ e))) + (176*c^4*cos(f*x + e)^3 - 122*c^4*cos(f*x + e)^2 + 66*c^4*cos(f*x
+ e) - 15*c^4)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos
(f*x + e)^4 + f*cos(f*x + e)^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^4\left(\int -4\sqrt{a\sec\left(e+fx\right)+a}\sec\left(e+fx\right)dx+\int 6\sqrt{a\sec\left(e+fx\right)+a}\sec^2\left(e+fx\right)dx+\int -4\sqrt{a\sec\left(e+fx\right)+a}\sec^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**4*(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] c**4*(Integral(-4*sqrt(a*sec(e + f*x) + a)*sec(e + f*x), x) + Integral(6*sq
rt(a*sec(e + f*x) + a)*sec(e + f*x)**2, x) + Integral(-4*sqrt(a*sec(e + f*x
) + a)*sec(e + f*x)**3, x) + Integral(sqrt(a*sec(e + f*x) + a)*sec(e + f*x)
**4, x) + Integral(sqrt(a*sec(e + f*x) + a), x))
```

<u>Giac</u> [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^4*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.43 $\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^3\,dx$

Optimal. Leaf size=140

$$-\frac{2a^3c^3\tan^5(e+fx)}{5f(a\sec(e+fx)+a)^{5/2}}+\frac{2a^2c^3\tan^3(e+fx)}{3f(a\sec(e+fx)+a)^{3/2}}+\frac{2\sqrt{a}c^3\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}-\frac{2ac^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

```
[Out] (2*Sqrt[a]*c^3*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f -
 (2*a*c^3*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) + (2*a^2*c^3*Tan[e + f
*x]^3)/(3*f*(a + a*Sec[e + f*x])^(3/2)) - (2*a^3*c^3*Tan[e + f*x]^5)/(5*f*(
a + a*Sec[e + f*x])^(5/2))
```

Rubi [A] time = 0.167614, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3904, 3887, 302, 203}

$$-\frac{2a^3c^3\tan^5(e+fx)}{5f(a\sec(e+fx)+a)^{5/2}}+\frac{2a^2c^3\tan^3(e+fx)}{3f(a\sec(e+fx)+a)^{3/2}}+\frac{2\sqrt{a}c^3\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}-\frac{2ac^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (2*Sqrt[a]*c^3*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f -
 (2*a*c^3*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) + (2*a^2*c^3*Tan[e + f
*x]^3)/(3*f*(a + a*Sec[e + f*x])^(3/2)) - (2*a^3*c^3*Tan[e + f*x]^5)/(5*f*(
a + a*Sec[e + f*x])^(5/2))
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
```

```
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^3\,dx = -\left(\left(a^3c^3\right)\int \frac{\tan^6(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx\right)$$

$$= \frac{\left(2a^4c^3\right)\text{Subst}\left(\int \frac{x^6}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= \frac{\left(2a^4c^3\right)\text{Subst}\left(\int \left(\frac{1}{a^3}-\frac{x^2}{a^2}+\frac{x^4}{a}-\frac{1}{a^3\left(1+ax^2\right)}\right)dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= -\frac{2ac^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2a^2c^3\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} - \frac{2a^3c^3\tan^5(e+fx}{5f(a+a\sec(e+fx}$$

$$= \frac{2\sqrt{a}c^3\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} - \frac{2ac^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2a^2c^3\tan^3(e+}{3f(a+a\sec(e+}$$

Mathematica [A] time = 1.20576, size = 111, normalized size = 0.79

$$\frac{c^3\tan\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\sqrt{a(\sec(e+fx)+1)}\left((22\cos(e+fx)-23\cos(2(e+fx))-29)\sqrt{\sec(e+fx)-1}+30\,\text{co}\right.}{15f\sqrt{\sec(e+fx)-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (c^3*(30*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Cos[e + f*x]^2 + (-29 + 22*Cos[e +
 f*x] - 23*Cos[2*(e + f*x)])*Sqrt[-1 + Sec[e + f*x]])*Sec[e + f*x]^2*Sqrt[a
*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(15*f*Sqrt[-1 + Sec[e + f*x]])
```

Maple [B] time = 0.291, size = 302, normalized size = 2.2

$$-\frac{c^3}{60\,f\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^2}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\left(15\,\sqrt{2}\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^2\mathrm{Artanh}\left(1/2\,\frac{\sqrt{2}\sin\left(fx+e\right)}{\cos\left(fx+e\right)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^3*(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/60*c^3/f*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(15*2^(1/2)*sin(f*x+e)*co
s(f*x+e)^2*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x
+e)/cos(f*x+e))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)+30*2^(1/2)*sin(f*x+e)*
cos(f*x+e)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x
+e)/cos(f*x+e))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)+15*2^(1/2)*arctanh(1/2
*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*(-2*co
s(f*x+e)/(1+cos(f*x+e)))^(5/2)*sin(f*x+e)-184*cos(f*x+e)^3+272*cos(f*x+e)^2
-112*cos(f*x+e)+24)/sin(f*x+e)/cos(f*x+e)^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^3*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.21018, size = 882, normalized size = 6.3

$$\left[\frac{15\left(c^3\cos\left(fx+e\right)^3+c^3\cos\left(fx+e\right)^2\right)\sqrt{-a}\log\left(\frac{2a\cos\left(fx+e\right)^2-2\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)\sin\left(fx+e\right)+a\cos\left(fx+e\right)-a}{\cos\left(fx+e\right)+1}\right)-2\left(23c^3\right.}{15\left(f\cos\left(fx+e\right)^3+f\cos\left(fx+e\right)^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^3*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/15*(15*(c^3*cos(f*x + e)^3 + c^3*cos(f*x + e)^2)*sqrt(-a)*log((2*a*cos(f
*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)
*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(23*c^3*cos(f*x
 + e)^2 - 11*c^3*cos(f*x + e) + 3*c^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x +
e))*sin(f*x + e))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2), -2/15*(15*(c^3*cos
(f*x + e)^3 + c^3*cos(f*x + e)^2)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/
cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + (23*c^3*cos(f*x + e)^2
 - 11*c^3*cos(f*x + e) + 3*c^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin
(f*x + e))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-c^3\left(\int 3\sqrt{a\sec\left(e+fx\right)+a}\sec\left(e+fx\right)dx+\int -3\sqrt{a\sec\left(e+fx\right)+a}\sec^2\left(e+fx\right)dx+\int\sqrt{a\sec\left(e+fx\right)+a}\sec^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**3*(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] -c**3*(Integral(3*sqrt(a*sec(e + f*x) + a)*sec(e + f*x), x) + Integral(-3*s
qrt(a*sec(e + f*x) + a)*sec(e + f*x)**2, x) + Integral(sqrt(a*sec(e + f*x)
+ a)*sec(e + f*x)**3, x) + Integral(-sqrt(a*sec(e + f*x) + a), x))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^3*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.44 $\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2\,dx$

Optimal. Leaf size=105

$$\frac{2a^2c^2\tan^3(e+fx)}{3f(a\sec(e+fx)+a)^{3/2}}+\frac{2\sqrt{a}c^2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}-\frac{2ac^2\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

```
[Out] (2*Sqrt[a]*c^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f -
 (2*a*c^2*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) + (2*a^2*c^2*Tan[e + f
*x]^3)/(3*f*(a + a*Sec[e + f*x])^(3/2))
```

Rubi [A] time = 0.158486, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3904, 3887, 302, 203}

$$\frac{2a^2c^2\tan^3(e+fx)}{3f(a\sec(e+fx)+a)^{3/2}}+\frac{2\sqrt{a}c^2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}-\frac{2ac^2\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (2*Sqrt[a]*c^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f -
 (2*a*c^2*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) + (2*a^2*c^2*Tan[e + f
*x]^3)/(3*f*(a + a*Sec[e + f*x])^(3/2))
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
```

```
tegerQ[n - 1/2]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2\,dx = \left(a^2c^2\right)\int \frac{\tan^4(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx$$
$$= -\frac{\left(2a^3c^2\right)\text{Subst}\left(\int \frac{x^4}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$
$$= -\frac{\left(2a^3c^2\right)\text{Subst}\left(\int \left(-\frac{1}{a^2}+\frac{x^2}{a}+\frac{1}{a^2\left(1+ax^2\right)}\right)dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$
$$= -\frac{2ac^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}+\frac{2a^2c^2\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}}-\frac{\left(2ac^2\right)\text{Subst}\left(\int \frac{1}{1+ax^2}\right.}{}$$
$$= \frac{2\sqrt{a}c^2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}-\frac{2ac^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}+\frac{2a^2c^2\tan^3(e+f}{3f(a+a\sec(e+f}$$

Mathematica [A] time = 0.772085, size = 97, normalized size = 0.92

$$-\frac{2c^2\tan\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\sqrt{a(\sec(e+fx)+1)}\left((4\cos(e+fx)-1)\sqrt{\sec(e+fx)-1}-3\cos(e+fx)\tan^{-1}\left(\sqrt{\sec(e\right.\right.}{3f\sqrt{\sec(e+fx)-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (-2*c^2*(-3*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Cos[e + f*x] + (-1 + 4*Cos[e +
f*x])*Sqrt[-1 + Sec[e + f*x]])*Sec[e + f*x]*Sqrt[a*(1 + Sec[e + f*x])]*Tan[
(e + f*x)/2])/(3*f*Sqrt[-1 + Sec[e + f*x]])
```

Maple [A] time = 0.263, size = 142, normalized size = 1.4

$$-\frac{c^2}{3 f \sin\left(fx+e\right) \cos\left(fx+e\right)} \left(3\sqrt{2}\cos\left(fx+e\right)\sin\left(fx+e\right)\sqrt{-2\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\operatorname{Artanh}\left(1/2\frac{\sqrt{2}\sin\left(fx+e\right)}{\cos\left(fx+e\right)}\sqrt{-2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^2*(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/3*c^2/f*(3*2^(1/2)*cos(f*x+e)*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^
(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/c
os(f*x+e))-8*cos(f*x+e)^2+10*cos(f*x+e)-2)*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^
(1/2)/sin(f*x+e)/cos(f*x+e)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^2*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.17203, size = 792, normalized size = 7.54

$$\left[\frac{3\left(c^2\cos\left(fx+e\right)^2+c^2\cos\left(fx+e\right)\right)\sqrt{-a}\log\left(\frac{2a\cos\left(fx+e\right)^2-2\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)\sin\left(fx+e\right)+a\cos\left(fx+e\right)-a}{\cos\left(fx+e\right)+1}\right)-2\left(4c^2\cos\right.}{3\left(f\cos\left(fx+e\right)^2+f\cos\left(fx+e\right)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^2*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(c^2*cos(f*x + e)^2 + c^2*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x +
 e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin
(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(4*c^2*cos(f*x + e)
 - c^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x +
e)^2 + f*cos(f*x + e)), -2/3*(3*(c^2*cos(f*x + e)^2 + c^2*cos(f*x + e))*sqr
t(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*s
in(f*x + e))) + (4*c^2*cos(f*x + e) - c^2)*sqrt((a*cos(f*x + e) + a)/cos(f*
x + e))*sin(f*x + e))/(f*cos(f*x + e)^2 + f*cos(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^2\left(\int -2\sqrt{a\sec\left(e+fx\right)+a}\sec\left(e+fx\right)dx+\int\sqrt{a\sec\left(e+fx\right)+a}\sec^2\left(e+fx\right)dx+\int\sqrt{a\sec\left(e+fx\right)+a}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**2*(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] c**2*(Integral(-2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x), x) + Integral(sqrt
(a*sec(e + f*x) + a)*sec(e + f*x)**2, x) + Integral(sqrt(a*sec(e + f*x) + a
), x))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^2*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.45 $\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))\,dx$

Optimal. Leaf size=66

$$\frac{2\sqrt{a}c\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}-\frac{2ac\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

```
[Out] (2*Sqrt[a]*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f - (
2*a*c*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]])
```

Rubi [A] time = 0.109402, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3904, 3887, 321, 203}

$$\frac{2\sqrt{a}c\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}-\frac{2ac\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x]),x]
```

```
[Out] (2*Sqrt[a]*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f - (
2*a*c*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))\,dx &= -\left((ac)\int \frac{\tan^2(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx\right)\\
&= \frac{\left(2a^2c\right)\text{Subst}\left(\int \frac{x^2}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}\\
&= -\frac{2ac\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{(2ac)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}\\
&= \frac{2\sqrt{a}c\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} - \frac{2ac\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.320407, size = 70, normalized size = 1.06

$$-\frac{2c\tan\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}\left(\sqrt{\sec(e+fx)-1}-\tan^{-1}\left(\sqrt{\sec(e+fx)-1}\right)\right)}{f\sqrt{\sec(e+fx)-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x]),x]
```

```
[Out] (-2*c*(-ArcTan[Sqrt[-1 + Sec[e + f*x]]] + Sqrt[-1 + Sec[e + f*x]])*Sqrt[a*(
1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(f*Sqrt[-1 + Sec[e + f*x]])
```

Maple [A] time = 0.212, size = 115, normalized size = 1.7

$$-\frac{c}{f\sin\left(fx+e\right)}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\left(\sqrt{2}\sin\left(fx+e\right)\mathrm{Artanh}\left(\frac{\sqrt{2}\sin\left(fx+e\right)}{2\cos\left(fx+e\right)}\sqrt{-2\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\right)\sqrt{-2\frac{\cos\left(\right.}{1+\cos}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))*(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] -c/f*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(2^(1/2)*sin(f*x+e)*arctanh(1/2*
2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*(-2*cos
(f*x+e)/(1+cos(f*x+e)))^(1/2)-2*cos(f*x+e)+2)/sin(f*x+e)
```

Maxima [B] time = 1.71311, size = 198, normalized size = 3.

$$\frac{\sqrt{a}c\arctan\left(\left(\cos\left(2\,fx+2\,e\right)^{2}+\sin\left(2\,fx+2\,e\right)^{2}+2\,\cos\left(2\,fx+2\,e\right)+1\right)^{\frac{1}{4}}\sin\left(\frac{1}{2}\arctan\left(\sin\left(2\,fx+2\,e\right),\cos\left(2\,fx\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(a)*c*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x +
2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) +
sin(f*x + e), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
 + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + cos(
f*x + e))/f
```

Fricas [A] time = 1.14571, size = 620, normalized size = 9.39

$$\left[\frac{\left(c\cos\left(fx+e\right)+c\right)\sqrt{-a}\log\left(\frac{2\,a\cos\left(fx+e\right)^{2}-2\,\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)\sin\left(fx+e\right)+a\cos\left(fx+e\right)-a}{\cos\left(fx+e\right)+1}\right)-2\,c\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sin\left(fx+\right.}{f\cos\left(fx+e\right)+f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [((c*cos(f*x + e) + c)*sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((
a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e
) - a)/(cos(f*x + e) + 1)) - 2*c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*si
n(f*x + e))/(f*cos(f*x + e) + f), -2*((c*cos(f*x + e) + c)*sqrt(a)*arctan(s
qrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))
 + c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)
+ f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-c\left(\int \sqrt{a\sec\left(e+fx\right)+a}\sec\left(e+fx\right)dx+\int -\sqrt{a\sec\left(e+fx\right)+a}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))*(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] -c*(Integral(sqrt(a*sec(e + f*x) + a)*sec(e + f*x), x) + Integral(-sqrt(a*s
ec(e + f*x) + a), x))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.46 $\int \frac{\sqrt{a+a\sec(e+fx)}}{c-c\sec(e+fx)}\,dx$

Optimal. Leaf size=69

$$\frac{2\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{cf}+\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{cf}$$

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c*f) +
 (2*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c*f)
```

Rubi [A] time = 0.147882, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3904, 3887, 325, 203}

$$\frac{2\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{cf}+\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{cf}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x]),x]
```

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c*f) +
 (2*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c*f)
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+a\sec(e+fx)}}{c-c\sec(e+fx)}\,dx &= -\frac{\int \cot^2(e+fx)(a+a\sec(e+fx))^{3/2}\,dx}{ac}\\ &= \frac{2\,\mathrm{Subst}\left(\int \frac{1}{x^2\left(1+ax^2\right)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf}\\ &= \frac{2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{cf} - \frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf}\\ &= \frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf} + \frac{2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{cf}\end{aligned}$$

Mathematica [A] time = 0.580177, size = 90, normalized size = 1.3

$$\frac{2a\tan(e+fx)\sec(e+fx)\left(\cos(e+fx)\sqrt{\sec(e+fx)-1}-(\cos(e+fx)-1)\tan^{-1}\left(\sqrt{\sec(e+fx)-1}\right)\right)}{cf(\sec(e+fx)-1)^{3/2}\sqrt{a(\sec(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x]),x]
```

```
[Out] (2*a*(-(ArcTan[Sqrt[-1 + Sec[e + f*x]]]*(-1 + Cos[e + f*x])) + Cos[e + f*x]
*Sqrt[-1 + Sec[e + f*x]])*Sec[e + f*x]*Tan[e + f*x])/(c*f*(-1 + Sec[e + f*x
])^(3/2)*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [A] time = 0.251, size = 116, normalized size = 1.7

$$-\frac{1}{fc\sin\left(fx+e\right)}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\left(\sqrt{2}\sin\left(fx+e\right)\mathrm{Artanh}\left(\frac{\sqrt{2}\sin\left(fx+e\right)}{2\cos\left(fx+e\right)}\sqrt{-2\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\right)\sqrt{-2\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x)
```

```
[Out] -1/c/f*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(2^(1/2)*sin(f*x+e)*arctanh(1/
2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*(-2*c
os(f*x+e)/(1+cos(f*x+e)))^(1/2)-2*cos(f*x+e))/sin(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{a\sec\left(fx+e\right)+a}}{c\sec\left(fx+e\right)-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(a*sec(f*x + e) + a)/(c*sec(f*x + e) - c), x)
```

Fricas [A] time = 1.46504, size = 680, normalized size = 9.86

$$\left[\frac{\sqrt{-a}\log\left(-\frac{8a\cos\left(fx+e\right)^{3}-4\left(2\cos\left(fx+e\right)^{2}-\cos\left(fx+e\right)\right)\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)-7a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)+1}\right)\sin\left(fx+e\right)+4\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\cos}{2cf\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a)*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(f*x + e)^2 - cos(f*x + e
))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(
f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) + 4*sqrt((a*cos(f*x + e) + a
)/cos(f*x + e))*cos(f*x + e))/(c*f*sin(f*x + e)), (sqrt(a)*arctan(2*sqrt(a)
*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos
(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) + 2*sqrt((a*cos(f*x + e) +
a)/cos(f*x + e))*cos(f*x + e))/(c*f*sin(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{\sqrt{a\sec\left(e+fx\right)+a}}{\sec\left(e+fx\right)-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e)),x)
```

```
[Out] -Integral(sqrt(a*sec(e + f*x) + a)/(sec(e + f*x) - 1), x)/c
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.47 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^2}\,dx$

Optimal. Leaf size=104

$$-\frac{2\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{3ac^2f}+\frac{2\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{c^2f}+\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{c^2f}$$

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c^2*f)
 + (2*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c^2*f) - (2*Cot[e + f*x]^3*(a
 + a*Sec[e + f*x])^(3/2))/(3*a*c^2*f)
```

Rubi [A] time = 0.157338, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3904, 3887, 325, 203}

$$-\frac{2\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{3ac^2f}+\frac{2\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{c^2f}+\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{c^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c^2*f)
 + (2*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c^2*f) - (2*Cot[e + f*x]^3*(a
 + a*Sec[e + f*x])^(3/2))/(3*a*c^2*f)
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
```

```
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^2}\,dx &= \frac{\int \cot^4(e+fx)(a+a\sec(e+fx))^{5/2}\,dx}{a^2c^2}\\
&= -\frac{2\,\mathrm{Subst}\left(\int \frac{1}{x^4\left(1+ax^2\right)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{ac^2f}\\
&= -\frac{2\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3ac^2f} + \frac{2\,\mathrm{Subst}\left(\int \frac{1}{x^2\left(1+ax^2\right)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^2f}\\
&= \frac{2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^2f} - \frac{2\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3ac^2f} - \frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{1+ax
\end{aligned}$$

$$= \frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^2f} + \frac{2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^2f} - \frac{2\cot^3(e+fx)(a+a\sec(e+}{3ac^2f}$$

Mathematica [C] time = 0.241069, size = 78, normalized size = 0.75

$$-\frac{2\sqrt{\cos(e+fx)}\tan\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{2},-\frac{1}{2},2\sin^2\left(\frac{1}{2}(e+fx)\right)\right)}{3c^2f(\cos(e+fx)-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (-2*Sqrt[Cos[e + f*x]]*Hypergeometric2F1[-3/2, -3/2, -1/2, 2*Sin[(e + f*x)/
2]^2]*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(3*c^2*f*(-1 + Cos[e + f
*x])^2)
```

Maple [B] time = 0.289, size = 214, normalized size = 2.1

$$-\frac{1}{3\,fc^2\sin\left(fx+e\right)\left(-1+\cos\left(fx+e\right)\right)}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\left(3\sqrt{2}\cos\left(fx+e\right)\sin\left(fx+e\right)\sqrt{-2\,\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\mathrm{Ar}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^2,x)
```

```
[Out] -1/3/c^2/f*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(3*2^(1/2)*cos(f*x+e)*sin(
f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x
+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))-3*2^(1/2)*sin(f*x+e)*arcta
nh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*
(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)-8*cos(f*x+e)^2+6*cos(f*x+e))/sin(f*x+e
)/(-1+cos(f*x+e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\sec\left(fx+e\right)+a}}{\left(c\sec\left(fx+e\right)-c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sec(f*x + e) + a)/(c*sec(f*x + e) - c)^2, x)
```

Fricas [A] time = 1.44907, size = 872, normalized size = 8.38

$$\left[\frac{3\sqrt{-a}\left(\cos\left(fx+e\right)-1\right)\log\left(-\frac{8\,a\cos\left(fx+e\right)^{3}-4\left(2\,\cos\left(fx+e\right)^{2}-\cos\left(fx+e\right)\right)\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)-7\,a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)+1}\right)\sin\left(fx+e\right)+4}{6\left(c^{2}f\cos\left(fx+e\right)-c^{2}f\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(-a)*(cos(f*x + e) - 1)*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(f*x
 + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*si
n(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) + 4*(4*
cos(f*x + e)^2 - 3*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/(
(c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e)), 1/3*(3*sqrt(a)*(cos(f*x + e) -
1)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*si
n(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) + 2*(4*c
os(f*x + e)^2 - 3*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((
c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{a\sec\left(e+fx\right)+a}}{\sec^{2}\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**2,x)
```

```
[Out] Integral(sqrt(a*sec(e + f*x) + a)/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x
)/c**2
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.48 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^3}\,dx$

Optimal. Leaf size=139

$$\frac{2\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}}{5a^2c^3f}-\frac{2\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{3ac^3f}+\frac{2\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{c^3f}+\frac{2\sqrt{a}\tan}{}$$

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c^3*f)
 + (2*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c^3*f) - (2*Cot[e + f*x]^3*(a
 + a*Sec[e + f*x])^(3/2))/(3*a*c^3*f) + (2*Cot[e + f*x]^5*(a + a*Sec[e + f*
x])^(5/2))/(5*a^2*c^3*f)
```

Rubi [A] time = 0.167437, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3904, 3887, 325, 203}

$$\frac{2\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}}{5a^2c^3f}-\frac{2\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{3ac^3f}+\frac{2\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{c^3f}+\frac{2\sqrt{a}\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c^3*f)
 + (2*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c^3*f) - (2*Cot[e + f*x]^3*(a
 + a*Sec[e + f*x])^(3/2))/(3*a*c^3*f) + (2*Cot[e + f*x]^5*(a + a*Sec[e + f*
x])^(5/2))/(5*a^2*c^3*f)
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
```

```
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^3}\,dx = -\frac{\int \cot^6(e+fx)(a+a\sec(e+fx))^{7/2}\,dx}{a^3c^3}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{1}{x^6\left(1+ax^2\right)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^2c^3f}$$

$$= \frac{2\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5a^2c^3f} - \frac{2\,\text{Subst}\left(\int \frac{1}{x^4\left(1+ax^2\right)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{ac^3f}$$

$$= -\frac{2\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3ac^3f} + \frac{2\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5a^2c^3f} + \frac{2\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^3f} - \frac{2\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3ac^3f} + \frac{2\cot^5(e+fx)(a}{5a}$$

$$= \frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^3f} + \frac{2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^3f} - \frac{2\cot^3(e+fx)(a+a\sec(e}{3ac^3f}$$

Mathematica [C] time = 0.259648, size = 78, normalized size = 0.56

$$-\frac{2\sqrt{\cos(e+fx)}\tan\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{5}{2},-\frac{3}{2},2\sin^2\left(\frac{1}{2}(e+fx)\right)\right)}{5c^3f(\cos(e+fx)-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (-2*Sqrt[Cos[e + f*x]]*Hypergeometric2F1[-5/2, -5/2, -3/2, 2*Sin[(e + f*x)/
2]^2]*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(5*c^3*f*(-1 + Cos[e + f
*x])^3)
```

Maple [B] time = 0.329, size = 311, normalized size = 2.2

$$-\frac{1+\cos(fx+e)}{15fc^3(\sin(fx+e))^3(-1+\cos(fx+e))}\sqrt{\frac{a(1+\cos(fx+e))}{\cos(fx+e)}}\left(-15(\cos(fx+e))^2\sin(fx+e)\sqrt{2}\sqrt{-2\frac{\cos(fx+}{1+\cos(fx}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^3,x)
```

```
[Out] -1/15/c^3/f*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(1+cos(f*x+e))*(-15*cos(f
*x+e)^2*sin(f*x+e)*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2
*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))+30*2^(
1/2)*cos(f*x+e)*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2
*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))-15*2^(
1/2)*sin(f*x+e)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*si
n(f*x+e)/cos(f*x+e))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)+46*cos(f*x+e)^3-7
0*cos(f*x+e)^2+30*cos(f*x+e))/sin(f*x+e)^3/(-1+cos(f*x+e))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.56565, size = 1056, normalized size = 7.6

$$\left[\frac{15\left(\cos\left(fx+e\right)^{2}-2\,\cos\left(fx+e\right)+1\right)\sqrt{-a}\log\left(-\frac{8\,a\cos\left(fx+e\right)^{3}-4\left(2\,\cos\left(fx+e\right)^{2}-\cos\left(fx+e\right)\right)\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)-7\,a\cos\left(fx+\cdots\right.}{\cos\left(fx+e\right)+1}\cdots\right.}{30\left(c^{3}f\cos\left(fx+e\right)^{2}-2\,c^{3}f\cos\left(fx+e\right)+\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/30*(15*(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*sqrt(-a)*log(-(8*a*cos(f*x
+ e)^3 - 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)
+ a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))
*sin(f*x + e) + 4*(23*cos(f*x + e)^3 - 35*cos(f*x + e)^2 + 15*cos(f*x + e))
*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((c^3*f*cos(f*x + e)^2 - 2*c^3*f*
cos(f*x + e) + c^3*f)*sin(f*x + e)), 1/15*(15*(cos(f*x + e)^2 - 2*cos(f*x +
 e) + 1)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*c
os(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x
 + e) + 2*(23*cos(f*x + e)^3 - 35*cos(f*x + e)^2 + 15*cos(f*x + e))*sqrt((a
*cos(f*x + e) + a)/cos(f*x + e)))/((c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x
+ e) + c^3*f)*sin(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{\sqrt{a\sec\left(e+fx\right)+a}}{\sec^{3}\left(e+fx\right)-3\sec^{2}\left(e+fx\right)+3\sec\left(e+fx\right)-1}\,dx}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**3,x)
```

```
[Out] -Integral(sqrt(a*sec(e + f*x) + a)/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3
*sec(e + f*x) - 1), x)/c**3
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.49 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^4}\,dx$

Optimal. Leaf size=174

$$-\frac{2\cot^7(e+fx)(a\sec(e+fx)+a)^{7/2}}{7a^3c^4f}+\frac{2\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}}{5a^2c^4f}-\frac{2\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{3ac^4f}+\frac{2}{}$$

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c^4*f)
 + (2*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c^4*f) - (2*Cot[e + f*x]^3*(a
 + a*Sec[e + f*x])^(3/2))/(3*a*c^4*f) + (2*Cot[e + f*x]^5*(a + a*Sec[e + f*
x])^(5/2))/(5*a^2*c^4*f) - (2*Cot[e + f*x]^7*(a + a*Sec[e + f*x])^(7/2))/(7
*a^3*c^4*f)
```

Rubi [A] time = 0.176576, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3904, 3887, 325, 203}

$$-\frac{2\cot^7(e+fx)(a\sec(e+fx)+a)^{7/2}}{7a^3c^4f}+\frac{2\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}}{5a^2c^4f}-\frac{2\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{3ac^4f}+\frac{2}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c^4*f)
 + (2*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c^4*f) - (2*Cot[e + f*x]^3*(a
 + a*Sec[e + f*x])^(3/2))/(3*a*c^4*f) + (2*Cot[e + f*x]^5*(a + a*Sec[e + f*
x])^(5/2))/(5*a^2*c^4*f) - (2*Cot[e + f*x]^7*(a + a*Sec[e + f*x])^(7/2))/(7
*a^3*c^4*f)
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^4}\,dx = \frac{\int \cot^8(e+fx)(a+a\sec(e+fx))^{9/2}\,dx}{a^4c^4}$$

$$= -\frac{2\,\mathrm{Subst}\left(\int \frac{1}{x^8(1+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^3c^4f}$$

$$= -\frac{2\cot^7(e+fx)(a+a\sec(e+fx))^{7/2}}{7a^3c^4f} + \frac{2\,\mathrm{Subst}\left(\int \frac{1}{x^6(1+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^2c^4f}$$

$$= \frac{2\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5a^2c^4f} - \frac{2\cot^7(e+fx)(a+a\sec(e+fx))^{7/2}}{7a^3c^4f} - \frac{2\,\mathrm{Subst}\left(\int \frac{}{x^4}\right.}{}$$

$$= -\frac{2\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3ac^4f} + \frac{2\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5a^2c^4f} - \frac{2\cot^7(e+f}{}$$

$$= \frac{2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^4f} - \frac{2\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3ac^4f} + \frac{2\cot^5(e+fx)(a}{5a}$$

$$= \frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^4f} + \frac{2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^4f} - \frac{2\cot^3(e+fx)(a+a\sec(e}{3ac^4f}$$

Mathematica [C] time = 0.249655, size = 78, normalized size = 0.45

$$-\frac{2\sqrt{\cos(e+fx)}\tan\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}\,\mathrm{Hypergeometric2F1}\left(-\frac{7}{2},-\frac{7}{2},-\frac{5}{2},2\sin^2\left(\frac{1}{2}(e+fx)\right)\right)}{7c^4f(\cos(e+fx)-1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (-2*Sqrt[Cos[e + f*x]]*Hypergeometric2F1[-7/2, -7/2, -5/2, 2*Sin[(e + f*x)/
2]^2]*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(7*c^4*f*(-1 + Cos[e + f
*x])^4)
```

Maple [B] time = 0.37, size = 402, normalized size = 2.3

$$-\frac{\left(1+\cos\left(fx+e\right)\right)^2}{105\,fc^4\left(\sin\left(fx+e\right)\right)^5\left(-1+\cos\left(fx+e\right)\right)}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\left(105\sqrt{2}\left(\cos\left(fx+e\right)\right)^3\sin\left(fx+e\right)\sqrt{-2\frac{\cos\left(f\right.}{1+\cos\left(\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^4,x)
```

```
[Out] -1/105/c^4/f*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(1+cos(f*x+e))^2*(105*2^
(1/2)*cos(f*x+e)^3*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(
1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))-315
*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arcta
nh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))+
315*2^(1/2)*cos(f*x+e)*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arct
anh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))
-105*2^(1/2)*sin(f*x+e)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^
(1/2)*sin(f*x+e)/cos(f*x+e))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)-352*cos(f
*x+e)^4+812*cos(f*x+e)^3-700*cos(f*x+e)^2+210*cos(f*x+e))/sin(f*x+e)^5/(-1+
cos(f*x+e))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.64564, size = 1245, normalized size = 7.16

$$\left[\frac{105\left(\cos\left(fx+e\right)^{3}-3\,\cos\left(fx+e\right)^{2}+3\,\cos\left(fx+e\right)-1\right)\sqrt{-a}\log\left(-\frac{8\,a\cos\left(fx+e\right)^{3}-4\left(2\,\cos\left(fx+e\right)^{2}-\cos\left(fx+e\right)\right)\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}}{\cos\left(fx+e\right)+1}\right.}{210\left(c^{4}f\cos\left(fx+e\right)^{3}-3\,c^{4}f\cos\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] [1/210*(105*(cos(f*x + e)^3 - 3*cos(f*x + e)^2 + 3*cos(f*x + e) - 1)*sqrt(-
a)*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*
sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a
)/(cos(f*x + e) + 1))*sin(f*x + e) + 4*(176*cos(f*x + e)^4 - 406*cos(f*x +
e)^3 + 350*cos(f*x + e)^2 - 105*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos
(f*x + e)))/((c^4*f*cos(f*x + e)^3 - 3*c^4*f*cos(f*x + e)^2 + 3*c^4*f*cos(f
*x + e) - c^4*f)*sin(f*x + e)), 1/105*(105*(cos(f*x + e)^3 - 3*cos(f*x + e)
^2 + 3*cos(f*x + e) - 1)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)
/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x +
e) - a))*sin(f*x + e) + 2*(176*cos(f*x + e)^4 - 406*cos(f*x + e)^3 + 350*co
s(f*x + e)^2 - 105*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/(
(c^4*f*cos(f*x + e)^3 - 3*c^4*f*cos(f*x + e)^2 + 3*c^4*f*cos(f*x + e) - c^4
*f)*sin(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{a\sec\left(e+fx\right)+a}}{\sec^4\left(e+fx\right)-4\sec^3\left(e+fx\right)+6\sec^2\left(e+fx\right)-4\sec\left(e+fx\right)+1}\,dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**4,x)
```

```
[Out] Integral(sqrt(a*sec(e + f*x) + a)/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*
sec(e + f*x)**2 - 4*sec(e + f*x) + 1), x)/c**4
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^4,x, algorithm="giac")
```

```
[Out] Timed out
```

3.50 $\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3\,dx$

Optimal. Leaf size=177

$$-\frac{2a^5c^3\tan^7(e+fx)}{7f(a\sec(e+fx)+a)^{7/2}}-\frac{2a^4c^3\tan^5(e+fx)}{5f(a\sec(e+fx)+a)^{5/2}}+\frac{2a^3c^3\tan^3(e+fx)}{3f(a\sec(e+fx)+a)^{3/2}}+\frac{2a^{3/2}c^3\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}-\frac{2a^2c^3}{f\sqrt{a\,\mathrm{s}}}$$

```
[Out] (2*a^(3/2)*c^3*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f -
 (2*a^2*c^3*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) + (2*a^3*c^3*Tan[e +
 f*x]^3)/(3*f*(a + a*Sec[e + f*x])^(3/2)) - (2*a^4*c^3*Tan[e + f*x]^5)/(5*f
*(a + a*Sec[e + f*x])^(5/2)) - (2*a^5*c^3*Tan[e + f*x]^7)/(7*f*(a + a*Sec[e
 + f*x])^(7/2))
```

Rubi [A] time = 0.183837, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3904, 3887, 459, 302, 203}

$$-\frac{2a^5c^3\tan^7(e+fx)}{7f(a\sec(e+fx)+a)^{7/2}}-\frac{2a^4c^3\tan^5(e+fx)}{5f(a\sec(e+fx)+a)^{5/2}}+\frac{2a^3c^3\tan^3(e+fx)}{3f(a\sec(e+fx)+a)^{3/2}}+\frac{2a^{3/2}c^3\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}-\frac{2a^2c^3}{f\sqrt{a\,\mathrm{s}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^3,x]
```


```
[Out] (2*a^(3/2)*c^3*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f -
 (2*a^2*c^3*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) + (2*a^3*c^3*Tan[e +
 f*x]^3)/(3*f*(a + a*Sec[e + f*x])^(3/2)) - (2*a^4*c^3*Tan[e + f*x]^5)/(5*f
*(a + a*Sec[e + f*x])^(5/2)) - (2*a^5*c^3*Tan[e + f*x]^7)/(7*f*(a + a*Sec[e
 + f*x])^(7/2))
```


Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3\,dx = -\left(\left(a^3c^3\right)\int \frac{\tan^6(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx\right)$$

$$= \frac{\left(2a^5c^3\right)\operatorname{Subst}\left(\int \frac{x^6(2+ax^2)}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= -\frac{2a^5c^3\tan^7(e+fx)}{7f(a+a\sec(e+fx))^{7/2}} + \frac{\left(2a^5c^3\right)\operatorname{Subst}\left(\int \frac{x^6}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right.}{f}$$

$$= -\frac{2a^5c^3\tan^7(e+fx)}{7f(a+a\sec(e+fx))^{7/2}} + \frac{\left(2a^5c^3\right)\operatorname{Subst}\left(\int\left(\frac{1}{a^3}-\frac{x^2}{a^2}+\frac{x^4}{a}-\frac{1}{a^3(1+ax^2)}\right)\right.}{f}$$

$$= -\frac{2a^2c^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2a^3c^3\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} - \frac{2a^4c^3\tan^5(e+fx}{5f(a+a\sec(e+fx}$$

$$= \frac{2a^{3/2}c^3\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} - \frac{2a^2c^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2a^3c^3\tan^3(e+}{3f(a+a\sec(e+}$$

Mathematica [A] time = 0.958505, size = 122, normalized size = 0.69

$$\frac{ac^3\tan\left(\frac{1}{2}(e+fx)\right)\sec^3(e+fx)\sqrt{a(\sec(e+fx)+1)}\left((-171\cos(e+fx)+32\cos(2(e+fx))-73\cos(3(e+fx))+2)\sqrt{\text{se}}\right.}{105f\sqrt{\sec(e+fx)-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (a*c^3*(210*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Cos[e + f*x]^3 + (2 - 171*Cos[e
 + f*x] + 32*Cos[2*(e + f*x)] - 73*Cos[3*(e + f*x)])*Sqrt[-1 + Sec[e + f*x]
])*Sec[e + f*x]^3*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(105*f*Sqrt[
-1 + Sec[e + f*x]])
```

Maple [B] time = 0.272, size = 392, normalized size = 2.2

$$\frac{ac^3}{840f\left(\cos\left(fx+e\right)\right)^3\sin\left(fx+e\right)}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\left(105\sqrt{2}\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^3\operatorname{Artanh}\left(1/2\frac{\sqrt{2}\sin\left(fx+\right.}{\cos\left(fx+e\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^3,x)
```

```
[Out] 1/840*c^3/f*a*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(105*2^(1/2)*sin(f*x+e)
*cos(f*x+e)^3*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(
f*x+e)/cos(f*x+e))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(7/2)+315*2^(1/2)*sin(f*x
+e)*cos(f*x+e)^2*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*s
in(f*x+e)/cos(f*x+e))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(7/2)+315*2^(1/2)*sin(
f*x+e)*cos(f*x+e)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*
sin(f*x+e)/cos(f*x+e))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(7/2)+105*2^(1/2)*arc
tanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e)
)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(7/2)*sin(f*x+e)+2336*cos(f*x+e)^4-2848*co
s(f*x+e)^3+128*cos(f*x+e)^2+624*cos(f*x+e)-240)/cos(f*x+e)^3/sin(f*x+e)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.27523, size = 990, normalized size = 5.59

$$\left[\frac{105\left(ac^{3}\cos\left(fx+e\right)^{4}+ac^{3}\cos\left(fx+e\right)^{3}\right)\sqrt{-a}\log\left(\frac{2\,a\cos\left(fx+e\right)^{2}-2\,\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)\sin\left(fx+e\right)+a\cos\left(fx+e\right)-a}{\cos\left(fx+e\right)+1}\right)-2\left(1\cdots\right.}{105\left(f\cos\left(fx+e\right)^{4}+f\cos\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/105*(105*(a*c^3*cos(f*x + e)^4 + a*c^3*cos(f*x + e)^3)*sqrt(-a)*log((2*a
*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*
x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(146*a*c^
3*cos(f*x + e)^3 - 32*a*c^3*cos(f*x + e)^2 - 24*a*c^3*cos(f*x + e) + 15*a*c
^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^4
 + f*cos(f*x + e)^3), -2/105*(105*(a*c^3*cos(f*x + e)^4 + a*c^3*cos(f*x + e
)^3)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(s
qrt(a)*sin(f*x + e))) + (146*a*c^3*cos(f*x + e)^3 - 32*a*c^3*cos(f*x + e)^2
 - 24*a*c^3*cos(f*x + e) + 15*a*c^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)
)*sin(f*x + e))/(f*cos(f*x + e)^4 + f*cos(f*x + e)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.51 $\int (a + a\sec(e + fx))^{3/2}(c - c\sec(e + fx))^2\,dx$

Optimal. Leaf size=142

$$\frac{2a^4c^2\tan^5(e+fx)}{5f(a\sec(e+fx)+a)^{5/2}}+\frac{2a^3c^2\tan^3(e+fx)}{3f(a\sec(e+fx)+a)^{3/2}}+\frac{2a^{3/2}c^2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}-\frac{2a^2c^2\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

```
[Out] (2*a^(3/2)*c^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f -
 (2*a^2*c^2*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) + (2*a^3*c^2*Tan[e +
 f*x]^3)/(3*f*(a + a*Sec[e + f*x])^(3/2)) + (2*a^4*c^2*Tan[e + f*x]^5)/(5*f
*(a + a*Sec[e + f*x])^(5/2))
```

Rubi [A] time = 0.170043, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3904, 3887, 459, 302, 203}

$$\frac{2a^4c^2\tan^5(e+fx)}{5f(a\sec(e+fx)+a)^{5/2}}+\frac{2a^3c^2\tan^3(e+fx)}{3f(a\sec(e+fx)+a)^{3/2}}+\frac{2a^{3/2}c^2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}-\frac{2a^2c^2\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (2*a^(3/2)*c^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f -
 (2*a^2*c^2*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) + (2*a^3*c^2*Tan[e +
 f*x]^3)/(3*f*(a + a*Sec[e + f*x])^(3/2)) + (2*a^4*c^2*Tan[e + f*x]^5)/(5*f
*(a + a*Sec[e + f*x])^(5/2))
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
```

```
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2\,dx = \left(a^2c^2\right)\int \frac{\tan^4(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx$$

$$= -\frac{\left(2a^4c^2\right)\operatorname{Subst}\left(\int \frac{x^4(2+ax^2)}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= \frac{2a^4c^2\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}} - \frac{\left(2a^4c^2\right)\operatorname{Subst}\left(\int \frac{x^4}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right.}{f}$$

$$= \frac{2a^4c^2\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}} - \frac{\left(2a^4c^2\right)\operatorname{Subst}\left(\int \left(-\frac{1}{a^2}+\frac{x^2}{a}+\frac{1}{a^2(1+ax^2)}\right)dx\right.}{f}$$

$$= -\frac{2a^2c^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2a^3c^2\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} + \frac{2a^4c^2\tan^5(e+}{5f(a+a\sec(e+f}$$

$$= \frac{2a^{3/2}c^2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} - \frac{2a^2c^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2a^3c^2\tan^3(e}{3f(a+a\sec(e}$$

Mathematica [A] time = 0.84158, size = 112, normalized size = 0.79

$$-\frac{ac^2\tan\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\sqrt{a(\sec(e+fx)+1)}\left((2\cos(e+fx)+17\cos(2(e+fx))+11)\sqrt{\sec(e+fx)-1}-30\,c\right.}{15f\sqrt{\sec(e+fx)-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] -(a*c^2*(-30*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Cos[e + f*x]^2 + (11 + 2*Cos[e
 + f*x] + 17*Cos[2*(e + f*x)])*Sqrt[-1 + Sec[e + f*x]])*Sec[e + f*x]^2*Sqrt
[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(15*f*Sqrt[-1 + Sec[e + f*x]])
```

Maple [A] time = 0.247, size = 232, normalized size = 1.6

$$\frac{c^2a}{30\,f\left(\cos\left(fx+e\right)\right)^2\sin\left(fx+e\right)}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\left(15\operatorname{Artanh}\left(1/2\,\frac{\sqrt{2}\sin\left(fx+e\right)}{\cos\left(fx+e\right)}\sqrt{-2\,\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\right)\left(-2\,\frac{}{1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^2,x)
```

```
[Out] 1/30*c^2/f*a*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(15*arctanh(1/2*2^(1/2)*
(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*(-2*cos(f*x+e)/
(1+cos(f*x+e)))^(3/2)*2^(1/2)*sin(f*x+e)*cos(f*x+e)^2+15*arctanh(1/2*2^(1/2
)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*(-2*cos(f*x+e
)/(1+cos(f*x+e)))^(3/2)*2^(1/2)*sin(f*x+e)*cos(f*x+e)+68*cos(f*x+e)^3-64*co
s(f*x+e)^2-16*cos(f*x+e)+12)/cos(f*x+e)^2/sin(f*x+e)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.19576, size = 900, normalized size = 6.34

$$\left[\frac{15\left(ac^2\cos\left(fx+e\right)^3+ac^2\cos\left(fx+e\right)^2\right)\sqrt{-a}\log\left(\frac{2a\cos\left(fx+e\right)^2-2\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)\sin\left(fx+e\right)+a\cos\left(fx+e\right)-a}{\cos\left(fx+e\right)+1}\right)-2\left(17a\right.}{15\left(f\cos\left(fx+e\right)^3+f\cos\left(fx+e\right)^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/15*(15*(a*c^2*cos(f*x + e)^3 + a*c^2*cos(f*x + e)^2)*sqrt(-a)*log((2*a*c
os(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x
+ e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(17*a*c^2*c
os(f*x + e)^2 + a*c^2*cos(f*x + e) - 3*a*c^2)*sqrt((a*cos(f*x + e) + a)/cos
(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2), -2/15*(15*(
a*c^2*cos(f*x + e)^3 + a*c^2*cos(f*x + e)^2)*sqrt(a)*arctan(sqrt((a*cos(f*x
```

```
 + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + (17*a*c^2*c
os(f*x + e)^2 + a*c^2*cos(f*x + e) - 3*a*c^2)*sqrt((a*cos(f*x + e) + a)/cos
(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^2\left(\int a\sqrt{a\sec\left(e+fx\right)+a}\,dx+\int -a\sqrt{a\sec\left(e+fx\right)+a}\sec\left(e+fx\right)dx+\int -a\sqrt{a\sec\left(e+fx\right)+a}\sec^2\left(e+fx\right)dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e))**2,x)
```

```
[Out] c**2*(Integral(a*sqrt(a*sec(e + f*x) + a), x) + Integral(-a*sqrt(a*sec(e +
f*x) + a)*sec(e + f*x), x) + Integral(-a*sqrt(a*sec(e + f*x) + a)*sec(e + f
*x)**2, x) + Integral(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**3, x))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.52 $\int (a + a\sec(e + fx))^{3/2}(c - c\sec(e + fx))\,dx$

Optimal. Leaf size=101

$$-\frac{2a^3c\tan^3(e+fx)}{3f(a\sec(e+fx)+a)^{3/2}}+\frac{2a^{3/2}c\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}-\frac{2a^2c\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

```
[Out] (2*a^(3/2)*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f - (
2*a^2*c*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) - (2*a^3*c*Tan[e + f*x]^
3)/(3*f*(a + a*Sec[e + f*x])^(3/2))
```

Rubi [A] time = 0.123469, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3904, 3887, 459, 321, 203}

$$-\frac{2a^3c\tan^3(e+fx)}{3f(a\sec(e+fx)+a)^{3/2}}+\frac{2a^{3/2}c\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}-\frac{2a^2c\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x]),x]
```

```
[Out] (2*a^(3/2)*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f - (
2*a^2*c*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) - (2*a^3*c*Tan[e + f*x]^
3)/(3*f*(a + a*Sec[e + f*x])^(3/2))
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
```

```
tegerQ[n - 1/2]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))\,dx = -\left((ac)\int \sqrt{a+a\sec(e+fx)}\tan^2(e+fx)\,dx\right)$$

$$= \frac{\left(2a^3c\right)\text{Subst}\left(\int \frac{x^2\left(2+ax^2\right)}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= -\frac{2a^3c\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} + \frac{\left(2a^3c\right)\text{Subst}\left(\int \frac{x^2}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right.}{f}$$

$$= -\frac{2a^2c\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{2a^3c\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} - \frac{\left(2a^2c\right)\text{Subst}\left(\int \frac{}{1+}\right.}{}$$

$$= \frac{2a^{3/2}c\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} - \frac{2a^2c\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{2a^3c\tan^3(e+}{3f(a+a\sec(e+}$$

Mathematica [A] time = 0.669873, size = 96, normalized size = 0.95

$$-\frac{2ac\tan\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\sqrt{a(\sec(e+fx)+1)}\left((2\cos(e+fx)+1)\sqrt{\sec(e+fx)-1}-3\cos(e+fx)\tan^{-1}\left(\sqrt{\sec(e+fx)-1}\right)\right)}{3f\sqrt{\sec(e+fx)-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x]),x]
```

```
[Out] (-2*a*c*(-3*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Cos[e + f*x] + (1 + 2*Cos[e + f
*x])*Sqrt[-1 + Sec[e + f*x]])*Sec[e + f*x]*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(
e + f*x)/2])/(3*f*Sqrt[-1 + Sec[e + f*x]])
```

Maple [B] time = 0.222, size = 212, normalized size = 2.1

$$\frac{ac}{6f\cos(fx+e)\sin(fx+e)}\sqrt{\frac{a(1+\cos(fx+e))}{\cos(fx+e)}}\left(3\,\mathrm{Artanh}\left(1/2\,\frac{\sqrt{2}\sin(fx+e)}{\cos(fx+e)}\sqrt{-2\,\frac{\cos(fx+e)}{1+\cos(fx+e)}}\right)\left(-2\,\frac{\cos(fx+e)}{1+\cos(fx+e)}\right)^{3/2}\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e)),x)
```

```
[Out] 1/6*c/f*a*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(3*arctanh(1/2*2^(1/2)*(-2*
cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*(-2*cos(f*x+e)/(1+c
os(f*x+e)))^(3/2)*2^(1/2)*sin(f*x+e)*cos(f*x+e)+3*2^(1/2)*arctanh(1/2*2^(1/
2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*(-2*cos(f*x+
e)/(1+cos(f*x+e)))^(3/2)*sin(f*x+e)+8*cos(f*x+e)^2-4*cos(f*x+e)-4)/cos(f*x+
e)/sin(f*x+e)
```

Maxima [B] time = 1.91235, size = 1347, normalized size = 13.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/2*((a*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*
e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(1/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - cos(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e)))), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
 + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*cos(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sin(1/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e)))) + 1) - a*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*co
s(2*f*x + 2*e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e)))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - cos(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e)))), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(
2*f*x + 2*e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
 + 1))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sin(1/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e)))) - 1) - a*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2
*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x
 + 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))
 + 1) + a*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x +
2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)), (
cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*cos
(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 1))*(cos(2*f*x + 2*
e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sqrt(a) + 4*(a*co
s(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e))) - (a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e))) - a)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1
)))*sqrt(a))*c/((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*
e) + 1)^(1/4)*f)
```

Fricas [A] time = 1.1727, size = 792, normalized size = 7.84

$$\left[\frac{3\left(ac\cos\left(fx+e\right)^2+ac\cos\left(fx+e\right)\right)\sqrt{-a}\log\left(\frac{2\,a\cos\left(fx+e\right)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)\sin\left(fx+e\right)+a\cos\left(fx+e\right)-a}{\cos\left(fx+e\right)+1}\right)-2\left(2\,ac\cos\right.}{3\left(f\cos\left(fx+e\right)^2+f\cos\left(fx+e\right)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(a*c*cos(f*x + e)^2 + a*c*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x +
 e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin
(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(2*a*c*cos(f*x + e)
 + a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x +
e)^2 + f*cos(f*x + e)), -2/3*(3*(a*c*cos(f*x + e)^2 + a*c*cos(f*x + e))*sqr
t(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*s
in(f*x + e))) + (2*a*c*cos(f*x + e) + a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*
x + e))*sin(f*x + e))/(f*cos(f*x + e)^2 + f*cos(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-c\left(\int -a\sqrt{a\sec\left(e+fx\right)+a}\,dx+\int a\sqrt{a\sec\left(e+fx\right)+a}\sec^2\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e)),x)
```

```
[Out] -c*(Integral(-a*sqrt(a*sec(e + f*x) + a), x) + Integral(a*sqrt(a*sec(e + f*
x) + a)*sec(e + f*x)**2, x))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.53 $\int \frac{(a+a\sec(e+fx))^{3/2}}{c-c\sec(e+fx)}\,dx$

Optimal. Leaf size=70

$$\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{cf}+\frac{4a\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{cf}$$

```
[Out] (2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c*f) +
 (4*a*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c*f)
```

Rubi [A] time = 0.156149, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3904, 3887, 453, 203}

$$\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{cf}+\frac{4a\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{cf}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x]),x]
```

```
[Out] (2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c*f) +
 (4*a*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c*f)
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sec(e+fx))^{3/2}}{c-c\sec(e+fx)}\,dx &= -\frac{\int \cot^2(e+fx)(a+a\sec(e+fx))^{5/2}\,dx}{ac}\\
&= \frac{(2a)\,\text{Subst}\left(\int \frac{2+ax^2}{x^2\left(1+ax^2\right)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf}\\
&= \frac{4a\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{cf}-\frac{\left(2a^2\right)\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf}\\
&= \frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf}+\frac{4a\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{cf}
\end{aligned}$$

Mathematica [A] time = 0.57662, size = 93, normalized size = 1.33

$$\frac{2a^2\tan(e+fx)\sec(e+fx)\left(2\cos(e+fx)\sqrt{\sec(e+fx)-1}-(\cos(e+fx)-1)\tan^{-1}\left(\sqrt{\sec(e+fx)-1}\right)\right)}{cf(\sec(e+fx)-1)^{3/2}\sqrt{a(\sec(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x]),x]
```

```
[Out] (2*a^2*(-(ArcTan[Sqrt[-1 + Sec[e + f*x]]]*(-1 + Cos[e + f*x])) + 2*Cos[e +
f*x]*Sqrt[-1 + Sec[e + f*x]])*Sec[e + f*x]*Tan[e + f*x])/(c*f*(-1 + Sec[e +
```

```
f*x])^(3/2)*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [B] time = 0.191, size = 194, normalized size = 2.8

$$-\frac{a}{fc\left(\left(\cos\left(fx+e\right)\right)^{2}-1\right)}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\left(\sqrt{2}\left(\cos\left(fx+e\right)\right)^{2}\sqrt{-2\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\operatorname{Artanh}\left(\frac{\sqrt{2}\sin\left(fx+e\right)}{2\cos\left(fx+e\right)}\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e)),x)
```

```
[Out] -1/c/f*a*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(2^(1/2)*cos(f*x+e)^2*(-2*co
s(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*
x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))-2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^
(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/c
os(f*x+e))+4*cos(f*x+e)*sin(f*x+e))/(cos(f*x+e)^2-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(a\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}}{c\sec\left(fx+e\right)-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] -integrate((a*sec(f*x + e) + a)^(3/2)/(c*sec(f*x + e) - c), x)
```

Fricas [A] time = 1.40933, size = 689, normalized size = 9.84

$$\left[\frac{\sqrt{-a}a\log\left(-\frac{8\,a\cos\left(fx+e\right)^{3}-4\left(2\,\cos\left(fx+e\right)^{2}-\cos\left(fx+e\right)\right)\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\sin\left(fx+e\right)-7\,a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)+1}\right)\sin\left(fx+e\right)+8\,a\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}}{2\,cf\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a)*a*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(f*x + e)^2 - cos(f*x +
 e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*co
s(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) + 8*a*sqrt((a*cos(f*x + e)
 + a)/cos(f*x + e))*cos(f*x + e))/(c*f*sin(f*x + e)), (a^(3/2)*arctan(2*sqr
t(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a
*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) + 4*a*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*cos(f*x + e))/(c*f*sin(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{a\sqrt{a\sec\left(e+fx\right)+a}}{\sec\left(e+fx\right)-1}\,dx + \int \frac{a\sqrt{a\sec\left(e+fx\right)+a}\sec\left(e+fx\right)}{\sec\left(e+fx\right)-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e)),x)
```

```
[Out] -(Integral(a*sqrt(a*sec(e + f*x) + a)/(sec(e + f*x) - 1), x) + Integral(a*s
qrt(a*sec(e + f*x) + a)*sec(e + f*x)/(sec(e + f*x) - 1), x))/c
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.54 $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^2}\,dx$

Optimal. Leaf size=102

$$\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{c^2f}-\frac{4\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{3c^2f}+\frac{2a\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{c^2f}$$

```
[Out] (2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c^2*f)
 + (2*a*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c^2*f) - (4*Cot[e + f*x]^3*
(a + a*Sec[e + f*x])^(3/2))/(3*c^2*f)
```

Rubi [A] time = 0.162159, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3904, 3887, 453, 325, 203}

$$\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{c^2f}-\frac{4\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{3c^2f}+\frac{2a\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{c^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^2,x]
```


```
[Out] (2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c^2*f)
 + (2*a*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c^2*f) - (4*Cot[e + f*x]^3*
(a + a*Sec[e + f*x])^(3/2))/(3*c^2*f)
```


Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
```

```
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^2}\,dx = \frac{\int \cot^4(e+fx)(a+a\sec(e+fx))^{7/2}\,dx}{a^2c^2}$$

$$= -\frac{2\,\text{Subst}\left(\int \frac{2+ax^2}{x^4(1+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^2f}$$

$$= -\frac{4\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3c^2f} + \frac{(2a)\,\text{Subst}\left(\int \frac{1}{x^2(1+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^2f}$$

$$= \frac{2a\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^2f} - \frac{4\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3c^2f} - \frac{(2a^2)\,\text{Subst}\left(\right.}{}$$

$$= \frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^2f} + \frac{2a\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^2f} - \frac{4\cot^3(e+fx)(a+a\,\text{se}}{3c^2f}$$

Mathematica [A] time = 0.65334, size = 113, normalized size = 1.11

$$-\frac{2a\sqrt{\cos(e+fx)}\tan\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}\left(\sqrt{\cos(e+fx)}(5\cos(e+fx)-3)-6\sqrt{2}\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(e+fx\right.\right.\right.}{3c^2f(\cos(e+fx)-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (-2*a*Sqrt[Cos[e + f*x]]*Sqrt[a*(1 + Sec[e + f*x])]*(Sqrt[Cos[e + f*x]]*(-3
 + 5*Cos[e + f*x]) - 6*Sqrt[2]*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]]*Sin[(e + f*
x)/2]^3)*Tan[(e + f*x)/2])/(3*c^2*f*(-1 + Cos[e + f*x])^2)
```

Maple [B] time = 0.255, size = 215, normalized size = 2.1

$$-\frac{a}{3fc^2\sin(fx+e)(-1+\cos(fx+e))}\sqrt{\frac{a(1+\cos(fx+e))}{\cos(fx+e)}}\left(3\sqrt{2}\cos(fx+e)\sin(fx+e)\sqrt{-2\frac{\cos(fx+e)}{1+\cos(fx+e)}}\text{Ar}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^2,x)
```

```
[Out] -1/3/c^2/f*a*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(3*2^(1/2)*cos(f*x+e)*si
n(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(f
*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))-3*2^(1/2)*sin(f*x+e)*arc
tanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e)
)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)-10*cos(f*x+e)^2+6*cos(f*x+e))/sin(f*
x+e)/(-1+cos(f*x+e))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.51704, size = 888, normalized size = 8.71

$$\left[\frac{3\left(a\cos\left(fx+e\right)-a\right)\sqrt{-a}\log\left(-\frac{8\,a\cos\left(fx+e\right)^{3}-4\left(2\cos\left(fx+e\right)^{2}-\cos\left(fx+e\right)\right)\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)-7\,a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)+1}\right)\sin\left(fx+e\right)+}{6\left(c^{2}f\cos\left(fx+e\right)-c^{2}f\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/6*(3*(a*cos(f*x + e) - a)*sqrt(-a)*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(f
*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*
sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) + 4*(
5*a*cos(f*x + e)^2 - 3*a*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x +
e)))/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e)), 1/3*(3*(a*cos(f*x + e) -
a)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x
 + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e)
+ 2*(5*a*cos(f*x + e)^2 - 3*a*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f
```

```
*x + e)))/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a\sqrt{a\sec\left(e+fx\right)+a}}{\sec^{2}\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx+\int \frac{a\sqrt{a\sec\left(e+fx\right)+a}\sec\left(e+fx\right)}{\sec^{2}\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**2,x)
```

```
[Out] (Integral(a*sqrt(a*sec(e + f*x) + a)/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1)
, x) + Integral(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)/(sec(e + f*x)**2 -
2*sec(e + f*x) + 1), x))/c**2
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.55 $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^3}\,dx$

Optimal. Leaf size=137

$$\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{c^3f}+\frac{4\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}}{5ac^3f}-\frac{2\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{3c^3f}+\frac{2a\cot(e+fx}{}$$

```
[Out] (2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c^3*f)
 + (2*a*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c^3*f) - (2*Cot[e + f*x]^3*
(a + a*Sec[e + f*x])^(3/2))/(3*c^3*f) + (4*Cot[e + f*x]^5*(a + a*Sec[e + f*
x])^(5/2))/(5*a*c^3*f)
```

Rubi [A] time = 0.177796, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3904, 3887, 453, 325, 203}

$$\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{c^3f}+\frac{4\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}}{5ac^3f}-\frac{2\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{3c^3f}+\frac{2a\cot(e+fx}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c^3*f)
 + (2*a*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c^3*f) - (2*Cot[e + f*x]^3*
(a + a*Sec[e + f*x])^(3/2))/(3*c^3*f) + (4*Cot[e + f*x]^5*(a + a*Sec[e + f*
x])^(5/2))/(5*a*c^3*f)
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
```

```
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^3}\,dx = -\frac{\int \cot^6(e+fx)(a+a\sec(e+fx))^{9/2}\,dx}{a^3c^3}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{2+ax^2}{x^6(1+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{ac^3f}$$

$$= \frac{4\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5ac^3f} - \frac{2\,\text{Subst}\left(\int \frac{1}{x^4(1+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^3f}$$

$$= -\frac{2\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3c^3f} + \frac{4\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5ac^3f} + \frac{(2a)\,\text{Subst}$$

$$= \frac{2a\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^3f} - \frac{2\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3c^3f} + \frac{4\cot^5(e+fx)($$

$$= \frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^3f} + \frac{2a\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^3f} - \frac{2\cot^3(e+fx)(a+a\sec}{3c^3f}$$

Mathematica [C] time = 0.770296, size = 102, normalized size = 0.74

$$\frac{2a\tan\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}\left(5(\cos(e+fx)-1)\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{2},-\frac{1}{2},2\sin^2\left(\frac{1}{2}(e+fx)\right)\right)+6\,\text{co}}{15c^3f\cos^{\frac{5}{2}}(e+fx)(\sec(e+fx)-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (2*a*(6*Cos[e + f*x]^(5/2) + 5*(-1 + Cos[e + f*x])*Hypergeometric2F1[-3/2,
-3/2, -1/2, 2*Sin[(e + f*x)/2]^2])*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)
/2])/(15*c^3*f*Cos[e + f*x]^(5/2)*(-1 + Sec[e + f*x])^3)
```

Maple [B] time = 0.276, size = 304, normalized size = 2.2

$$-\frac{a}{15\,fc^3\sin(fx+e)\left(-1+\cos(fx+e)\right)^2}\sqrt{\frac{a\left(1+\cos(fx+e)\right)}{\cos(fx+e)}}\left(15\left(\cos(fx+e)\right)^2\sin(fx+e)\sqrt{2}\sqrt{-2\frac{\cos(fx+e)}{1+\cos(fx+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^3,x)
```

```
[Out] -1/15/c^3/f*a*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(15*cos(f*x+e)^2*sin(f*
x+e)*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*2^(1/2)*(-2*c
os(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))-30*2^(1/2)*cos(f*x+e
)*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*2^(1/2)*(-2*c
os(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))+15*2^(1/2)*sin(f*x+e
)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f
*x+e))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)-52*cos(f*x+e)^3+70*cos(f*x+e)^2
-30*cos(f*x+e))/sin(f*x+e)/(-1+cos(f*x+e))^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.57714, size = 1083, normalized size = 7.91

$$\left[\frac{15\left(a\cos\left(fx+e\right)^{2}-2a\cos\left(fx+e\right)+a\right)\sqrt{-a}\log\left(-\frac{8a\cos\left(fx+e\right)^{3}-4\left(2\cos\left(fx+e\right)^{2}-\cos\left(fx+e\right)\right)\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)-7a\cos\cdots}{\cos\left(fx+e\right)+1}\cdots\right)}{30\left(c^{3}f\cos\left(fx+e\right)^{2}-2c^{3}f\cos\left(fx+\cdots\right)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/30*(15*(a*cos(f*x + e)^2 - 2*a*cos(f*x + e) + a)*sqrt(-a)*log(-(8*a*cos(
f*x + e)^3 - 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x +
```

```
e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) +
 1))*sin(f*x + e) + 4*(26*a*cos(f*x + e)^3 - 35*a*cos(f*x + e)^2 + 15*a*cos
(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((c^3*f*cos(f*x + e)^2
- 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e)), 1/15*(15*(a*cos(f*x + e)^2 -
 2*a*cos(f*x + e) + a)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/c
os(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e)
 - a))*sin(f*x + e) + 2*(26*a*cos(f*x + e)^3 - 35*a*cos(f*x + e)^2 + 15*a*c
os(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((c^3*f*cos(f*x + e)^
2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{a\sqrt{a\sec\left(e+fx\right)+a}}{\sec^3\left(e+fx\right)-3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)-1}\,dx+\int \frac{a\sqrt{a\sec\left(e+fx\right)+a}\sec\left(e+fx\right)}{\sec^3\left(e+fx\right)-3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)-1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**3,x)
```

```
[Out] -(Integral(a*sqrt(a*sec(e + f*x) + a)/(sec(e + f*x)**3 - 3*sec(e + f*x)**2
+ 3*sec(e + f*x) - 1), x) + Integral(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x
)/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*sec(e + f*x) - 1), x))/c**3
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.56 $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^4}\,dx$

Optimal. Leaf size=172

$$-\frac{4\cot^7(e+fx)(a\sec(e+fx)+a)^{7/2}}{7a^2c^4f}+\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{c^4f}+\frac{2\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}}{5ac^4f}-\frac{2\cot^3(e+$$

```
[Out] (2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c^4*f)
 + (2*a*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c^4*f) - (2*Cot[e + f*x]^3*
(a + a*Sec[e + f*x])^(3/2))/(3*c^4*f) + (2*Cot[e + f*x]^5*(a + a*Sec[e + f*
x])^(5/2))/(5*a*c^4*f) - (4*Cot[e + f*x]^7*(a + a*Sec[e + f*x])^(7/2))/(7*a
^2*c^4*f)
```

Rubi [A] time = 0.186627, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3904, 3887, 453, 325, 203}

$$-\frac{4\cot^7(e+fx)(a\sec(e+fx)+a)^{7/2}}{7a^2c^4f}+\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{c^4f}+\frac{2\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}}{5ac^4f}-\frac{2\cot^3(e+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c^4*f)
 + (2*a*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c^4*f) - (2*Cot[e + f*x]^3*
(a + a*Sec[e + f*x])^(3/2))/(3*c^4*f) + (2*Cot[e + f*x]^5*(a + a*Sec[e + f*
x])^(5/2))/(5*a*c^4*f) - (4*Cot[e + f*x]^7*(a + a*Sec[e + f*x])^(7/2))/(7*a
^2*c^4*f)
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^4}\,dx = \frac{\int \cot^8(e+fx)(a+a\sec(e+fx))^{11/2}\,dx}{a^4c^4}$$
$$= -\frac{2\,\mathrm{Subst}\left(\int \frac{2+ax^2}{x^8(1+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^2c^4f}$$
$$= -\frac{4\cot^7(e+fx)(a+a\sec(e+fx))^{7/2}}{7a^2c^4f} + \frac{2\,\mathrm{Subst}\left(\int \frac{1}{x^6(1+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{ac^4f}$$
$$= \frac{2\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5ac^4f} - \frac{4\cot^7(e+fx)(a+a\sec(e+fx))^{7/2}}{7a^2c^4f} - \frac{2\,\mathrm{Subst}\left(\int\right.}{}$$
$$= -\frac{2\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3c^4f} + \frac{2\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5ac^4f} - \frac{4\cot^7(e+}{}$$
$$= \frac{2a\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^4f} - \frac{2\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3c^4f} + \frac{2\cot^5(e+fx}{}$$
$$= \frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^4f} + \frac{2a\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^4f} - \frac{2\cot^3(e+fx)(a+a\,\mathrm{se}}{3c^4f}$$

Mathematica [C] time = 1.32685, size = 102, normalized size = 0.59

$$-\frac{2a\sqrt{\cos(e+fx)}\tan\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}\left(7(\cos(e+fx)-1)\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{5}{2},-\frac{3}{2},2\sin^2\left(\frac{1}{2}\right.\right.\right.}{35c^4f(\cos(e+fx)-1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (-2*a*Sqrt[Cos[e + f*x]]*(10*Cos[e + f*x]^(7/2) + 7*(-1 + Cos[e + f*x])*Hyp
ergeometric2F1[-5/2, -5/2, -3/2, 2*Sin[(e + f*x)/2]^2])*Sqrt[a*(1 + Sec[e +
 f*x])]*Tan[(e + f*x)/2])/(35*c^4*f*(-1 + Cos[e + f*x])^4)
```

Maple [B] time = 0.314, size = 401, normalized size = 2.3

$$\frac{a\left(1+\cos\left(fx+e\right)\right)}{105\,fc^4\left(\sin\left(fx+e\right)\right)^3\left(-1+\cos\left(fx+e\right)\right)^2}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\left(105\sqrt{2}\left(\cos\left(fx+e\right)\right)^3\sin\left(fx+e\right)\sqrt{-2\frac{\cos\left(f\right.}{1+\cos\left(\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^4,x)
```

```
[Out] 1/105/c^4/f*a*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(1+cos(f*x+e))*(105*2^(
1/2)*cos(f*x+e)^3*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1
/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))-315*
cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctan
h(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))+3
15*2^(1/2)*cos(f*x+e)*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arcta
nh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))-
105*2^(1/2)*sin(f*x+e)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(
1/2)*sin(f*x+e)/cos(f*x+e))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)-382*cos(f*
x+e)^4+812*cos(f*x+e)^3-700*cos(f*x+e)^2+210*cos(f*x+e))/sin(f*x+e)^3/(-1+c
os(f*x+e))^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.62543, size = 1283, normalized size = 7.46

$$\left[\frac{105\left(a\cos\left(fx+e\right)^{3}-3\,a\cos\left(fx+e\right)^{2}+3\,a\cos\left(fx+e\right)-a\right)\sqrt{-a}\log\left(-\frac{8\,a\cos\left(fx+e\right)^{3}-4\left(2\cos\left(fx+e\right)^{2}-\cos\left(fx+e\right)\right)\sqrt{-a}\sqrt{\frac{a\cos\left(fx\right.}{\cos\left(f\right.}}}{\cos\left(fx+e\right)+1}\right.}{210\left(c^{4}f\cos\left(fx+e\right)^{3}-3\,c^{4}f\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] [1/210*(105*(a*cos(f*x + e)^3 - 3*a*cos(f*x + e)^2 + 3*a*cos(f*x + e) - a)*
sqrt(-a)*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqr
t(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x +
e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) + 4*(191*a*cos(f*x + e)^4 - 406*a*
cos(f*x + e)^3 + 350*a*cos(f*x + e)^2 - 105*a*cos(f*x + e))*sqrt((a*cos(f*x
 + e) + a)/cos(f*x + e)))/((c^4*f*cos(f*x + e)^3 - 3*c^4*f*cos(f*x + e)^2 +
 3*c^4*f*cos(f*x + e) - c^4*f)*sin(f*x + e)), 1/105*(105*(a*cos(f*x + e)^3
- 3*a*cos(f*x + e)^2 + 3*a*cos(f*x + e) - a)*sqrt(a)*arctan(2*sqrt(a)*sqrt(
(a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x +
 e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) + 2*(191*a*cos(f*x + e)^4 - 406*a
*cos(f*x + e)^3 + 350*a*cos(f*x + e)^2 - 105*a*cos(f*x + e))*sqrt((a*cos(f*
x + e) + a)/cos(f*x + e)))/((c^4*f*cos(f*x + e)^3 - 3*c^4*f*cos(f*x + e)^2
+ 3*c^4*f*cos(f*x + e) - c^4*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**4,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^4,x, algorithm="giac")
```

```
[Out] Timed out
```

3.57 $\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^3\,dx$

Optimal. Leaf size=212

$$-\frac{2a^7c^3\tan^9(e+fx)}{9f(a\sec(e+fx)+a)^{9/2}}-\frac{6a^6c^3\tan^7(e+fx)}{7f(a\sec(e+fx)+a)^{7/2}}-\frac{2a^5c^3\tan^5(e+fx)}{5f(a\sec(e+fx)+a)^{5/2}}+\frac{2a^4c^3\tan^3(e+fx)}{3f(a\sec(e+fx)+a)^{3/2}}+\frac{2a^{5/2}c^3\tan}{}$$

```
[Out] (2*a^(5/2)*c^3*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f -
 (2*a^3*c^3*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) + (2*a^4*c^3*Tan[e +
 f*x]^3)/(3*f*(a + a*Sec[e + f*x])^(3/2)) - (2*a^5*c^3*Tan[e + f*x]^5)/(5*f
*(a + a*Sec[e + f*x])^(5/2)) - (6*a^6*c^3*Tan[e + f*x]^7)/(7*f*(a + a*Sec[e
 + f*x])^(7/2)) - (2*a^7*c^3*Tan[e + f*x]^9)/(9*f*(a + a*Sec[e + f*x])^(9/2
))
```

Rubi [A] time = 0.190908, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3904, 3887, 461, 203}

$$-\frac{2a^7c^3\tan^9(e+fx)}{9f(a\sec(e+fx)+a)^{9/2}}-\frac{6a^6c^3\tan^7(e+fx)}{7f(a\sec(e+fx)+a)^{7/2}}-\frac{2a^5c^3\tan^5(e+fx)}{5f(a\sec(e+fx)+a)^{5/2}}+\frac{2a^4c^3\tan^3(e+fx)}{3f(a\sec(e+fx)+a)^{3/2}}+\frac{2a^{5/2}c^3\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^3,x]
```


```
[Out] (2*a^(5/2)*c^3*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f -
 (2*a^3*c^3*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) + (2*a^4*c^3*Tan[e +
 f*x]^3)/(3*f*(a + a*Sec[e + f*x])^(3/2)) - (2*a^5*c^3*Tan[e + f*x]^5)/(5*f
*(a + a*Sec[e + f*x])^(5/2)) - (6*a^6*c^3*Tan[e + f*x]^7)/(7*f*(a + a*Sec[e
 + f*x])^(7/2)) - (2*a^7*c^3*Tan[e + f*x]^9)/(9*f*(a + a*Sec[e + f*x])^(9/2
))
```


Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^3\,dx = -\left(\left(a^3c^3\right)\int \frac{\tan^6(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx\right)$$

$$= \frac{\left(2a^6c^3\right)\mathrm{Subst}\left(\int \frac{x^6\left(2+ax^2\right)^2}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= \frac{\left(2a^6c^3\right)\mathrm{Subst}\left(\int \left(\frac{1}{a^3}-\frac{x^2}{a^2}+\frac{x^4}{a}+3x^6+ax^8-\frac{1}{a^3\left(1+ax^2\right)}\right)dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= -\frac{2a^3c^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}+\frac{2a^4c^3\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}}-\frac{2a^5c^3\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}}$$

$$= \frac{2a^{5/2}c^3\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}-\frac{2a^3c^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}+\frac{2a^4c^3\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}}$$

Mathematica [A] time = 1.21355, size = 134, normalized size = 0.63

$$-\frac{a^2c^3\tan\left(\frac{1}{2}(e+fx)\right)\sec^4(e+fx)\sqrt{a(\sec(e+fx)+1)}\left((164\cos(e+fx)+1004\cos(2(e+fx))+68\cos(3(e+fx))+38\right.}{1260f\sqrt{\sec(e+fx)-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^3,x]
```

```
[Out] -(a^2*c^3*(-2520*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Cos[e + f*x]^4 + (901 + 16
4*Cos[e + f*x] + 1004*Cos[2*(e + f*x)] + 68*Cos[3*(e + f*x)] + 383*Cos[4*(e
 + f*x)])*Sqrt[-1 + Sec[e + f*x]])*Sec[e + f*x]^4*Sqrt[a*(1 + Sec[e + f*x])
]*Tan[(e + f*x)/2])/(1260*f*Sqrt[-1 + Sec[e + f*x]])
```

Maple [B] time = 0.28, size = 483, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^3,x)
```

```
[Out] -1/5040*c^3/f*a^2*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(315*sin(f*x+e)*cos
(f*x+e)^4*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*
sin(f*x+e)/cos(f*x+e))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(9/2)+1260*sin(f*x+e)
*cos(f*x+e)^3*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1
/2)*sin(f*x+e)/cos(f*x+e))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(9/2)+1890*sin(f*
x+e)*cos(f*x+e)^2*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e))
)^(1/2)*sin(f*x+e)/cos(f*x+e))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(9/2)+1260*si
n(f*x+e)*cos(f*x+e)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e
)))^(1/2)*sin(f*x+e)/cos(f*x+e))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(9/2)+315*2
^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/
cos(f*x+e))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(9/2)*sin(f*x+e)-12256*cos(f*x+e
)^5+11168*cos(f*x+e)^4+5312*cos(f*x+e)^3-4064*cos(f*x+e)^2-1280*cos(f*x+e)+
1120)/cos(f*x+e)^4/sin(f*x+e)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.30711, size = 1098, normalized size = 5.18

$$\left[\frac{315\left(a^2c^3\cos\left(fx+e\right)^5+a^2c^3\cos\left(fx+e\right)^4\right)\sqrt{-a}\log\left(\frac{2\,a\cos\left(fx+e\right)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)\sin\left(fx+e\right)+a\cos\left(fx+e\right)-a}{\cos\left(fx+e\right)+1}\right)-2}{315\left(f\cos\left(f\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/315*(315*(a^2*c^3*cos(f*x + e)^5 + a^2*c^3*cos(f*x + e)^4)*sqrt(-a)*log(
(2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*co
s(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(383*
a^2*c^3*cos(f*x + e)^4 + 34*a^2*c^3*cos(f*x + e)^3 - 132*a^2*c^3*cos(f*x +
e)^2 - 5*a^2*c^3*cos(f*x + e) + 35*a^2*c^3)*sqrt((a*cos(f*x + e) + a)/cos(f
*x + e))*sin(f*x + e))/(f*cos(f*x + e)^5 + f*cos(f*x + e)^4), -2/315*(315*(
a^2*c^3*cos(f*x + e)^5 + a^2*c^3*cos(f*x + e)^4)*sqrt(a)*arctan(sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + (383*a^
2*c^3*cos(f*x + e)^4 + 34*a^2*c^3*cos(f*x + e)^3 - 132*a^2*c^3*cos(f*x + e)
^2 - 5*a^2*c^3*cos(f*x + e) + 35*a^2*c^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x
 + e))*sin(f*x + e))/(f*cos(f*x + e)^5 + f*cos(f*x + e)^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e))**3,x)
```

```
[Out] Timed out
```

<u>Giac</u> [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.58 $\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2\,dx$

Optimal. Leaf size=177

$$\frac{2a^6c^2\tan^7(e+fx)}{7f(a\sec(e+fx)+a)^{7/2}}+\frac{6a^5c^2\tan^5(e+fx)}{5f(a\sec(e+fx)+a)^{5/2}}+\frac{2a^4c^2\tan^3(e+fx)}{3f(a\sec(e+fx)+a)^{3/2}}+\frac{2a^{5/2}c^2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}-\frac{2a^3c}{f\sqrt{a\,\mathrm{s}}}$$

```
[Out] (2*a^(5/2)*c^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f -
 (2*a^3*c^2*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) + (2*a^4*c^2*Tan[e +
 f*x]^3)/(3*f*(a + a*Sec[e + f*x])^(3/2)) + (6*a^5*c^2*Tan[e + f*x]^5)/(5*f
*(a + a*Sec[e + f*x])^(5/2)) + (2*a^6*c^2*Tan[e + f*x]^7)/(7*f*(a + a*Sec[e
 + f*x])^(7/2))
```

Rubi [A] time = 0.174306, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3904, 3887, 461, 203}

$$\frac{2a^6c^2\tan^7(e+fx)}{7f(a\sec(e+fx)+a)^{7/2}}+\frac{6a^5c^2\tan^5(e+fx)}{5f(a\sec(e+fx)+a)^{5/2}}+\frac{2a^4c^2\tan^3(e+fx)}{3f(a\sec(e+fx)+a)^{3/2}}+\frac{2a^{5/2}c^2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}-\frac{2a^3c}{f\sqrt{a\,\mathrm{s}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (2*a^(5/2)*c^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f -
 (2*a^3*c^2*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) + (2*a^4*c^2*Tan[e +
 f*x]^3)/(3*f*(a + a*Sec[e + f*x])^(3/2)) + (6*a^5*c^2*Tan[e + f*x]^5)/(5*f
*(a + a*Sec[e + f*x])^(5/2)) + (2*a^6*c^2*Tan[e + f*x]^7)/(7*f*(a + a*Sec[e
 + f*x])^(7/2))
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2\,dx = \left(a^2c^2\right)\int \sqrt{a+a\sec(e+fx)}\tan^4(e+fx)\,dx$$

$$= -\frac{\left(2a^5c^2\right)\text{Subst}\left(\int \frac{x^4\left(2+ax^2\right)^2}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= -\frac{\left(2a^5c^2\right)\text{Subst}\left(\int \left(-\frac{1}{a^2}+\frac{x^2}{a}+3x^4+ax^6+\frac{1}{a^2\left(1+ax^2\right)}\right)dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+}}\right.}{f}$$

$$= -\frac{2a^3c^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}+\frac{2a^4c^2\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}}+\frac{6a^5c^2\tan^5(e+f}{5f(a+a\sec(e+fx}$$

$$= \frac{2a^{5/2}c^2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}-\frac{2a^3c^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}+\frac{2a^4c^2\tan^3(e+}{3f(a+a\sec(e+}$$

Mathematica [A] time = 0.921237, size = 124, normalized size = 0.7

$$-\frac{2a^2c^2\tan\left(\frac{1}{2}(e+fx)\right)\sec^3(e+fx)\sqrt{a(\sec(e+fx)+1)}\left((51\cos(e+fx)+23\cos(2(e+fx))+23\cos(3(e+fx))+8)\sqrt{s}\right.}{105f\sqrt{\sec(e+fx)-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (-2*a^2*c^2*(-105*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Cos[e + f*x]^3 + (8 + 51*
Cos[e + f*x] + 23*Cos[2*(e + f*x)] + 23*Cos[3*(e + f*x)])*Sqrt[-1 + Sec[e +
 f*x]])*Sec[e + f*x]^3*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(105*f*
Sqrt[-1 + Sec[e + f*x]])
```

Maple [B] time = 0.26, size = 323, normalized size = 1.8

$$-\frac{c^2a^2}{420\,f\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^3}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\left(105\sqrt{2}\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^3\left(-2\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}\right)^{5/}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^2,x)
```

```
[Out] -1/420*c^2/f*a^2*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(105*2^(1/2)*sin(f*x
+e)*cos(f*x+e)^3*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)*arctanh(1/2*2^(1/2)*(
-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))+210*2^(1/2)*sin(
f*x+e)*cos(f*x+e)^2*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2
)*sin(f*x+e)/cos(f*x+e))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)+105*2^(1/2)*s
in(f*x+e)*cos(f*x+e)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/
2)*sin(f*x+e)/cos(f*x+e))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)-736*cos(f*x+
e)^4+368*cos(f*x+e)^3+512*cos(f*x+e)^2-24*cos(f*x+e)-120)/sin(f*x+e)/cos(f*
x+e)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.23743, size = 1019, normalized size = 5.76

$$\left[\frac{105\left(a^{2}c^{2}\cos\left(fx+e\right)^{4}+a^{2}c^{2}\cos\left(fx+e\right)^{3}\right)\sqrt{-a}\log\left(\frac{2\,a\cos\left(fx+e\right)^{2}-2\,\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)\sin\left(fx+e\right)+a\cos\left(fx+e\right)-a}{\cos\left(fx+e\right)+1}\right)-2\left(9\right.}{105\left(f\cos\left(fx+e\right)^{4}+f\cos\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/105*(105*(a^2*c^2*cos(f*x + e)^4 + a^2*c^2*cos(f*x + e)^3)*sqrt(-a)*log(
(2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*co
s(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(92*a
^2*c^2*cos(f*x + e)^3 + 46*a^2*c^2*cos(f*x + e)^2 - 18*a^2*c^2*cos(f*x + e)
 - 15*a^2*c^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos
(f*x + e)^4 + f*cos(f*x + e)^3), -2/105*(105*(a^2*c^2*cos(f*x + e)^4 + a^2*
c^2*cos(f*x + e)^3)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*
cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + (92*a^2*c^2*cos(f*x + e)^3 + 46*a^2*
c^2*cos(f*x + e)^2 - 18*a^2*c^2*cos(f*x + e) - 15*a^2*c^2)*sqrt((a*cos(f*x
+ e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^4 + f*cos(f*x + e)^3)
]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.59 $\int (a + a\sec(e + fx))^{5/2}(c - c\sec(e + fx))\,dx$

Optimal. Leaf size=132

$$-\frac{2a^5c\tan^5(e+fx)}{5f(a\sec(e+fx)+a)^{5/2}}-\frac{2a^4c\tan^3(e+fx)}{f(a\sec(e+fx)+a)^{3/2}}+\frac{2a^{5/2}c\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}-\frac{2a^3c\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

```
[Out] (2*a^(5/2)*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f - (
2*a^3*c*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) - (2*a^4*c*Tan[e + f*x]^
3)/(f*(a + a*Sec[e + f*x])^(3/2)) - (2*a^5*c*Tan[e + f*x]^5)/(5*f*(a + a*Se
c[e + f*x])^(5/2))
```

Rubi [A] time = 0.140682, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3904, 3887, 461, 203}

$$-\frac{2a^5c\tan^5(e+fx)}{5f(a\sec(e+fx)+a)^{5/2}}-\frac{2a^4c\tan^3(e+fx)}{f(a\sec(e+fx)+a)^{3/2}}+\frac{2a^{5/2}c\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}-\frac{2a^3c\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x]),x]
```

```
[Out] (2*a^(5/2)*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f - (
2*a^3*c*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) - (2*a^4*c*Tan[e + f*x]^
3)/(f*(a + a*Sec[e + f*x])^(3/2)) - (2*a^5*c*Tan[e + f*x]^5)/(5*f*(a + a*Se
c[e + f*x])^(5/2))
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
```

```
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))\,dx = -\left((ac)\int (a+a\sec(e+fx))^{3/2}\tan^2(e+fx)\,dx\right)$$

$$= \frac{\left(2a^4c\right)\text{Subst}\left(\int \frac{x^2\left(2+ax^2\right)^2}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= \frac{\left(2a^4c\right)\text{Subst}\left(\int \left(\frac{1}{a}+3x^2+ax^4-\frac{1}{a\left(1+ax^2\right)}\right)dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= -\frac{2a^3c\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{2a^4c\tan^3(e+fx)}{f(a+a\sec(e+fx))^{3/2}} - \frac{2a^5c\tan^5(e+fx)}{5f(a+a\sec(e+fx)}$$

$$= \frac{2a^{5/2}c\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} - \frac{2a^3c\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{2a^4c\tan^3(e+}{f(a+a\sec(e+f}$$

Mathematica [A] time = 0.802766, size = 110, normalized size = 0.83

$$-\frac{a^2c\tan\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\sqrt{a(\sec(e+fx)+1)}\left((6\cos(e+fx)+\cos(2(e+fx))+3)\sqrt{\sec(e+fx)-1}-10\cos^2(}{5f\sqrt{\sec(e+fx)-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x]),x]
```

```
[Out] -(a^2*c*(-10*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Cos[e + f*x]^2 + (3 + 6*Cos[e
+ f*x] + Cos[2*(e + f*x)])*Sqrt[-1 + Sec[e + f*x]])*Sec[e + f*x]^2*Sqrt[a*(
1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(5*f*Sqrt[-1 + Sec[e + f*x]])
```

Maple [B] time = 0.229, size = 303, normalized size = 2.3

$$-\frac{a^2c}{20f\left(\cos\left(fx+e\right)\right)^2\sin\left(fx+e\right)}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\left(5\sqrt{2}\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^2\mathrm{Artanh}\left(1/2\frac{\sqrt{2}\sin\left(fx+e\right)}{\cos\left(fx+e\right)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e)),x)
```

```
[Out] -1/20*c/f*a^2*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(5*2^(1/2)*sin(f*x+e)*c
os(f*x+e)^2*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*
x+e)/cos(f*x+e))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)+10*2^(1/2)*sin(f*x+e)
*cos(f*x+e)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*
x+e)/cos(f*x+e))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)+5*2^(1/2)*arctanh(1/2
*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*(-2*co
s(f*x+e)/(1+cos(f*x+e)))^(5/2)*sin(f*x+e)-8*cos(f*x+e)^3-16*cos(f*x+e)^2+16
*cos(f*x+e)+8)/cos(f*x+e)^2/sin(f*x+e)
```

Maxima [B] time = 2.03237, size = 1885, normalized size = 14.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/6*(30*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^
(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 2*
(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*((
12*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(2*f*x + 2*e
) - 3*a^2*sin(2*f*x + 2*e) - 4*(3*a^2*cos(2*f*x + 2*e) + 4*a^2)*sin(3/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arctan2(sin(2*f*x + 2*e)
```

```
, cos(2*f*x + 2*e) + 1)) + (12*a^2*sin(2*f*x + 2*e)*sin(3/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e))) + 3*a^2*cos(2*f*x + 2*e) - a^2 + 4*(3*a^2*cos
(2*f*x + 2*e) + 4*a^2)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))*sqrt(a) + 3*((a
^2*cos(2*f*x + 2*e)^2 + a^2*sin(2*f*x + 2*e)^2 + 2*a^2*cos(2*f*x + 2*e) + a
^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) +
 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - cos(1/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e)))), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1
)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*cos(1/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sin(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e)))) + 1) - (a^2*cos(2*f*x + 2*e)^2 + a^2*sin(2*f*x + 2*e)^2 + 2*a^2*cos(
2*f*x + 2*e) + a^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*co
s(2*f*x + 2*e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e)))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - cos(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e)))), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(
2*f*x + 2*e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
 + 1))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sin(1/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e)))) - 1) - (a^2*cos(2*f*x + 2*e)^2 + a^2*sin(2*f*x + 2*e
)^2 + 2*a^2*cos(2*f*x + 2*e) + a^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x
 + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2
*f*x + 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) +
 1)) + 1) + (a^2*cos(2*f*x + 2*e)^2 + a^2*sin(2*f*x + 2*e)^2 + 2*a^2*cos(2*
f*x + 2*e) + a^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(
2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
+ 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(
1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 1))*sqrt(a)
)*c/((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*f)
```

Fricas [A] time = 1.24532, size = 887, normalized size = 6.72

$$\left[\frac{5\left(a^2c\cos\left(fx+e\right)^3+a^2c\cos\left(fx+e\right)^2\right)\sqrt{-a}\log\left(\frac{2\,a\cos\left(fx+e\right)^2-2\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)\sin\left(fx+e\right)+a\cos\left(fx+e\right)-a}{\cos\left(fx+e\right)+1}\right)-2\left(a^2c\,c\right.}{5\left(f\cos\left(fx+e\right)^3+f\cos\left(fx+e\right)^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/5*(5*(a^2*c*cos(f*x + e)^3 + a^2*c*cos(f*x + e)^2)*sqrt(-a)*log((2*a*cos
(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x +
e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(a^2*c*cos(f*
x + e)^2 + 3*a^2*c*cos(f*x + e) + a^2*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x
+ e))*sin(f*x + e))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2), -2/5*(5*(a^2*c*c
os(f*x + e)^3 + a^2*c*cos(f*x + e)^2)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) +
 a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + (a^2*c*cos(f*x + e
)^2 + 3*a^2*c*cos(f*x + e) + a^2*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))
*sin(f*x + e))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.60 $\int \frac{(a+a\sec(e+fx))^{5/2}}{c-c\sec(e+fx)}\,dx$

Optimal. Leaf size=103

$$\frac{8a^2\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{cf}+\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{cf}-\frac{2a^3\tan(e+fx)}{cf\sqrt{a\sec(e+fx)+a}}$$

```
[Out] (2*a^(5/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c*f) +
 (8*a^2*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c*f) - (2*a^3*Tan[e + f*x])
/(c*f*Sqrt[a + a*Sec[e + f*x]])
```

Rubi [A] time = 0.171966, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3904, 3887, 461, 203}

$$\frac{8a^2\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{cf}+\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{cf}-\frac{2a^3\tan(e+fx)}{cf\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x]),x]
```

```
[Out] (2*a^(5/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c*f) +
 (8*a^2*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c*f) - (2*a^3*Tan[e + f*x])
/(c*f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
```

```
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{c-c\sec(e+fx)}\,dx = -\frac{\int \cot^2(e+fx)(a+a\sec(e+fx))^{7/2}\,dx}{ac}$$

$$= \frac{\left(2a^2\right)\text{Subst}\left(\int \frac{\left(2+ax^2\right)^2}{x^2\left(1+ax^2\right)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf}$$

$$= \frac{\left(2a^2\right)\text{Subst}\left(\int \left(a+\frac{4}{x^2}-\frac{a}{1+ax^2}\right)dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf}$$

$$= \frac{8a^2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{cf} - \frac{2a^3\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}} - \frac{\left(2a^3\right)\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x\right.}{cf}$$

$$= \frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf} + \frac{8a^2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{cf} - \frac{2a^3\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}}$$

Mathematica [A] time = 0.680392, size = 96, normalized size = 0.93

$$\frac{2a^3\tan(e+fx)\sec(e+fx)\left((5\cos(e+fx)-1)\sqrt{\sec(e+fx)-1}-(\cos(e+fx)-1)\tan^{-1}\left(\sqrt{\sec(e+fx)-1}\right)\right)}{cf(\sec(e+fx)-1)^{3/2}\sqrt{a(\sec(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x]),x]
```

```
[Out] (2*a^3*(-(ArcTan[Sqrt[-1 + Sec[e + f*x]]]*(-1 + Cos[e + f*x])) + (-1 + 5*Co
s[e + f*x])*Sqrt[-1 + Sec[e + f*x]])*Sec[e + f*x]*Tan[e + f*x])/(c*f*(-1 +
Sec[e + f*x])^(3/2)*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [A] time = 0.227, size = 120, normalized size = 1.2

$$-\frac{a^2}{fc\sin\left(fx+e\right)}\left(\sqrt{2}\sin\left(fx+e\right)\operatorname{Artanh}\left(\frac{\sqrt{2}\sin\left(fx+e\right)}{2\cos\left(fx+e\right)}\sqrt{-2\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\right)\sqrt{-2\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}-10\cos\left(fx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e)),x)
```

```
[Out] -1/c/f*a^2*(2^(1/2)*sin(f*x+e)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*
x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)-10
*cos(f*x+e)+2)*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)/sin(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(a\sec\left(fx+e\right)+a\right)^{\frac{5}{2}}}{c\sec\left(fx+e\right)-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] -integrate((a*sec(f*x + e) + a)^(5/2)/(c*sec(f*x + e) - c), x)
```

Fricas [A] time = 1.47104, size = 724, normalized size = 7.03

$$\left[\frac{\sqrt{-a}a^{2}\log\left(-\frac{8\,a\cos\left(fx+e\right)^{3}-4\left(2\,\cos\left(fx+e\right)^{2}-\cos\left(fx+e\right)\right)\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)-7\,a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)+1}\right)\sin\left(fx+e\right)+4\left(5\,a^{2}\cos\left(fx+\right.\right.}{2\,cf\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a)*a^2*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(f*x + e)^2 - cos(f*x
 + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*
cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) + 4*(5*a^2*cos(f*x + e)
- a^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/(c*f*sin(f*x + e)), (a^(5/2
)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin
(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) + 2*(5*a^
2*cos(f*x + e) - a^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/(c*f*sin(f*x
 + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.61 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^2}\,dx$

Optimal. Leaf size=74

$$\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{c^2f}-\frac{8a\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{3c^2f}$$

```
[Out] (2*a^(5/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c^2*f)
 - (8*a*Cot[e + f*x]^3*(a + a*Sec[e + f*x])^(3/2))/(3*c^2*f)
```

Rubi [A] time = 0.167922, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3904, 3887, 461, 203}

$$\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{c^2f}-\frac{8a\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{3c^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (2*a^(5/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c^2*f)
 - (8*a*Cot[e + f*x]^3*(a + a*Sec[e + f*x])^(3/2))/(3*c^2*f)
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^2}\,dx &= \frac{\int \cot^4(e+fx)(a+a\sec(e+fx))^{9/2}\,dx}{a^2c^2}\\ &= -\frac{(2a)\operatorname{Subst}\left(\int \frac{\left(2+ax^2\right)^2}{x^4\left(1+ax^2\right)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^2f}\\ &= -\frac{(2a)\operatorname{Subst}\left(\int \left(\frac{4}{x^4}+\frac{a^2}{1+ax^2}\right)dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^2f}\\ &= -\frac{8a\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3c^2f}-\frac{\left(2a^3\right)\operatorname{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^2f}\\ &= \frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^2f}-\frac{8a\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3c^2f}\end{aligned}$$

Mathematica [A] time = 4.22586, size = 102, normalized size = 1.38

$$-\frac{\cos^{\frac{5}{2}}(e+fx)\csc^3\left(\frac{1}{2}(e+fx)\right)\sec^5\left(\frac{1}{2}(e+fx)\right)(a(\sec(e+fx)+1))^{5/2}\left(4\cos^{\frac{3}{2}}(e+fx)-3(1-\cos(e+fx))^{3/2}\sin^{-1}\left(\sqrt{1\right.\right.}}{24c^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^2,x]
```

```
[Out] -(Cos[e + f*x]^(5/2)*(-3*ArcSin[Sqrt[1 - Cos[e + f*x]]]*(1 - Cos[e + f*x])^
(3/2) + 4*Cos[e + f*x]^(3/2))*Csc[(e + f*x)/2]^3*Sec[(e + f*x)/2]^5*(a*(1 +
```

```
Sec[e + f*x]))^(5/2))/(24*c^2*f)
```

Maple [B] time = 0.24, size = 351, normalized size = 4.7

$$-\frac{a^2}{3fc^2\left(-1+\cos\left(fx+e\right)\right)^2\left(1+\cos\left(fx+e\right)\right)}\left(3\sqrt{2}\left(\cos\left(fx+e\right)\right)^3\sqrt{-2\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\mathrm{Artanh}\left(1/2\frac{\sqrt{2}\sin\left(fx+e\right)}{\cos\left(fx+e\right)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^2,x)
```

```
[Out] -1/3/c^2/f*a^2*(3*2^(1/2)*cos(f*x+e)^3*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)
*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*
x+e))-3*2^(1/2)*cos(f*x+e)^2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1
/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))-3*2^
(1/2)*cos(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*2^(1/2)*(
-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))+3*2^(1/2)*(-2*co
s(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*
x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))+8*cos(f*x+e)^2*sin(f*x+e))*(1/cos(f*x+e
)*a*(1+cos(f*x+e)))^(1/2)/(-1+cos(f*x+e))^2/(1+cos(f*x+e))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.49101, size = 844, normalized size = 11.41

$$\left[\frac{16\,a^2\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)^2+3\left(a^2\cos\left(fx+e\right)-a^2\right)\sqrt{-a}\log\left(-\frac{8\,a\cos\left(fx+e\right)^3-4\left(2\cos\left(fx+e\right)^2-\cos\left(fx+e\right)\right)\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}}{\cos\left(fx+e\right)+1}\right.}{6\left(c^2f\cos\left(fx+e\right)-c^2f\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/6*(16*a^2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)^2 + 3*(a^
2*cos(f*x + e) - a^2)*sqrt(-a)*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(f*x + e)
^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x
 + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e))/((c^2*f*cos
(f*x + e) - c^2*f)*sin(f*x + e)), 1/3*(8*a^2*sqrt((a*cos(f*x + e) + a)/cos(
f*x + e))*cos(f*x + e)^2 + 3*(a^2*cos(f*x + e) - a^2)*sqrt(a)*arctan(2*sqrt
(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*
cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e))/((c^2*f*cos(f*x + e) -
c^2*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.62 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^3}\,dx$

Optimal. Leaf size=104

$$\frac{2a^2\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{c^3f}+\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{c^3f}+\frac{8\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}}{5c^3f}$$

```
[Out] (2*a^(5/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c^3*f)
 + (2*a^2*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c^3*f) + (8*Cot[e + f*x]^
5*(a + a*Sec[e + f*x])^(5/2))/(5*c^3*f)
```

Rubi [A] time = 0.17705, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3904, 3887, 461, 203}

$$\frac{2a^2\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{c^3f}+\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{c^3f}+\frac{8\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}}{5c^3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (2*a^(5/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c^3*f)
 + (2*a^2*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c^3*f) + (8*Cot[e + f*x]^
5*(a + a*Sec[e + f*x])^(5/2))/(5*c^3*f)
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
```

```
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^3}\,dx = -\frac{\int \cot^6(e+fx)(a+a\sec(e+fx))^{11/2}\,dx}{a^3c^3}$$

$$= \frac{2\,\mathrm{Subst}\left(\int \frac{\left(2+ax^2\right)^2}{x^6\left(1+ax^2\right)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^3f}$$

$$= \frac{2\,\mathrm{Subst}\left(\int \left(\frac{4}{x^6}+\frac{a^2}{x^2}-\frac{a^3}{1+ax^2}\right)dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^3f}$$

$$= \frac{2a^2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^3f} + \frac{8\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5c^3f} - \frac{\left(2a^3\right)\mathrm{Subst}}{}$$

$$= \frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^3f} + \frac{2a^2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^3f} + \frac{8\cot^5(e+fx)(a+a}{5c^3f}$$

Mathematica [C] time = 5.40929, size = 196, normalized size = 1.88

$$\frac{a^2\sqrt{\cos(e+fx)}\tan\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}\left(4(20\cos(e+fx)-15\cos(2(e+fx))-29)\mathrm{Hypergeometric2F1}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (a^2*Sqrt[Cos[e + f*x]]*Sqrt[a*(1 + Sec[e + f*x])]*(30*ArcSin[Sqrt[1 - Cos[
e + f*x]]]*Cos[(e + f*x)/2]^2*Sqrt[1 - Cos[e + f*x]]*(-1 + 7*Cos[e + f*x])
+ 4*(-29 + 20*Cos[e + f*x] - 15*Cos[2*(e + f*x)])*Hypergeometric2F1[-5/2, -
1/2, 1/2, 2*Sin[(e + f*x)/2]^2] + 5*Sqrt[Cos[e + f*x]]*(11*Cos[e + f*x] + 3
*Cos[2*(e + f*x)])*Sin[e + f*x]^2)*Tan[(e + f*x)/2])/(60*c^3*f*(-1 + Cos[e
+ f*x])^3)
```

Maple [B] time = 0.27, size = 306, normalized size = 2.9

$$-\frac{a^2}{5 f c^3 \sin\left(fx+e\right)\left(-1+\cos\left(fx+e\right)\right)^2}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\left(5\left(\cos\left(fx+e\right)\right)^2\sin\left(fx+e\right)\sqrt{2}\sqrt{-2\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^3,x)
```

```
[Out] -1/5/c^3/f*a^2*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(5*cos(f*x+e)^2*sin(f*
x+e)*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*2^(1/2)*(-2*c
os(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))-10*2^(1/2)*cos(f*x+e
)*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*2^(1/2)*(-2*c
os(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))+5*2^(1/2)*sin(f*x+e)
*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*
x+e))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)-18*cos(f*x+e)^3+20*cos(f*x+e)^2-
10*cos(f*x+e))/sin(f*x+e)/(-1+cos(f*x+e))^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.55039, size = 1106, normalized size = 10.63

$$\left[\frac{5\left(a^2\cos\left(fx+e\right)^2-2\,a^2\cos\left(fx+e\right)+a^2\right)\sqrt{-a}\log\left(-\frac{8\,a\cos\left(fx+e\right)^3-4\left(2\,\cos\left(fx+e\right)^2-\cos\left(fx+e\right)\right)\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)-7\,a\cos\ldots}{\cos\left(fx+e\right)+1}\ldots\right.}{10\left(c^3f\cos\left(fx+e\right)^2-2\,c^3f\cos\left(fx+\ldots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/10*(5*(a^2*cos(f*x + e)^2 - 2*a^2*cos(f*x + e) + a^2)*sqrt(-a)*log(-(8*a
*cos(f*x + e)^3 - 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x +
 e) + 1))*sin(f*x + e) + 4*(9*a^2*cos(f*x + e)^3 - 10*a^2*cos(f*x + e)^2 +
5*a^2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((c^3*f*cos(f*
x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e)), 1/5*(5*(a^2*cos(f*x
 + e)^2 - 2*a^2*cos(f*x + e) + a^2)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(f*
x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a
*cos(f*x + e) - a))*sin(f*x + e) + 2*(9*a^2*cos(f*x + e)^3 - 10*a^2*cos(f*x
 + e)^2 + 5*a^2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((c^
3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.63 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^4}\,dx$

Optimal. Leaf size=140

$$\frac{2a^2\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{c^4f}+\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{c^4f}-\frac{8\cot^7(e+fx)(a\sec(e+fx)+a)^{7/2}}{7ac^4f}-\frac{2a\cot^3(e+f}{}$$

```
[Out] (2*a^(5/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c^4*f)
 + (2*a^2*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c^4*f) - (2*a*Cot[e + f*x
]^3*(a + a*Sec[e + f*x])^(3/2))/(3*c^4*f) - (8*Cot[e + f*x]^7*(a + a*Sec[e
+ f*x])^(7/2))/(7*a*c^4*f)
```

Rubi [A] time = 0.184025, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3904, 3887, 461, 203}

$$\frac{2a^2\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{c^4f}+\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{c^4f}-\frac{8\cot^7(e+fx)(a\sec(e+fx)+a)^{7/2}}{7ac^4f}-\frac{2a\cot^3(e+f}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (2*a^(5/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c^4*f)
 + (2*a^2*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c^4*f) - (2*a*Cot[e + f*x
]^3*(a + a*Sec[e + f*x])^(3/2))/(3*c^4*f) - (8*Cot[e + f*x]^7*(a + a*Sec[e
+ f*x])^(7/2))/(7*a*c^4*f)
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
```

```
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^4}\,dx = \frac{\int \cot^8(e+fx)(a+a\sec(e+fx))^{13/2}\,dx}{a^4c^4}$$

$$= -\frac{2\,\mathrm{Subst}\left(\int \frac{\left(2+ax^2\right)^2}{x^8\left(1+ax^2\right)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{ac^4f}$$

$$= -\frac{2\,\mathrm{Subst}\left(\int \left(\frac{4}{x^8}+\frac{a^2}{x^4}-\frac{a^3}{x^2}+\frac{a^4}{1+ax^2}\right)dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{ac^4f}$$

$$= \frac{2a^2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^4f} - \frac{2a\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3c^4f} - \frac{8\cot^7(e+fx \ldots}{\ldots}$$

$$= \frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^4f} + \frac{2a^2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^4f} - \frac{2a\cot^3(e+fx)(a+a\,\mathrm{s}\ldots}{3c^4f}$$

Mathematica [C] time = 7.93817, size = 361, normalized size = 2.58

$$-\frac{\sin^8\left(\frac{e}{2}+\frac{fx}{2}\right)\sqrt{\frac{1}{1-2\sin^2\left(\frac{1}{2}(e+fx)\right)}}\sqrt{1-2\sin^2\left(\frac{1}{2}(e+fx)\right)}\csc^7\left(\frac{1}{2}(e+fx)\right)\sec^5\left(\frac{1}{2}(e+fx)\right)\sec^{\frac{3}{2}}(e+fx)(a(\sec(e+fx)+\ldots}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^4,x]
```

```
[Out] -(Csc[(e + f*x)/2]^7*Sec[(e + f*x)/2]^5*Sec[e + f*x]^(3/2)*(a*(1 + Sec[e +
f*x]))^(5/2)*Sin[e/2 + (f*x)/2]^8*Sqrt[(1 - 2*Sin[(e + f*x)/2]^2)^(-1)]*Sqr
t[1 - 2*Sin[(e + f*x)/2]^2]*(336*Hypergeometric2F1[-5/2, -1/2, 1/2, 2*Sin[(
e + f*x)/2]^2]*Sin[(e + f*x)/2]^2*(3 - 8*Sin[(e + f*x)/2]^2 + 5*Sin[(e + f*
x)/2]^4) + 4*Hypergeometric2F1[-7/2, -3/2, -1/2, 2*Sin[(e + f*x)/2]^2]*(15
- 42*Sin[(e + f*x)/2]^2 + 35*Sin[(e + f*x)/2]^4) - 105*Cos[(e + f*x)/2]^4*(
3*Sqrt[2]*ArcSin[Sqrt[2]*Sqrt[Sin[(e + f*x)/2]^2]]*(Sin[(e + f*x)/2]^2)^(3/
2) + 2*Sin[(e + f*x)/2]^4*(5 - 4*Sin[(e + f*x)/2]^2)*Sqrt[1 - 2*Sin[(e + f*
x)/2]^2])))/(210*f*(c - c*Sec[e + f*x])^4)
```

Maple [B] time = 0.295, size = 395, normalized size = 2.8

$$-\frac{a^2}{21\,fc^4\sin\left(fx+e\right)\left(-1+\cos\left(fx+e\right)\right)^3}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\left(21\,\sqrt{2}\left(\cos\left(fx+e\right)\right)^3\sin\left(fx+e\right)\sqrt{-2\,\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^4,x)
```

```
[Out] -1/21/c^4/f*a^2*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(21*2^(1/2)*cos(f*x+e
)^3*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*2^(1/2)*(-2
*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))-63*cos(f*x+e)^2*si
n(f*x+e)*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*2^(1/2)*(
-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))+63*2^(1/2)*cos(f
*x+e)*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*2^(1/2)*(
-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))-21*2^(1/2)*sin(f
*x+e)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/c
os(f*x+e))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)-80*cos(f*x+e)^4+154*cos(f*x
+e)^3-140*cos(f*x+e)^2+42*cos(f*x+e))/sin(f*x+e)/(-1+cos(f*x+e))^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.61115, size = 1310, normalized size = 9.36

$$\left[\frac{21\left(a^2\cos\left(fx+e\right)^3-3a^2\cos\left(fx+e\right)^2+3a^2\cos\left(fx+e\right)-a^2\right)\sqrt{-a}\log\left(-\frac{8a\cos\left(fx+e\right)^3-4\left(2\cos\left(fx+e\right)^2-\cos\left(fx+e\right)\right)\sqrt{-a}\sqrt{\frac{a\cos}{\cos}}}{\cos\left(fx+e\right)+1}\right.}{42\left(c^4f\cos\left(fx+e\right)^3-3c^4f\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] [1/42*(21*(a^2*cos(f*x + e)^3 - 3*a^2*cos(f*x + e)^2 + 3*a^2*cos(f*x + e) -
 a^2)*sqrt(-a)*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(f*x + e)^2 - cos(f*x + e
))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(
f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) + 4*(40*a^2*cos(f*x + e)^4 -
 77*a^2*cos(f*x + e)^3 + 70*a^2*cos(f*x + e)^2 - 21*a^2*cos(f*x + e))*sqrt(
(a*cos(f*x + e) + a)/cos(f*x + e)))/((c^4*f*cos(f*x + e)^3 - 3*c^4*f*cos(f*
x + e)^2 + 3*c^4*f*cos(f*x + e) - c^4*f)*sin(f*x + e)), 1/21*(21*(a^2*cos(f
*x + e)^3 - 3*a^2*cos(f*x + e)^2 + 3*a^2*cos(f*x + e) - a^2)*sqrt(a)*arctan
(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e
)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) + 2*(40*a^2*cos(f
*x + e)^4 - 77*a^2*cos(f*x + e)^3 + 70*a^2*cos(f*x + e)^2 - 21*a^2*cos(f*x
+ e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((c^4*f*cos(f*x + e)^3 - 3*c
^4*f*cos(f*x + e)^2 + 3*c^4*f*cos(f*x + e) - c^4*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**4,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^4,x, algorithm="giac")
```

```
[Out] Timed out
```

3.64 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^5}\,dx$

Optimal. Leaf size=172

$$\frac{8\cot^9(e+fx)(a\sec(e+fx)+a)^{9/2}}{9a^2c^5f}+\frac{2a^2\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{c^5f}+\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{c^5f}+\frac{2\cot^5(e+fx)}{}$$

```
[Out] (2*a^(5/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c^5*f)
 + (2*a^2*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c^5*f) - (2*a*Cot[e + f*x
]^3*(a + a*Sec[e + f*x])^(3/2))/(3*c^5*f) + (2*Cot[e + f*x]^5*(a + a*Sec[e
+ f*x])^(5/2))/(5*c^5*f) + (8*Cot[e + f*x]^9*(a + a*Sec[e + f*x])^(9/2))/(9
*a^2*c^5*f)
```

Rubi [A] time = 0.198272, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3904, 3887, 461, 203}

$$\frac{8\cot^9(e+fx)(a\sec(e+fx)+a)^{9/2}}{9a^2c^5f}+\frac{2a^2\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{c^5f}+\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{c^5f}+\frac{2\cot^5(e+fx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^5,x]
```

```
[Out] (2*a^(5/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c^5*f)
 + (2*a^2*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c^5*f) - (2*a*Cot[e + f*x
]^3*(a + a*Sec[e + f*x])^(3/2))/(3*c^5*f) + (2*Cot[e + f*x]^5*(a + a*Sec[e
+ f*x])^(5/2))/(5*c^5*f) + (8*Cot[e + f*x]^9*(a + a*Sec[e + f*x])^(9/2))/(9
*a^2*c^5*f)
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^5}\,dx = -\frac{\int \cot^{10}(e+fx)(a+a\sec(e+fx))^{15/2}\,dx}{a^5c^5}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{\left(2+ax^2\right)^2}{x^{10}\left(1+ax^2\right)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^2c^5f}$$

$$= \frac{2\,\text{Subst}\left(\int \left(\frac{4}{x^{10}}+\frac{a^2}{x^6}-\frac{a^3}{x^4}+\frac{a^4}{x^2}-\frac{a^5}{1+ax^2}\right)dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^2c^5f}$$

$$= \frac{2a^2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^5f} - \frac{2a\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3c^5f} + \frac{2\cot^5(e+\ldots}{\ldots}$$

$$= \frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^5f} + \frac{2a^2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^5f} - \frac{2a\cot^3(e+fx)(a+a\ldots}{3c^5f}$$

Mathematica [C] time = 3.47695, size = 205, normalized size = 1.19

$$\frac{a^2\tan\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}\left(-15(\cos(e+fx)-2\cos(2(e+fx))-\cos(3(e+fx))+2)\text{HypergeometricPF}\ldots\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^5,x]
```

```
[Out] (a^2*Sqrt[a*(1 + Sec[e + f*x])]*((109 + 108*Cos[e + f*x] + 63*Cos[2*(e + f*
x)])*Hypergeometric2F1[-9/2, -5/2, -3/2, 2*Sin[(e + f*x)/2]^2] - 15*(2 + Co
s[e + f*x] - 2*Cos[2*(e + f*x)] - Cos[3*(e + f*x)])*HypergeometricPFQ[{-7/2
, -3/2, 2}, {-1/2, 1}, 2*Sin[(e + f*x)/2]^2] + 240*(1 + 2*Cos[e + f*x])*Hyp
ergeometric2F1[-7/2, -3/2, -1/2, 2*Sin[(e + f*x)/2]^2]*Sin[e + f*x]^2)*Tan[
(e + f*x)/2])/(315*c^5*f*Cos[e + f*x]^(9/2)*(-1 + Sec[e + f*x])^5)
```

Maple [B] time = 0.357, size = 492, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^5,x)
```

```
[Out] 1/45/c^5/f*a^2*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(1+cos(f*x+e))*(45*(-2
*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*cos(f*x+e)^4*sin(f*x+e)*2^(1/2)*arctanh(1
/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))-180*
2^(1/2)*cos(f*x+e)^3*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctan
h(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))+2
70*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arc
tanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e)
)-180*2^(1/2)*cos(f*x+e)*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ar
ctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e
))-178*cos(f*x+e)^5+45*2^(1/2)*sin(f*x+e)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e
)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*(-2*cos(f*x+e)/(1+cos(f*x+e)
))^(1/2)+486*cos(f*x+e)^4-648*cos(f*x+e)^3+390*cos(f*x+e)^2-90*cos(f*x+e))/
sin(f*x+e)^3/(-1+cos(f*x+e))^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^5,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.6696, size = 1512, normalized size = 8.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^5,x, algorithm="fricas")
```

```
[Out] [1/90*(45*(a^2*cos(f*x + e)^4 - 4*a^2*cos(f*x + e)^3 + 6*a^2*cos(f*x + e)^2
 - 4*a^2*cos(f*x + e) + a^2)*sqrt(-a)*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(f
*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*
sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) + 4*(
89*a^2*cos(f*x + e)^5 - 243*a^2*cos(f*x + e)^4 + 324*a^2*cos(f*x + e)^3 - 1
95*a^2*cos(f*x + e)^2 + 45*a^2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(
f*x + e)))/((c^5*f*cos(f*x + e)^4 - 4*c^5*f*cos(f*x + e)^3 + 6*c^5*f*cos(f*
x + e)^2 - 4*c^5*f*cos(f*x + e) + c^5*f)*sin(f*x + e)), 1/45*(45*(a^2*cos(f
*x + e)^4 - 4*a^2*cos(f*x + e)^3 + 6*a^2*cos(f*x + e)^2 - 4*a^2*cos(f*x + e
) + a^2)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*c
os(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x
 + e) + 2*(89*a^2*cos(f*x + e)^5 - 243*a^2*cos(f*x + e)^4 + 324*a^2*cos(f*x
 + e)^3 - 195*a^2*cos(f*x + e)^2 + 45*a^2*cos(f*x + e))*sqrt((a*cos(f*x + e
) + a)/cos(f*x + e)))/((c^5*f*cos(f*x + e)^4 - 4*c^5*f*cos(f*x + e)^3 + 6*c
^5*f*cos(f*x + e)^2 - 4*c^5*f*cos(f*x + e) + c^5*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**5,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^5,x, algorithm="giac")
```

```
[Out] Timed out
```

3.65 $\int \frac{(c-c\sec(e+fx))^4}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal. Leaf size=185

$$\frac{2a^2c^4\tan^5(e+fx)}{5f(a\sec(e+fx)+a)^{5/2}}-\frac{2ac^4\tan^3(e+fx)}{f(a\sec(e+fx)+a)^{3/2}}+\frac{2c^4\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f}-\frac{16\sqrt{2}c^4\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f}+\frac{1\ldots}{f\sqrt{\ldots}}$$

```
[Out] (2*c^4*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(Sqrt[a]*f)
 - (16*Sqrt[2]*c^4*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e
+ f*x]])])/(Sqrt[a]*f) + (14*c^4*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]])
 - (2*a*c^4*Tan[e + f*x]^3)/(f*(a + a*Sec[e + f*x])^(3/2)) + (2*a^2*c^4*Tan
[e + f*x]^5)/(5*f*(a + a*Sec[e + f*x])^(5/2))
```

Rubi [A] time = 0.276144, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3904, 3887, 479, 582, 522, 203}

$$\frac{2a^2c^4\tan^5(e+fx)}{5f(a\sec(e+fx)+a)^{5/2}}-\frac{2ac^4\tan^3(e+fx)}{f(a\sec(e+fx)+a)^{3/2}}+\frac{2c^4\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f}-\frac{16\sqrt{2}c^4\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f}+\frac{1\ldots}{f\sqrt{\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^4/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*c^4*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(Sqrt[a]*f)
 - (16*Sqrt[2]*c^4*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e
+ f*x]])])/(Sqrt[a]*f) + (14*c^4*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]])
 - (2*a*c^4*Tan[e + f*x]^3)/(f*(a + a*Sec[e + f*x])^(3/2)) + (2*a^2*c^4*Tan
[e + f*x]^5)/(5*f*(a + a*Sec[e + f*x])^(5/2))
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(c-c\sec(e+fx))^4}{\sqrt{a+a\sec(e+fx)}}\,dx = \left(a^4c^4\right)\int \frac{\tan^8(e+fx)}{(a+a\sec(e+fx))^{9/2}}\,dx$$

$$= -\frac{\left(2a^4c^4\right)\text{Subst}\left(\int \frac{x^8}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= \frac{2a^2c^4\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}} + \frac{\left(2a^2c^4\right)\text{Subst}\left(\int \frac{x^4(10+15ax^2)}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{5f}$$

$$= -\frac{2ac^4\tan^3(e+fx)}{f(a+a\sec(e+fx))^{3/2}} + \frac{2a^2c^4\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}} - \frac{\left(2c^4\right)\text{Subst}\left(\int \frac{x^2(90a+105a^2x^2)}{(1+ax^2)(2+ax^2)}\,dx, x, \ldots\right.}{15f}$$

$$= \frac{14c^4\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{2ac^4\tan^3(e+fx)}{f(a+a\sec(e+fx))^{3/2}} + \frac{2a^2c^4\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}} + \frac{\left(2c^4\right)\text{Subst}\left(\ldots\right.}{\ldots}$$

$$= \frac{14c^4\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{2ac^4\tan^3(e+fx)}{f(a+a\sec(e+fx))^{3/2}} + \frac{2a^2c^4\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}} - \frac{\left(2c^4\right)\text{Subst}\left(\ldots\right.}{\ldots}$$

$$= \frac{2c^4\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}f} - \frac{16\sqrt{2}c^4\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}f} + \frac{14c^4\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{2\ldots}{f(a\ldots}$$

Mathematica [A] time = 1.42351, size = 153, normalized size = 0.83

$$\frac{c^4\cot\left(\frac{1}{2}(e+fx)\right)\sec^3(e+fx)\left(-155\cos(e+fx)+96\cos(2(e+fx))-41\cos(3(e+fx))+20\cos^3(e+fx)\sqrt{\sec(e+f\ldots}\right.}{10f\sqrt{a(\sec(e+fx)+\ldots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^4/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (c^4*Cot[(e + f*x)/2]*(100 - 155*Cos[e + f*x] + 96*Cos[2*(e + f*x)] - 41*Co
s[3*(e + f*x)] + 20*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Cos[e + f*x]^3*Sqrt[-1
+ Sec[e + f*x]] - 160*Sqrt[2]*ArcTan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]]*Cos[e
 + f*x]^3*Sqrt[-1 + Sec[e + f*x]])*Sec[e + f*x]^3)/(10*f*Sqrt[a*(1 + Sec[e
+ f*x])])
```

Maple [B] time = 0.321, size = 544, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/20*c^4/f/a*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(5*2^(1/2)*sin(f*x+e)*c
os(f*x+e)^2*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*
x+e)/cos(f*x+e))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)+80*sin(f*x+e)*cos(f*x
+e)^2*ln(((-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin
(f*x+e))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)+10*2^(1/2)*sin(f*x+e)*cos(f*x
+e)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos
(f*x+e))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)+160*sin(f*x+e)*cos(f*x+e)*ln(
((-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))*
(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)+5*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(
f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*(-2*cos(f*x+e)/(1+cos(f
*x+e)))^(5/2)*sin(f*x+e)+80*ln(((-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*
x+e)-cos(f*x+e)+1)/sin(f*x+e))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)*sin(f*x
+e)+328*cos(f*x+e)^3-384*cos(f*x+e)^2+64*cos(f*x+e)-8)/cos(f*x+e)^2/sin(f*x
+e)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 5.2609, size = 1420, normalized size = 7.68

$$\left[\frac{40\sqrt{2}\left(ac^4\cos\left(fx+e\right)^3+ac^4\cos\left(fx+e\right)^2\right)\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{-\frac{1}{a}}\cos(fx+e)\sin(fx+e)+3\cos(fx+e)^2+2\cos(fx+e)-1}{\cos(fx+e)^2+2\cos(fx+e)+1}\right)\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/5*(40*sqrt(2)*(a*c^4*cos(f*x + e)^3 + a*c^4*cos(f*x + e)^2)*sqrt(-1/a)*l
og((2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x +
e)*sin(f*x + e) + 3*cos(f*x + e)^2 + 2*cos(f*x + e) - 1)/(cos(f*x + e)^2 +
2*cos(f*x + e) + 1)) - 5*(c^4*cos(f*x + e)^3 + c^4*cos(f*x + e)^2)*sqrt(-a)
*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e
))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*
(41*c^4*cos(f*x + e)^2 - 7*c^4*cos(f*x + e) + c^4)*sqrt((a*cos(f*x + e) + a
)/cos(f*x + e))*sin(f*x + e))/(a*f*cos(f*x + e)^3 + a*f*cos(f*x + e)^2), -2
/5*(5*(c^4*cos(f*x + e)^3 + c^4*cos(f*x + e)^2)*sqrt(a)*arctan(sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - (41*c^4*
cos(f*x + e)^2 - 7*c^4*cos(f*x + e) + c^4)*sqrt((a*cos(f*x + e) + a)/cos(f*
x + e))*sin(f*x + e) - 40*sqrt(2)*(a*c^4*cos(f*x + e)^3 + a*c^4*cos(f*x + e
)^2)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(s
qrt(a)*sin(f*x + e)))/sqrt(a))/(a*f*cos(f*x + e)^3 + a*f*cos(f*x + e)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^4\left(\int -\frac{4\sec\left(e+fx\right)}{\sqrt{a\sec\left(e+fx\right)+a}}\,dx+\int \frac{6\sec^2\left(e+fx\right)}{\sqrt{a\sec\left(e+fx\right)+a}}\,dx+\int -\frac{4\sec^3\left(e+fx\right)}{\sqrt{a\sec\left(e+fx\right)+a}}\,dx+\int \frac{\sec^4\left(e+fx\right)}{\sqrt{a\sec\left(e+fx\right)+a}}\,dx\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**4/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] c**4*(Integral(-4*sec(e + f*x)/sqrt(a*sec(e + f*x) + a), x) + Integral(6*se
c(e + f*x)**2/sqrt(a*sec(e + f*x) + a), x) + Integral(-4*sec(e + f*x)**3/sq
rt(a*sec(e + f*x) + a), x) + Integral(sec(e + f*x)**4/sqrt(a*sec(e + f*x) +
 a), x) + Integral(1/sqrt(a*sec(e + f*x) + a), x))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.66 $\int \frac{(c-c\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal. Leaf size=152

$$-\frac{2ac^3\tan^3(e+fx)}{3f(a\sec(e+fx)+a)^{3/2}}+\frac{2c^3\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f}-\frac{8\sqrt{2}c^3\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f}+\frac{6c^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

```
[Out] (2*c^3*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(Sqrt[a]*f)
 - (8*Sqrt[2]*c^3*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e +
 f*x]])])/(Sqrt[a]*f) + (6*c^3*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) -
 (2*a*c^3*Tan[e + f*x]^3)/(3*f*(a + a*Sec[e + f*x])^(3/2))
```

Rubi [A] time = 0.228497, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3904, 3887, 479, 582, 522, 203}

$$-\frac{2ac^3\tan^3(e+fx)}{3f(a\sec(e+fx)+a)^{3/2}}+\frac{2c^3\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f}-\frac{8\sqrt{2}c^3\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f}+\frac{6c^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^3/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*c^3*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(Sqrt[a]*f)
 - (8*Sqrt[2]*c^3*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e +
 f*x]])])/(Sqrt[a]*f) + (6*c^3*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) -
 (2*a*c^3*Tan[e + f*x]^3)/(3*f*(a + a*Sec[e + f*x])^(3/2))
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(c-c\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx = -\left(\left(a^3c^3\right)\int \frac{\tan^6(e+fx)}{(a+a\sec(e+fx))^{7/2}}\,dx\right)$$

$$= \frac{\left(2a^3c^3\right)\text{Subst}\left(\int \frac{x^6}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= -\frac{2ac^3\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} - \frac{\left(2ac^3\right)\text{Subst}\left(\int \frac{x^2(6+9ax^2)}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{3f}$$

$$= \frac{6c^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{2ac^3\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} + \frac{\left(2c^3\right)\text{Subst}\left(\int \frac{18a+21a^2x^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\text{t}}{\sqrt{a+}}\right.}{3af}$$

$$= \frac{6c^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{2ac^3\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} - \frac{\left(2c^3\right)\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+f}}\right.}{f}$$

$$= \frac{2c^3\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}f} - \frac{8\sqrt{2}c^3\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}f} + \frac{6c^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{2a}{3f(a}$$

Mathematica [A] time = 1.31573, size = 166, normalized size = 1.09

$$\frac{4c^3\cos\left(\frac{e}{2}\right)\cos(e)\cot\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\left(11\cos(e+fx)-5\cos(2(e+fx))+3\cos^2(e+fx)\sqrt{\sec(e+fx)-1}\tan\right.}{3f\left(\cos\left(\frac{e}{2}\right)+\cos\left(\frac{3e}{2}\right)\right)\sqrt{a(\sec(e+fx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^3/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (4*c^3*Cos[e/2]*Cos[e]*Cot[(e + f*x)/2]*(-6 + 11*Cos[e + f*x] - 5*Cos[2*(e
+ f*x)] + 3*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Cos[e + f*x]^2*Sqrt[-1 + Sec[e
+ f*x]] - 12*Sqrt[2]*ArcTan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]]*Cos[e + f*x]^2
*Sqrt[-1 + Sec[e + f*x]])*Sec[e + f*x]^2)/(3*f*(Cos[e/2] + Cos[(3*e)/2])*Sq
rt[a*(1 + Sec[e + f*x])])
```

Maple [B] time = 0.299, size = 372, normalized size = 2.5

$$-\frac{c^3}{6af\sin(fx+e)\cos(fx+e)}\sqrt{\frac{a(1+\cos(fx+e))}{\cos(fx+e)}}\left(-3\,\text{Artanh}\left(1/2\,\frac{\sqrt{2}\sin(fx+e)}{\cos(fx+e)}\sqrt{-2\,\frac{\cos(fx+e)}{1+\cos(fx+e)}}\right)\left(-2\,\frac{\text{c}}{1+}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/6*c^3/f/a*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(-3*arctanh(1/2*2^(1/2)*
(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*(-2*cos(f*x+e)/
(1+cos(f*x+e)))^(3/2)*2^(1/2)*sin(f*x+e)*cos(f*x+e)-24*cos(f*x+e)*sin(f*x+e
)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)*ln(((-2*cos(f*x+e)/(1+cos(f*x+e)))^(
1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))-3*2^(1/2)*arctanh(1/2*2^(1/2)*(-2
*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*(-2*cos(f*x+e)/(1+
cos(f*x+e)))^(3/2)*sin(f*x+e)-24*ln(((-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*s
in(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)*si
n(f*x+e)+40*cos(f*x+e)^2-44*cos(f*x+e)+4)/sin(f*x+e)/cos(f*x+e)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 4.49942, size = 1342, normalized size = 8.83

$$\left[\frac{12\sqrt{2}\left(ac^3\cos\left(fx+e\right)^2+ac^3\cos\left(fx+e\right)\right)\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{-\frac{1}{a}}\cos(fx+e)\sin(fx+e)+3\cos(fx+e)^2+2\cos(fx+e)-1}{\cos(fx+e)^2+2\cos(fx+e)+1}\right)-3}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/3*(12*sqrt(2)*(a*c^3*cos(f*x + e)^2 + a*c^3*cos(f*x + e))*sqrt(-1/a)*log
((2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x + e)
*sin(f*x + e) + 3*cos(f*x + e)^2 + 2*cos(f*x + e) - 1)/(cos(f*x + e)^2 + 2*
cos(f*x + e) + 1)) - 3*(c^3*cos(f*x + e)^2 + c^3*cos(f*x + e))*sqrt(-a)*log
((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*c
os(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*(10*
c^3*cos(f*x + e) - c^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e
))/(a*f*cos(f*x + e)^2 + a*f*cos(f*x + e)), -2/3*(3*(c^3*cos(f*x + e)^2 + c
^3*cos(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos
(f*x + e)/(sqrt(a)*sin(f*x + e))) - (10*c^3*cos(f*x + e) - c^3)*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 12*sqrt(2)*(a*c^3*cos(f*x + e)^
2 + a*c^3*cos(f*x + e))*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x +
e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))/sqrt(a))/(a*f*cos(f*x + e)^2 + a*f
*cos(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-c^3\left(\int \frac{3\sec\left(e+fx\right)}{\sqrt{a\sec\left(e+fx\right)+a}}\,dx+\int -\frac{3\sec^2\left(e+fx\right)}{\sqrt{a\sec\left(e+fx\right)+a}}\,dx+\int \frac{\sec^3\left(e+fx\right)}{\sqrt{a\sec\left(e+fx\right)+a}}\,dx+\int -\frac{1}{\sqrt{a\sec\left(e+fx\right)+a}}\,d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**3/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] -c**3*(Integral(3*sec(e + f*x)/sqrt(a*sec(e + f*x) + a), x) + Integral(-3*s
ec(e + f*x)**2/sqrt(a*sec(e + f*x) + a), x) + Integral(sec(e + f*x)**3/sqrt
(a*sec(e + f*x) + a), x) + Integral(-1/sqrt(a*sec(e + f*x) + a), x))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.67 $\int \frac{(c-c\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal. Leaf size=119

$$\frac{2c^2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f}-\frac{4\sqrt{2}c^2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f}+\frac{2c^2\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

```
[Out] (2*c^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(Sqrt[a]*f)
 - (4*Sqrt[2]*c^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e +
 f*x]])])/(Sqrt[a]*f) + (2*c^2*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]])
```

Rubi [A] time = 0.18764, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3904, 3887, 479, 522, 203}

$$\frac{2c^2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f}-\frac{4\sqrt{2}c^2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f}+\frac{2c^2\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^2/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*c^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(Sqrt[a]*f)
 - (4*Sqrt[2]*c^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e +
 f*x]])])/(Sqrt[a]*f) + (2*c^2*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
```

```
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(c-c\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx = \left(a^2c^2\right)\int \frac{\tan^4(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx$$

$$= -\frac{\left(2a^2c^2\right)\operatorname{Subst}\left(\int \frac{x^4}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= \frac{2c^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{\left(2c^2\right)\operatorname{Subst}\left(\int \frac{2+3ax^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= \frac{2c^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{\left(2c^2\right)\operatorname{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} + \frac{\left(8c^2\right)\operatorname{Subst}\left(\int \frac{1}{2+ax^2}\right.}{}$$

$$= \frac{2c^2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}f} - \frac{4\sqrt{2}c^2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}f} + \frac{2c^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}$$

Mathematica [A] time = 0.475033, size = 124, normalized size = 1.04

$$\frac{2c^2\cot\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\left(-\cos(e+fx)+\cos(e+fx)\sqrt{\sec(e+fx)-1}\tan^{-1}\left(\sqrt{\sec(e+fx)-1}\right)-2\sqrt{2}\cos(e+\right.}{f\sqrt{a(\sec(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^2/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*c^2*Cot[(e + f*x)/2]*(1 - Cos[e + f*x] + ArcTan[Sqrt[-1 + Sec[e + f*x]]]
*Cos[e + f*x]*Sqrt[-1 + Sec[e + f*x]] - 2*Sqrt[2]*ArcTan[Sqrt[-1 + Sec[e +
f*x]]/Sqrt[2]]*Cos[e + f*x]*Sqrt[-1 + Sec[e + f*x]])*Sec[e + f*x])/(f*Sqrt[
a*(1 + Sec[e + f*x])])
```

Maple [B] time = 0.277, size = 330, normalized size = 2.8

$$\frac{c^2}{af\left(1+\cos\left(fx+e\right)\right)}\left(-\sqrt{2}\cos\left(fx+e\right)\sqrt{-2\,\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\mathrm{Artanh}\left(\frac{\sqrt{2}\sin\left(fx+e\right)}{2\,\cos\left(fx+e\right)}\sqrt{-2\,\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\right)-4\,\mathrm{co}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] c^2/f/a*(-2^(1/2)*cos(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1
/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))-4*co
s(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(((-2*cos(f*x+e)/(1+cos(f*x
+e)))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))-2^(1/2)*(-2*cos(f*x+e)/(1+
cos(f*x+e)))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)
*sin(f*x+e)/cos(f*x+e))-4*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(((-2*cos(
f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))+2*sin(f*x
+e))*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)/(1+cos(f*x+e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.75899, size = 1138, normalized size = 9.56

$$\left[\frac{2\,c^{2}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)+2\sqrt{2}\left(ac^{2}\cos\left(fx+e\right)+ac^{2}\right)\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{-\frac{1}{a}}\cos\left(fx+e\right)\sin\left(fx+e\right)+3\cos\left(fx+e\right)^{2}}{\cos\left(fx+e\right)^{2}+2\cos\left(fx+e\right)+1}\right.}{af\cos\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [(2*c^2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) + 2*sqrt(2)*(a
*c^2*cos(f*x + e) + a*c^2)*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(f*x + e) +
 a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) + 3*cos(f*x + e)^2 +
 2*cos(f*x + e) - 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) - (c^2*cos(f*x
+ e) + c^2)*sqrt(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos
(f*x + e) + 1)))/(a*f*cos(f*x + e) + a*f), 2*(c^2*sqrt((a*cos(f*x + e) + a)
/cos(f*x + e))*sin(f*x + e) - (c^2*cos(f*x + e) + c^2)*sqrt(a)*arctan(sqrt(
(a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + 2
*sqrt(2)*(a*c^2*cos(f*x + e) + a*c^2)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) +
 a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))/sqrt(a))/(a*f*cos(f*
x + e) + a*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^2\left(\int -\frac{2\sec\left(e+fx\right)}{\sqrt{a\sec\left(e+fx\right)+a}}\,dx+\int \frac{\sec^2\left(e+fx\right)}{\sqrt{a\sec\left(e+fx\right)+a}}\,dx+\int \frac{1}{\sqrt{a\sec\left(e+fx\right)+a}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**2/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] c**2*(Integral(-2*sec(e + f*x)/sqrt(a*sec(e + f*x) + a), x) + Integral(sec(
e + f*x)**2/sqrt(a*sec(e + f*x) + a), x) + Integral(1/sqrt(a*sec(e + f*x) +
 a), x))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.68 $\int \frac{c-c\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal. Leaf size=87

$$\frac{2c\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f}-\frac{2\sqrt{2}c\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f}$$

```
[Out] (2*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(Sqrt[a]*f) -
 (2*Sqrt[2]*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x
]])])/(Sqrt[a]*f)
```

Rubi [A] time = 0.137723, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3904, 3887, 481, 203}

$$\frac{2c\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f}-\frac{2\sqrt{2}c\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(Sqrt[a]*f) -
 (2*Sqrt[2]*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x
]])])/(Sqrt[a]*f)
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
```

```
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{c-c\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx &= -\left((ac)\int \frac{\tan^2(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx\right)\\
&= \frac{(2ac)\,\text{Subst}\left(\int \frac{x^2}{\left(1+ax^2\right)\left(2+ax^2\right)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}\\
&= -\frac{(2c)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}+\frac{(4c)\,\text{Subst}\left(\int \frac{1}{2+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}\\
&= \frac{2c\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}f}-\frac{2\sqrt{2}c\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}f}
\end{aligned}$$

Mathematica [A] time = 0.276408, size = 82, normalized size = 0.94

$$\frac{2c\cot\left(\frac{1}{2}(e+fx)\right)\sqrt{\sec(e+fx)-1}\left(\tan^{-1}\left(\sqrt{\sec(e+fx)-1}\right)-\sqrt{2}\tan^{-1}\left(\frac{\sqrt{\sec(e+fx)-1}}{\sqrt{2}}\right)\right)}{f\sqrt{a(\sec(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*c*(ArcTan[Sqrt[-1 + Sec[e + f*x]]] - Sqrt[2]*ArcTan[Sqrt[-1 + Sec[e + f*
x]]/Sqrt[2]])*Cot[(e + f*x)/2]*Sqrt[-1 + Sec[e + f*x]])/(f*Sqrt[a*(1 + Sec[
e + f*x])])
```

Maple [A] time = 0.219, size = 144, normalized size = 1.7

$$-\frac{c}{af}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\sqrt{-2\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\left(\sqrt{2}\mathrm{Artanh}\left(\frac{\sqrt{2}\sin\left(fx+e\right)}{2\cos\left(fx+e\right)}\sqrt{-2\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\right)+2\ln\left(\frac{1}{\sin\left(fx\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] -c/f/a*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))
^(1/2)*(2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*si
n(f*x+e)/cos(f*x+e))+2*ln(((-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-
cos(f*x+e)+1)/sin(f*x+e)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.11928, size = 787, normalized size = 9.05

$$\left[\frac{\sqrt{2}ac\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{-\frac{1}{a}}\cos\left(fx+e\right)\sin\left(fx+e\right)+3\cos\left(fx+e\right)^{2}+2\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)^{2}+2\cos\left(fx+e\right)+1}\right)-\sqrt{-a}c\log\left(\frac{2a\cos\left(fx+e\right)^{2}+2\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}}{\cos\left(f\right.}\right.}{af}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(2)*a*c*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x +
 e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) + 3*cos(f*x + e)^2 + 2*cos(f*x +
e) - 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) - sqrt(-a)*c*log((2*a*cos(f*
x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*
sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/(a*f), 2*(sqrt(2)*s
qrt(a)*c*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e
)/(sqrt(a)*sin(f*x + e))) - sqrt(a)*c*arctan(sqrt((a*cos(f*x + e) + a)/cos(
f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))))/(a*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-c\left(\int \frac{\sec\left(e+fx\right)}{\sqrt{a\sec\left(e+fx\right)+a}}\,dx+\int -\frac{1}{\sqrt{a\sec\left(e+fx\right)+a}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] -c*(Integral(sec(e + f*x)/sqrt(a*sec(e + f*x) + a), x) + Integral(-1/sqrt(a
*sec(e + f*x) + a), x))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.69 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx$

Optimal. Leaf size=121

$$\frac{\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{acf}+\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}cf}-\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{2}\sqrt{a}cf}$$

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(Sqrt[a]*c*f) -
 ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])]/(Sqrt[2]
*Sqrt[a]*c*f) + (Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(a*c*f)
```

Rubi [A] time = 0.192333, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3904, 3887, 480, 522, 203}

$$\frac{\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{acf}+\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}cf}-\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{2}\sqrt{a}cf}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])),x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(Sqrt[a]*c*f) -
 ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])]/(Sqrt[2]
*Sqrt[a]*c*f) + (Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(a*c*f)
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
```

```
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = -\frac{\int \cot^2(e+fx)\sqrt{a+a\sec(e+fx)}\,dx}{ac}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{1}{x^2(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{acf}$$

$$= \frac{\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{acf} + \frac{\text{Subst}\left(\int \frac{-3a-a^2x^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{acf}$$

$$= \frac{\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{acf} + \frac{\text{Subst}\left(\int \frac{1}{2+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf} -$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}cf} - \frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{2}\sqrt{a}cf} + \frac{\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{acf}$$

Mathematica [A] time = 0.552949, size = 101, normalized size = 0.83

$$\frac{\cot\left(\frac{1}{2}(e+fx)\right)\left(4\sqrt{\sec(e+fx)-1}\tan^{-1}\left(\sqrt{\sec(e+fx)-1}\right)-\sqrt{2}\sqrt{\sec(e+fx)-1}\tan^{-1}\left(\frac{\sqrt{\sec(e+fx)-1}}{\sqrt{2}}\right)+2\right)}{2cf\sqrt{a(\sec(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])),x]
```

```
[Out] (Cot[(e + f*x)/2]*(2 + 4*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Sqrt[-1 + Sec[e +
f*x]] - Sqrt[2]*ArcTan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]]*Sqrt[-1 + Sec[e + f
*x]]))/(2*c*f*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [A] time = 0.267, size = 194, normalized size = 1.6

$$-\frac{1}{2\,fca\sin(fx+e)}\sqrt{\frac{a(1+\cos(fx+e))}{\cos(fx+e)}}\left(2\sqrt{2}\sin(fx+e)\,\text{Artanh}\left(1/2\,\frac{\sqrt{2}\sin(fx+e)}{\cos(fx+e)}\sqrt{-2\,\frac{\cos(fx+e)}{1+\cos(fx+e)}}\right)\sqrt{-2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/2/c/f/a*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(2*2^(1/2)*sin(f*x+e)*arct
anh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))
*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)+sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+
e)))^(1/2)*ln(((-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-cos(f*x+e)+1
)/sin(f*x+e))-2*cos(f*x+e))/sin(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\sqrt{a\sec\left(fx+e\right)+a}\left(c\sec\left(fx+e\right)-c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(a*sec(f*x + e) + a)*(c*sec(f*x + e) - c)), x)
```

Fricas [A] time = 2.35921, size = 1143, normalized size = 9.45

$$\left[\frac{\sqrt{2}a\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{-\frac{1}{a}}\cos\left(fx+e\right)\sin\left(fx+e\right)+3\cos\left(fx+e\right)^{2}+2\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)^{2}+2\cos\left(fx+e\right)+1}\right)\sin\left(fx+e\right)-2\sqrt{-a}\log\left(-\frac{8a\cos\left(fx+e\right)^{3}+4\cdots}{\cdots}\right.\cdots}{4acf\sin\left(fx+e\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(2)*a*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x
 + e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) + 3*cos(f*x + e)^2 + 2*cos(f*x
+ e) - 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1))*sin(f*x + e) - 2*sqrt(-a)*
log(-(8*a*cos(f*x + e)^3 + 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqr
t((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(
cos(f*x + e) + 1))*sin(f*x + e) + 4*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))
*cos(f*x + e))/(a*c*f*sin(f*x + e)), 1/2*(sqrt(2)*sqrt(a)*arctan(sqrt(2)*sq
```

```
rt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))*
sin(f*x + e) + 2*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x
 + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))
*sin(f*x + e) + 2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e))/(a*
c*f*sin(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{1}{\sqrt{a\sec\left(e+fx\right)+a}\sec\left(e+fx\right)-\sqrt{a\sec\left(e+fx\right)+a}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] -Integral(1/(sqrt(a*sec(e + f*x) + a)*sec(e + f*x) - sqrt(a*sec(e + f*x) +
a)), x)/c
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.70 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2}\,dx$

Optimal. Leaf size=161

$$-\frac{\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{3a^2c^2f}+\frac{3\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{2ac^2f}+\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}c^2f}-\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{2\sqrt{2}\sqrt{a}c^2f}$$

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(Sqrt[a]*c^2*f)
 - ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])]/(2*Sqr
t[2]*Sqrt[a]*c^2*f) + (3*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(2*a*c^2*f)
 - (Cot[e + f*x]^3*(a + a*Sec[e + f*x])^(3/2))/(3*a^2*c^2*f)
```

Rubi [A] time = 0.234524, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3904, 3887, 480, 583, 522, 203}

$$-\frac{\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{3a^2c^2f}+\frac{3\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{2ac^2f}+\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}c^2f}-\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{2\sqrt{2}\sqrt{a}c^2f}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^2),x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(Sqrt[a]*c^2*f)
 - ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])]/(2*Sqr
t[2]*Sqrt[a]*c^2*f) + (3*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(2*a*c^2*f)
 - (Cot[e + f*x]^3*(a + a*Sec[e + f*x])^(3/2))/(3*a^2*c^2*f)
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2}\,dx = \frac{\int \cot^4(e+fx)(a+a\sec(e+fx))^{3/2}\,dx}{a^2c^2}$$

$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{x^4(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^2c^2f}$$

$$= -\frac{\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3a^2c^2f} - \frac{\text{Subst}\left(\int \frac{-9a-3a^2x^2}{x^2(1+ax^2)(2+ax^2)}\,dx, x, -\right.}{3a^2c^2f}$$

$$= \frac{3\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{2ac^2f} - \frac{\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3a^2c^2f} +$$

$$= \frac{3\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{2ac^2f} - \frac{\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3a^2c^2f} +$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}c^2f} - \frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{2\sqrt{2}\sqrt{a}c^2f} + \frac{3\cot(e+fx)\sqrt{a+}}{2ac^2}$$

Mathematica [C] time = 23.7911, size = 5586, normalized size = 34.7

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.303, size = 377, normalized size = 2.3

$$\frac{1}{12\,fc^2a\left(\sin\left(fx+e\right)\right)^3}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\left(12\,\left(\cos\left(fx+e\right)\right)^2\sin\left(fx+e\right)\sqrt{2}\sqrt{-2\,\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\text{Artanh}\left(1/2\,\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/12/c^2/f/a*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(12*cos(f*x+e)^2*sin(f*x
+e)*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*2^(1/2)*(-2*co
s(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))+3*sin(f*x+e)*cos(f*x+
e)^2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(((-2*cos(f*x+e)/(1+cos(f*x+e))
)^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))-12*2^(1/2)*sin(f*x+e)*arctanh(
1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*(-2
*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)-3*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)
))^(1/2)*ln(((-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/
sin(f*x+e))-22*cos(f*x+e)^3-4*cos(f*x+e)^2+18*cos(f*x+e))/sin(f*x+e)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\sec\left(fx+e\right)+a}\left(c\sec\left(fx+e\right)-c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/(sqrt(a*sec(f*x + e) + a)*(c*sec(f*x + e) - c)^2), x)
```

Fricas [A] time = 2.32107, size = 1395, normalized size = 8.66

$$\left[-\frac{3\sqrt{2}\sqrt{-a}\left(\cos\left(fx+e\right)-1\right)\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)\sin\left(fx+e\right)-3a\cos\left(fx+e\right)^2-2a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)^2+2\cos\left(fx+e\right)+1}\right)\sin\left(fx+e\right)+12\sqrt{-}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas"
)
```

```
[Out] [-1/24*(3*sqrt(2)*sqrt(-a)*(cos(f*x + e) - 1)*log(-(2*sqrt(2)*sqrt(-a)*sqrt
((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 3*a*cos(f*x
```

```
 + e)^2 - 2*a*cos(f*x + e) + a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1))*sin(
f*x + e) + 12*sqrt(-a)*(cos(f*x + e) - 1)*log(-(8*a*cos(f*x + e)^3 + 4*(2*c
os(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x +
e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) -
 4*(11*cos(f*x + e)^2 - 9*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x +
 e)))/((a*c^2*f*cos(f*x + e) - a*c^2*f)*sin(f*x + e)), 1/12*(3*sqrt(2)*sqrt
(a)*(cos(f*x + e) - 1)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e
))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))*sin(f*x + e) + 12*sqrt(a)*(cos(f*x
+ e) - 1)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x
+ e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) +
 2*(11*cos(f*x + e)^2 - 9*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x +
 e)))/((a*c^2*f*cos(f*x + e) - a*c^2*f)*sin(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{\sqrt{a\sec\left(e+fx\right)+a}\sec^2\left(e+fx\right)-2\sqrt{a\sec\left(e+fx\right)+a}\sec\left(e+fx\right)+\sqrt{a\sec\left(e+fx\right)+a}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-c*sec(f*x+e))**2/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**2 - 2*sqrt(a*sec(e + f*x
) + a)*sec(e + f*x) + sqrt(a*sec(e + f*x) + a)), x)/c**2
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.71 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^3}\,dx$

Optimal. Leaf size=196

$$\frac{\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}}{5a^3c^3f}-\frac{\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{2a^2c^3f}+\frac{7\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{4ac^3f}+\frac{2\tan^{-1}\left(\frac{\sqrt{\cdots}}{\sqrt{a\cdots}}\right)}{\sqrt{a}\cdots}$$

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(Sqrt[a]*c^3*f)
 - ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])]/(4*Sqr
t[2]*Sqrt[a]*c^3*f) + (7*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(4*a*c^3*f)
 - (Cot[e + f*x]^3*(a + a*Sec[e + f*x])^(3/2))/(2*a^2*c^3*f) + (Cot[e + f*x
]^5*(a + a*Sec[e + f*x])^(5/2))/(5*a^3*c^3*f)
```

Rubi [A] time = 0.284238, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3904, 3887, 480, 583, 522, 203}

$$\frac{\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}}{5a^3c^3f}-\frac{\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{2a^2c^3f}+\frac{7\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{4ac^3f}+\frac{2\tan^{-1}\left(\frac{\sqrt{\cdots}}{\sqrt{a\cdots}}\right)}{\sqrt{a}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^3),x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(Sqrt[a]*c^3*f)
 - ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])]/(4*Sqr
t[2]*Sqrt[a]*c^3*f) + (7*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(4*a*c^3*f)
 - (Cot[e + f*x]^3*(a + a*Sec[e + f*x])^(3/2))/(2*a^2*c^3*f) + (Cot[e + f*x
]^5*(a + a*Sec[e + f*x])^(5/2))/(5*a^3*c^3*f)
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^3}\,dx = -\frac{\int \cot^6(e+fx)(a+a\sec(e+fx))^{5/2}\,dx}{a^3c^3}$$

$$= \frac{2\,\mathrm{Subst}\left(\int \frac{1}{x^6(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^3c^3f}$$

$$= \frac{\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5a^3c^3f} + \frac{\mathrm{Subst}\left(\int \frac{-15a-5a^2x^2}{x^4(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\mathrm{ta}}{\sqrt{a+}}\right.}{5a^3c^3f}$$

$$= -\frac{\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{2a^2c^3f} + \frac{\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5a^3c^3f}$$

$$= \frac{7\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{4ac^3f} - \frac{\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{2a^2c^3f} + \frac{\mathrm{c}}{}$$

$$= \frac{7\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{4ac^3f} - \frac{\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{2a^2c^3f} + \frac{\mathrm{c}}{}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}c^3f} - \frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{4\sqrt{2}\sqrt{a}c^3f} + \frac{7\cot(e+fx)\sqrt{a+a}}{4ac^3f}$$

Mathematica [C] time = 23.5009, size = 5602, normalized size = 28.58

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^3),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.359, size = 545, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/40/c^3/f/a*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(1+cos(f*x+e))^2*(40*co
s(f*x+e)^2*sin(f*x+e)*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(
1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))+5*s
in(f*x+e)*cos(f*x+e)^2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(((-2*cos(f*x
+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))-80*2^(1/2)*c
os(f*x+e)*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*2^(1/
2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))-10*sin(f*x+e
)*cos(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(((-2*cos(f*x+e)/(1+cos
(f*x+e)))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))+40*2^(1/2)*sin(f*x+e)*
arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x
+e))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)+5*sin(f*x+e)*(-2*cos(f*x+e)/(1+co
s(f*x+e)))^(1/2)*ln(((-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-cos(f*
x+e)+1)/sin(f*x+e))-98*cos(f*x+e)^3+160*cos(f*x+e)^2-70*cos(f*x+e))/sin(f*x
+e)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\sqrt{a \sec\left(fx+e\right)+a}\left(c \sec\left(fx+e\right)-c\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima"
)
```

```
[Out] -integrate(1/(sqrt(a*sec(f*x + e) + a)*(c*sec(f*x + e) - c)^3), x)
```

Fricas [A] time = 2.62276, size = 1627, normalized size = 8.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas"
)
```

```
[Out] [-1/80*(5*sqrt(2)*(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*sqrt(-a)*log(-(2*sq
rt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x
 + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f*x + e) + a)/(cos(f*x + e)^2 + 2*cos(
f*x + e) + 1))*sin(f*x + e) + 40*(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*sqrt
(-a)*log(-(8*a*cos(f*x + e)^3 + 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a
)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x + e) +
 a)/(cos(f*x + e) + 1))*sin(f*x + e) - 4*(49*cos(f*x + e)^3 - 80*cos(f*x +
e)^2 + 35*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((a*c^3*f*
cos(f*x + e)^2 - 2*a*c^3*f*cos(f*x + e) + a*c^3*f)*sin(f*x + e)), 1/40*(5*s
qrt(2)*(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt((a
*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))*sin(f
*x + e) + 40*(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*sqrt(a)*arctan(2*sqrt(a)
*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos
(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) + 2*(49*cos(f*x + e)^3 - 80
*cos(f*x + e)^2 + 35*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))
/((a*c^3*f*cos(f*x + e)^2 - 2*a*c^3*f*cos(f*x + e) + a*c^3*f)*sin(f*x + e))
]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{1}{\sqrt{a\sec\left(e+fx\right)+a}\sec^3\left(e+fx\right)-3\sqrt{a\sec\left(e+fx\right)+a}\sec^2\left(e+fx\right)+3\sqrt{a\sec\left(e+fx\right)+a}\sec\left(e+fx\right)-\sqrt{a\sec\left(e+fx\right)+a}}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-c*sec(f*x+e))**3/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] -Integral(1/(sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**3 - 3*sqrt(a*sec(e + f*
x) + a)*sec(e + f*x)**2 + 3*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) - sqrt(a*
sec(e + f*x) + a)), x)/c**3
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.72 $\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=203

$$\frac{2c^4\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{3/2}f}+\frac{12\sqrt{2}c^4\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{a^{3/2}f}+\frac{8c^4\tan^3(e+fx)}{3f(a\sec(e+fx)+a)^{3/2}}-\frac{14c^4\tan(e+fx)}{af\sqrt{a\sec(e+fx)+a}}-\frac{ac^4\sin$$

```
[Out] (2*c^4*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(3/2)*f)
 + (12*Sqrt[2]*c^4*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e
+ f*x]])])/(a^(3/2)*f) - (14*c^4*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e + f*x]
]) + (8*c^4*Tan[e + f*x]^3)/(3*f*(a + a*Sec[e + f*x])^(3/2)) - (a*c^4*Sec[(
e + f*x)/2]^2*Sin[e + f*x]*Tan[e + f*x]^4)/(f*(a + a*Sec[e + f*x])^(5/2))
```

Rubi [A] time = 0.286233, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3904, 3887, 470, 582, 522, 203}

$$\frac{2c^4\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{3/2}f}+\frac{12\sqrt{2}c^4\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{a^{3/2}f}+\frac{8c^4\tan^3(e+fx)}{3f(a\sec(e+fx)+a)^{3/2}}-\frac{14c^4\tan(e+fx)}{af\sqrt{a\sec(e+fx)+a}}-\frac{ac^4\sin$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^4/(a + a*Sec[e + f*x])^(3/2),x]
```


```
[Out] (2*c^4*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(3/2)*f)
 + (12*Sqrt[2]*c^4*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e
+ f*x]])])/(a^(3/2)*f) - (14*c^4*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e + f*x]
]) + (8*c^4*Tan[e + f*x]^3)/(3*f*(a + a*Sec[e + f*x])^(3/2)) - (a*c^4*Sec[(
e + f*x)/2]^2*Sin[e + f*x]*Tan[e + f*x]^4)/(f*(a + a*Sec[e + f*x])^(5/2))
```


Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{3/2}}\,dx = \left(a^4c^4\right)\int \frac{\tan^8(e+fx)}{(a+a\sec(e+fx))^{11/2}}\,dx$$

$$= -\frac{\left(2a^3c^4\right)\text{Subst}\left(\int \frac{x^8}{(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= -\frac{ac^4\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)\tan^4(e+fx)}{f(a+a\sec(e+fx))^{5/2}} - \frac{\left(ac^4\right)\text{Subst}\left(\int \frac{x^4(10+8ax^2)}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\text{ta}}{\sqrt{a+}}\right.}{f}$$

$$= \frac{8c^4\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} - \frac{ac^4\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)\tan^4(e+fx)}{f(a+a\sec(e+fx))^{5/2}} + \frac{c^4\,\text{Subst}\left(\int \frac{x^2}{(1}\right.}{}$$

$$= -\frac{14c^4\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}} + \frac{8c^4\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} - \frac{ac^4\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)\tan^4}{f(a+a\sec(e+fx))^{5/2}}$$

$$= -\frac{14c^4\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}} + \frac{8c^4\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} - \frac{ac^4\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)\tan^4}{f(a+a\sec(e+fx))^{5/2}}$$

$$= \frac{2c^4\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}f} + \frac{12\sqrt{2}c^4\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}f} - \frac{14c^4\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}} + \frac{}{3f}$$

Mathematica [A] time = 1.53023, size = 196, normalized size = 0.97

$$\frac{c^4\csc\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\left(20\cos(e+fx)-26\cos(2(e+fx))+28\cos(3(e+fx))+6\left(\cos\left(\frac{1}{2}(e+f\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^4/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (c^4*Csc[(e + f*x)/2]*Sec[(e + f*x)/2]*(-22 + 20*Cos[e + f*x] - 26*Cos[2*(e
 + f*x)] + 28*Cos[3*(e + f*x)] + 6*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*(Cos[(e
+ f*x)/2] + Cos[(3*(e + f*x))/2])^2*Sqrt[-1 + Sec[e + f*x]] + 36*Sqrt[2]*Ar
cTan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]]*(Cos[(e + f*x)/2] + Cos[(3*(e + f*x))
/2])^2*Sqrt[-1 + Sec[e + f*x]])*Sec[e + f*x]^2)/(12*a*f*Sqrt[a*(1 + Sec[e +
 f*x])])
```

Maple [B] time = 0.28, size = 552, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(3/2),x)
```

```
[Out] -1/6*c^4/f/a^2*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(-1+cos(f*x+e))*(3*arc
tanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e)
)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)*2^(1/2)*sin(f*x+e)*cos(f*x+e)^2-36*c
os(f*x+e)^2*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)*ln(((-2*cos(f*x
+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))+6*arctanh(1/
2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*(-2*c
os(f*x+e)/(1+cos(f*x+e)))^(3/2)*2^(1/2)*sin(f*x+e)*cos(f*x+e)-72*cos(f*x+e)
*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)*ln(((-2*cos(f*x+e)/(1+cos(
f*x+e)))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))+3*2^(1/2)*arctanh(1/2*2
^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*(-2*cos(
f*x+e)/(1+cos(f*x+e)))^(3/2)*sin(f*x+e)-36*ln(((-2*cos(f*x+e)/(1+cos(f*x+e)
))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))*(-2*cos(f*x+e)/(1+cos(f*x+e))
)^(3/2)*sin(f*x+e)+112*cos(f*x+e)^3-52*cos(f*x+e)^2-64*cos(f*x+e)+4)/cos(f*
x+e)/sin(f*x+e)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 10.8018, size = 1616, normalized size = 7.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/3*(18*sqrt(2)*(a*c^4*cos(f*x + e)^3 + 2*a*c^4*cos(f*x + e)^2 + a*c^4*cos
(f*x + e))*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e
))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) - 3*cos(f*x + e)^2 - 2*cos(f*x + e)
 + 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) - 3*(c^4*cos(f*x + e)^3 + 2*c^
4*cos(f*x + e)^2 + c^4*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x + e)^2 + 2*s
qrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) +
 a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(28*c^4*cos(f*x + e)^2 + 15*c^
4*cos(f*x + e) - c^4)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))
/(a^2*f*cos(f*x + e)^3 + 2*a^2*f*cos(f*x + e)^2 + a^2*f*cos(f*x + e)), -2/3
*(3*(c^4*cos(f*x + e)^3 + 2*c^4*cos(f*x + e)^2 + c^4*cos(f*x + e))*sqrt(a)*
arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*
x + e))) + (28*c^4*cos(f*x + e)^2 + 15*c^4*cos(f*x + e) - c^4)*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*sin(f*x + e) + 18*sqrt(2)*(a*c^4*cos(f*x + e)^3
 + 2*a*c^4*cos(f*x + e)^2 + a*c^4*cos(f*x + e))*arctan(sqrt(2)*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))/sqrt(a))/(
a^2*f*cos(f*x + e)^3 + 2*a^2*f*cos(f*x + e)^2 + a^2*f*cos(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^4\left(\int -\frac{4\sec\left(e+fx\right)}{a\sqrt{a\sec\left(e+fx\right)+a}\sec\left(e+fx\right)+a\sqrt{a\sec\left(e+fx\right)+a}}\,dx+\int \frac{6\sec^2\left(e+fx\right)}{a\sqrt{a\sec\left(e+fx\right)+a}\sec\left(e+fx\right)+a\sqrt{a\sec\left(\cdots\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**4/(a+a*sec(f*x+e))**(3/2),x)
```

```
[Out] c**4*(Integral(-4*sec(e + f*x)/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a
*sqrt(a*sec(e + f*x) + a)), x) + Integral(6*sec(e + f*x)**2/(a*sqrt(a*sec(e
 + f*x) + a)*sec(e + f*x) + a*sqrt(a*sec(e + f*x) + a)), x) + Integral(-4*s
ec(e + f*x)**3/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a*sqrt(a*sec(e +
f*x) + a)), x) + Integral(sec(e + f*x)**4/(a*sqrt(a*sec(e + f*x) + a)*sec(e
 + f*x) + a*sqrt(a*sec(e + f*x) + a)), x) + Integral(1/(a*sqrt(a*sec(e + f*
x) + a)*sec(e + f*x) + a*sqrt(a*sec(e + f*x) + a)), x))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.73 $\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=169

$$\frac{2c^3\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{3/2}f}+\frac{2\sqrt{2}c^3\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{a^{3/2}f}-\frac{4c^3\tan(e+fx)}{af\sqrt{a\sec(e+fx)+a}}+\frac{c^3\sin(e+fx)\tan^2(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)}{f(a\sec(e+fx)+a)^{3/2}}$$

```
[Out] (2*c^3*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(3/2)*f)
 + (2*Sqrt[2]*c^3*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e +
 f*x]])])/(a^(3/2)*f) - (4*c^3*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e + f*x]])
 + (c^3*Sec[(e + f*x)/2]^2*Sin[e + f*x]*Tan[e + f*x]^2)/(f*(a + a*Sec[e + f
*x])^(3/2))
```

Rubi [A] time = 0.240393, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3904, 3887, 470, 582, 522, 203}

$$\frac{2c^3\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{3/2}f}+\frac{2\sqrt{2}c^3\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{a^{3/2}f}-\frac{4c^3\tan(e+fx)}{af\sqrt{a\sec(e+fx)+a}}+\frac{c^3\sin(e+fx)\tan^2(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)}{f(a\sec(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*c^3*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(3/2)*f)
 + (2*Sqrt[2]*c^3*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e +
 f*x]])])/(a^(3/2)*f) - (4*c^3*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e + f*x]])
 + (c^3*Sec[(e + f*x)/2]^2*Sin[e + f*x]*Tan[e + f*x]^2)/(f*(a + a*Sec[e + f
*x])^(3/2))
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx = -\left(\left(a^3c^3\right)\int \frac{\tan^6(e+fx)}{(a+a\sec(e+fx))^{9/2}}\,dx\right)$$
$$= \frac{\left(2a^2c^3\right)\text{Subst}\left(\int \frac{x^6}{\left(1+ax^2\right)\left(2+ax^2\right)^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$
$$= \frac{c^3\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)\tan^2(e+fx)}{f(a+a\sec(e+fx))^{3/2}} + \frac{c^3\,\text{Subst}\left(\int \frac{x^2\left(6+4ax^2\right)}{\left(1+ax^2\right)\left(2+ax^2\right)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+}}\right.}{f}$$
$$= -\frac{4c^3\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}} + \frac{c^3\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)\tan^2(e+fx)}{f(a+a\sec(e+fx))^{3/2}} - \frac{c^3\,\text{Subst}\left(\int \frac{8a}{(1+a}\right.}{}$$
$$= -\frac{4c^3\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}} + \frac{c^3\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)\tan^2(e+fx)}{f(a+a\sec(e+fx))^{3/2}} - \frac{\left(2c^3\right)\text{Subst}\left(\int \frac{}{1}\right.}{}$$
$$= \frac{2c^3\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}f} + \frac{2\sqrt{2}c^3\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}f} - \frac{4c^3\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}} + \frac{c^3\,\text{s}}{}$$

Mathematica [A] time = 1.73902, size = 132, normalized size = 0.78

$$\frac{2c^3\tan\left(\frac{1}{2}(e+fx)\right)\left(-\sec(e+fx)+\cot^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\sec(e+fx)-1}\tan^{-1}\left(\sqrt{\sec(e+fx)-1}\right)+\sqrt{2}\cot^2\left(\frac{1}{2}(e+fx)\right)\right.}{af\sqrt{a(\sec(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*c^3*(-3 + ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Cot[(e + f*x)/2]^2*Sqrt[-1 + S
ec[e + f*x]] + Sqrt[2]*ArcTan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]]*Cot[(e + f*x
)/2]^2*Sqrt[-1 + Sec[e + f*x]] - Sec[e + f*x])*Tan[(e + f*x)/2])/(a*f*Sqrt[
a*(1 + Sec[e + f*x])])
```

Maple [B] time = 0.257, size = 376, normalized size = 2.2

$$\frac{c^3}{fa^2\left(\sin\left(fx+e\right)\right)^3}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\left(\left(\cos\left(fx+e\right)\right)^2\sin\left(fx+e\right)\sqrt{2}\sqrt{-2\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\text{Artanh}\left(\frac{\sqrt{2}\sin\left(fx+\right.}{2\cos\left(fx+\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(3/2),x)
```

```
[Out] c^3/f/a^2*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)^2*sin(f*x+e)*2^
(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+
e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))-2*sin(f*x+e)*cos(f*x+e)^2*(
-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(((-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2
)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))-2^(1/2)*sin(f*x+e)*arctanh(1/2*2^(1/
2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*(-2*cos(f*x+
e)/(1+cos(f*x+e)))^(1/2)+2*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*
ln(((-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e
))-6*cos(f*x+e)^3+10*cos(f*x+e)^2-2*cos(f*x+e)-2)/sin(f*x+e)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 5.0206, size = 1405, normalized size = 8.31

$$\left[\frac{\sqrt{2}\left(ac^3\cos\left(fx+e\right)^2+2\,ac^3\cos\left(fx+e\right)+ac^3\right)\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{-\frac{1}{a}}\cos(fx+e)\sin(fx+e)-3\cos(fx+e)^2-2\cos(fx+e)}{\cos(fx+e)^2+2\cos(fx+e)+1}\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(2)*(a*c^3*cos(f*x + e)^2 + 2*a*c^3*cos(f*x + e) + a*c^3)*sqrt(-1/a)*
log(-(2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x
+ e)*sin(f*x + e) - 3*cos(f*x + e)^2 - 2*cos(f*x + e) + 1)/(cos(f*x + e)^2
+ 2*cos(f*x + e) + 1)) - (c^3*cos(f*x + e)^2 + 2*c^3*cos(f*x + e) + c^3)*sq
rt(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f
*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)
) - 2*(3*c^3*cos(f*x + e) + c^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*si
n(f*x + e))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f), -2*((c^3
*cos(f*x + e)^2 + 2*c^3*cos(f*x + e) + c^3)*sqrt(a)*arctan(sqrt((a*cos(f*x
+ e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + (3*c^3*cos(f
*x + e) + c^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) + sqrt(
2)*(a*c^3*cos(f*x + e)^2 + 2*a*c^3*cos(f*x + e) + a*c^3)*arctan(sqrt(2)*sqr
t((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))/s
qrt(a))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-c^3\left(\int\frac{3\sec\left(e+fx\right)}{a\sqrt{a\sec\left(e+fx\right)+a}\sec\left(e+fx\right)+a\sqrt{a\sec\left(e+fx\right)+a}}\,dx+\int-\frac{3\sec^2\left(e+fx\right)}{a\sqrt{a\sec\left(e+fx\right)+a}\sec\left(e+fx\right)+a\sqrt{a\sec}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**3/(a+a*sec(f*x+e))**(3/2),x)
```

```
[Out] -c**3*(Integral(3*sec(e + f*x)/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a
*sqrt(a*sec(e + f*x) + a)), x) + Integral(-3*sec(e + f*x)**2/(a*sqrt(a*sec(
e + f*x) + a)*sec(e + f*x) + a*sqrt(a*sec(e + f*x) + a)), x) + Integral(sec
(e + f*x)**3/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a*sqrt(a*sec(e + f*
x) + a)), x) + Integral(-1/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a*sqr
t(a*sec(e + f*x) + a)), x))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.74 $\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=119

$$\frac{2c^2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{3/2}f}-\frac{\sqrt{2}c^2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{a^{3/2}f}-\frac{2c^2\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}}$$

```
[Out] (2*c^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(3/2)*f)
 - (Sqrt[2]*c^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f
*x]])])/(a^(3/2)*f) - (2*c^2*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])^(3/2))
```

Rubi [A] time = 0.191734, antiderivative size = 134, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3904, 3887, 470, 12, 391, 203}

$$\frac{2c^2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{3/2}f}-\frac{\sqrt{2}c^2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{a^{3/2}f}-\frac{c^2\sin(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)}{af\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*c^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(3/2)*f)
 - (Sqrt[2]*c^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f
*x]])])/(a^(3/2)*f) - (c^2*Sec[(e + f*x)/2]^2*Sin[e + f*x])/(a*f*Sqrt[a + a
*Sec[e + f*x]])
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
```

```
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx = \left(a^2c^2\right)\int \frac{\tan^4(e+fx)}{(a+a\sec(e+fx))^{7/2}}\,dx$$

$$= -\frac{\left(2ac^2\right)\mathrm{Subst}\left(\int \frac{x^4}{(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= -\frac{c^2\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{af\sqrt{a+a\sec(e+fx)}} - \frac{c^2\,\mathrm{Subst}\left(\int \frac{2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{af}$$

$$= -\frac{c^2\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{af\sqrt{a+a\sec(e+fx)}} - \frac{\left(2c^2\right)\mathrm{Subst}\left(\int \frac{1}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{af}$$

$$= -\frac{c^2\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{af\sqrt{a+a\sec(e+fx)}} - \frac{\left(2c^2\right)\mathrm{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{af} + \frac{\left(2c^2\right)\mathrm{Su}}{}$$

$$= \frac{2c^2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}f} - \frac{\sqrt{2}c^2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}f} - \frac{c^2\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx}{af\sqrt{a+a\sec(e+fx)}}$$

Mathematica [A] time = 1.06782, size = 128, normalized size = 1.08

$$\frac{c^2\cot\left(\frac{1}{2}(e+fx)\right)\left(\sec^2\left(\frac{1}{2}(e+fx)\right)\left(\cos(e+fx)+(\cos(e+fx)+1)\sqrt{\sec(e+fx)-1}\tan^{-1}\left(\sqrt{\sec(e+fx)-1}\right)-1\right)-\sqrt{}\right.}{af\sqrt{a(\sec(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (c^2*Cot[(e + f*x)/2]*(Sec[(e + f*x)/2]^2*(-1 + Cos[e + f*x] + ArcTan[Sqrt[
-1 + Sec[e + f*x]]]*(1 + Cos[e + f*x])*Sqrt[-1 + Sec[e + f*x]]) - Sqrt[2]*A
rcTan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]]*Sqrt[-1 + Sec[e + f*x]]))/(a*f*Sqrt[
a*(1 + Sec[e + f*x])])
```

Maple [B] time = 0.189, size = 369, normalized size = 3.1

$$-\frac{c^2}{fa^2(1+\cos(fx+e))\sin(fx+e)}\sqrt{\frac{a(1+\cos(fx+e))}{\cos(fx+e)}}\left(\sqrt{2}\cos(fx+e)\sin(fx+e)\sqrt{-2\frac{\cos(fx+e)}{1+\cos(fx+e)}}\mathrm{Artanh}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(3/2),x)
```

```
[Out] -c^2/f/a^2*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(2^(1/2)*cos(f*x+e)*sin(f*
x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e
)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))+2^(1/2)*sin(f*x+e)*arctanh(1
/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*(-2*
cos(f*x+e)/(1+cos(f*x+e)))^(1/2)+sin(f*x+e)*cos(f*x+e)*(-2*cos(f*x+e)/(1+co
s(f*x+e)))^(1/2)*ln(((-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-cos(f*
x+e)+1)/sin(f*x+e))+sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(((-2
*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))-2*co
s(f*x+e)^2+2*cos(f*x+e))/(1+cos(f*x+e))/sin(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\sec\left(fx+e\right)-c\right)^2}{\left(a\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sec(f*x + e) - c)^2/(a*sec(f*x + e) + a)^(3/2), x)
```

Fricas [B] time = 3.8022, size = 1389, normalized size = 11.67

$$\left[-\frac{4\,c^2\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)\sin\left(fx+e\right)-\sqrt{2}\left(ac^2\cos\left(fx+e\right)^2+2\,ac^2\cos\left(fx+e\right)+ac^2\right)\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(fx\cdots}{\cos(fx\cdots}}}{\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(4*c^2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x +
 e) - sqrt(2)*(a*c^2*cos(f*x + e)^2 + 2*a*c^2*cos(f*x + e) + a*c^2)*sqrt(-1
/a)*log((2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f
*x + e)*sin(f*x + e) + 3*cos(f*x + e)^2 + 2*cos(f*x + e) - 1)/(cos(f*x + e)
^2 + 2*cos(f*x + e) + 1)) + 2*(c^2*cos(f*x + e)^2 + 2*c^2*cos(f*x + e) + c^
2)*sqrt(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/
cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e)
 + 1)))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f), -(2*c^2*sqrt
((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + 2*(c^2*cos(
f*x + e)^2 + 2*c^2*cos(f*x + e) + c^2)*sqrt(a)*arctan(sqrt((a*cos(f*x + e)
+ a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - sqrt(2)*(a*c^2*co
s(f*x + e)^2 + 2*a*c^2*cos(f*x + e) + a*c^2)*arctan(sqrt(2)*sqrt((a*cos(f*x
 + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))/sqrt(a))/(a^2
*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^2\left(\int -\frac{2\sec\left(e+fx\right)}{a\sqrt{a\sec\left(e+fx\right)+a}\sec\left(e+fx\right)+a\sqrt{a\sec\left(e+fx\right)+a}}\,dx+\int \frac{\sec^2\left(e+fx\right)}{a\sqrt{a\sec\left(e+fx\right)+a}\sec\left(e+fx\right)+a\sqrt{a\sec(\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**2/(a+a*sec(f*x+e))**(3/2),x)
```

```
[Out] c**2*(Integral(-2*sec(e + f*x)/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a
*sqrt(a*sec(e + f*x) + a)), x) + Integral(sec(e + f*x)**2/(a*sqrt(a*sec(e +
 f*x) + a)*sec(e + f*x) + a*sqrt(a*sec(e + f*x) + a)), x) + Integral(1/(a*s
qrt(a*sec(e + f*x) + a)*sec(e + f*x) + a*sqrt(a*sec(e + f*x) + a)), x))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.75 $\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=113

$$\frac{2c\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{3/2}f}-\frac{3c\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{2}a^{3/2}f}-\frac{c\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}}$$

```
[Out] (2*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(3/2)*f) -
 (3*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])/(S
qrt[2]*a^(3/2)*f) - (c*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])^(3/2))
```

Rubi [A] time = 0.16639, antiderivative size = 130, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3904, 3887, 471, 522, 203}

$$\frac{2c\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{3/2}f}-\frac{3c\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{2}a^{3/2}f}-\frac{c\sin(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)}{2af\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(3/2)*f) -
 (3*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])/(S
qrt[2]*a^(3/2)*f) - (c*Sec[(e + f*x)/2]^2*Sin[e + f*x])/(2*a*f*Sqrt[a + a*S
ec[e + f*x]])
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
```

```
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx = -\left((ac)\int \frac{\tan^2(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx\right)$$
$$= \frac{(2c)\,\text{Subst}\left(\int \frac{x^2}{(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$
$$= -\frac{c\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{2af\sqrt{a+a\sec(e+fx)}} - \frac{c\,\text{Subst}\left(\int \frac{1-ax^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{af}$$
$$= -\frac{c\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{2af\sqrt{a+a\sec(e+fx)}} - \frac{(2c)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{af} + \frac{(3c)\,\text{Subst}}{}$$
$$= \frac{2c\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}f} - \frac{3c\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{2}a^{3/2}f} - \frac{c\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{2af\sqrt{a+a\sec(e+fx)}}$$

Mathematica [A] time = 0.993428, size = 130, normalized size = 1.15

$$\frac{c\cot\left(\frac{1}{2}(e+fx)\right)\left(\sec^2\left(\frac{1}{2}(e+fx)\right)\left(\cos(e+fx)+2(\cos(e+fx)+1)\sqrt{\sec(e+fx)-1}\tan^{-1}\left(\sqrt{\sec(e+fx)-1}\right)-1\right)-3}{2af\sqrt{a(\sec(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (c*Cot[(e + f*x)/2]*(Sec[(e + f*x)/2]^2*(-1 + Cos[e + f*x] + 2*ArcTan[Sqrt[
-1 + Sec[e + f*x]]]*(1 + Cos[e + f*x])*Sqrt[-1 + Sec[e + f*x]]) - 3*Sqrt[2]
*ArcTan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]]*Sqrt[-1 + Sec[e + f*x]]))/(2*a*f*S
qrt[a*(1 + Sec[e + f*x])])
```

Maple [B] time = 0.181, size = 371, normalized size = 3.3

$$-\frac{c}{2fa^2\left(1+\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\left(2\sqrt{2}\cos\left(fx+e\right)\sin\left(fx+e\right)\sqrt{-2\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\text{Arta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(3/2),x)
```

```
[Out] -1/2*c/f/a^2*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(2*2^(1/2)*cos(f*x+e)*si
n(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(f
*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))+3*sin(f*x+e)*cos(f*x+e)*
(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(((-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/
2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))+2*2^(1/2)*sin(f*x+e)*arctanh(1/2*2^
(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*(-2*cos(f
*x+e)/(1+cos(f*x+e)))^(1/2)+3*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/
2)*ln(((-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*
x+e))-2*cos(f*x+e)^2+2*cos(f*x+e))/(1+cos(f*x+e))/sin(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{c\sec\left(fx+e\right)-c}{\left(a\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate((c*sec(f*x + e) - c)/(a*sec(f*x + e) + a)^(3/2), x)
```

Fricas [B] time = 4.3266, size = 1347, normalized size = 11.92

$$\left[-\frac{4\,c\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)\sin\left(fx+e\right)+3\sqrt{2}\left(c\cos\left(fx+e\right)^{2}+2\,c\cos\left(fx+e\right)+c\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)}{\cos\left(fx+e\right.}}}{}\right.}{4\left(a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e
) + 3*sqrt(2)*(c*cos(f*x + e)^2 + 2*c*cos(f*x + e) + c)*sqrt(-a)*log(-(2*sq
rt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x
 + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f*x + e) + a)/(cos(f*x + e)^2 + 2*cos(
f*x + e) + 1)) + 4*(c*cos(f*x + e)^2 + 2*c*cos(f*x + e) + c)*sqrt(-a)*log((
2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos
(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/(a^2*f*co
s(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f), -1/2*(2*c*sqrt((a*cos(f*x + e
) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 3*sqrt(2)*(c*cos(f*x + e)^
2 + 2*c*cos(f*x + e) + c)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/
cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + 4*(c*cos(f*x + e)^2 +
2*c*cos(f*x + e) + c)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e)
)*cos(f*x + e)/(sqrt(a)*sin(f*x + e))))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos
(f*x + e) + a^2*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-c\left(\int\frac{\sec\left(e+fx\right)}{a\sqrt{a\sec\left(e+fx\right)+a}\sec\left(e+fx\right)+a\sqrt{a\sec\left(e+fx\right)+a}}\,dx+\int-\frac{1}{a\sqrt{a\sec\left(e+fx\right)+a}\sec\left(e+fx\right)+a\sqrt{a\sec$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))**(3/2),x)
```

```
[Out] -c*(Integral(sec(e + f*x)/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a*sqrt
(a*sec(e + f*x) + a)), x) + Integral(-1/(a*sqrt(a*sec(e + f*x) + a)*sec(e +
 f*x) + a*sqrt(a*sec(e + f*x) + a)), x))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.76 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))}\,dx$

Optimal. Leaf size=177

$$\frac{\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{4a^2cf}+\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{3/2}cf}-\frac{7\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{4\sqrt{2}a^{3/2}cf}+\frac{\cos(e+fx)\cot(e+fx)\sec^2\left(\right.}{4a^2}$$

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(3/2)*c*f) -
 (7*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])/(4*S
qrt[2]*a^(3/2)*c*f) + (Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(4*a^2*c*f) +
 (Cos[e + f*x]*Cot[e + f*x]*Sec[(e + f*x)/2]^2*Sqrt[a + a*Sec[e + f*x]])/(4
*a^2*c*f)
```

Rubi [A] time = 0.244441, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3904, 3887, 472, 583, 522, 203}

$$\frac{\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{4a^2cf}+\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{3/2}cf}-\frac{7\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{4\sqrt{2}a^{3/2}cf}+\frac{\cos(e+fx)\cot(e+fx)\sec^2\left(\right.}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])),x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(3/2)*c*f) -
 (7*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])/(4*S
qrt[2]*a^(3/2)*c*f) + (Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(4*a^2*c*f) +
 (Cos[e + f*x]*Cot[e + f*x]*Sec[(e + f*x)/2]^2*Sqrt[a + a*Sec[e + f*x]])/(4
*a^2*c*f)
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))}\,dx = -\frac{\int \frac{\cot^2(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx}{ac}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{1}{x^2(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^2cf}$$

$$= \frac{\cos(e+fx)\cot(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)\sqrt{a+a\sec(e+fx)}}{4a^2cf} + \frac{\text{Subst}\left(\right.}{}$$

$$= \frac{\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{4a^2cf} + \frac{\cos(e+fx)\cot(e+fx)\sec^2\left(\frac{1}{2}(e+\right.}{4a^2cf}$$

$$= \frac{\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{4a^2cf} + \frac{\cos(e+fx)\cot(e+fx)\sec^2\left(\frac{1}{2}(e+\right.}{4a^2cf}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}cf} - \frac{7\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{4\sqrt{2}a^{3/2}cf} + \frac{\cot(e+fx)\sqrt{a}}{4a^2}$$

Mathematica [A] time = 1.1741, size = 154, normalized size = 0.87

$$\frac{\sin^2\left(\frac{1}{2}(e+fx)\right)\tan\left(\frac{1}{2}(e+fx)\right)\left(3\cos(e+fx)-7\sqrt{2}\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\sec(e+fx)-1}\tan^{-1}\left(\frac{\sqrt{\sec(e+fx)-1}}{\sqrt{2}}\right)+8(\cos(e\right.}{2acf(\cos(e+fx)-1)^2\sqrt{a(\sec(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])),x]
```

```
[Out] ((1 + 3*Cos[e + f*x] - 7*Sqrt[2]*ArcTan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]]*Co
s[(e + f*x)/2]^2*Sqrt[-1 + Sec[e + f*x]] + 8*ArcTan[Sqrt[-1 + Sec[e + f*x]]
]*(1 + Cos[e + f*x])*Sqrt[-1 + Sec[e + f*x]])*Sin[(e + f*x)/2]^2*Tan[(e + f
*x)/2])/(2*a*c*f*(-1 + Cos[e + f*x])^2*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [B] time = 0.254, size = 377, normalized size = 2.1

$$-\frac{1}{8fca^2(\sin(fx+e))^3}\sqrt{\frac{a(1+\cos(fx+e))}{\cos(fx+e)}}\left(-8(\cos(fx+e))^2\sin(fx+e)\sqrt{2}\sqrt{-2\frac{\cos(fx+e)}{1+\cos(fx+e)}}\text{Artanh}\left(1/2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e)),x)
```

```
[Out] -1/8/c/f/a^2*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(-8*cos(f*x+e)^2*sin(f*x
+e)*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*2^(1/2)*(-2*co
s(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))-7*sin(f*x+e)*cos(f*x+
e)^2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(((-2*cos(f*x+e)/(1+cos(f*x+e))
)^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))+8*2^(1/2)*sin(f*x+e)*arctanh(1
/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*(-2*
cos(f*x+e)/(1+cos(f*x+e)))^(1/2)+6*cos(f*x+e)^3+7*sin(f*x+e)*(-2*cos(f*x+e)
/(1+cos(f*x+e)))^(1/2)*ln(((-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-
cos(f*x+e)+1)/sin(f*x+e))-4*cos(f*x+e)^2-2*cos(f*x+e))/sin(f*x+e)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(a\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(c\sec\left(fx+e\right)-c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] -integrate(1/((a*sec(f*x + e) + a)^(3/2)*(c*sec(f*x + e) - c)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{1}{a\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)-a\sqrt{a\sec(e+fx)+a}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e)),x)
```

```
[Out] -Integral(1/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**2 - a*sqrt(a*sec(e +
f*x) + a)), x)/c
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.77 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2}\,dx$

Optimal. Leaf size=214

$$\frac{\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{12a^3c^2f}+\frac{7\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{8a^2c^2f}+\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{3/2}c^2f}-\frac{9\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)}}\right.}{8\sqrt{2}a^{3/2}c^2f}$$

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(3/2)*c^2*f)
 - (9*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])/(8
*Sqrt[2]*a^(3/2)*c^2*f) + (7*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(8*a^2*
c^2*f) + (Cot[e + f*x]^3*(a + a*Sec[e + f*x])^(3/2))/(12*a^3*c^2*f) - (Cos[
e + f*x]*Cot[e + f*x]^3*Sec[(e + f*x)/2]^2*(a + a*Sec[e + f*x])^(3/2))/(4*a
^3*c^2*f)
```

Rubi [A] time = 0.278458, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3904, 3887, 472, 583, 522, 203}

$$\frac{\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{12a^3c^2f}+\frac{7\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{8a^2c^2f}+\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{3/2}c^2f}-\frac{9\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)}}\right.}{8\sqrt{2}a^{3/2}c^2f}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^2),x]
```


```
[Out] (2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(3/2)*c^2*f)
 - (9*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])/(8
*Sqrt[2]*a^(3/2)*c^2*f) + (7*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(8*a^2*
c^2*f) + (Cot[e + f*x]^3*(a + a*Sec[e + f*x])^(3/2))/(12*a^3*c^2*f) - (Cos[
e + f*x]*Cot[e + f*x]^3*Sec[(e + f*x)/2]^2*(a + a*Sec[e + f*x])^(3/2))/(4*a
^3*c^2*f)
```


Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(I
```

```
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2}\,dx = \frac{\int \cot^4(e+fx)\sqrt{a+a\sec(e+fx)}\,dx}{a^2c^2}$$

$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{x^4(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^3c^2f}$$

$$= -\frac{\cos(e+fx)\cot^3(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)(a+a\sec(e+fx))^{3/2}}{4a^3c^2f} - \frac{\text{Sul}}{}$$

$$= \frac{\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{12a^3c^2f} - \frac{\cos(e+fx)\cot^3(e+fx)\sec^2\left(\frac{1}{2}(e\right.}{4a^3c^2}$$

$$= \frac{7\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{8a^2c^2f} + \frac{\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{12a^3c^2f} -$$

$$= \frac{7\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{8a^2c^2f} + \frac{\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{12a^3c^2f} -$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}c^2f} - \frac{9\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{8\sqrt{2}a^{3/2}c^2f} + \frac{7\cot(e+fx)\sqrt{a}}{8a^2}$$

Mathematica [C] time = 23.7926, size = 5622, normalized size = 26.27

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.293, size = 387, normalized size = 1.8

$$-\frac{(\cos(fx+e))^2-1}{48\,fc^2a^2(\sin(fx+e))^5}\sqrt{\frac{a(1+\cos(fx+e))}{\cos(fx+e)}}\left(48\,(\cos(fx+e))^2\sin(fx+e)\sqrt{2}\sqrt{-2\,\frac{\cos(fx+e)}{1+\cos(fx+e)}}\text{Artanh}\left(1/2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^2,x)
```

```
[Out] -1/48/c^2/f/a^2*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(48*cos(f*x+e)^2*sin(
f*x+e)*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*2^(1/2)*(-2
*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))+27*sin(f*x+e)*cos(
f*x+e)^2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(((-2*cos(f*x+e)/(1+cos(f*x
+e)))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))-48*2^(1/2)*sin(f*x+e)*arct
anh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))
*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)-27*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f
*x+e)))^(1/2)*ln(((-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-cos(f*x+e
)+1)/sin(f*x+e))-62*cos(f*x+e)^3+4*cos(f*x+e)^2+42*cos(f*x+e))/sin(f*x+e)^5
*(cos(f*x+e)^2-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(c\sec\left(fx+e\right)-c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^2,x, algorithm="maxima"
)
```

```
[Out] integrate(1/((a*sec(f*x + e) + a)^(3/2)*(c*sec(f*x + e) - c)^2), x)
```

Fricas [A] time = 2.00765, size = 1478, normalized size = 6.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^2,x, algorithm="fricas"
)
```

```
[Out] [-1/96*(27*sqrt(2)*(cos(f*x + e)^2 - 1)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*s
qrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 3*a*cos(
f*x + e)^2 - 2*a*cos(f*x + e) + a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1))*s
in(f*x + e) + 48*(cos(f*x + e)^2 - 1)*sqrt(-a)*log(-(8*a*cos(f*x + e)^3 + 4
*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f
```

```
*x + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x +
 e) - 4*(31*cos(f*x + e)^3 - 2*cos(f*x + e)^2 - 21*cos(f*x + e))*sqrt((a*co
s(f*x + e) + a)/cos(f*x + e)))/((a^2*c^2*f*cos(f*x + e)^2 - a^2*c^2*f)*sin(
f*x + e)), 1/48*(27*sqrt(2)*(cos(f*x + e)^2 - 1)*sqrt(a)*arctan(sqrt(2)*sqr
t((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))*s
in(f*x + e) + 48*(cos(f*x + e)^2 - 1)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 +
 a*cos(f*x + e) - a))*sin(f*x + e) + 2*(31*cos(f*x + e)^3 - 2*cos(f*x + e)^
2 - 21*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((a^2*c^2*f*c
os(f*x + e)^2 - a^2*c^2*f)*sin(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a\sqrt{a\sec\left(e+fx\right)+a}\sec^{3}\left(e+fx\right)-a\sqrt{a\sec\left(e+fx\right)+a}\sec^{2}\left(e+fx\right)-a\sqrt{a\sec\left(e+fx\right)+a}\sec\left(e+fx\right)+a\sqrt{a\sec\left(e+fx\right)+a}}\,dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**2,x)
```

```
[Out] Integral(1/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**3 - a*sqrt(a*sec(e + f
*x) + a)*sec(e + f*x)**2 - a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a*sqrt
(a*sec(e + f*x) + a)), x)/c**2
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.78 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3}\,dx$

Optimal. Leaf size=249

$$-\frac{3\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}}{20a^4c^3f}-\frac{5\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{24a^3c^3f}+\frac{21\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{16a^2c^3f}+\frac{2\,\mathrm{ta}}{}$$

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(3/2)*c^3*f)
 - (11*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])/(
16*Sqrt[2]*a^(3/2)*c^3*f) + (21*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(16*
a^2*c^3*f) - (5*Cot[e + f*x]^3*(a + a*Sec[e + f*x])^(3/2))/(24*a^3*c^3*f) -
 (3*Cot[e + f*x]^5*(a + a*Sec[e + f*x])^(5/2))/(20*a^4*c^3*f) + (Cos[e + f*
x]*Cot[e + f*x]^5*Sec[(e + f*x)/2]^2*(a + a*Sec[e + f*x])^(5/2))/(4*a^4*c^3
*f)
```

Rubi [A] time = 0.339961, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3904, 3887, 472, 583, 522, 203}

$$-\frac{3\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}}{20a^4c^3f}-\frac{5\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{24a^3c^3f}+\frac{21\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{16a^2c^3f}+\frac{2\,\mathrm{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^3),x]
```


```
[Out] (2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(3/2)*c^3*f)
 - (11*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])/(
16*Sqrt[2]*a^(3/2)*c^3*f) + (21*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(16*
a^2*c^3*f) - (5*Cot[e + f*x]^3*(a + a*Sec[e + f*x])^(3/2))/(24*a^3*c^3*f) -
 (3*Cot[e + f*x]^5*(a + a*Sec[e + f*x])^(5/2))/(20*a^4*c^3*f) + (Cos[e + f*
x]*Cot[e + f*x]^5*Sec[(e + f*x)/2]^2*(a + a*Sec[e + f*x])^(5/2))/(4*a^4*c^3
*f)
```


Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
```

```
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3}\,dx = -\frac{\int \cot^6(e+fx)(a+a\sec(e+fx))^{3/2}\,dx}{a^3c^3}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{1}{x^6(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^4c^3f}$$

$$= \frac{\cos(e+fx)\cot^5(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)(a+a\sec(e+fx))^{5/2}}{4a^4c^3f} + \text{Su}$$

$$= -\frac{3\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{20a^4c^3f} + \frac{\cos(e+fx)\cot^5(e+fx)\sec^2}{4}$$

$$= -\frac{5\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{24a^3c^3f} - \frac{3\cot^5(e+fx)(a+a\sec(e+}{20a^4c^3f}$$

$$= \frac{21\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{16a^2c^3f} - \frac{5\cot^3(e+fx)(a+a\sec(e+fx))}{24a^3c^3f}$$

$$= \frac{21\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{16a^2c^3f} - \frac{5\cot^3(e+fx)(a+a\sec(e+fx))}{24a^3c^3f}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}c^3f} - \frac{11\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{16\sqrt{2}a^{3/2}c^3f} + \frac{21\cot(e+fx}{1}$$

Mathematica [C] time = 23.7951, size = 5639, normalized size = 22.65

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^3),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.362, size = 725, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^3,x)
```

```
[Out] 1/480/c^3/f/a^2*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(-1+cos(f*x+e))*(1+co
s(f*x+e))^2*(480*2^(1/2)*cos(f*x+e)^3*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+
e)))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x
+e)/cos(f*x+e))+165*sin(f*x+e)*cos(f*x+e)^3*(-2*cos(f*x+e)/(1+cos(f*x+e)))^
(1/2)*ln(((-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin
(f*x+e))-480*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))
^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/
cos(f*x+e))-165*sin(f*x+e)*cos(f*x+e)^2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2
)*ln(((-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x
+e))-480*2^(1/2)*cos(f*x+e)*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)
*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*
x+e))-165*sin(f*x+e)*cos(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(((-
2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))+480
*2^(1/2)*sin(f*x+e)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2
)*sin(f*x+e)/cos(f*x+e))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)-898*cos(f*x+e
)^4+165*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(((-2*cos(f*x+e)/
(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))+702*cos(f*x+e)^3
+730*cos(f*x+e)^2-630*cos(f*x+e))/sin(f*x+e)^7
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(a\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(c\sec\left(fx+e\right)-c\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^3,x, algorithm="maxima"
)
```

```
[Out] -integrate(1/((a*sec(f*x + e) + a)^(3/2)*(c*sec(f*x + e) - c)^3), x)
```

Fricas [A] time = 2.18013, size = 1867, normalized size = 7.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^3,x, algorithm="fricas"
)
```

```
[Out] [-1/960*(165*sqrt(2)*(cos(f*x + e)^3 - cos(f*x + e)^2 - cos(f*x + e) + 1)*s
qrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*co
s(f*x + e)*sin(f*x + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f*x + e) + a)/(cos(f
*x + e)^2 + 2*cos(f*x + e) + 1))*sin(f*x + e) + 480*(cos(f*x + e)^3 - cos(f
*x + e)^2 - cos(f*x + e) + 1)*sqrt(-a)*log(-(8*a*cos(f*x + e)^3 + 4*(2*cos(
f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))
*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) - 4*
(449*cos(f*x + e)^4 - 351*cos(f*x + e)^3 - 365*cos(f*x + e)^2 + 315*cos(f*x
 + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((a^2*c^3*f*cos(f*x + e)^3
- a^2*c^3*f*cos(f*x + e)^2 - a^2*c^3*f*cos(f*x + e) + a^2*c^3*f)*sin(f*x +
e)), 1/480*(165*sqrt(2)*(cos(f*x + e)^3 - cos(f*x + e)^2 - cos(f*x + e) + 1
)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x +
e)/(sqrt(a)*sin(f*x + e)))*sin(f*x + e) + 480*(cos(f*x + e)^3 - cos(f*x + e
)^2 - cos(f*x + e) + 1)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/
cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e
) - a))*sin(f*x + e) + 2*(449*cos(f*x + e)^4 - 351*cos(f*x + e)^3 - 365*cos
(f*x + e)^2 + 315*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((
a^2*c^3*f*cos(f*x + e)^3 - a^2*c^3*f*cos(f*x + e)^2 - a^2*c^3*f*cos(f*x + e
) + a^2*c^3*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.79 $\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=260

$$\frac{2c^5\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{5/2}f}-\frac{23\sqrt{2}c^5\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{a^{5/2}f}+\frac{21c^5\tan(e+fx)}{a^2f\sqrt{a\sec(e+fx)+a}}-\frac{19c^5\tan^3(e+fx)}{6af(a\sec(e+fx)+a)^{3/2}}+\frac{ac}{-}$$

```
[Out] (2*c^5*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(5/2)*f)
 - (23*Sqrt[2]*c^5*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e
+ f*x]])])/(a^(5/2)*f) + (21*c^5*Tan[e + f*x])/(a^2*f*Sqrt[a + a*Sec[e + f*
x]]) - (19*c^5*Tan[e + f*x]^3)/(6*a*f*(a + a*Sec[e + f*x])^(3/2)) + (3*c^5*
Sec[(e + f*x)/2]^2*Sin[e + f*x]*Tan[e + f*x]^4)/(4*f*(a + a*Sec[e + f*x])^(
5/2)) + (a*c^5*Sec[(e + f*x)/2]^4*Sin[e + f*x]^2*Tan[e + f*x]^5)/(4*f*(a +
a*Sec[e + f*x])^(7/2))
```

Rubi [A] time = 0.341487, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3904, 3887, 470, 578, 582, 522, 203}

$$\frac{2c^5\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{5/2}f}-\frac{23\sqrt{2}c^5\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{a^{5/2}f}+\frac{21c^5\tan(e+fx)}{a^2f\sqrt{a\sec(e+fx)+a}}-\frac{19c^5\tan^3(e+fx)}{6af(a\sec(e+fx)+a)^{3/2}}+\frac{ac}{-}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^5/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] (2*c^5*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(5/2)*f)
 - (23*Sqrt[2]*c^5*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e
+ f*x]])])/(a^(5/2)*f) + (21*c^5*Tan[e + f*x])/(a^2*f*Sqrt[a + a*Sec[e + f*
x]]) - (19*c^5*Tan[e + f*x]^3)/(6*a*f*(a + a*Sec[e + f*x])^(3/2)) + (3*c^5*
Sec[(e + f*x)/2]^2*Sin[e + f*x]*Tan[e + f*x]^4)/(4*f*(a + a*Sec[e + f*x])^(
5/2)) + (a*c^5*Sec[(e + f*x)/2]^4*Sin[e + f*x]^2*Tan[e + f*x]^5)/(4*f*(a +
a*Sec[e + f*x])^(7/2))
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
```

```
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^{5/2}}\,dx = -\left(\left(a^5c^5\right)\int \frac{\tan^{10}(e+fx)}{(a+a\sec(e+fx))^{15/2}}\,dx\right)$$

$$= \frac{\left(2a^3c^5\right)\text{Subst}\left(\int \frac{x^{10}}{\left(1+ax^2\right)\left(2+ax^2\right)^3}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= \frac{ac^5\sec^4\left(\frac{1}{2}(e+fx)\right)\sin^2(e+fx)\tan^5(e+fx)}{4f(a+a\sec(e+fx))^{7/2}} + \frac{\left(ac^5\right)\text{Subst}\left(\int \frac{x^6\left(14+10ax^2\right)}{\left(1+ax^2\right)\left(2+ax^2\right)^2}\,dx, x, -\frac{\quad}{\sqrt{\quad}}\right.}{2f}$$

$$= \frac{3c^5\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)\tan^4(e+fx)}{4f(a+a\sec(e+fx))^{5/2}} + \frac{ac^5\sec^4\left(\frac{1}{2}(e+fx)\right)\sin^2(e+fx)\tan^5(e}{4f(a+a\sec(e+fx))^{7/2}}$$

$$= -\frac{19c^5\tan^3(e+fx)}{6af(a+a\sec(e+fx))^{3/2}} + \frac{3c^5\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)\tan^4(e+fx)}{4f(a+a\sec(e+fx))^{5/2}} + \frac{ac^5\sec^4\left(\frac{1}{2}\right.}{}$$

$$= \frac{21c^5\tan(e+fx)}{a^2f\sqrt{a+a\sec(e+fx)}} - \frac{19c^5\tan^3(e+fx)}{6af(a+a\sec(e+fx))^{3/2}} + \frac{3c^5\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)\,\text{ta}}{4f(a+a\sec(e+fx))^{5/2}}$$

$$= \frac{21c^5\tan(e+fx)}{a^2f\sqrt{a+a\sec(e+fx)}} - \frac{19c^5\tan^3(e+fx)}{6af(a+a\sec(e+fx))^{3/2}} + \frac{3c^5\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)\,\text{ta}}{4f(a+a\sec(e+fx))^{5/2}}$$

$$= \frac{2c^5\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}f} - \frac{23\sqrt{2}c^5\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}f} + \frac{21c^5\tan(e+fx)}{a^2f\sqrt{a+a\sec(e+fx)}} -$$

Mathematica [A] time = 3.48483, size = 180, normalized size = 0.69

$$\frac{c^5 \cot\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\left((-30\cos(e+fx)+52\cos(2(e+fx))-66\cos(3(e+fx))-37\cos(4(e+fx))+81)\sec^4\right.}{48a^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^5/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] (c^5*Cot[(e + f*x)/2]*((81 - 30*Cos[e + f*x] + 52*Cos[2*(e + f*x)] - 66*Cos
[3*(e + f*x)] - 37*Cos[4*(e + f*x)])*Sec[(e + f*x)/2]^4 + 96*ArcTan[Sqrt[-1
 + Sec[e + f*x]]]*Cos[e + f*x]^2*Sqrt[-1 + Sec[e + f*x]] - 1104*Sqrt[2]*Arc
Tan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]]*Cos[e + f*x]^2*Sqrt[-1 + Sec[e + f*x]]
)*Sec[e + f*x]^2)/(48*a^2*f*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [B] time = 0.308, size = 726, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^(5/2),x)
```

```
[Out] 1/6*c^5/f/a^3*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(-1+cos(f*x+e))^2*(3*si
n(f*x+e)*cos(f*x+e)^3*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1
/2)*sin(f*x+e)/cos(f*x+e))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)*2^(1/2)+69*
sin(f*x+e)*cos(f*x+e)^3*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)*ln(((-2*cos(f*
x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))+9*arctanh(1
/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*(-2*
cos(f*x+e)/(1+cos(f*x+e)))^(3/2)*2^(1/2)*sin(f*x+e)*cos(f*x+e)^2+207*cos(f*
x+e)^2*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)*ln(((-2*cos(f*x+e)/(
1+cos(f*x+e)))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))+9*arctanh(1/2*2^(
1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*(-2*cos(f*
x+e)/(1+cos(f*x+e)))^(3/2)*2^(1/2)*sin(f*x+e)*cos(f*x+e)+207*cos(f*x+e)*sin
(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)*ln(((-2*cos(f*x+e)/(1+cos(f*x+
e)))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))+3*2^(1/2)*arctanh(1/2*2^(1/
2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*(-2*cos(f*x+
e)/(1+cos(f*x+e)))^(3/2)*sin(f*x+e)+69*ln(((-2*cos(f*x+e)/(1+cos(f*x+e)))^(
1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3
/2)*sin(f*x+e)-148*cos(f*x+e)^4-132*cos(f*x+e)^3+200*cos(f*x+e)^2+84*cos(f*
```

```
x+e)-4)/sin(f*x+e)^5/cos(f*x+e)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")

[Out] Timed out
```

Fricas [A] time = 14.5397, size = 1879, normalized size = 7.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^(5/2),x, algorithm="fricas")

[Out] [1/6*(69*sqrt(2)*(a*c^5*cos(f*x + e)^4 + 3*a*c^5*cos(f*x + e)^3 + 3*a*c^5*c
os(f*x + e)^2 + a*c^5*cos(f*x + e))*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(f
*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) + 3*cos(f*x
 + e)^2 + 2*cos(f*x + e) - 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) - 6*(c
^5*cos(f*x + e)^4 + 3*c^5*cos(f*x + e)^3 + 3*c^5*cos(f*x + e)^2 + c^5*cos(f
*x + e))*sqrt(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e)
 + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*
x + e) + 1)) + 4*(37*c^5*cos(f*x + e)^3 + 70*c^5*cos(f*x + e)^2 + 20*c^5*co
s(f*x + e) - c^5)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(a^
3*f*cos(f*x + e)^4 + 3*a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + a^3*
f*cos(f*x + e)), -1/3*(6*(c^5*cos(f*x + e)^4 + 3*c^5*cos(f*x + e)^3 + 3*c^5
*cos(f*x + e)^2 + c^5*cos(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a
)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 2*(37*c^5*cos(f*x +
e)^3 + 70*c^5*cos(f*x + e)^2 + 20*c^5*cos(f*x + e) - c^5)*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*sin(f*x + e) - 69*sqrt(2)*(a*c^5*cos(f*x + e)^4 + 3*
a*c^5*cos(f*x + e)^3 + 3*a*c^5*cos(f*x + e)^2 + a*c^5*cos(f*x + e))*arctan(
sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f
*x + e)))/sqrt(a))/(a^3*f*cos(f*x + e)^4 + 3*a^3*f*cos(f*x + e)^3 + 3*a^3*f
*cos(f*x + e)^2 + a^3*f*cos(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**5/(a+a*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.80 $\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=229

$$\frac{2c^4\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{5/2}f}-\frac{11c^4\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{2}a^{5/2}f}+\frac{7c^4\tan(e+fx)}{2a^2f\sqrt{a\sec(e+fx)+a}}-\frac{c^4\sin^2(e+fx)\tan^3(e+fx)\sec^4}{4f(a\sec(e+fx)+a)^5}$$

```
[Out] (2*c^4*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(5/2)*f)
 - (11*c^4*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])
])/(Sqrt[2]*a^(5/2)*f) + (7*c^4*Tan[e + f*x])/(2*a^2*f*Sqrt[a + a*Sec[e + f
*x]]) - (c^4*Sec[(e + f*x)/2]^2*Sin[e + f*x]*Tan[e + f*x]^2)/(4*a*f*(a + a*
Sec[e + f*x])^(3/2)) - (c^4*Sec[(e + f*x)/2]^4*Sin[e + f*x]^2*Tan[e + f*x]^
3)/(4*f*(a + a*Sec[e + f*x])^(5/2))
```

Rubi [A] time = 0.295997, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3904, 3887, 470, 578, 582, 522, 203}

$$\frac{2c^4\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{5/2}f}-\frac{11c^4\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{2}a^{5/2}f}+\frac{7c^4\tan(e+fx)}{2a^2f\sqrt{a\sec(e+fx)+a}}-\frac{c^4\sin^2(e+fx)\tan^3(e+fx)\sec^4}{4f(a\sec(e+fx)+a)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^4/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] (2*c^4*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(5/2)*f)
 - (11*c^4*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])
])/(Sqrt[2]*a^(5/2)*f) + (7*c^4*Tan[e + f*x])/(2*a^2*f*Sqrt[a + a*Sec[e + f
*x]]) - (c^4*Sec[(e + f*x)/2]^2*Sin[e + f*x]*Tan[e + f*x]^2)/(4*a*f*(a + a*
Sec[e + f*x])^(3/2)) - (c^4*Sec[(e + f*x)/2]^4*Sin[e + f*x]^2*Tan[e + f*x]^
3)/(4*f*(a + a*Sec[e + f*x])^(5/2))
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
```

```
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
```

```
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{5/2}}\,dx = \left(a^4c^4\right)\int \frac{\tan^8(e+fx)}{(a+a\sec(e+fx))^{13/2}}\,dx$$

$$= -\frac{\left(2a^2c^4\right)\text{Subst}\left(\int \frac{x^8}{(1+ax^2)(2+ax^2)^3}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= -\frac{c^4\sec^4\left(\frac{1}{2}(e+fx)\right)\sin^2(e+fx)\tan^3(e+fx)}{4f(a+a\sec(e+fx))^{5/2}} - \frac{c^4\,\text{Subst}\left(\int \frac{x^4(10+6ax^2)}{(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan}{\sqrt{a+a}}\right.}{2f}$$

$$= -\frac{c^4\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)\tan^2(e+fx)}{4af(a+a\sec(e+fx))^{3/2}} - \frac{c^4\sec^4\left(\frac{1}{2}(e+fx)\right)\sin^2(e+fx)\tan^3(e}{4f(a+a\sec(e+fx))^{5/2}}$$

$$= \frac{7c^4\tan(e+fx)}{2a^2f\sqrt{a+a\sec(e+fx)}} - \frac{c^4\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)\tan^2(e+fx)}{4af(a+a\sec(e+fx))^{3/2}} - \frac{c^4\sec^4\left(\frac{1}{2}(e+\right.}{4f(}$$

$$= \frac{7c^4\tan(e+fx)}{2a^2f\sqrt{a+a\sec(e+fx)}} - \frac{c^4\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)\tan^2(e+fx)}{4af(a+a\sec(e+fx))^{3/2}} - \frac{c^4\sec^4\left(\frac{1}{2}(e+\right.}{4f(}$$

$$= \frac{2c^4\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}f} - \frac{11c^4\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{2}a^{5/2}f} + \frac{7c^4\tan(e+fx)}{2a^2f\sqrt{a+a\sec(e+fx)}} - \frac{c^4}{}$$

Mathematica [A] time = 2.59924, size = 164, normalized size = 0.72

$$\frac{c^4\cot\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\left((19\cos(e+fx)-12\cos(2(e+fx))-3\cos(3(e+fx))-4)\sec^4\left(\frac{1}{2}(e+fx)\right)+32\cos(e\right.}{16a^2f\sqrt{a(\sec(e+f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^4/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] (c^4*Cot[(e + f*x)/2]*((-4 + 19*Cos[e + f*x] - 12*Cos[2*(e + f*x)] - 3*Cos[
3*(e + f*x)])*Sec[(e + f*x)/2]^4 + 32*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Cos[e
 + f*x]*Sqrt[-1 + Sec[e + f*x]] - 88*Sqrt[2]*ArcTan[Sqrt[-1 + Sec[e + f*x]]
/Sqrt[2]]*Cos[e + f*x]*Sqrt[-1 + Sec[e + f*x]])*Sec[e + f*x])/(16*a^2*f*Sqr
t[a*(1 + Sec[e + f*x])])
```

Maple [B] time = 0.272, size = 550, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(5/2),x)
```

```
[Out] -1/2*c^4/f/a^3*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(2*(-2*cos(f*x+e)/(1+c
os(f*x+e)))^(1/2)*cos(f*x+e)^4*sin(f*x+e)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*c
os(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))+11*cos(f*x+e)^4*sin(
f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(((-2*cos(f*x+e)/(1+cos(f*x+e
)))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))-4*cos(f*x+e)^2*sin(f*x+e)*2^
(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+
e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))+6*cos(f*x+e)^5-22*sin(f*x+e
)*cos(f*x+e)^2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(((-2*cos(f*x+e)/(1+c
os(f*x+e)))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))+2*2^(1/2)*sin(f*x+e)
*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*
x+e))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)-32*cos(f*x+e)^3+11*sin(f*x+e)*(-
2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(((-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)
*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))+36*cos(f*x+e)^2-6*cos(f*x+e)-4)/sin(f
*x+e)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 11.8767, size = 1673, normalized size = 7.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(11*sqrt(2)*(c^4*cos(f*x + e)^3 + 3*c^4*cos(f*x + e)^2 + 3*c^4*cos(f*
x + e) + c^4)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/c
os(f*x + e))*cos(f*x + e)*sin(f*x + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f*x +
 e) + a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 4*(c^4*cos(f*x + e)^3 + 3
*c^4*cos(f*x + e)^2 + 3*c^4*cos(f*x + e) + c^4)*sqrt(-a)*log((2*a*cos(f*x +
 e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin
(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 4*(3*c^4*cos(f*x + e)
^2 + 9*c^4*cos(f*x + e) + 2*c^4)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*si
n(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*
x + e) + a^3*f), 1/2*(11*sqrt(2)*(c^4*cos(f*x + e)^3 + 3*c^4*cos(f*x + e)^2
 + 3*c^4*cos(f*x + e) + c^4)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) +
a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 4*(c^4*cos(f*x + e)
^3 + 3*c^4*cos(f*x + e)^2 + 3*c^4*cos(f*x + e) + c^4)*sqrt(a)*arctan(sqrt((
a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + 2*
(3*c^4*cos(f*x + e)^2 + 9*c^4*cos(f*x + e) + 2*c^4)*sqrt((a*cos(f*x + e) +
a)/cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)
^2 + 3*a^3*f*cos(f*x + e) + a^3*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^4\left(\int -\frac{4\sec\left(e+fx\right)}{a^2\sqrt{a\sec\left(e+fx\right)+a}\sec^2\left(e+fx\right)+2a^2\sqrt{a\sec\left(e+fx\right)+a}\sec\left(e+fx\right)+a^2\sqrt{a\sec\left(e+fx\right)+a}}\,dx+\int \frac{}{a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**4/(a+a*sec(f*x+e))**(5/2),x)
```

```
[Out] c**4*(Integral(-4*sec(e + f*x)/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)*
*2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a*sec(e + f*x
```

```
) + a)), x) + Integral(6*sec(e + f*x)**2/(a**2*sqrt(a*sec(e + f*x) + a)*sec
(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a*s
ec(e + f*x) + a)), x) + Integral(-4*sec(e + f*x)**3/(a**2*sqrt(a*sec(e + f*
x) + a)*sec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a*
*2*sqrt(a*sec(e + f*x) + a)), x) + Integral(sec(e + f*x)**4/(a**2*sqrt(a*se
c(e + f*x) + a)*sec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f
*x) + a**2*sqrt(a*sec(e + f*x) + a)), x) + Integral(1/(a**2*sqrt(a*sec(e +
f*x) + a)*sec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) +
a**2*sqrt(a*sec(e + f*x) + a)), x))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.81 $\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=191

$$-\frac{c^3\sin(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)}{4a^2f\sqrt{a\sec(e+fx)+a}}+\frac{2c^3\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{5/2}f}-\frac{7c^3\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{2\sqrt{2}a^{5/2}f}+\frac{c^3\sin^2(e+fx)\tan(e+f}{4af(a\sec(e+f}$$

```
[Out] (2*c^3*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(5/2)*f)
 - (7*c^3*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])]
)/(2*Sqrt[2]*a^(5/2)*f) - (c^3*Sec[(e + f*x)/2]^2*Sin[e + f*x])/(4*a^2*f*Sq
rt[a + a*Sec[e + f*x]]) + (c^3*Sec[(e + f*x)/2]^4*Sin[e + f*x]^2*Tan[e + f*
x])/(4*a*f*(a + a*Sec[e + f*x])^(3/2))
```

Rubi [A] time = 0.252055, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3904, 3887, 470, 578, 522, 203}

$$-\frac{c^3\sin(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)}{4a^2f\sqrt{a\sec(e+fx)+a}}+\frac{2c^3\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{5/2}f}-\frac{7c^3\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{2\sqrt{2}a^{5/2}f}+\frac{c^3\sin^2(e+fx)\tan(e+f}{4af(a\sec(e+f}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] (2*c^3*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(5/2)*f)
 - (7*c^3*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])]
)/(2*Sqrt[2]*a^(5/2)*f) - (c^3*Sec[(e + f*x)/2]^2*Sin[e + f*x])/(4*a^2*f*Sq
rt[a + a*Sec[e + f*x]]) + (c^3*Sec[(e + f*x)/2]^4*Sin[e + f*x]^2*Tan[e + f*
x])/(4*a*f*(a + a*Sec[e + f*x])^(3/2))
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx = -\left(\left(a^3c^3\right)\int \frac{\tan^6(e+fx)}{(a+a\sec(e+fx))^{11/2}}\,dx\right)$$

$$= \frac{\left(2ac^3\right)\text{Subst}\left(\int \frac{x^6}{\left(1+ax^2\right)\left(2+ax^2\right)^3}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= \frac{c^3\sec^4\left(\frac{1}{2}(e+fx)\right)\sin^2(e+fx)\tan(e+fx)}{4af(a+a\sec(e+fx))^{3/2}} + \frac{c^3\,\text{Subst}\left(\int \frac{x^2\left(6+2ax^2\right)}{\left(1+ax^2\right)\left(2+ax^2\right)^2}\,dx, x, -\frac{\tan(e+}{\sqrt{a+a\sec}}\right.}{2af}$$

$$= -\frac{c^3\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{4a^2f\sqrt{a+a\sec(e+fx)}} + \frac{c^3\sec^4\left(\frac{1}{2}(e+fx)\right)\sin^2(e+fx)\tan(e+fx)}{4af(a+a\sec(e+fx))^{3/2}} - \frac{c^3\,\text{Sub}}{}$$

$$= -\frac{c^3\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{4a^2f\sqrt{a+a\sec(e+fx)}} + \frac{c^3\sec^4\left(\frac{1}{2}(e+fx)\right)\sin^2(e+fx)\tan(e+fx)}{4af(a+a\sec(e+fx))^{3/2}} - \frac{\left(2c^3\right)}{}$$

$$= \frac{2c^3\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}f} - \frac{7c^3\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{2\sqrt{2}a^{5/2}f} - \frac{c^3\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx}{4a^2f\sqrt{a+a\sec(e+fx)}}$$

Mathematica [A] time = 1.53731, size = 136, normalized size = 0.71

$$-\frac{c^3\cot\left(\frac{1}{2}(e+fx)\right)\left((8\cos(e+fx)-3\cos(2(e+fx))-5)\sec^4\left(\frac{1}{2}(e+fx)\right)-32\sqrt{\sec(e+fx)-1}\tan^{-1}\left(\sqrt{\sec(e+fx)}\right.\right.}{16a^2f\sqrt{a(\sec(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] -(c^3*Cot[(e + f*x)/2]*((-5 + 8*Cos[e + f*x] - 3*Cos[2*(e + f*x)])*Sec[(e +
 f*x)/2]^4 - 32*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Sqrt[-1 + Sec[e + f*x]] + 2
8*Sqrt[2]*ArcTan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]]*Sqrt[-1 + Sec[e + f*x]]))
/(16*a^2*f*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [B] time = 0.252, size = 553, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(5/2),x)
```

```
[Out] -1/4*c^3/f/a^3*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(-1+cos(f*x+e))*(-4*co
s(f*x+e)^2*sin(f*x+e)*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(
1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))-7*s
in(f*x+e)*cos(f*x+e)^2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(((-2*cos(f*x
+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))-8*2^(1/2)*co
s(f*x+e)*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*2^(1/2
)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))-14*sin(f*x+e)
*cos(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(((-2*cos(f*x+e)/(1+cos(
f*x+e)))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))-4*2^(1/2)*sin(f*x+e)*ar
ctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e
))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)+6*cos(f*x+e)^3-7*sin(f*x+e)*(-2*cos
(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(((-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(
f*x+e)-cos(f*x+e)+1)/sin(f*x+e))-8*cos(f*x+e)^2+2*cos(f*x+e))/(1+cos(f*x+e)
)/sin(f*x+e)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 9.10625, size = 1643, normalized size = 8.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(7*sqrt(2)*(c^3*cos(f*x + e)^3 + 3*c^3*cos(f*x + e)^2 + 3*c^3*cos(f*x
 + e) + c^3)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/co
```

```
s(f*x + e))*cos(f*x + e)*sin(f*x + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f*x +
e) + a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 8*(c^3*cos(f*x + e)^3 + 3*
c^3*cos(f*x + e)^2 + 3*c^3*cos(f*x + e) + c^3)*sqrt(-a)*log((2*a*cos(f*x +
e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(
f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 4*(3*c^3*cos(f*x + e)^
2 - c^3*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))
/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^
3*f), 1/4*(7*sqrt(2)*(c^3*cos(f*x + e)^3 + 3*c^3*cos(f*x + e)^2 + 3*c^3*cos
(f*x + e) + c^3)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x +
 e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 8*(c^3*cos(f*x + e)^3 + 3*c^3*c
os(f*x + e)^2 + 3*c^3*cos(f*x + e) + c^3)*sqrt(a)*arctan(sqrt((a*cos(f*x +
e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 2*(3*c^3*cos(f
*x + e)^2 - c^3*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f
*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x +
 e) + a^3*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-c^3\left(\int \frac{3\sec\left(e+fx\right)}{a^2\sqrt{a\sec\left(e+fx\right)+a}\sec^2\left(e+fx\right)+2a^2\sqrt{a\sec\left(e+fx\right)+a}\sec\left(e+fx\right)+a^2\sqrt{a\sec\left(e+fx\right)+a}}\,dx+\int -\frac{}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**3/(a+a*sec(f*x+e))**(5/2),x)
```

```
[Out] -c**3*(Integral(3*sec(e + f*x)/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)*
*2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a*sec(e + f*x
) + a)), x) + Integral(-3*sec(e + f*x)**2/(a**2*sqrt(a*sec(e + f*x) + a)*se
c(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a*
sec(e + f*x) + a)), x) + Integral(sec(e + f*x)**3/(a**2*sqrt(a*sec(e + f*x)
 + a)*sec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2
*sqrt(a*sec(e + f*x) + a)), x) + Integral(-1/(a**2*sqrt(a*sec(e + f*x) + a)
*sec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt
(a*sec(e + f*x) + a)), x))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.82 $\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=189

$$-\frac{3c^2\sin(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)}{8a^2f\sqrt{a\sec(e+fx)+a}}+\frac{2c^2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{5/2}f}-\frac{11c^2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{4\sqrt{2}a^{5/2}f}-\frac{c^2\sin(e+fx)\cos(e+}{4a^2f\sqrt{a\sec(e}}$$

```
[Out] (2*c^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(5/2)*f)
 - (11*c^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])
])/(4*Sqrt[2]*a^(5/2)*f) - (3*c^2*Sec[(e + f*x)/2]^2*Sin[e + f*x])/(8*a^2*f
*Sqrt[a + a*Sec[e + f*x]]) - (c^2*Cos[e + f*x]*Sec[(e + f*x)/2]^4*Sin[e + f
*x])/(4*a^2*f*Sqrt[a + a*Sec[e + f*x]])
```

Rubi [A] time = 0.229578, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3904, 3887, 470, 527, 522, 203}

$$-\frac{3c^2\sin(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)}{8a^2f\sqrt{a\sec(e+fx)+a}}+\frac{2c^2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{5/2}f}-\frac{11c^2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{4\sqrt{2}a^{5/2}f}-\frac{c^2\sin(e+fx)\cos(e+}{4a^2f\sqrt{a\sec(e}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] (2*c^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(5/2)*f)
 - (11*c^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])
])/(4*Sqrt[2]*a^(5/2)*f) - (3*c^2*Sec[(e + f*x)/2]^2*Sin[e + f*x])/(8*a^2*f
*Sqrt[a + a*Sec[e + f*x]]) - (c^2*Cos[e + f*x]*Sec[(e + f*x)/2]^4*Sin[e + f
*x])/(4*a^2*f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(c - c\sec(e + fx))^2}{(a + a\sec(e + fx))^{5/2}}\,dx = \left(a^2c^2\right)\int \frac{\tan^4(e + fx)}{(a + a\sec(e + fx))^{9/2}}\,dx$$

$$= -\frac{\left(2c^2\right)\text{Subst}\left(\int \frac{x^4}{\left(1+ax^2\right)\left(2+ax^2\right)^3}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= -\frac{c^2\cos(e + fx)\sec^4\left(\frac{1}{2}(e + fx)\right)\sin(e + fx)}{4a^2f\sqrt{a + a\sec(e + fx)}} - \frac{c^2\,\text{Subst}\left(\int \frac{2-2ax^2}{\left(1+ax^2\right)\left(2+ax^2\right)^2}\,dx, x, -\frac{\tan(e+}{\sqrt{a+a\sec}}\right.}{2a^2f}$$

$$= -\frac{3c^2\sec^2\left(\frac{1}{2}(e + fx)\right)\sin(e + fx)}{8a^2f\sqrt{a + a\sec(e + fx)}} - \frac{c^2\cos(e + fx)\sec^4\left(\frac{1}{2}(e + fx)\right)\sin(e + fx)}{4a^2f\sqrt{a + a\sec(e + fx)}} - \frac{c^2\,\text{Sub}}{}$$

$$= -\frac{3c^2\sec^2\left(\frac{1}{2}(e + fx)\right)\sin(e + fx)}{8a^2f\sqrt{a + a\sec(e + fx)}} - \frac{c^2\cos(e + fx)\sec^4\left(\frac{1}{2}(e + fx)\right)\sin(e + fx)}{4a^2f\sqrt{a + a\sec(e + fx)}} - \frac{\left(2c^2\right)\,\text{S}}{}$$

$$= \frac{2c^2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}f} - \frac{11c^2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{4\sqrt{2}a^{5/2}f} - \frac{3c^2\sec^2\left(\frac{1}{2}(e + fx)\right)\sin(e +}{8a^2f\sqrt{a + a\sec(e + fx)}}$$

Mathematica [A] time = 1.4824, size = 136, normalized size = 0.72

$$-\frac{c^2\cot\left(\frac{1}{2}(e + fx)\right)\left((8\cos(e + fx) - 7\cos(2(e + fx)) - 1)\sec^4\left(\frac{1}{2}(e + fx)\right) - 64\sqrt{\sec(e + fx) - 1}\tan^{-1}\left(\sqrt{\sec(e + fx)}\right.\right.}{32a^2f\sqrt{a(\sec(e + fx) + 1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] -(c^2*Cot[(e + f*x)/2]*((-1 + 8*Cos[e + f*x] - 7*Cos[2*(e + f*x)])*Sec[(e +
 f*x)/2]^4 - 64*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Sqrt[-1 + Sec[e + f*x]] + 4
4*Sqrt[2]*ArcTan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]]*Sqrt[-1 + Sec[e + f*x]]))
/(32*a^2*f*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [B] time = 0.194, size = 545, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(5/2),x)
```

```
[Out] -1/8*c^2/f/a^3*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(8*cos(f*x+e)^2*sin(f*
x+e)*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*2^(1/2)*(-2*c
os(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))+16*2^(1/2)*cos(f*x+e
)*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*2^(1/2)*(-2*c
os(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))+11*sin(f*x+e)*cos(f*
x+e)^2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(((-2*cos(f*x+e)/(1+cos(f*x+e
)))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))+8*2^(1/2)*sin(f*x+e)*arctanh
(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*(-
2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)+22*sin(f*x+e)*cos(f*x+e)*(-2*cos(f*x+e)/
(1+cos(f*x+e)))^(1/2)*ln(((-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-c
os(f*x+e)+1)/sin(f*x+e))+11*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)
*ln(((-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+
e))-14*cos(f*x+e)^3+8*cos(f*x+e)^2+6*cos(f*x+e))/(1+cos(f*x+e))^2/sin(f*x+e
)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 6.55813, size = 1655, normalized size = 8.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(11*sqrt(2)*(c^2*cos(f*x + e)^3 + 3*c^2*cos(f*x + e)^2 + 3*c^2*cos(f
*x + e) + c^2)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/
```

```
cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f*x
+ e) + a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 16*(c^2*cos(f*x + e)^3 +
 3*c^2*cos(f*x + e)^2 + 3*c^2*cos(f*x + e) + c^2)*sqrt(-a)*log((2*a*cos(f*x
 + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*s
in(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 4*(7*c^2*cos(f*x +
e)^2 + 3*c^2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x
+ e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e)
 + a^3*f), 1/8*(11*sqrt(2)*(c^2*cos(f*x + e)^3 + 3*c^2*cos(f*x + e)^2 + 3*c
^2*cos(f*x + e) + c^2)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos
(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 16*(c^2*cos(f*x + e)^3 +
3*c^2*cos(f*x + e)^2 + 3*c^2*cos(f*x + e) + c^2)*sqrt(a)*arctan(sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 2*(7*c^
2*cos(f*x + e)^2 + 3*c^2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x +
e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*
cos(f*x + e) + a^3*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^2\left(\int -\frac{2\sec\left(e+fx\right)}{a^2\sqrt{a\sec\left(e+fx\right)+a}\sec^2\left(e+fx\right)+2a^2\sqrt{a\sec\left(e+fx\right)+a}\sec\left(e+fx\right)+a^2\sqrt{a\sec\left(e+fx\right)+a}}\,dx+\int \frac{}{a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**2/(a+a*sec(f*x+e))**(5/2),x)
```

```
[Out] c**2*(Integral(-2*sec(e + f*x)/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)*
*2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a*sec(e + f*x
) + a)), x) + Integral(sec(e + f*x)**2/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e
 + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a*sec
(e + f*x) + a)), x) + Integral(1/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x
)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a*sec(e + f
*x) + a)), x))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.83 $\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=148

$$\frac{2c\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{5/2}f}-\frac{23c\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{8\sqrt{2}a^{5/2}f}-\frac{7c\tan(e+fx)}{8af(a\sec(e+fx)+a)^{3/2}}-\frac{c\tan(e+fx)}{2f(a\sec(e+fx)+a)^{5/2}}$$

```
[Out] (2*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(5/2)*f) -
 (23*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])/(
8*Sqrt[2]*a^(5/2)*f) - (c*Tan[e + f*x])/(2*f*(a + a*Sec[e + f*x])^(5/2)) -
(7*c*Tan[e + f*x])/(8*a*f*(a + a*Sec[e + f*x])^(3/2))
```

Rubi [A] time = 0.191576, antiderivative size = 181, normalized size of antiderivative = 1.22, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3904, 3887, 471, 527, 522, 203}

$$-\frac{7c\sin(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)}{16a^2f\sqrt{a\sec(e+fx)+a}}+\frac{2c\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{5/2}f}-\frac{23c\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{8\sqrt{2}a^{5/2}f}-\frac{c\sin(e+fx)\cos(e+fx)\,s\cdots}{8a^2f\sqrt{a\sec(e+\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] (2*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(5/2)*f) -
 (23*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])/(
8*Sqrt[2]*a^(5/2)*f) - (7*c*Sec[(e + f*x)/2]^2*Sin[e + f*x])/(16*a^2*f*Sqrt
[a + a*Sec[e + f*x]]) - (c*Cos[e + f*x]*Sec[(e + f*x)/2]^4*Sin[e + f*x])/(8
*a^2*f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx = -\left((ac)\int \frac{\tan^2(e+fx)}{(a+a\sec(e+fx))^{7/2}}\,dx\right)$$

$$= \frac{(2c)\,\mathrm{Subst}\left(\int \frac{x^2}{(1+ax^2)(2+ax^2)^3}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{af}$$

$$= -\frac{c\cos(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{8a^2f\sqrt{a+a\sec(e+fx)}} - \frac{c\,\mathrm{Subst}\left(\int \frac{1-3ax^2}{(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan(e+f}{\sqrt{a+a\sec(e}}\right.}{2a^2f}$$

$$= -\frac{7c\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{16a^2f\sqrt{a+a\sec(e+fx)}} - \frac{c\cos(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{8a^2f\sqrt{a+a\sec(e+fx)}} - \frac{c\,\mathrm{Subst}}{}$$

$$= -\frac{7c\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{16a^2f\sqrt{a+a\sec(e+fx)}} - \frac{c\cos(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{8a^2f\sqrt{a+a\sec(e+fx)}} - \frac{(2c)\,\mathrm{Sub}}{}$$

$$= \frac{2c\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}f} - \frac{23c\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{8\sqrt{2}a^{5/2}f} - \frac{7c\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{16a^2f\sqrt{a+a\sec(e+fx)}}$$

Mathematica [A] time = 1.45808, size = 134, normalized size = 0.91

$$-\frac{c\cot\left(\frac{1}{2}(e+fx)\right)\left((8\cos(e+fx)-11\cos(2(e+fx))+3)\sec^4\left(\frac{1}{2}(e+fx)\right)-128\sqrt{\sec(e+fx)-1}\tan^{-1}\left(\sqrt{\sec(e+fx}\right.\right.}{64a^2f\sqrt{a(\sec(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] -(c*Cot[(e + f*x)/2]*((3 + 8*Cos[e + f*x] - 11*Cos[2*(e + f*x)])*Sec[(e + f
*x)/2]^4 - 128*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Sqrt[-1 + Sec[e + f*x]] + 92
*Sqrt[2]*ArcTan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]]*Sqrt[-1 + Sec[e + f*x]]))/
(64*a^2*f*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [B] time = 0.179, size = 543, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(5/2),x)
```

```
[Out] -1/16*c/f/a^3*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(16*cos(f*x+e)^2*sin(f*
x+e)*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*2^(1/2)*(-2*c
os(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))+23*sin(f*x+e)*cos(f*
x+e)^2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(((-2*cos(f*x+e)/(1+cos(f*x+e
)))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))+32*2^(1/2)*cos(f*x+e)*sin(f*
x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e
)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))+46*sin(f*x+e)*cos(f*x+e)*(-2
*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(((-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*
sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))+16*2^(1/2)*sin(f*x+e)*arctanh(1/2*2^(1
/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*(-2*cos(f*x
+e)/(1+cos(f*x+e)))^(1/2)+23*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2
)*ln(((-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x
+e))-22*cos(f*x+e)^3+8*cos(f*x+e)^2+14*cos(f*x+e))/(1+cos(f*x+e))^2/sin(f*x
+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{c\sec\left(fx+e\right)-c}{\left(a\sec\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] -integrate((c*sec(f*x + e) - c)/(a*sec(f*x + e) + a)^(5/2), x)
```

Fricas [B] time = 3.9638, size = 1605, normalized size = 10.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/32*(23*sqrt(2)*(c*cos(f*x + e)^3 + 3*c*cos(f*x + e)^2 + 3*c*cos(f*x + e
) + c)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x
+ e))*cos(f*x + e)*sin(f*x + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f*x + e) + a
)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 32*(c*cos(f*x + e)^3 + 3*c*cos(f
*x + e)^2 + 3*c*cos(f*x + e) + c)*sqrt(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt
(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*
cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 4*(11*c*cos(f*x + e)^2 + 7*c*cos(f*
x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*
x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f), 1/16*(23
*sqrt(2)*(c*cos(f*x + e)^3 + 3*c*cos(f*x + e)^2 + 3*c*cos(f*x + e) + c)*sqr
t(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(s
qrt(a)*sin(f*x + e))) - 32*(c*cos(f*x + e)^3 + 3*c*cos(f*x + e)^2 + 3*c*cos
(f*x + e) + c)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f
*x + e)/(sqrt(a)*sin(f*x + e))) - 2*(11*c*cos(f*x + e)^2 + 7*c*cos(f*x + e)
)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)
^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-c\left(\int \frac{\sec\left(e+fx\right)}{a^2\sqrt{a\sec\left(e+fx\right)+a}\sec^2\left(e+fx\right)+2a^2\sqrt{a\sec\left(e+fx\right)+a}\sec\left(e+fx\right)+a^2\sqrt{a\sec\left(e+fx\right)+a}}\,dx+\int -\frac{}{a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))**(5/2),x)
```

```
[Out] -c*(Integral(sec(e + f*x)/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**2 +
2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a*sec(e + f*x) + a
)), x) + Integral(-1/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**2 + 2*a**
2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a*sec(e + f*x) + a)), x
))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.84 $\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))}\,dx$

Optimal. Leaf size=230

$$-\frac{7\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{32a^3cf}+\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{5/2}cf}-\frac{71\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{32\sqrt{2}a^{5/2}cf}+\frac{\cos^2(e+fx)\cot(e+fx)\,\mathrm{s}}{}$$

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(5/2)*c*f) -
 (71*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])/(32
*Sqrt[2]*a^(5/2)*c*f) - (7*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(32*a^3*c
*f) + (13*Cos[e + f*x]*Cot[e + f*x]*Sec[(e + f*x)/2]^2*Sqrt[a + a*Sec[e + f
*x]])/(32*a^3*c*f) + (Cos[e + f*x]^2*Cot[e + f*x]*Sec[(e + f*x)/2]^4*Sqrt[a
 + a*Sec[e + f*x]])/(16*a^3*c*f)
```

Rubi [A] time = 0.306451, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3904, 3887, 472, 579, 583, 522, 203}

$$-\frac{7\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{32a^3cf}+\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{5/2}cf}-\frac{71\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{32\sqrt{2}a^{5/2}cf}+\frac{\cos^2(e+fx)\cot(e+fx)\,\mathrm{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])),x]
```



```
[Out] (2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(5/2)*c*f) -
 (71*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])/(32
*Sqrt[2]*a^(5/2)*c*f) - (7*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(32*a^3*c
*f) + (13*Cos[e + f*x]*Cot[e + f*x]*Sec[(e + f*x)/2]^2*Sqrt[a + a*Sec[e + f
*x]])/(32*a^3*c*f) + (Cos[e + f*x]^2*Cot[e + f*x]*Sec[(e + f*x)/2]^4*Sqrt[a
 + a*Sec[e + f*x]])/(16*a^3*c*f)
```


Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
```

```
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
```

```
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))}\,dx = -\frac{\int \frac{\cot^2(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx}{ac}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{1}{x^2(1+ax^2)(2+ax^2)^3}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^3cf}$$

$$= \frac{\cos^2(e+fx)\cot(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)\sqrt{a+a\sec(e+fx)}}{16a^3cf} + \frac{\text{Subst}}{}$$

$$= \frac{13\cos(e+fx)\cot(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)\sqrt{a+a\sec(e+fx)}}{32a^3cf} + \frac{\cos^2}{}$$

$$= -\frac{7\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{32a^3cf} + \frac{13\cos(e+fx)\cot(e+fx)\sec^2\left(\frac{1}{2}\right.}{32a^3c}$$

$$= -\frac{7\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{32a^3cf} + \frac{13\cos(e+fx)\cot(e+fx)\sec^2\left(\frac{1}{2}\right.}{32a^3c}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}cf} - \frac{71\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{32\sqrt{2}a^{5/2}cf} - \frac{7\cot(e+fx)\sqrt{}}{32}$$

Mathematica [A] time = 1.44271, size = 158, normalized size = 0.69

$$\frac{\tan^3\left(\frac{1}{2}(e+fx)\right)\left(24\cos(e+fx)+27\cos(2(e+fx))+512\cos^4\left(\frac{1}{2}(e+fx)\right)\sqrt{\sec(e+fx)-1}\tan^{-1}\left(\sqrt{\sec(e+fx)-1}\right.\right.}{64a^2cf(\cos(e+fx)-1)^2\sqrt{a(\sec(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])),x]
```

```
[Out] ((13 + 24*Cos[e + f*x] + 27*Cos[2*(e + f*x)] + 512*ArcTan[Sqrt[-1 + Sec[e +
 f*x]]]*Cos[(e + f*x)/2]^4*Sqrt[-1 + Sec[e + f*x]] - 284*Sqrt[2]*ArcTan[Sqr
t[-1 + Sec[e + f*x]]/Sqrt[2]]*Cos[(e + f*x)/2]^4*Sqrt[-1 + Sec[e + f*x]])*T
an[(e + f*x)/2]^3)/(64*a^2*c*f*(-1 + Cos[e + f*x])^2*Sqrt[a*(1 + Sec[e + f*
x])])
```

Maple [B] time = 0.265, size = 545, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e)),x)
```

```
[Out] -1/64/c/f/a^3*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(-1+cos(f*x+e))^2*(64*c
os(f*x+e)^2*sin(f*x+e)*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh
(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))+12
8*2^(1/2)*cos(f*x+e)*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctan
h(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))+7
1*sin(f*x+e)*cos(f*x+e)^2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(((-2*cos(
f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))+64*2^(1/2
)*sin(f*x+e)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f
*x+e)/cos(f*x+e))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)+142*sin(f*x+e)*cos(f
*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(((-2*cos(f*x+e)/(1+cos(f*x+e)
))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))+71*sin(f*x+e)*(-2*cos(f*x+e)/
(1+cos(f*x+e)))^(1/2)*ln(((-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-c
os(f*x+e)+1)/sin(f*x+e))-54*cos(f*x+e)^3-24*cos(f*x+e)^2+14*cos(f*x+e))/sin
(f*x+e)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(a \sec\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(c \sec\left(fx+e\right)-c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] -integrate(1/((a*sec(f*x + e) + a)^(5/2)*(c*sec(f*x + e) - c)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{1}{a^2\sqrt{a\sec\left(e+fx\right)+a}\sec^3\left(e+fx\right)+a^2\sqrt{a\sec\left(e+fx\right)+a}\sec^2\left(e+fx\right)-a^2\sqrt{a\sec\left(e+fx\right)+a}\sec\left(e+fx\right)-a^2\sqrt{a\sec\left(e+fx\right)+a}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e)),x)
```

```
[Out] -Integral(1/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**3 + a**2*sqrt(a*se
c(e + f*x) + a)*sec(e + f*x)**2 - a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x
) - a**2*sqrt(a*sec(e + f*x) + a)), x)/c
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.85 $\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2}\,dx$

Optimal. Leaf size=269

$$\frac{43\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{96a^4c^2f}+\frac{21\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{64a^3c^2f}+\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{5/2}c^2f}-\frac{107\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{64\sqrt{2}a^{5/2}c^2f}$$

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(5/2)*c^2*f)
 - (107*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])/
(64*Sqrt[2]*a^(5/2)*c^2*f) + (21*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(64
*a^3*c^2*f) + (43*Cot[e + f*x]^3*(a + a*Sec[e + f*x])^(3/2))/(96*a^4*c^2*f)
 - (15*Cos[e + f*x]*Cot[e + f*x]^3*Sec[(e + f*x)/2]^2*(a + a*Sec[e + f*x])^
(3/2))/(32*a^4*c^2*f) - (Cos[e + f*x]^2*Cot[e + f*x]^3*Sec[(e + f*x)/2]^4*(
a + a*Sec[e + f*x])^(3/2))/(16*a^4*c^2*f)
```

Rubi [A] time = 0.335235, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3904, 3887, 472, 579, 583, 522, 203}

$$\frac{43\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{96a^4c^2f}+\frac{21\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{64a^3c^2f}+\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{5/2}c^2f}-\frac{107\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{64\sqrt{2}a^{5/2}c^2f}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^2),x]
```


```
[Out] (2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(5/2)*c^2*f)
 - (107*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])/
(64*Sqrt[2]*a^(5/2)*c^2*f) + (21*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(64
*a^3*c^2*f) + (43*Cot[e + f*x]^3*(a + a*Sec[e + f*x])^(3/2))/(96*a^4*c^2*f)
 - (15*Cos[e + f*x]*Cot[e + f*x]^3*Sec[(e + f*x)/2]^2*(a + a*Sec[e + f*x])^
(3/2))/(32*a^4*c^2*f) - (Cos[e + f*x]^2*Cot[e + f*x]^3*Sec[(e + f*x)/2]^4*(
a + a*Sec[e + f*x])^(3/2))/(16*a^4*c^2*f)
```


Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
```

```
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
```

```
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2}\,dx = \frac{\int \frac{\cot^4(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx}{a^2c^2}$$

$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{x^4(1+ax^2)(2+ax^2)^3}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^4c^2f}$$

$$= -\frac{\cos^2(e+fx)\cot^3(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)(a+a\sec(e+fx))^{3/2}}{16a^4c^2f} - \text{Su}$$

$$= -\frac{15\cos(e+fx)\cot^3(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)(a+a\sec(e+fx))^{3/2}}{32a^4c^2f} - $$

$$= \frac{43\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{96a^4c^2f} - \frac{15\cos(e+fx)\cot^3(e+fx)\sec}{32}$$

$$= \frac{21\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{64a^3c^2f} + \frac{43\cot^3(e+fx)(a+a\sec(e+fx))}{96a^4c^2f}$$

$$= \frac{21\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{64a^3c^2f} + \frac{43\cot^3(e+fx)(a+a\sec(e+fx))}{96a^4c^2f}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}c^2f} - \frac{107\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{64\sqrt{2}a^{5/2}c^2f} + \frac{21\cot(e+fx}{6}$$

Mathematica [C] time = 24.0523, size = 5660, normalized size = 21.04

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.326, size = 725, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^2,x)
```

```
[Out] 1/384/c^2/f/a^3*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(1+cos(f*x+e))*(-1+co
s(f*x+e))^2*(384*2^(1/2)*cos(f*x+e)^3*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+
e)))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x
+e)/cos(f*x+e))+321*sin(f*x+e)*cos(f*x+e)^3*(-2*cos(f*x+e)/(1+cos(f*x+e)))^
(1/2)*ln(((-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin
(f*x+e))+384*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))
^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/
cos(f*x+e))+321*sin(f*x+e)*cos(f*x+e)^2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2
)*ln(((-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x
+e))-384*2^(1/2)*cos(f*x+e)*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)
*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*
x+e))-321*sin(f*x+e)*cos(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(((-
2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))-384
*2^(1/2)*sin(f*x+e)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2
)*sin(f*x+e)/cos(f*x+e))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)-410*cos(f*x+e
)^4-321*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(((-2*cos(f*x+e)/
(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))-142*cos(f*x+e)^3
+298*cos(f*x+e)^2+126*cos(f*x+e))/sin(f*x+e)^7
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^2,x, algorithm="maxima"
)
```

```
[Out] Timed out
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^2,x, algorithm="fricas"
)
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.86 $\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}\,dx$

Optimal. Leaf size=185

$$-\frac{ac^3\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}-\frac{ac^2\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}+\frac{ac^4\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-$$

```
[Out] (a*c^4*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
 c*Sec[e + f*x]]) - (a*c^3*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a
 + a*Sec[e + f*x]]) - (a*c^2*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(2*f*
Sqrt[a + a*Sec[e + f*x]]) - (a*c*(c - c*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(
3*f*Sqrt[a + a*Sec[e + f*x]])
```

Rubi [A] time = 0.372323, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3906, 3905, 3475}

$$-\frac{ac^3\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}-\frac{ac^2\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}+\frac{ac^4\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(7/2),x]
```


```
[Out] (a*c^4*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
 c*Sec[e + f*x]]) - (a*c^3*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a
 + a*Sec[e + f*x]]) - (a*c^2*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(2*f*
Sqrt[a + a*Sec[e + f*x]]) - (a*c*(c - c*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(
3*f*Sqrt[a + a*Sec[e + f*x]])
```


Rule 3906

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e + f*x]*(c + d*Csc[e + f*x]
)^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[c, Int[Sqrt[a
+ b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && GtQ[n, 1/2]
```

Rule 3905

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Dist[((-(a*c))^(m + 1/2)*Cot[e + f*x])/(Sqrt
```

```
[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int[Cot[e + f*x]^(2*m), x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IntegerQ[m + 1/2]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}\,dx &= -\frac{ac(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}}+c\int \sqrt{a+a\sec(e+fx)}(c-c\sec\\ &= -\frac{ac^2(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}}-\frac{ac(c-c\sec(e+fx))^{5/2}\tan(e+}{3f\sqrt{a+a\sec(e+fx)}}\\ &= -\frac{ac^3\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}-\frac{ac^2(c-c\sec(e+fx))^{3/2}\tan(e+f}{2f\sqrt{a+a\sec(e+fx)}}\\ &= -\frac{ac^3\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}-\frac{ac^2(c-c\sec(e+fx))^{3/2}\tan(e+f}{2f\sqrt{a+a\sec(e+fx)}}\\ &= \frac{ac^4\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}-\frac{ac^3\sqrt{c-c\sec(e+fx)}\tan(e+}{f\sqrt{a+a\sec(e+fx)}}\end{aligned}$$

Mathematica [C] time = 5.81818, size = 149, normalized size = 0.81

$$\frac{c^3\csc\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\sqrt{a(\sec(e+fx)+1)}\sqrt{c-c\sec(e+fx)}\left(-18\cos(2(e+fx))+3ifx\cos(3(e\right.}{24f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(7/2),x]
```

```
[Out] (c^3*Csc[(e + f*x)/2]*(-22 - 18*Cos[2*(e + f*x)] + (3*I)*f*x*Cos[3*(e + f*x
)] + 9*Cos[e + f*x]*(2 + I*f*x - Log[1 + E^((2*I)*(e + f*x))]) - 3*Cos[3*(e
 + f*x)]*Log[1 + E^((2*I)*(e + f*x))])*Sec[(e + f*x)/2]*Sec[e + f*x]^2*Sqrt
[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])/(24*f)
```

Maple [A] time = 0.356, size = 194, normalized size = 1.1

$$\frac{\cos\left(fx+e\right)}{6f\sin\left(fx+e\right)\left(-1+\cos\left(fx+e\right)\right)^{3}}\left(6\left(\cos\left(fx+e\right)\right)^{3}\ln\left(2\left(1+\cos\left(fx+e\right)\right)^{-1}\right)-6\left(\cos\left(fx+e\right)\right)^{3}\ln\left(\frac{1-\cos\left(fx\right.}{\mathrm{si}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^(7/2)*(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/6/f*(6*cos(f*x+e)^3*ln(2/(1+cos(f*x+e)))-6*cos(f*x+e)^3*ln((1-cos(f*x+e)+
sin(f*x+e))/sin(f*x+e))-6*cos(f*x+e)^3*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f
*x+e))-29*cos(f*x+e)^3-18*cos(f*x+e)^2+9*cos(f*x+e)-2)*cos(f*x+e)*(c*(-1+co
s(f*x+e))/cos(f*x+e))^(7/2)*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)/sin(f*x+e
)/(-1+cos(f*x+e))^3
```

Maxima [B] time = 2.14074, size = 1740, normalized size = 9.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(7/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] -1/3*(3*(f*x + e)*c^3*cos(6*f*x + 6*e)^2 + 27*(f*x + e)*c^3*cos(4*f*x + 4*e
)^2 + 27*(f*x + e)*c^3*cos(2*f*x + 2*e)^2 + 3*(f*x + e)*c^3*sin(6*f*x + 6*e
)^2 + 27*(f*x + e)*c^3*sin(4*f*x + 4*e)^2 + 27*(f*x + e)*c^3*sin(2*f*x + 2*
e)^2 + 18*(f*x + e)*c^3*cos(2*f*x + 2*e) + 3*(f*x + e)*c^3 + 18*c^3*sin(2*f
*x + 2*e) - 3*(c^3*cos(6*f*x + 6*e)^2 + 9*c^3*cos(4*f*x + 4*e)^2 + 9*c^3*co
s(2*f*x + 2*e)^2 + c^3*sin(6*f*x + 6*e)^2 + 9*c^3*sin(4*f*x + 4*e)^2 + 18*c
^3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 9*c^3*sin(2*f*x + 2*e)^2 + 6*c^3*cos
(2*f*x + 2*e) + c^3 + 2*(3*c^3*cos(4*f*x + 4*e) + 3*c^3*cos(2*f*x + 2*e) +
c^3)*cos(6*f*x + 6*e) + 6*(3*c^3*cos(2*f*x + 2*e) + c^3)*cos(4*f*x + 4*e) +
 6*(c^3*sin(4*f*x + 4*e) + c^3*sin(2*f*x + 2*e))*sin(6*f*x + 6*e))*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) + 6*(3*(f*x + e)*c^3*cos(4*f*x + 4*
e) + 3*(f*x + e)*c^3*cos(2*f*x + 2*e) + (f*x + e)*c^3 + 3*c^3*sin(4*f*x + 4
*e) + 3*c^3*sin(2*f*x + 2*e))*cos(6*f*x + 6*e) + 18*(3*(f*x + e)*c^3*cos(2*
f*x + 2*e) + (f*x + e)*c^3)*cos(4*f*x + 4*e) + 18*(c^3*sin(6*f*x + 6*e) + 3
*c^3*sin(4*f*x + 4*e) + 3*c^3*sin(2*f*x + 2*e))*cos(5/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e))) + 44*(c^3*sin(6*f*x + 6*e) + 3*c^3*sin(4*f*x + 4*
```

```
e) + 3*c^3*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e))) + 18*(c^3*sin(6*f*x + 6*e) + 3*c^3*sin(4*f*x + 4*e) + 3*c^3*sin(2*f*
x + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 18*((f*x +
 e)*c^3*sin(4*f*x + 4*e) + (f*x + e)*c^3*sin(2*f*x + 2*e) - c^3*cos(4*f*x +
 4*e) - c^3*cos(2*f*x + 2*e))*sin(6*f*x + 6*e) + 18*(3*(f*x + e)*c^3*sin(2*
f*x + 2*e) + c^3)*sin(4*f*x + 4*e) - 18*(c^3*cos(6*f*x + 6*e) + 3*c^3*cos(4
*f*x + 4*e) + 3*c^3*cos(2*f*x + 2*e) + c^3)*sin(5/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))) - 44*(c^3*cos(6*f*x + 6*e) + 3*c^3*cos(4*f*x + 4*e) +
 3*c^3*cos(2*f*x + 2*e) + c^3)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e))) - 18*(c^3*cos(6*f*x + 6*e) + 3*c^3*cos(4*f*x + 4*e) + 3*c^3*cos(2*
f*x + 2*e) + c^3)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqr
t(a)*sqrt(c)/((2*(3*cos(4*f*x + 4*e) + 3*cos(2*f*x + 2*e) + 1)*cos(6*f*x +
6*e) + cos(6*f*x + 6*e)^2 + 6*(3*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 9
*cos(4*f*x + 4*e)^2 + 9*cos(2*f*x + 2*e)^2 + 6*(sin(4*f*x + 4*e) + sin(2*f*
x + 2*e))*sin(6*f*x + 6*e) + sin(6*f*x + 6*e)^2 + 9*sin(4*f*x + 4*e)^2 + 18
*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 9*sin(2*f*x + 2*e)^2 + 6*cos(2*f*x + 2
*e) + 1)*f)
```

Fricas [A] time = 1.69636, size = 1135, normalized size = 6.14

$$\left[-\frac{\left(11\,c^{3}\cos\left(fx+e\right)^{2}-7\,c^{3}\cos\left(fx+e\right)+2\,c^{3}\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)-3\left(c^{3}\cos\left(fx+e\right)^{3}+c^{3}\cos\left(\right.\right.}{6\left(f\cos\left(fx+e\right)^{3}+f\cos\left(fx+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(7/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] [-1/6*((11*c^3*cos(f*x + e)^2 - 7*c^3*cos(f*x + e) + 2*c^3)*sqrt((a*cos(f*x
 + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x +
e) - 3*(c^3*cos(f*x + e)^3 + c^3*cos(f*x + e)^2)*sqrt(-a*c)*log(1/2*(a*c*co
s(f*x + e)^4 - (cos(f*x + e)^3 + cos(f*x + e))*sqrt(-a*c)*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e)
 + a*c)/cos(f*x + e)^2))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2), -1/6*((11*c
^3*cos(f*x + e)^2 - 7*c^3*cos(f*x + e) + 2*c^3)*sqrt((a*cos(f*x + e) + a)/c
os(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) - 6*(c^3*
cos(f*x + e)^3 + c^3*cos(f*x + e)^2)*sqrt(a*c)*arctan(sqrt(a*c)*sqrt((a*cos
```

```
(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*
x + e)*sin(f*x + e)/(a*c*cos(f*x + e)^2 + a*c)))/(f*cos(f*x + e)^3 + f*cos(
f*x + e)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**(7/2)*(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(7/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

3.87 $\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}\,dx$

Optimal. Leaf size=139

$$-\frac{ac^2\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}+\frac{ac^3\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}$$

```
[Out] (a*c^3*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
 c*Sec[e + f*x]]) - (a*c^2*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a
 + a*Sec[e + f*x]]) - (a*c*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(2*f*Sq
rt[a + a*Sec[e + f*x]])
```

Rubi [A] time = 0.272815, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3906, 3905, 3475}

$$-\frac{ac^2\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}+\frac{ac^3\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (a*c^3*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
 c*Sec[e + f*x]]) - (a*c^2*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a
 + a*Sec[e + f*x]]) - (a*c*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(2*f*Sq
rt[a + a*Sec[e + f*x]])
```

Rule 3906

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e + f*x]*(c + d*Csc[e + f*x]
)^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[c, Int[Sqrt[a
+ b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && GtQ[n, 1/2]
```

Rule 3905

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Dist[((-(a*c))^(m + 1/2)*Cot[e + f*x])/(Sqrt
[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int[Cot[e + f*x]^(2*m), x],
```

```
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IntegerQ[m + 1/2]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}\,dx = -\frac{ac(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}} + c\int \sqrt{a+a\sec(e+fx)}(c-c\sec$$

$$= -\frac{ac^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{ac(c-c\sec(e+fx))^{3/2}\tan(e+}{2f\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{ac^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{ac(c-c\sec(e+fx))^{3/2}\tan(e+}{2f\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{ac^3\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{ac^2\sqrt{c-c\sec(e+fx)}\tan(e}{f\sqrt{a+a\sec(e+fx)}}$$

Mathematica [C] time = 2.16014, size = 162, normalized size = 1.17

$$-\frac{c^2e^{-3i(e+fx)}\left(1+e^{2i(e+fx)}\right)^3\left(\cot\left(\frac{1}{2}(e+fx)\right)+i\right)\sec^4(e+fx)\sqrt{a(\sec(e+fx)+1)}\sqrt{c-c\sec(e+fx)}\left(\log\left(1+e^{2i(e+fx)}\right)\right.}{16f\left(1+e^{i(e+fx)}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] -(c^2*(1 + E^((2*I)*(e + f*x)))^3*(I + Cot[(e + f*x)/2])*(-1 - I*f*x + 4*Co
s[e + f*x] + Log[1 + E^((2*I)*(e + f*x))] + Cos[2*(e + f*x)]*((-I)*f*x + Lo
g[1 + E^((2*I)*(e + f*x))]))*Sec[e + f*x]^4*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt
[c - c*Sec[e + f*x]])/(16*E^((3*I)*(e + f*x))*(1 + E^(I*(e + f*x)))*f)
```

Maple [A] time = 0.309, size = 184, normalized size = 1.3

$$-\frac{\cos(fx+e)}{2f\sin(fx+e)\left(-1+\cos(fx+e)\right)^2}\left(2\ln\left(\frac{1-\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)\left(\cos(fx+e)\right)^2+2\ln\left(-\frac{-1+\cos(fx+}{\sin(}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^(5/2)*(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/2/f*(2*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2+2*ln(-(-1+c
os(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-2*ln(2/(1+cos(f*x+e)))*cos(f
*x+e)^2+5*cos(f*x+e)^2+4*cos(f*x+e)-1)*cos(f*x+e)*(c*(-1+cos(f*x+e))/cos(f*
x+e))^(5/2)*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)/sin(f*x+e)/(-1+cos(f*x+e)
)^2
```

Maxima [B] time = 1.88241, size = 959, normalized size = 6.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(5/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] -((f*x + e)*c^2*cos(4*f*x + 4*e)^2 + 4*(f*x + e)*c^2*cos(2*f*x + 2*e)^2 + (
f*x + e)*c^2*sin(4*f*x + 4*e)^2 + 4*(f*x + e)*c^2*sin(2*f*x + 2*e)^2 + 4*(f
*x + e)*c^2*cos(2*f*x + 2*e) + (f*x + e)*c^2 + 2*c^2*sin(2*f*x + 2*e) - (c^
2*cos(4*f*x + 4*e)^2 + 4*c^2*cos(2*f*x + 2*e)^2 + c^2*sin(4*f*x + 4*e)^2 +
4*c^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*c^2*sin(2*f*x + 2*e)^2 + 4*c^2*
cos(2*f*x + 2*e) + c^2 + 2*(2*c^2*cos(2*f*x + 2*e) + c^2)*cos(4*f*x + 4*e))
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) + 2*(2*(f*x + e)*c^2*cos(2
*f*x + 2*e) + (f*x + e)*c^2 + c^2*sin(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(c
^2*sin(4*f*x + 4*e) + 2*c^2*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) + 4*(c^2*sin(4*f*x + 4*e) + 2*c^2*sin(2*f*x + 2*e))
*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*(2*(f*x + e)*c^2*
sin(2*f*x + 2*e) - c^2*cos(2*f*x + 2*e))*sin(4*f*x + 4*e) - 4*(c^2*cos(4*f*
x + 4*e) + 2*c^2*cos(2*f*x + 2*e) + c^2)*sin(3/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e))) - 4*(c^2*cos(4*f*x + 4*e) + 2*c^2*cos(2*f*x + 2*e) + c^2
)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((2
*(2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + cos(4*f*x + 4*e)^2 + 4*cos(2*f
*x + 2*e)^2 + sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*
sin(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*f)
```

Fricas [A] time = 1.68976, size = 1052, normalized size = 7.57

$$\left[-\frac{\left(3c^2\cos\left(fx+e\right)-c^2\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sin\left(fx+e\right)-\left(c^2\cos\left(fx+e\right)^2+c^2\cos\left(fx+e\right)\right)\sqrt{-ac}\log\left(\frac{ac\cos\cdots}{\cdots}\right.}{2\left(f\cos\left(fx+e\right)^2+f\cos\left(fx+e\right)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(5/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] [-1/2*((3*c^2*cos(f*x + e) - c^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*s
qrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) - (c^2*cos(f*x + e)^2 +
 c^2*cos(f*x + e))*sqrt(-a*c)*log(1/2*(a*c*cos(f*x + e)^4 - (cos(f*x + e)^3
 + cos(f*x + e))*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c
*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) + a*c)/cos(f*x + e)^2))/(f*co
s(f*x + e)^2 + f*cos(f*x + e)), -1/2*((3*c^2*cos(f*x + e) - c^2)*sqrt((a*co
s(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f
*x + e) - 2*(c^2*cos(f*x + e)^2 + c^2*cos(f*x + e))*sqrt(a*c)*arctan(sqrt(a
*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f
*x + e))*cos(f*x + e)*sin(f*x + e)/(a*c*cos(f*x + e)^2 + a*c)))/(f*cos(f*x
+ e)^2 + f*cos(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**(5/2)*(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(5/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

3.88 $\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=93

$$\frac{ac^2\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}$$

```
[Out] (a*c^2*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
 c*Sec[e + f*x]]) - (a*c*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a +
 a*Sec[e + f*x]])
```

Rubi [A] time = 0.178326, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3906, 3905, 3475}

$$\frac{ac^2\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (a*c^2*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
 c*Sec[e + f*x]]) - (a*c*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a +
 a*Sec[e + f*x]])
```

Rule 3906

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e + f*x]*(c + d*Csc[e + f*x]
)^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[c, Int[Sqrt[a
+ b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && GtQ[n, 1/2]
```

Rule 3905

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Dist[((-(a*c))^(m + 1/2)*Cot[e + f*x])/(Sqrt
[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int[Cot[e + f*x]^(2*m), x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IntegerQ[m + 1/2]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}\,dx = -\frac{ac\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + c\int \sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}$$
$$= -\frac{ac\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{\left(ac^2\tan(e+fx)\right)\int\tan(e+fx)\,d}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$
$$= \frac{ac^2\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{ac\sqrt{c-c\sec(e+fx)}\tan(e+f}{f\sqrt{a+a\sec(e+fx)}}$$

Mathematica [C] time = 1.22626, size = 99, normalized size = 1.06

$$\frac{ic\left(\cot\left(\frac{1}{2}(e+fx)\right)+i\right)\sqrt{a(\sec(e+fx)+1)}\sqrt{c-c\sec(e+fx)}\left(\left(fx+i\log\left(1+e^{2i(e+fx)}\right)\right)\cos(e+fx)+i\right)}{f\left(1+e^{i(e+fx)}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (I*c*(I + Cot[(e + f*x)/2])*(I + Cos[e + f*x]*(f*x + I*Log[1 + E^((2*I)*(e
+ f*x))]))*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])/((1 + E^(I*
(e + f*x)))*f)
```

Maple [A] time = 0.297, size = 164, normalized size = 1.8

$$-\frac{\cos(fx+e)}{f\sin(fx+e)(-1+\cos(fx+e))}\left(\cos(fx+e)\ln\left(\frac{1-\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)+\cos(fx+e)\ln\left(-\frac{-1+\cos(fx}{\sin}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/f*(cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+cos(f*x+e)*ln(-(-
1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-cos(f*x+e)*ln(2/(1+cos(f*x+e)))+cos(f*
x+e)+1)*cos(f*x+e)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(3/2)*(1/cos(f*x+e)*a*(1+
cos(f*x+e)))^(1/2)/sin(f*x+e)/(-1+cos(f*x+e))
```

Maxima [B] time = 1.79602, size = 328, normalized size = 3.53

$$-\frac{\left(\left(fx+e\right)c\cos\left(2\,fx+2\,e\right)^{2}+\left(fx+e\right)c\sin\left(2\,fx+2\,e\right)^{2}+2\left(fx+e\right)c\cos\left(2\,fx+2\,e\right)+2\,c\cos\left(\frac{1}{2}\arctan\left(\sin\left(2\,fx\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] -((f*x + e)*c*cos(2*f*x + 2*e)^2 + (f*x + e)*c*sin(2*f*x + 2*e)^2 + 2*(f*x
+ e)*c*cos(2*f*x + 2*e) + 2*c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e)))*sin(2*f*x + 2*e) + (f*x + e)*c - (c*cos(2*f*x + 2*e)^2 + c*sin(2*f*
x + 2*e)^2 + 2*c*cos(2*f*x + 2*e) + c)*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e) + 1) - 2*(c*cos(2*f*x + 2*e) + c)*sin(1/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)
^2 + 2*cos(2*f*x + 2*e) + 1)*f)
```

Fricas [A] time = 1.65703, size = 887, normalized size = 9.54

$$\left[-\frac{2\,c\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)-\sqrt{-ac}\left(c\cos\left(fx+e\right)+c\right)\log\left(\frac{ac\cos\left(fx+e\right)^{4}-\left(\cos\left(fx+e\right)^{3}+\cos\left(fx+e\right)\right)\sqrt{-ac}\sqrt{\frac{a\cos}{\cos}}}{2\cos\left(fx+e\right)^{2}}\right.}{2\left(f\cos\left(fx+e\right)+f\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] [-1/2*(2*c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c
)/cos(f*x + e))*sin(f*x + e) - sqrt(-a*c)*(c*cos(f*x + e) + c)*log(1/2*(a*c
*cos(f*x + e)^4 - (cos(f*x + e)^3 + cos(f*x + e))*sqrt(-a*c)*sqrt((a*cos(f*
x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x +
 e) + a*c)/cos(f*x + e)^2))/(f*cos(f*x + e) + f), -(c*sqrt((a*cos(f*x + e)
+ a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) - s
qrt(a*c)*(c*cos(f*x + e) + c)*arctan(sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/co
s(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)*sin(f*x +
e)/(a*c*cos(f*x + e)^2 + a*c)))/(f*cos(f*x + e) + f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**(3/2)*(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

3.89 $\int \sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx$

Optimal. Leaf size=48

$$\frac{ac\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] (a*c*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c
*Sec[e + f*x]])
```

Rubi [A] time = 0.0838858, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3905, 3475}

$$\frac{ac\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (a*c*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c
*Sec[e + f*x]])
```

Rule 3905

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Dist[((-(a*c))^(m + 1/2)*Cot[e + f*x])/(Sqrt
[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int[Cot[e + f*x]^(2*m), x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IntegerQ[m + 1/2]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx = -\frac{(ac\tan(e+fx))\int\tan(e+fx)\,dx}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$
$$= \frac{ac\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

Mathematica [C] time = 0.573025, size = 102, normalized size = 2.12

$$\frac{ie^{\frac{1}{2}i(e+fx)}\left(fx+i\log\left(1+e^{2i(e+fx)}\right)\right)\cos(e+fx)\csc\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}\sqrt{c-c\sec(e+fx)}}{f\left(1+e^{i(e+fx)}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (I*E^((I/2)*(e + f*x))*Cos[e + f*x]*Csc[(e + f*x)/2]*(f*x + I*Log[1 + E^((2
*I)*(e + f*x))])*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])/((1 +
 E^(I*(e + f*x)))*f)
```

Maple [B] time = 0.308, size = 127, normalized size = 2.7

$$-\frac{\cos\left(fx+e\right)}{f\sin\left(fx+e\right)}\left(-\ln\left(2\left(1+\cos\left(fx+e\right)\right)^{-1}\right)+\ln\left(\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)+\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx\right.}{\sin\left(fx+e\right)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/f*(-ln(2/(1+cos(f*x+e)))+ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+ln(-(-
1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e)))*cos(f*x+e)*(c*(-1+cos(f*x+e))/cos(f*x
+e))^(1/2)*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)/sin(f*x+e)
```

Maxima [A] time = 1.8378, size = 53, normalized size = 1.1

$$-\frac{\left(fx+e-\arctan\left(\sin\left(2\,fx+2\,e\right),\cos\left(2\,fx+2\,e\right)+1\right)\right)\sqrt{a}\sqrt{c}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] -(f*x + e - arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sqrt(a)*sqrt(c
)/f
```

Fricas [B] time = 1.61973, size = 506, normalized size = 10.54

$$\left[\frac{\sqrt{-ac}\log\left(\frac{ac\cos\left(fx+e\right)^4-\left(\cos\left(fx+e\right)^3+\cos\left(fx+e\right)\right)\sqrt{-ac}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)+ac}{2\cos\left(fx+e\right)^2}\right)}{2f}, \frac{\sqrt{ac}\arctan\left(\frac{\sqrt{ac}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)\sin\left(fx+e\right)}{ac\cos\left(fx+e\right)^2+ac}\right)}{f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] [1/2*sqrt(-a*c)*log(1/2*(a*c*cos(f*x + e)^4 - (cos(f*x + e)^3 + cos(f*x + e
))*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e)
- c)/cos(f*x + e))*sin(f*x + e) + a*c)/cos(f*x + e)^2)/f, sqrt(a*c)*arctan(
sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)
/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(a*c*cos(f*x + e)^2 + a*c))/f]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\left(\sec\left(e+fx\right)+1\right)}\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**(1/2)*(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*sqrt(-c*(sec(e + f*x) - 1)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

3.90 $\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal. Leaf size=51

$$\frac{a\tan(e+fx)\log(1-\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] (a*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
 c*Sec[e + f*x]])
```

Rubi [A] time = 0.0863869, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3911, 31}

$$\frac{a\tan(e+fx)\log(1-\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]/Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (a*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
 c*Sec[e + f*x]])
```

Rule 3911

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> -Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((b + a*x)^(m - 1/2)*(d + c*x
)^(n - 1/2))/x^(m + n), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && EqQ
[m + n, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{(ac\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{-c+cx}\,dx,x,\cos(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$
$$= \frac{a\log(1-\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

Mathematica [C] time = 0.877877, size = 86, normalized size = 1.69

$$-\frac{\left(-1+e^{i(e+fx)}\right)\left(fx+2i\log\left(1-e^{i(e+fx)}\right)\right)\sqrt{a(\sec(e+fx)+1)}}{f\left(1+e^{i(e+fx)}\right)\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]/Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] -(((-1 + E^(I*(e + f*x)))*(f*x + (2*I)*Log[1 - E^(I*(e + f*x))])*Sqrt[a*(1
+ Sec[e + f*x])])/((1 + E^(I*(e + f*x)))*f*Sqrt[c - c*Sec[e + f*x]]))
```

Maple [B] time = 0.276, size = 97, normalized size = 1.9

$$\frac{\cos(fx+e)}{f\sin(fx+e)c}\sqrt{\frac{a(1+\cos(fx+e))}{\cos(fx+e)}}\left(\ln\left(2\left(1+\cos(fx+e)\right)^{-1}\right)-2\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)\right)\sqrt{\frac{c(-1+\cos(fx+e)}{\cos(fx+e)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/f*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(ln(2/(1+cos(f*x+e)))-2*ln(-(-1+c
os(f*x+e))/sin(f*x+e)))*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)*cos(f*x+e)/sin
(f*x+e)/c
```

Maxima [A] time = 1.52011, size = 88, normalized size = 1.73

$$\frac{\frac{2\sqrt{-a}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{\sqrt{c}}-\frac{\sqrt{-a}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{\sqrt{c}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] (2*sqrt(-a)*log(sin(f*x + e)/(cos(f*x + e) + 1))/sqrt(c) - sqrt(-a)*log(sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/sqrt(c))/f
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{a\sec\left(fx+e\right)+a}\sqrt{-c\sec\left(fx+e\right)+c}}{c\sec\left(fx+e\right)-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(c*sec(f*x + e
) - c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}}{\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))/sqrt(-c*(sec(e + f*x) - 1)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

3.91 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=96

$$\frac{a\tan(e+fx)\log(1-\cos(e+fx))}{cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}$$

```
[Out] -((a*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2)))
 + (a*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c*f*Sqrt[a + a*Sec[e + f*x]]*Sqr
t[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.178692, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3907, 3911, 31}

$$\frac{a\tan(e+fx)\log(1-\cos(e+fx))}{cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] -((a*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2)))
 + (a*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c*f*Sqrt[a + a*Sec[e + f*x]]*Sqr
t[c - c*Sec[e + f*x]])
```

Rule 3907

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_), x_Symbol] :> Simp[(-2*a*Cot[e + f*x]*(c + d*Csc[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[1/c, Int[Sqrt[a + b*Cs
c[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3911

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> -Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((b + a*x)^(m - 1/2)*(d + c*x
)^(n - 1/2))/x^(m + n), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && EqQ
```

```
[m + n, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx=-\frac{a\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}+\frac{\int\frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx}{c}$$
$$=-\frac{a\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}+\frac{(a\tan(e+fx))\,\mathrm{Subst}\left(\int\frac{1}{-c+cx}\,dx,x,\cos(e+$$
$$=-\frac{a\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}+\frac{a\log(1-\cos(e+fx))\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

Mathematica [C] time = 1.00142, size = 107, normalized size = 1.11

$$\frac{\tan\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\sqrt{a(\sec(e+fx)+1)}\left(-2\log\left(1-e^{i(e+fx)}\right)+\left(2\log\left(1-e^{i(e+fx)}\right)-ifx\right)\cos(e+fx)+ifx-1\right)}{f(c-c\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] ((-1 + I*f*x - 2*Log[1 - E^(I*(e + f*x))] + Cos[e + f*x]*((-I)*f*x + 2*Log[
1 - E^(I*(e + f*x))]))*Sec[e + f*x]*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x
)/2])/(f*(c - c*Sec[e + f*x])^(3/2))
```

Maple [A] time = 0.293, size = 164, normalized size = 1.7

$$-\frac{-1+\cos(fx+e)}{2f\cos(fx+e)\sin(fx+e)}\left(4\cos(fx+e)\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)-2\cos(fx+e)\ln\left(2\left(1+\cos(fx+e)\right)^{-1}\right)-\mathrm{co}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2),x)
```

```
[Out] -1/2/f*(-1+cos(f*x+e))*(4*cos(f*x+e)*ln(-(-1+cos(f*x+e))/sin(f*x+e))-2*cos(
f*x+e)*ln(2/(1+cos(f*x+e)))-cos(f*x+e)-4*ln(-(-1+cos(f*x+e))/sin(f*x+e))+2*
ln(2/(1+cos(f*x+e)))-1)*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)/cos(f*x+e)/(c
*(-1+cos(f*x+e))/cos(f*x+e))^(3/2)/sin(f*x+e)
```

Maxima [B] time = 1.80299, size = 539, normalized size = 5.61

$$-\frac{\left(\left(fx+e\right)\cos\left(2\,fx+2\,e\right)^{2}+4\left(fx+e\right)\cos\left(fx+e\right)^{2}+\left(fx+e\right)\sin\left(2\,fx+2\,e\right)^{2}+4\left(fx+e\right)\sin\left(fx+e\right)^{2}+fx+2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] -((f*x + e)*cos(2*f*x + 2*e)^2 + 4*(f*x + e)*cos(f*x + e)^2 + (f*x + e)*sin
(2*f*x + 2*e)^2 + 4*(f*x + e)*sin(f*x + e)^2 + f*x + 2*(2*(2*cos(f*x + e) -
 1)*cos(2*f*x + 2*e) - cos(2*f*x + 2*e)^2 - 4*cos(f*x + e)^2 - sin(2*f*x +
2*e)^2 + 4*sin(2*f*x + 2*e)*sin(f*x + e) - 4*sin(f*x + e)^2 + 4*cos(f*x + e
) - 1)*arctan2(sin(f*x + e), cos(f*x + e) - 1) + 2*(f*x - 2*(f*x + e)*cos(f
*x + e) + e + sin(f*x + e))*cos(2*f*x + 2*e) - 4*(f*x + e)*cos(f*x + e) - 2
*(2*(f*x + e)*sin(f*x + e) + cos(f*x + e))*sin(2*f*x + 2*e) + e + 2*sin(f*x
 + e))*sqrt(a)*sqrt(c)/((c^2*cos(2*f*x + 2*e)^2 + 4*c^2*cos(f*x + e)^2 + c^
2*sin(2*f*x + 2*e)^2 - 4*c^2*sin(2*f*x + 2*e)*sin(f*x + e) + 4*c^2*sin(f*x
+ e)^2 - 4*c^2*cos(f*x + e) + c^2 - 2*(2*c^2*cos(f*x + e) - c^2)*cos(2*f*x
+ 2*e))*f)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{a\sec\left(fx+e\right)+a}\sqrt{-c\sec\left(fx+e\right)+c}}{c^{2}\sec\left(fx+e\right)^{2}-2\,c^{2}\sec\left(fx+e\right)+c^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(c^2*sec(f*x +
e)^2 - 2*c^2*sec(f*x + e) + c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}}{\left(-c\left(\sec\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))/(-c*(sec(e + f*x) - 1))**(3/2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

3.92 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=142

$$\frac{a\tan(e+fx)\log(1-\cos(e+fx))}{c^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)}{cf\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\,\mathrm{sec}}$$

```
[Out] -(a*Tan[e + f*x])/(2*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2))
 - (a*Tan[e + f*x])/(c*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2
)) + (a*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c^2*f*Sqrt[a + a*Sec[e + f*x]]
*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.271143, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3907, 3911, 31}

$$\frac{a\tan(e+fx)\log(1-\cos(e+fx))}{c^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)}{cf\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\,\mathrm{sec}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] -(a*Tan[e + f*x])/(2*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2))
 - (a*Tan[e + f*x])/(c*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2
)) + (a*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c^2*f*Sqrt[a + a*Sec[e + f*x]]
*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3907

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_), x_Symbol] :> Simp[(-2*a*Cot[e + f*x]*(c + d*Csc[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[1/c, Int[Sqrt[a + b*Cs
c[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3911

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> -Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((b + a*x)^(m - 1/2)*(d + c*x
```

```
)^(n - 1/2))/x^(m + n), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && EqQ
[m + n, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{a\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}+\frac{\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx}{c}$$

$$= -\frac{a\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}-\frac{a\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}+$$

$$= -\frac{a\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}-\frac{a\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}+$$

$$= -\frac{a\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}-\frac{a\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}+$$

Mathematica [C] time = 1.23054, size = 152, normalized size = 1.07

$$\frac{\tan\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}\left(6\log\left(1-e^{i(e+fx)}\right)+\left(-8\log\left(1-e^{i(e+fx)}\right)+4ifx-4\right)\cos(e+fx)+\left(2\log\left(1-e^{i(e+}\right.\right.\right.}{2c^2f(\cos(e+fx)-1)^2\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] ((3 - (3*I)*f*x + Cos[e + f*x]*(-4 + (4*I)*f*x - 8*Log[1 - E^(I*(e + f*x))]
) + 6*Log[1 - E^(I*(e + f*x))] + Cos[2*(e + f*x)]*((-I)*f*x + 2*Log[1 - E^(
I*(e + f*x))]))*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(2*c^2*f*(-1 +
 Cos[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] time = 0.303, size = 226, normalized size = 1.6

$$-\frac{-1+\cos\left(fx+e\right)}{8f\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^{2}}\left(16\ln\left(-\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right)\left(\cos\left(fx+e\right)\right)^{2}-8\ln\left(2\left(1+\cos\left(fx+e\right)\right)^{-1}\right)\left(\cos\left(fx\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x)
```

```
[Out] -1/8/f*(-1+cos(f*x+e))*(16*ln(-(-1+cos(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-8*l
n(2/(1+cos(f*x+e)))*cos(f*x+e)^2-7*cos(f*x+e)^2-32*cos(f*x+e)*ln(-(-1+cos(f
*x+e))/sin(f*x+e))+16*cos(f*x+e)*ln(2/(1+cos(f*x+e)))-2*cos(f*x+e)+16*ln(-(
-1+cos(f*x+e))/sin(f*x+e))-8*ln(2/(1+cos(f*x+e)))+5)*(1/cos(f*x+e)*a*(1+cos
(f*x+e)))^(1/2)/(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)/sin(f*x+e)/cos(f*x+e)^
2
```

Maxima [B] time = 2.41535, size = 1584, normalized size = 11.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] -((f*x + e)*cos(4*f*x + 4*e)^2 + 16*(f*x + e)*cos(3*f*x + 3*e)^2 + 36*(f*x
+ e)*cos(2*f*x + 2*e)^2 + 16*(f*x + e)*cos(f*x + e)^2 + (f*x + e)*sin(4*f*x
 + 4*e)^2 + 16*(f*x + e)*sin(3*f*x + 3*e)^2 + 36*(f*x + e)*sin(2*f*x + 2*e)
^2 + 16*(f*x + e)*sin(f*x + e)^2 + f*x + 2*(2*(4*cos(3*f*x + 3*e) - 6*cos(2
*f*x + 2*e) + 4*cos(f*x + e) - 1)*cos(4*f*x + 4*e) - cos(4*f*x + 4*e)^2 + 8
*(6*cos(2*f*x + 2*e) - 4*cos(f*x + e) + 1)*cos(3*f*x + 3*e) - 16*cos(3*f*x
+ 3*e)^2 + 12*(4*cos(f*x + e) - 1)*cos(2*f*x + 2*e) - 36*cos(2*f*x + 2*e)^2
 - 16*cos(f*x + e)^2 + 4*(2*sin(3*f*x + 3*e) - 3*sin(2*f*x + 2*e) + 2*sin(f
*x + e))*sin(4*f*x + 4*e) - sin(4*f*x + 4*e)^2 + 16*(3*sin(2*f*x + 2*e) - 2
*sin(f*x + e))*sin(3*f*x + 3*e) - 16*sin(3*f*x + 3*e)^2 - 36*sin(2*f*x + 2*
e)^2 + 48*sin(2*f*x + 2*e)*sin(f*x + e) - 16*sin(f*x + e)^2 + 8*cos(f*x + e
) - 1)*arctan2(sin(f*x + e), cos(f*x + e) - 1) + 2*(f*x - 4*(f*x + e)*cos(3
*f*x + 3*e) + 6*(f*x + e)*cos(2*f*x + 2*e) - 4*(f*x + e)*cos(f*x + e) + e +
 2*sin(3*f*x + 3*e) - 3*sin(2*f*x + 2*e) + 2*sin(f*x + e))*cos(4*f*x + 4*e)
 - 8*(f*x + 6*(f*x + e)*cos(2*f*x + 2*e) - 4*(f*x + e)*cos(f*x + e) + e)*co
```

```
s(3*f*x + 3*e) + 12*(f*x - 4*(f*x + e)*cos(f*x + e) + e)*cos(2*f*x + 2*e) -
 8*(f*x + e)*cos(f*x + e) - 2*(4*(f*x + e)*sin(3*f*x + 3*e) - 6*(f*x + e)*s
in(2*f*x + 2*e) + 4*(f*x + e)*sin(f*x + e) + 2*cos(3*f*x + 3*e) - 3*cos(2*f
*x + 2*e) + 2*cos(f*x + e))*sin(4*f*x + 4*e) - 4*(12*(f*x + e)*sin(2*f*x +
2*e) - 8*(f*x + e)*sin(f*x + e) - 1)*sin(3*f*x + 3*e) - 6*(8*(f*x + e)*sin(
f*x + e) + 1)*sin(2*f*x + 2*e) + e + 4*sin(f*x + e))*sqrt(a)*sqrt(c)/((c^3*
cos(4*f*x + 4*e)^2 + 16*c^3*cos(3*f*x + 3*e)^2 + 36*c^3*cos(2*f*x + 2*e)^2
+ 16*c^3*cos(f*x + e)^2 + c^3*sin(4*f*x + 4*e)^2 + 16*c^3*sin(3*f*x + 3*e)^
2 + 36*c^3*sin(2*f*x + 2*e)^2 - 48*c^3*sin(2*f*x + 2*e)*sin(f*x + e) + 16*c
^3*sin(f*x + e)^2 - 8*c^3*cos(f*x + e) + c^3 - 2*(4*c^3*cos(3*f*x + 3*e) -
6*c^3*cos(2*f*x + 2*e) + 4*c^3*cos(f*x + e) - c^3)*cos(4*f*x + 4*e) - 8*(6*
c^3*cos(2*f*x + 2*e) - 4*c^3*cos(f*x + e) + c^3)*cos(3*f*x + 3*e) - 12*(4*c
^3*cos(f*x + e) - c^3)*cos(2*f*x + 2*e) - 4*(2*c^3*sin(3*f*x + 3*e) - 3*c^3
*sin(2*f*x + 2*e) + 2*c^3*sin(f*x + e))*sin(4*f*x + 4*e) - 16*(3*c^3*sin(2*
f*x + 2*e) - 2*c^3*sin(f*x + e))*sin(3*f*x + 3*e))*f)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{a\sec\left(fx+e\right)+a}\sqrt{-c\sec\left(fx+e\right)+c}}{c^3\sec\left(fx+e\right)^3-3\,c^3\sec\left(fx+e\right)^2+3\,c^3\sec\left(fx+e\right)-c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(c^3*sec(f*x +
 e)^3 - 3*c^3*sec(f*x + e)^2 + 3*c^3*sec(f*x + e) - c^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

3.93 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=188

$$-\frac{a\tan(e+fx)}{c^2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}+\frac{a\tan(e+fx)\log(1-\cos(e+fx))}{c^3f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)}{2cf\sqrt{a\sec(e+fx)+a}(c-c\s$$

```
[Out] -(a*Tan[e + f*x])/(3*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(7/2))
 - (a*Tan[e + f*x])/(2*c*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5
/2)) - (a*Tan[e + f*x])/(c^2*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x]
)^(3/2)) + (a*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c^3*f*Sqrt[a + a*Sec[e +
 f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.366465, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3907, 3911, 31}

$$-\frac{a\tan(e+fx)}{c^2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}+\frac{a\tan(e+fx)\log(1-\cos(e+fx))}{c^3f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)}{2cf\sqrt{a\sec(e+fx)+a}(c-c\s$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^(7/2),x]
```


```
[Out] -(a*Tan[e + f*x])/(3*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(7/2))
 - (a*Tan[e + f*x])/(2*c*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5
/2)) - (a*Tan[e + f*x])/(c^2*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x]
)^(3/2)) + (a*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c^3*f*Sqrt[a + a*Sec[e +
 f*x]]*Sqrt[c - c*Sec[e + f*x]])
```


Rule 3907

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_), x_Symbol] :> Simp[(-2*a*Cot[e + f*x]*(c + d*Csc[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[1/c, Int[Sqrt[a + b*Cs
c[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3911

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> -Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((b + a*x)^(m - 1/2)*(d + c*x
)^(n - 1/2))/x^(m + n), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && EqQ
[m + n, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{7/2}}\,dx = -\frac{a\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}} + \frac{\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx}{c}$$

$$= -\frac{a\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}} - \frac{a\tan(e+fx)}{2cf\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/}}$$

$$= -\frac{a\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}} - \frac{a\tan(e+fx)}{2cf\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/}}$$

$$= -\frac{a\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}} - \frac{a\tan(e+fx)}{2cf\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/}}$$

$$= -\frac{a\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}} - \frac{a\tan(e+fx)}{2cf\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/}}$$

Mathematica [C] time = 1.78405, size = 198, normalized size = 1.05

$$\frac{\tan\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}\left(-60\log\left(1-e^{i(e+fx)}\right)-3ifx\cos(3(e+fx))+18i\left(2i\log\left(1-e^{i(e+fx)}\right)+fx+i\right)\text{co}\right.}{12c^3f(\cos(e+fx)-1)^3\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^(7/2),x]
```

```
[Out] ((-40 + (30*I)*f*x - (3*I)*f*x*Cos[3*(e + f*x)] + (18*I)*Cos[2*(e + f*x)]*(
I + f*x + (2*I)*Log[1 - E^(I*(e + f*x))]) - 60*Log[1 - E^(I*(e + f*x))] + 6
```

```
*Cos[3*(e + f*x)]*Log[1 - E^(I*(e + f*x))] + 9*Cos[e + f*x]*(6 - (5*I)*f*x
+ 10*Log[1 - E^(I*(e + f*x))]))*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2]
)/(12*c^3*f*(-1 + Cos[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] time = 0.306, size = 288, normalized size = 1.5

$$-\frac{-1+\cos\left(fx+e\right)}{6 f \sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^{3}}\left(12\left(\cos\left(fx+e\right)\right)^{3}\ln\left(-\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-6\left(\cos\left(fx+e\right)\right)^{3}\ln\left(2\left(1+\cos\left(fx+e\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(7/2),x)
```

```
[Out] -1/6/f*(-1+cos(f*x+e))*(12*cos(f*x+e)^3*ln(-(-1+cos(f*x+e))/sin(f*x+e))-6*c
os(f*x+e)^3*ln(2/(1+cos(f*x+e)))-36*ln(-(-1+cos(f*x+e))/sin(f*x+e))*cos(f*x
+e)^2-7*cos(f*x+e)^3+18*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^2+36*cos(f*x+e)*ln(
-(-1+cos(f*x+e))/sin(f*x+e))+3*cos(f*x+e)^2-18*cos(f*x+e)*ln(2/(1+cos(f*x+e
)))-12*ln(-(-1+cos(f*x+e))/sin(f*x+e))+6*cos(f*x+e)+6*ln(2/(1+cos(f*x+e)))-
4)*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)/(c*(-1+cos(f*x+e))/cos(f*x+e))^(7/
2)/sin(f*x+e)/cos(f*x+e)^3
```

Maxima [B] time = 11.0069, size = 3299, normalized size = 17.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="maxim
a")
```

```
[Out] -1/3*(3*(f*x + e)*cos(6*f*x + 6*e)^2 + 108*(f*x + e)*cos(5*f*x + 5*e)^2 + 6
75*(f*x + e)*cos(4*f*x + 4*e)^2 + 1200*(f*x + e)*cos(3*f*x + 3*e)^2 + 675*(
f*x + e)*cos(2*f*x + 2*e)^2 + 108*(f*x + e)*cos(f*x + e)^2 + 3*(f*x + e)*si
n(6*f*x + 6*e)^2 + 108*(f*x + e)*sin(5*f*x + 5*e)^2 + 675*(f*x + e)*sin(4*f
*x + 4*e)^2 + 1200*(f*x + e)*sin(3*f*x + 3*e)^2 + 675*(f*x + e)*sin(2*f*x +
 2*e)^2 + 108*(f*x + e)*sin(f*x + e)^2 + 3*f*x + 6*(2*(6*cos(5*f*x + 5*e) -
 15*cos(4*f*x + 4*e) + 20*cos(3*f*x + 3*e) - 15*cos(2*f*x + 2*e) + 6*cos(f*
x + e) - 1)*cos(6*f*x + 6*e) - cos(6*f*x + 6*e)^2 + 12*(15*cos(4*f*x + 4*e)
```

```
 - 20*cos(3*f*x + 3*e) + 15*cos(2*f*x + 2*e) - 6*cos(f*x + e) + 1)*cos(5*f*
x + 5*e) - 36*cos(5*f*x + 5*e)^2 + 30*(20*cos(3*f*x + 3*e) - 15*cos(2*f*x +
 2*e) + 6*cos(f*x + e) - 1)*cos(4*f*x + 4*e) - 225*cos(4*f*x + 4*e)^2 + 40*
(15*cos(2*f*x + 2*e) - 6*cos(f*x + e) + 1)*cos(3*f*x + 3*e) - 400*cos(3*f*x
 + 3*e)^2 + 30*(6*cos(f*x + e) - 1)*cos(2*f*x + 2*e) - 225*cos(2*f*x + 2*e)
^2 - 36*cos(f*x + e)^2 + 2*(6*sin(5*f*x + 5*e) - 15*sin(4*f*x + 4*e) + 20*s
in(3*f*x + 3*e) - 15*sin(2*f*x + 2*e) + 6*sin(f*x + e))*sin(6*f*x + 6*e) -
sin(6*f*x + 6*e)^2 + 12*(15*sin(4*f*x + 4*e) - 20*sin(3*f*x + 3*e) + 15*sin
(2*f*x + 2*e) - 6*sin(f*x + e))*sin(5*f*x + 5*e) - 36*sin(5*f*x + 5*e)^2 +
30*(20*sin(3*f*x + 3*e) - 15*sin(2*f*x + 2*e) + 6*sin(f*x + e))*sin(4*f*x +
 4*e) - 225*sin(4*f*x + 4*e)^2 + 120*(5*sin(2*f*x + 2*e) - 2*sin(f*x + e))*
sin(3*f*x + 3*e) - 400*sin(3*f*x + 3*e)^2 - 225*sin(2*f*x + 2*e)^2 + 180*si
n(2*f*x + 2*e)*sin(f*x + e) - 36*sin(f*x + e)^2 + 12*cos(f*x + e) - 1)*arct
an2(sin(f*x + e), cos(f*x + e) - 1) + 2*(3*f*x - 18*(f*x + e)*cos(5*f*x + 5
*e) + 45*(f*x + e)*cos(4*f*x + 4*e) - 60*(f*x + e)*cos(3*f*x + 3*e) + 45*(f
*x + e)*cos(2*f*x + 2*e) - 18*(f*x + e)*cos(f*x + e) + 3*e + 9*sin(5*f*x +
5*e) - 27*sin(4*f*x + 4*e) + 40*sin(3*f*x + 3*e) - 27*sin(2*f*x + 2*e) + 9*
sin(f*x + e))*cos(6*f*x + 6*e) - 6*(6*f*x + 90*(f*x + e)*cos(4*f*x + 4*e) -
 120*(f*x + e)*cos(3*f*x + 3*e) + 90*(f*x + e)*cos(2*f*x + 2*e) - 36*(f*x +
 e)*cos(f*x + e) + 6*e - 9*sin(4*f*x + 4*e) + 20*sin(3*f*x + 3*e) - 9*sin(2
*f*x + 2*e))*cos(5*f*x + 5*e) + 6*(15*f*x - 300*(f*x + e)*cos(3*f*x + 3*e)
+ 225*(f*x + e)*cos(2*f*x + 2*e) - 90*(f*x + e)*cos(f*x + e) + 15*e + 20*si
n(3*f*x + 3*e) - 9*sin(f*x + e))*cos(4*f*x + 4*e) - 120*(f*x + 15*(f*x + e)
*cos(2*f*x + 2*e) - 6*(f*x + e)*cos(f*x + e) + e + sin(2*f*x + 2*e) - sin(f
*x + e))*cos(3*f*x + 3*e) + 18*(5*f*x - 30*(f*x + e)*cos(f*x + e) + 5*e - 3
*sin(f*x + e))*cos(2*f*x + 2*e) - 36*(f*x + e)*cos(f*x + e) - 2*(18*(f*x +
e)*sin(5*f*x + 5*e) - 45*(f*x + e)*sin(4*f*x + 4*e) + 60*(f*x + e)*sin(3*f*
x + 3*e) - 45*(f*x + e)*sin(2*f*x + 2*e) + 18*(f*x + e)*sin(f*x + e) + 9*co
s(5*f*x + 5*e) - 27*cos(4*f*x + 4*e) + 40*cos(3*f*x + 3*e) - 27*cos(2*f*x +
 2*e) + 9*cos(f*x + e))*sin(6*f*x + 6*e) - 6*(90*(f*x + e)*sin(4*f*x + 4*e)
 - 120*(f*x + e)*sin(3*f*x + 3*e) + 90*(f*x + e)*sin(2*f*x + 2*e) - 36*(f*x
 + e)*sin(f*x + e) + 9*cos(4*f*x + 4*e) - 20*cos(3*f*x + 3*e) + 9*cos(2*f*x
 + 2*e) - 3)*sin(5*f*x + 5*e) - 6*(300*(f*x + e)*sin(3*f*x + 3*e) - 225*(f*
x + e)*sin(2*f*x + 2*e) + 90*(f*x + e)*sin(f*x + e) + 20*cos(3*f*x + 3*e) -
 9*cos(f*x + e) + 9)*sin(4*f*x + 4*e) - 40*(45*(f*x + e)*sin(2*f*x + 2*e) -
 18*(f*x + e)*sin(f*x + e) - 3*cos(2*f*x + 2*e) + 3*cos(f*x + e) - 2)*sin(3
*f*x + 3*e) - 54*(10*(f*x + e)*sin(f*x + e) - cos(f*x + e) + 1)*sin(2*f*x +
 2*e) + 3*e + 18*sin(f*x + e))*sqrt(a)*sqrt(c)/((c^4*cos(6*f*x + 6*e)^2 + 3
6*c^4*cos(5*f*x + 5*e)^2 + 225*c^4*cos(4*f*x + 4*e)^2 + 400*c^4*cos(3*f*x +
 3*e)^2 + 225*c^4*cos(2*f*x + 2*e)^2 + 36*c^4*cos(f*x + e)^2 + c^4*sin(6*f*
x + 6*e)^2 + 36*c^4*sin(5*f*x + 5*e)^2 + 225*c^4*sin(4*f*x + 4*e)^2 + 400*c
^4*sin(3*f*x + 3*e)^2 + 225*c^4*sin(2*f*x + 2*e)^2 - 180*c^4*sin(2*f*x + 2*
e)*sin(f*x + e) + 36*c^4*sin(f*x + e)^2 - 12*c^4*cos(f*x + e) + c^4 - 2*(6*
c^4*cos(5*f*x + 5*e) - 15*c^4*cos(4*f*x + 4*e) + 20*c^4*cos(3*f*x + 3*e) -
15*c^4*cos(2*f*x + 2*e) + 6*c^4*cos(f*x + e) - c^4)*cos(6*f*x + 6*e) - 12*(
```

```
15*c^4*cos(4*f*x + 4*e) - 20*c^4*cos(3*f*x + 3*e) + 15*c^4*cos(2*f*x + 2*e)
 - 6*c^4*cos(f*x + e) + c^4)*cos(5*f*x + 5*e) - 30*(20*c^4*cos(3*f*x + 3*e)
 - 15*c^4*cos(2*f*x + 2*e) + 6*c^4*cos(f*x + e) - c^4)*cos(4*f*x + 4*e) - 4
0*(15*c^4*cos(2*f*x + 2*e) - 6*c^4*cos(f*x + e) + c^4)*cos(3*f*x + 3*e) - 3
0*(6*c^4*cos(f*x + e) - c^4)*cos(2*f*x + 2*e) - 2*(6*c^4*sin(5*f*x + 5*e) -
 15*c^4*sin(4*f*x + 4*e) + 20*c^4*sin(3*f*x + 3*e) - 15*c^4*sin(2*f*x + 2*e
) + 6*c^4*sin(f*x + e))*sin(6*f*x + 6*e) - 12*(15*c^4*sin(4*f*x + 4*e) - 20
*c^4*sin(3*f*x + 3*e) + 15*c^4*sin(2*f*x + 2*e) - 6*c^4*sin(f*x + e))*sin(5
*f*x + 5*e) - 30*(20*c^4*sin(3*f*x + 3*e) - 15*c^4*sin(2*f*x + 2*e) + 6*c^4
*sin(f*x + e))*sin(4*f*x + 4*e) - 120*(5*c^4*sin(2*f*x + 2*e) - 2*c^4*sin(f
*x + e))*sin(3*f*x + 3*e))*f)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a\sec\left(fx+e\right)+a}\sqrt{-c\sec\left(fx+e\right)+c}}{c^4\sec\left(fx+e\right)^4-4\,c^4\sec\left(fx+e\right)^3+6\,c^4\sec\left(fx+e\right)^2-4\,c^4\sec\left(fx+e\right)+c^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(c^4*sec(f*x +
e)^4 - 4*c^4*sec(f*x + e)^3 + 6*c^4*sec(f*x + e)^2 - 4*c^4*sec(f*x + e) + c
^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

3.94 $\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}\,dx$

Optimal. Leaf size=190

$$-\frac{a^2c^2\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}+\frac{a^2c^3\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a^2c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}+$$

```
[Out] (a^2*c^3*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c
 - c*Sec[e + f*x]]) - (a^2*c^2*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(f*Sq
rt[a + a*Sec[e + f*x]]) - (a^2*c*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(
2*f*Sqrt[a + a*Sec[e + f*x]]) + (a^2*(c - c*Sec[e + f*x])^(5/2)*Tan[e + f*x
])/(3*f*Sqrt[a + a*Sec[e + f*x]])
```

Rubi [A] time = 0.363996, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3909, 3906, 3905, 3475}

$$-\frac{a^2c^2\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}+\frac{a^2c^3\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a^2c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (a^2*c^3*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c
 - c*Sec[e + f*x]]) - (a^2*c^2*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(f*Sq
rt[a + a*Sec[e + f*x]]) - (a^2*c*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(
2*f*Sqrt[a + a*Sec[e + f*x]]) + (a^2*(c - c*Sec[e + f*x])^(5/2)*Tan[e + f*x
])/(3*f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 3909

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[(-2*a^2*Cot[e + f*x]*(c + d*Csc[e + f
*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[a, Int[Sqrt[a + b
*Csc[e + f*x]]*(c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LeQ[n, -2^(-1)]
```

Rule 3906

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e + f*x]*(c + d*Csc[e + f*x]
```

```
)^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[c, Int[Sqrt[a
+ b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && GtQ[n, 1/2]
```

Rule 3905

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Dist[((-(a*c))^(m + 1/2)*Cot[e + f*x])/(Sqrt
[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int[Cot[e + f*x]^(2*m), x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IntegerQ[m + 1/2]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}\,dx = \frac{a^2(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}} + a\int \sqrt{a+a\sec(e+fx)}(c-c\,\mathrm{se}$$

$$= -\frac{a^2c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}} + \frac{a^2(c-c\sec(e+fx))^{5/2}\tan(}{3f\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{a^2c^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{a^2c(c-c\sec(e+fx))^{3/2}\tan(}{2f\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{a^2c^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{a^2c(c-c\sec(e+fx))^{3/2}\tan(}{2f\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{a^2c^3\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{a^2c^2\sqrt{c-c\sec(e+fx)}\tan}{f\sqrt{a+a\sec(e+fx}}$$

Mathematica [C] time = 1.25331, size = 157, normalized size = 0.83

$$\frac{iac^2\csc\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\sqrt{a(\sec(e+fx)+1)}\sqrt{c-c\sec(e+fx)}\left(6i\cos(2(e+fx))+3fx\cos(3(\right.}{24f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] ((I/24)*a*c^2*Csc[(e + f*x)/2]*(2*I + (6*I)*Cos[2*(e + f*x)] + 3*f*x*Cos[3*
(e + f*x)] + Cos[e + f*x]*(6*I + 9*f*x + (9*I)*Log[1 + E^((2*I)*(e + f*x))]
) + (3*I)*Cos[3*(e + f*x)]*Log[1 + E^((2*I)*(e + f*x))])*Sec[(e + f*x)/2]*S
ec[e + f*x]^2*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])/f
```

Maple [A] time = 0.29, size = 187, normalized size = 1.

$$-\frac{a}{6 f \sin\left(fx+e\right)\left(-1+\cos\left(fx+e\right)\right)^{2}}\left(6\left(\cos\left(fx+e\right)\right)^{3}\ln\left(\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)+6\left(\cos\left(fx+e\right)\right)^{3}\ln\left(-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(5/2),x)
```

```
[Out] -1/6/f*a*(6*cos(f*x+e)^3*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+6*cos(f*x
+e)^3*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-6*cos(f*x+e)^3*ln(2/(1+cos
(f*x+e)))+cos(f*x+e)^3+6*cos(f*x+e)^2+3*cos(f*x+e)-2)*(c*(-1+cos(f*x+e))/co
s(f*x+e))^(5/2)*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)/sin(f*x+e)/(-1+cos(f*
x+e))^2
```

Maxima [B] time = 2.17331, size = 1831, normalized size = 9.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] -1/3*(3*(f*x + e)*a*c^2*cos(6*f*x + 6*e)^2 + 27*(f*x + e)*a*c^2*cos(4*f*x +
 4*e)^2 + 27*(f*x + e)*a*c^2*cos(2*f*x + 2*e)^2 + 3*(f*x + e)*a*c^2*sin(6*f
*x + 6*e)^2 + 27*(f*x + e)*a*c^2*sin(4*f*x + 4*e)^2 + 27*(f*x + e)*a*c^2*si
n(2*f*x + 2*e)^2 + 18*(f*x + e)*a*c^2*cos(2*f*x + 2*e) + 3*(f*x + e)*a*c^2
- 6*a*c^2*sin(2*f*x + 2*e) - 3*(a*c^2*cos(6*f*x + 6*e)^2 + 9*a*c^2*cos(4*f*
x + 4*e)^2 + 9*a*c^2*cos(2*f*x + 2*e)^2 + a*c^2*sin(6*f*x + 6*e)^2 + 9*a*c^
2*sin(4*f*x + 4*e)^2 + 18*a*c^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 9*a*c^2
*sin(2*f*x + 2*e)^2 + 6*a*c^2*cos(2*f*x + 2*e) + a*c^2 + 2*(3*a*c^2*cos(4*f
*x + 4*e) + 3*a*c^2*cos(2*f*x + 2*e) + a*c^2)*cos(6*f*x + 6*e) + 6*(3*a*c^2
```

```
*cos(2*f*x + 2*e) + a*c^2)*cos(4*f*x + 4*e) + 6*(a*c^2*sin(4*f*x + 4*e) + a
*c^2*sin(2*f*x + 2*e))*sin(6*f*x + 6*e))*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e) + 1) + 6*(3*(f*x + e)*a*c^2*cos(4*f*x + 4*e) + 3*(f*x + e)*a*c^2*c
os(2*f*x + 2*e) + (f*x + e)*a*c^2 - a*c^2*sin(4*f*x + 4*e) - a*c^2*sin(2*f*
x + 2*e))*cos(6*f*x + 6*e) + 18*(3*(f*x + e)*a*c^2*cos(2*f*x + 2*e) + (f*x
+ e)*a*c^2)*cos(4*f*x + 4*e) + 6*(a*c^2*sin(6*f*x + 6*e) + 3*a*c^2*sin(4*f*
x + 4*e) + 3*a*c^2*sin(2*f*x + 2*e))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e))) + 4*(a*c^2*sin(6*f*x + 6*e) + 3*a*c^2*sin(4*f*x + 4*e) + 3*a
*c^2*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
 + 6*(a*c^2*sin(6*f*x + 6*e) + 3*a*c^2*sin(4*f*x + 4*e) + 3*a*c^2*sin(2*f*x
 + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 6*(3*(f*x +
 e)*a*c^2*sin(4*f*x + 4*e) + 3*(f*x + e)*a*c^2*sin(2*f*x + 2*e) + a*c^2*cos
(4*f*x + 4*e) + a*c^2*cos(2*f*x + 2*e))*sin(6*f*x + 6*e) + 6*(9*(f*x + e)*a
*c^2*sin(2*f*x + 2*e) - a*c^2)*sin(4*f*x + 4*e) - 6*(a*c^2*cos(6*f*x + 6*e)
 + 3*a*c^2*cos(4*f*x + 4*e) + 3*a*c^2*cos(2*f*x + 2*e) + a*c^2)*sin(5/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*(a*c^2*cos(6*f*x + 6*e) + 3*a
*c^2*cos(4*f*x + 4*e) + 3*a*c^2*cos(2*f*x + 2*e) + a*c^2)*sin(3/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e))) - 6*(a*c^2*cos(6*f*x + 6*e) + 3*a*c^2*c
os(4*f*x + 4*e) + 3*a*c^2*cos(2*f*x + 2*e) + a*c^2)*sin(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((2*(3*cos(4*f*x + 4*e) + 3*
cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + cos(6*f*x + 6*e)^2 + 6*(3*cos(2*f*
x + 2*e) + 1)*cos(4*f*x + 4*e) + 9*cos(4*f*x + 4*e)^2 + 9*cos(2*f*x + 2*e)^
2 + 6*(sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + sin(6*f*x +
6*e)^2 + 9*sin(4*f*x + 4*e)^2 + 18*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 9*si
n(2*f*x + 2*e)^2 + 6*cos(2*f*x + 2*e) + 1)*f)
```

Fricas [A] time = 1.69682, size = 1154, normalized size = 6.07

$$\left[-\frac{\left(7\,ac^2\cos\left(fx+e\right)^2 + ac^2\cos\left(fx+e\right) - 2\,ac^2\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\sin\left(fx+e\right) - 3\left(ac^2\cos\left(fx+e\right)^3 + ac^2\right.}{6\left(f\cos\left(fx+e\right)^3 + f\cos\left(f\right.\right.} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] [-1/6*((7*a*c^2*cos(f*x + e)^2 + a*c^2*cos(f*x + e) - 2*a*c^2)*sqrt((a*cos(
```

```
f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x
 + e) - 3*(a*c^2*cos(f*x + e)^3 + a*c^2*cos(f*x + e)^2)*sqrt(-a*c)*log(1/2*
(a*c*cos(f*x + e)^4 - (cos(f*x + e)^3 + cos(f*x + e))*sqrt(-a*c)*sqrt((a*co
s(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f
*x + e) + a*c)/cos(f*x + e)^2))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2), -1/6
*((7*a*c^2*cos(f*x + e)^2 + a*c^2*cos(f*x + e) - 2*a*c^2)*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e)
 - 6*(a*c^2*cos(f*x + e)^3 + a*c^2*cos(f*x + e)^2)*sqrt(a*c)*arctan(sqrt(a*
c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*
x + e))*cos(f*x + e)*sin(f*x + e)/(a*c*cos(f*x + e)^2 + a*c)))/(f*cos(f*x +
 e)^3 + f*cos(f*x + e)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

3.95 $\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=103

$$\frac{a^2c^2\tan^3(e+fx)}{2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{a^2c^2\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] (a^2*c^2*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c
 - c*Sec[e + f*x]]) + (a^2*c^2*Tan[e + f*x]^3)/(2*f*Sqrt[a + a*Sec[e + f*x]
]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.109507, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3905, 3473, 3475}

$$\frac{a^2c^2\tan^3(e+fx)}{2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{a^2c^2\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (a^2*c^2*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c
 - c*Sec[e + f*x]]) + (a^2*c^2*Tan[e + f*x]^3)/(2*f*Sqrt[a + a*Sec[e + f*x]
]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3905

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Dist[((-(a*c))^(m + 1/2)*Cot[e + f*x])/(Sqrt
[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int[Cot[e + f*x]^(2*m), x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IntegerQ[m + 1/2]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}\,dx = \frac{\left(a^2c^2\tan(e+fx)\right)\int \tan^3(e+fx)\,dx}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= \frac{a^2c^2\tan^3(e+fx)}{2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{\left(a^2c^2\tan(e+fx)\right)\int \tan(e}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\,\mathrm{se}}}$$

$$= \frac{a^2c^2\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{a^2c^2\tan^3(e+fx}{2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c}}$$

Mathematica [C] time = 1.37087, size = 159, normalized size = 1.54

$$\frac{iace^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2\left(\cot\left(\frac{1}{2}(e+fx)\right)+i\right)\sec^3(e+fx)\sqrt{a(\sec(e+fx)+1)}\sqrt{c-c\sec(e+fx)}\left(i\log\left(1+e^{2i(e+fx)}\right)+\right.}{8f\left(1+e^{i(e+fx)}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] ((I/8)*a*c*(1 + E^((2*I)*(e + f*x)))^2*(I + Cot[(e + f*x)/2])*(I + f*x + Co
s[2*(e + f*x)]*(f*x + I*Log[1 + E^((2*I)*(e + f*x))]) + I*Log[1 + E^((2*I)*
(e + f*x))])*Sec[e + f*x]^3*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f
*x]])/(E^((2*I)*(e + f*x))*(1 + E^(I*(e + f*x)))*f)
```

Maple [A] time = 0.276, size = 171, normalized size = 1.7

$$-\frac{a}{2f\sin\left(fx+e\right)\left(-1+\cos\left(fx+e\right)\right)}\left(2\ln\left(\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)\left(\cos\left(fx+e\right)\right)^2+2\ln\left(-\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(3/2),x)
```

```
[Out] -1/2/f*a*(2*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2+2*ln(-(-1
+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-2*ln(2/(1+cos(f*x+e)))*cos
(f*x+e)^2-cos(f*x+e)^2+1)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(3/2)*(1/cos(f*x+e
)*a*(1+cos(f*x+e)))^(1/2)/sin(f*x+e)/(-1+cos(f*x+e))
```

Maxima [B] time = 1.82381, size = 644, normalized size = 6.25

$$-\frac{\left(\left(fx+e\right)ac\cos\left(4\,fx+4\,e\right)^{2}+4\left(fx+e\right)ac\cos\left(2\,fx+2\,e\right)^{2}+\left(fx+e\right)ac\sin\left(4\,fx+4\,e\right)^{2}+4\left(fx+e\right)ac\sin\left(2\,fx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] -((f*x + e)*a*c*cos(4*f*x + 4*e)^2 + 4*(f*x + e)*a*c*cos(2*f*x + 2*e)^2 + (
f*x + e)*a*c*sin(4*f*x + 4*e)^2 + 4*(f*x + e)*a*c*sin(2*f*x + 2*e)^2 + 4*(f
*x + e)*a*c*cos(2*f*x + 2*e) + (f*x + e)*a*c - 2*a*c*sin(2*f*x + 2*e) - (a*
c*cos(4*f*x + 4*e)^2 + 4*a*c*cos(2*f*x + 2*e)^2 + a*c*sin(4*f*x + 4*e)^2 +
4*a*c*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*a*c*sin(2*f*x + 2*e)^2 + 4*a*c*
cos(2*f*x + 2*e) + a*c + 2*(2*a*c*cos(2*f*x + 2*e) + a*c)*cos(4*f*x + 4*e))
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) + 2*(2*(f*x + e)*a*c*cos(2
*f*x + 2*e) + (f*x + e)*a*c - a*c*sin(2*f*x + 2*e))*cos(4*f*x + 4*e) + 2*(2
*(f*x + e)*a*c*sin(2*f*x + 2*e) + a*c*cos(2*f*x + 2*e))*sin(4*f*x + 4*e))*s
qrt(a)*sqrt(c)/((2*(2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + cos(4*f*x +
4*e)^2 + 4*cos(2*f*x + 2*e)^2 + sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin
(2*f*x + 2*e) + 4*sin(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*f)
```

Fricas [A] time = 1.68808, size = 873, normalized size = 8.48

$$\left[\frac{\sqrt{-ac}ac\cos\left(fx+e\right)\log\left(\frac{ac\cos\left(fx+e\right)^{4}-\left(\cos\left(fx+e\right)^{3}+\cos\left(fx+e\right)\right)\sqrt{-ac}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)+ac}{2\cos\left(fx+e\right)^{2}}\right)-ac\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{}}{2\,f\cos\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] [1/2*(sqrt(-a*c)*a*c*cos(f*x + e)*log(1/2*(a*c*cos(f*x + e)^4 - (cos(f*x +
e)^3 + cos(f*x + e))*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqr
t((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) + a*c)/cos(f*x + e)^2) -
a*c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f
*x + e))*sin(f*x + e))/(f*cos(f*x + e)), 1/2*(2*sqrt(a*c)*a*c*arctan(sqrt(a
*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f
*x + e))*cos(f*x + e)*sin(f*x + e)/(a*c*cos(f*x + e)^2 + a*c))*cos(f*x + e)
 - a*c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/co
s(f*x + e))*sin(f*x + e))/(f*cos(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

3.96 $\int (a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\,dx$

Optimal. Leaf size=93

$$\frac{a^2c\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}$$

```
[Out] (a^2*c*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
 c*Sec[e + f*x]]) - (a*c*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[c -
 c*Sec[e + f*x]])
```

Rubi [A] time = 0.169297, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3906, 3905, 3475}

$$\frac{a^2c\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (a^2*c*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
 c*Sec[e + f*x]]) - (a*c*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[c -
 c*Sec[e + f*x]])
```

Rule 3906

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e + f*x]*(c + d*Csc[e + f*x]
)^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[c, Int[Sqrt[a
+ b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && GtQ[n, 1/2]
```

Rule 3905

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Dist[((-(a*c))^(m + 1/2)*Cot[e + f*x])/(Sqrt
[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int[Cot[e + f*x]^(2*m), x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IntegerQ[m + 1/2]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a + a\sec(e+fx))^{3/2}\sqrt{c - c\sec(e+fx)}\,dx = -\frac{ac\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} + a\int \sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec($$
$$= -\frac{ac\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} - \frac{\left(a^2c\tan(e+fx)\right)\int \tan(e+fx)\,d}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+}}$$
$$= \frac{a^2c\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{ac\sqrt{a+a\sec(e+fx)}\tan(e+}{f\sqrt{c-c\sec(e+fx)}}$$

Mathematica [C] time = 0.72738, size = 128, normalized size = 1.38

$$\frac{ae^{-i(e+fx)}\left(1+e^{2i(e+fx)}\right)\left(\cot\left(\frac{1}{2}(e+fx)\right)+i\right)\sec(e+fx)\sqrt{a(\sec(e+fx)+1)}\sqrt{c-c\sec(e+fx)}\left(1+\left(ifx-\log\left(1+e^{2i(e+}\right.\right.\right.}{2f\left(1+e^{i(e+fx)}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (a*(1 + E^((2*I)*(e + f*x)))*(I + Cot[(e + f*x)/2])*(1 + Cos[e + f*x]*(I*f*
x - Log[1 + E^((2*I)*(e + f*x))]))*Sec[e + f*x]*Sqrt[a*(1 + Sec[e + f*x])]*
Sqrt[c - c*Sec[e + f*x]])/(2*E^(I*(e + f*x))*(1 + E^(I*(e + f*x)))*f)
```

Maple [A] time = 0.309, size = 151, normalized size = 1.6

$$-\frac{a}{f\sin(fx+e)}\left(\cos(fx+e)\ln\left(\frac{1-\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)+\cos(fx+e)\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/f*a*(cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+cos(f*x+e)*ln(-
(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-cos(f*x+e)*ln(2/(1+cos(f*x+e)))-cos(
f*x+e)-1)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)*(1/cos(f*x+e)*a*(1+cos(f*x+e
)))^(1/2)/sin(f*x+e)
```

Maxima [B] time = 1.78084, size = 328, normalized size = 3.53

$$-\frac{\left((fx+e)a\cos\left(2fx+2e\right)^{2}+(fx+e)a\sin\left(2fx+2e\right)^{2}+2\left(fx+e\right)a\cos\left(2fx+2e\right)-2a\cos\left(\frac{1}{2}\arctan\left(\sin\left(2fx\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] -((f*x + e)*a*cos(2*f*x + 2*e)^2 + (f*x + e)*a*sin(2*f*x + 2*e)^2 + 2*(f*x
+ e)*a*cos(2*f*x + 2*e) - 2*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e)))*sin(2*f*x + 2*e) + (f*x + e)*a - (a*cos(2*f*x + 2*e)^2 + a*sin(2*f*
x + 2*e)^2 + 2*a*cos(2*f*x + 2*e) + a)*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e) + 1) + 2*(a*cos(2*f*x + 2*e) + a)*sin(1/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)
^2 + 2*cos(2*f*x + 2*e) + 1)*f)
```

Fricas [A] time = 1.66006, size = 884, normalized size = 9.51

$$\left[\frac{2a\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sin\left(fx+e\right)+\sqrt{-ac}\left(a\cos\left(fx+e\right)+a\right)\log\left(\frac{ac\cos(fx+e)^{4}-\left(\cos(fx+e)^{3}+\cos(fx+e)\right)\sqrt{-ac}\sqrt{\frac{a\cos}{\cos}}}{2\cos(fx+e)^{2}}\right.}{2\left(f\cos\left(fx+e\right)+f\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] [1/2*(2*a*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)
/cos(f*x + e))*sin(f*x + e) + sqrt(-a*c)*(a*cos(f*x + e) + a)*log(1/2*(a*c*
cos(f*x + e)^4 - (cos(f*x + e)^3 + cos(f*x + e))*sqrt(-a*c)*sqrt((a*cos(f*x
 + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x +
e) + a*c)/cos(f*x + e)^2))/(f*cos(f*x + e) + f), (a*sqrt((a*cos(f*x + e) +
a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) + sqr
t(a*c)*(a*cos(f*x + e) + a)*arctan(sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(
f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)
/(a*c*cos(f*x + e)^2 + a*c)))/(f*cos(f*x + e) + f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

3.97 $\int \frac{(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal. Leaf size=104

$$\frac{2a^2\tan(e+fx)\log(1-\sec(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{a^2\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] (a^2*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c
*Sec[e + f*x]]) + (2*a^2*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*
Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.10332, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3912, 72}

$$\frac{2a^2\tan(e+fx)\log(1-\sec(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{a^2\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)/Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (a^2*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c
*Sec[e + f*x]]) + (2*a^2*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*
Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
x)^(n - 1/2))/x, x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{(ac\tan(e+fx))\operatorname{Subst}\left(\int \frac{a+ax}{x(c-cx)}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$
$$= -\frac{(ac\tan(e+fx))\operatorname{Subst}\left(\int \left(-\frac{2a}{c(-1+x)}+\frac{a}{cx}\right)dx,x,\sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$
$$= \frac{a^2\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}+\frac{2a^2\log(1-\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

Mathematica [C] time = 1.18517, size = 105, normalized size = 1.01

$$-\frac{a\left(-1+e^{i(e+fx)}\right)\left(4i\log\left(1-e^{i(e+fx)}\right)-i\log\left(1+e^{2i(e+fx)}\right)+fx\right)\sqrt{a(\sec(e+fx)+1)}}{f\left(1+e^{i(e+fx)}\right)\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)/Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] -((a*(-1 + E^(I*(e + f*x)))*(f*x + (4*I)*Log[1 - E^(I*(e + f*x))] - I*Log[1
 + E^((2*I)*(e + f*x))])*Sqrt[a*(1 + Sec[e + f*x])])/((1 + E^(I*(e + f*x)))
*f*Sqrt[c - c*Sec[e + f*x]]))
```

Maple [A] time = 0.262, size = 149, normalized size = 1.4

$$\frac{a\cos\left(fx+e\right)}{f\sin\left(fx+e\right)c}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\left(\ln\left(2\left(1+\cos\left(fx+e\right)\right)^{-1}\right)+\ln\left(\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)+\ln\left(-\frac{-1+c}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/f*a*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(ln(2/(1+cos(f*x+e)))+ln((1-cos
(f*x+e)+sin(f*x+e))/sin(f*x+e))+ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-
4*ln(-(-1+cos(f*x+e))/sin(f*x+e)))*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)*cos
(f*x+e)/sin(f*x+e)/c
```

Maxima [A] time = 1.84575, size = 81, normalized size = 0.78

$$-\frac{\left(\left(fx+e\right)a+a\arctan\left(\sin\left(2\,fx+2\,e\right),\cos\left(2\,fx+2\,e\right)+1\right)-4\,a\arctan\left(\sin\left(fx+e\right),\cos\left(fx+e\right)-1\right)\right)\sqrt{a}}{\sqrt{c}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] -((f*x + e)*a + a*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) - 4*a*arc
tan2(sin(f*x + e), cos(f*x + e) - 1))*sqrt(a)/(sqrt(c)*f)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(a\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}\sqrt{-c\sec\left(fx+e\right)+c}}{c\sec\left(fx+e\right)-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-(a*sec(f*x + e) + a)^(3/2)*sqrt(-c*sec(f*x + e) + c)/(c*sec(f*x +
 e) - c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

3.98 $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=100

$$\frac{a^2\tan(e+fx)\log(1-\cos(e+fx))}{cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{2a^2\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}$$

```
[Out] (-2*a^2*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2
)) + (a^2*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c*f*Sqrt[a + a*Sec[e + f*x]]
*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.185599, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3908, 3911, 31}

$$\frac{a^2\tan(e+fx)\log(1-\cos(e+fx))}{cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{2a^2\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-2*a^2*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2
)) + (a^2*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c*f*Sqrt[a + a*Sec[e + f*x]]
*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3908

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[(-4*a^2*Cot[e + f*x]*(c + d*Csc[e + f
*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[a/c, Int[Sqrt[a +
 b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3911

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> -Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((b + a*x)^(m - 1/2)*(d + c*x
)^(n - 1/2))/x^(m + n), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && EqQ
```

```
[m + n, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{2a^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}} + \frac{a\int\frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx}{c}$$
$$= -\frac{2a^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}} + \frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int\frac{1}{-c+cx}\,dx,x,\cos(e\right.}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx}}$$
$$= -\frac{2a^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}} + \frac{a^2\log(1-\cos(e+fx))\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

Mathematica [C] time = 0.670817, size = 115, normalized size = 1.15

$$\frac{a\tan\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}\left(-2\log\left(1-e^{i(e+fx)}\right)+\left(2\log\left(1-e^{i(e+fx)}\right)-ifx\right)\cos(e+fx)+ifx-2\right)}{cf(\cos(e+fx)-1)\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (a*(-2 + I*f*x - 2*Log[1 - E^(I*(e + f*x))] + Cos[e + f*x]*((-I)*f*x + 2*Lo
g[1 - E^(I*(e + f*x))]))*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(c*f*
(-1 + Cos[e + f*x])*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] time = 0.268, size = 163, normalized size = 1.6

$$-\frac{a\left(-1+\cos\left(fx+e\right)\right)}{f\cos\left(fx+e\right)\sin\left(fx+e\right)}\left(2\cos\left(fx+e\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-\cos\left(fx+e\right)\ln\left(2\left(1+\cos\left(fx+e\right)\right)^{-1}\right)-\cos\left(f\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x)
```

```
[Out] -1/f*a*(-1+cos(f*x+e))*(2*cos(f*x+e)*ln(-(-1+cos(f*x+e))/sin(f*x+e))-cos(f*
x+e)*ln(2/(1+cos(f*x+e)))-cos(f*x+e)-2*ln(-(-1+cos(f*x+e))/sin(f*x+e))+ln(2
/(1+cos(f*x+e)))-1)*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)/cos(f*x+e)/(c*(-1
+cos(f*x+e))/cos(f*x+e))^(3/2)/sin(f*x+e)
```

Maxima [A] time = 1.53536, size = 128, normalized size = 1.28

$$\frac{\frac{2\sqrt{-a}a\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^{\frac{3}{2}}} - \frac{\sqrt{-a}a\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{c^{\frac{3}{2}}} + \frac{\sqrt{-a}a(\cos(fx+e)+1)^2}{c^{\frac{3}{2}}\sin(fx+e)^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] (2*sqrt(-a)*a*log(sin(f*x + e)/(cos(f*x + e) + 1))/c^(3/2) - sqrt(-a)*a*log
(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(3/2) + sqrt(-a)*a*(cos(f*x + e
) + 1)^2/(c^(3/2)*sin(f*x + e)^2))/f
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(a\sec(fx+e)+a)^{\frac{3}{2}}\sqrt{-c\sec(fx+e)+c}}{c^2\sec(fx+e)^2-2\,c^2\sec(fx+e)+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] integral((a*sec(f*x + e) + a)^(3/2)*sqrt(-c*sec(f*x + e) + c)/(c^2*sec(f*x
+ e)^2 - 2*c^2*sec(f*x + e) + c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

3.99 $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=146

$$\frac{a^2\tan(e+fx)\log(1-\cos(e+fx))}{c^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a^2\tan(e+fx)}{cf\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}-\frac{a^2\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}$$

```
[Out] -((a^2*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2)
)) - (a^2*Tan[e + f*x])/(c*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^
(3/2)) + (a^2*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c^2*f*Sqrt[a + a*Sec[e +
 f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.282822, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3908, 3907, 3911, 31}

$$\frac{a^2\tan(e+fx)\log(1-\cos(e+fx))}{c^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a^2\tan(e+fx)}{cf\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}-\frac{a^2\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^(5/2),x]
```


```
[Out] -((a^2*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2)
)) - (a^2*Tan[e + f*x])/(c*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^
(3/2)) + (a^2*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c^2*f*Sqrt[a + a*Sec[e +
 f*x]]*Sqrt[c - c*Sec[e + f*x]])
```


Rule 3908

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[(-4*a^2*Cot[e + f*x]*(c + d*Csc[e + f
*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[a/c, Int[Sqrt[a +
 b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3907

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_), x_Symbol] :> Simp[(-2*a*Cot[e + f*x]*(c + d*Csc[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[1/c, Int[Sqrt[a + b*Cs
```

```
c[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3911

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> -Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((b + a*x)^(m - 1/2)*(d + c*x
)^(n - 1/2))/x^(m + n), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && EqQ
[m + n, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{a^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}+\frac{a\int\frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx}{c}$$

$$= -\frac{a^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}-\frac{a^2\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}+$$

$$= -\frac{a^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}-\frac{a^2\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}+$$

$$= -\frac{a^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}-\frac{a^2\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}+$$

Mathematica [C] time = 1.20968, size = 153, normalized size = 1.05

$$\frac{a\tan\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}\left(6\log\left(1-e^{i(e+fx)}\right)+\left(-8\log\left(1-e^{i(e+fx)}\right)+4ifx-6\right)\cos(e+fx)+\left(2\log\left(1-e^{i(}\right.\right.\right.}{2c^2f(\cos(e+fx)-1)^2\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (a*(4 - (3*I)*f*x + Cos[e + f*x]*(-6 + (4*I)*f*x - 8*Log[1 - E^(I*(e + f*x)
)]) + 6*Log[1 - E^(I*(e + f*x))] + Cos[2*(e + f*x)]*((-I)*f*x + 2*Log[1 - E
^(I*(e + f*x))]))*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(2*c^2*f*(-1
 + Cos[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] time = 0.265, size = 227, normalized size = 1.6

$$-\frac{a\left(-1+\cos\left(fx+e\right)\right)}{4\,f\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^{2}}\left(8\,\ln\left(-\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right)\left(\cos\left(fx+e\right)\right)^{2}-4\,\ln\left(2\,\left(1+\cos\left(fx+e\right)\right)^{-1}\right)\left(\cos\left(fx+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x)
```

```
[Out] -1/4/f*a*(-1+cos(f*x+e))*(8*ln(-(-1+cos(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-4*
ln(2/(1+cos(f*x+e)))*cos(f*x+e)^2-16*cos(f*x+e)*ln(-(-1+cos(f*x+e))/sin(f*x
+e))-5*cos(f*x+e)^2+8*cos(f*x+e)*ln(2/(1+cos(f*x+e)))+8*ln(-(-1+cos(f*x+e))
/sin(f*x+e))-2*cos(f*x+e)-4*ln(2/(1+cos(f*x+e)))+3)*(1/cos(f*x+e)*a*(1+cos(
f*x+e)))^(1/2)/sin(f*x+e)/cos(f*x+e)^2/(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)
```

Maxima [B] time = 2.51214, size = 2411, normalized size = 16.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] -((f*x + e)*a*cos(4*f*x + 4*e)^2 + 36*(f*x + e)*a*cos(2*f*x + 2*e)^2 + 16*(
f*x + e)*a*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16*(f*x
 + e)*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + (f*x + e)*
a*sin(4*f*x + 4*e)^2 + 36*(f*x + e)*a*sin(2*f*x + 2*e)^2 + 16*(f*x + e)*a*s
in(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16*(f*x + e)*a*sin(
1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 12*(f*x + e)*a*cos(2*f
*x + 2*e) + (f*x + e)*a - 2*(a*cos(4*f*x + 4*e)^2 + 36*a*cos(2*f*x + 2*e)^2
 + 16*a*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16*a*cos(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + a*sin(4*f*x + 4*e)^2 +
```

```
12*a*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 36*a*sin(2*f*x + 2*e)^2 + 16*a*sin
(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16*a*sin(1/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*(6*a*cos(2*f*x + 2*e) + a)*cos(4
*f*x + 4*e) + 12*a*cos(2*f*x + 2*e) - 8*(a*cos(4*f*x + 4*e) + 6*a*cos(2*f*x
 + 2*e) - 4*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + a)*cos
(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 8*(a*cos(4*f*x + 4*e) +
 6*a*cos(2*f*x + 2*e) + a)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))) - 8*(a*sin(4*f*x + 4*e) + 6*a*sin(2*f*x + 2*e) - 4*a*sin(1/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e))) - 8*(a*sin(4*f*x + 4*e) + 6*a*sin(2*f*x + 2*e))*sin(1/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + a)*arctan2(sin(1/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))) - 1) + 2*(6*(f*x + e)*a*cos(2*f*x + 2*e) + (f*x + e)*a - 4*a*sin
(2*f*x + 2*e))*cos(4*f*x + 4*e) - 2*(4*(f*x + e)*a*cos(4*f*x + 4*e) + 24*(f
*x + e)*a*cos(2*f*x + 2*e) - 16*(f*x + e)*a*cos(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))) + 4*(f*x + e)*a + 3*a*sin(4*f*x + 4*e) + 2*a*sin(2*f*
x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*(4*(f*x
+ e)*a*cos(4*f*x + 4*e) + 24*(f*x + e)*a*cos(2*f*x + 2*e) + 4*(f*x + e)*a +
 3*a*sin(4*f*x + 4*e) + 2*a*sin(2*f*x + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) + 4*(3*(f*x + e)*a*sin(2*f*x + 2*e) + 2*a*cos(2*f*x
 + 2*e))*sin(4*f*x + 4*e) - 8*a*sin(2*f*x + 2*e) - 2*(4*(f*x + e)*a*sin(4*f
*x + 4*e) + 24*(f*x + e)*a*sin(2*f*x + 2*e) - 16*(f*x + e)*a*sin(1/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 3*a*cos(4*f*x + 4*e) - 2*a*cos(2*f
*x + 2*e) - 3*a)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*(
4*(f*x + e)*a*sin(4*f*x + 4*e) + 24*(f*x + e)*a*sin(2*f*x + 2*e) - 3*a*cos(
4*f*x + 4*e) - 2*a*cos(2*f*x + 2*e) - 3*a)*sin(1/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((c^3*cos(4*f*x + 4*e)^2 + 36*c^3*cos
(2*f*x + 2*e)^2 + 16*c^3*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
))^2 + 16*c^3*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + c^3*
sin(4*f*x + 4*e)^2 + 12*c^3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 36*c^3*sin(
2*f*x + 2*e)^2 + 16*c^3*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
)^2 + 16*c^3*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 12*c^
3*cos(2*f*x + 2*e) + c^3 + 2*(6*c^3*cos(2*f*x + 2*e) + c^3)*cos(4*f*x + 4*e
) - 8*(c^3*cos(4*f*x + 4*e) + 6*c^3*cos(2*f*x + 2*e) - 4*c^3*cos(1/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c^3)*cos(3/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) - 8*(c^3*cos(4*f*x + 4*e) + 6*c^3*cos(2*f*x + 2*e)
+ c^3)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 8*(c^3*sin(4*
f*x + 4*e) + 6*c^3*sin(2*f*x + 2*e) - 4*c^3*sin(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
 - 8*(c^3*sin(4*f*x + 4*e) + 6*c^3*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))))*f)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(a\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}\sqrt{-c\sec\left(fx+e\right)+c}}{c^3\sec\left(fx+e\right)^3-3\,c^3\sec\left(fx+e\right)^2+3\,c^3\sec\left(fx+e\right)-c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral(-(a*sec(f*x + e) + a)^(3/2)*sqrt(-c*sec(f*x + e) + c)/(c^3*sec(f*x
 + e)^3 - 3*c^3*sec(f*x + e)^2 + 3*c^3*sec(f*x + e) - c^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

3.100 $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=196

$$-\frac{a^2\tan(e+fx)}{c^2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}+\frac{a^2\tan(e+fx)\log(1-\cos(e+fx))}{c^3f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a^2\tan(e+fx)}{2cf\sqrt{a\sec(e+fx)+a}(c-c\text{s}}$$

```
[Out] (-2*a^2*Tan[e + f*x])/(3*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(7
/2)) - (a^2*Tan[e + f*x])/(2*c*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*
x])^(5/2)) - (a^2*Tan[e + f*x])/(c^2*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[
e + f*x])^(3/2)) + (a^2*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c^3*f*Sqrt[a +
 a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.382437, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3908, 3907, 3911, 31}

$$-\frac{a^2\tan(e+fx)}{c^2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}+\frac{a^2\tan(e+fx)\log(1-\cos(e+fx))}{c^3f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a^2\tan(e+fx)}{2cf\sqrt{a\sec(e+fx)+a}(c-c\text{s}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^(7/2),x]
```


```
[Out] (-2*a^2*Tan[e + f*x])/(3*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(7
/2)) - (a^2*Tan[e + f*x])/(2*c*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*
x])^(5/2)) - (a^2*Tan[e + f*x])/(c^2*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[
e + f*x])^(3/2)) + (a^2*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c^3*f*Sqrt[a +
 a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```


Rule 3908

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[(-4*a^2*Cot[e + f*x]*(c + d*Csc[e + f
*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[a/c, Int[Sqrt[a +
 b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3907

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_), x_Symbol] :> Simp[(-2*a*Cot[e + f*x]*(c + d*Csc[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[1/c, Int[Sqrt[a + b*Cs
c[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3911

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> -Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((b + a*x)^(m - 1/2)*(d + c*x
)^(n - 1/2))/x^(m + n), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && EqQ
[m + n, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = -\frac{2a^2\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}}+\frac{a\int\frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx}{c}$$

$$= -\frac{2a^2\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}}-\frac{a^2\tan(e+fx)}{2cf\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5}}$$

$$= -\frac{2a^2\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}}-\frac{a^2\tan(e+fx)}{2cf\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5}}$$

$$= -\frac{2a^2\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}}-\frac{a^2\tan(e+fx)}{2cf\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5}}$$

$$= -\frac{2a^2\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}}-\frac{a^2\tan(e+fx)}{2cf\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5}}$$

Mathematica [C] time = 2.0955, size = 199, normalized size = 1.02

$$\frac{a\tan\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}\left(-60\log\left(1-e^{i(e+fx)}\right)-3ifx\cos(3(e+fx))+6i\left(6i\log\left(1-e^{i(e+fx)}\right)+3fx+4i\right)\right.}{12c^3f(\cos(e+fx)-1)^{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^(7/2),x]
```

```
[Out] (a*(-50 + (30*I)*f*x - (3*I)*f*x*Cos[3*(e + f*x)] + (6*I)*Cos[2*(e + f*x)]*
(4*I + 3*f*x + (6*I)*Log[1 - E^(I*(e + f*x))]) - 60*Log[1 - E^(I*(e + f*x))
] + 6*Cos[3*(e + f*x)]*Log[1 - E^(I*(e + f*x))] + Cos[e + f*x]*(66 - (45*I)
*f*x + 90*Log[1 - E^(I*(e + f*x))]))*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*
x)/2])/(12*c^3*f*(-1 + Cos[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] time = 0.275, size = 289, normalized size = 1.5

$$-\frac{a\left(-1+\cos\left(fx+e\right)\right)}{24\,f\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^{3}}\left(48\,\left(\cos\left(fx+e\right)\right)^{3}\ln\left(-\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-24\,\left(\cos\left(fx+e\right)\right)^{3}\ln\left(2\,\left(1+\cos\left(fx+\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(7/2),x)
```

```
[Out] -1/24/f*a*(-1+cos(f*x+e))*(48*cos(f*x+e)^3*ln(-(-1+cos(f*x+e))/sin(f*x+e))-
24*cos(f*x+e)^3*ln(2/(1+cos(f*x+e)))-144*ln(-(-1+cos(f*x+e))/sin(f*x+e))*co
s(f*x+e)^2-35*cos(f*x+e)^3+72*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^2+144*cos(f*x
+e)*ln(-(-1+cos(f*x+e))/sin(f*x+e))+9*cos(f*x+e)^2-72*cos(f*x+e)*ln(2/(1+co
s(f*x+e)))-48*ln(-(-1+cos(f*x+e))/sin(f*x+e))+27*cos(f*x+e)+24*ln(2/(1+cos(
f*x+e)))-17)*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)/sin(f*x+e)/cos(f*x+e)^3/
(c*(-1+cos(f*x+e))/cos(f*x+e))^(7/2)
```

Maxima [B] time = 11.3007, size = 4698, normalized size = 23.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="maxim
a")
```

```
[Out] -1/3*(3*(f*x + e)*a*cos(6*f*x + 6*e)^2 + 675*(f*x + e)*a*cos(4*f*x + 4*e)^2
 + 675*(f*x + e)*a*cos(2*f*x + 2*e)^2 + 108*(f*x + e)*a*cos(5/2*arctan2(sin
```

```
(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 1200*(f*x + e)*a*cos(3/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 108*(f*x + e)*a*cos(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e)))^2 + 3*(f*x + e)*a*sin(6*f*x + 6*e)^2 + 675*(
f*x + e)*a*sin(4*f*x + 4*e)^2 + 675*(f*x + e)*a*sin(2*f*x + 2*e)^2 + 108*(f
*x + e)*a*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 1200*(f*
x + e)*a*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 108*(f*x
+ e)*a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 90*(f*x + e
)*a*cos(2*f*x + 2*e) + 3*(f*x + e)*a - 6*(a*cos(6*f*x + 6*e)^2 + 225*a*cos(
4*f*x + 4*e)^2 + 225*a*cos(2*f*x + 2*e)^2 + 36*a*cos(5/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e)))^2 + 400*a*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e)))^2 + 36*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
)^2 + a*sin(6*f*x + 6*e)^2 + 225*a*sin(4*f*x + 4*e)^2 + 450*a*sin(4*f*x + 4
*e)*sin(2*f*x + 2*e) + 225*a*sin(2*f*x + 2*e)^2 + 36*a*sin(5/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 400*a*sin(3/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e)))^2 + 36*a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e)))^2 + 2*(15*a*cos(4*f*x + 4*e) + 15*a*cos(2*f*x + 2*e) + a)*cos(6*f*x
 + 6*e) + 30*(15*a*cos(2*f*x + 2*e) + a)*cos(4*f*x + 4*e) + 30*a*cos(2*f*x
+ 2*e) - 12*(a*cos(6*f*x + 6*e) + 15*a*cos(4*f*x + 4*e) + 15*a*cos(2*f*x +
2*e) - 20*a*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 6*a*cos(
1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + a)*cos(5/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))) - 40*(a*cos(6*f*x + 6*e) + 15*a*cos(4*f*x +
 4*e) + 15*a*cos(2*f*x + 2*e) - 6*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e))) + a)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 1
2*(a*cos(6*f*x + 6*e) + 15*a*cos(4*f*x + 4*e) + 15*a*cos(2*f*x + 2*e) + a)*
cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 30*(a*sin(4*f*x + 4*
e) + a*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) - 12*(a*sin(6*f*x + 6*e) + 15*a*s
in(4*f*x + 4*e) + 15*a*sin(2*f*x + 2*e) - 20*a*sin(3/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))) - 6*a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e))))*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 40*(a*sin(
6*f*x + 6*e) + 15*a*sin(4*f*x + 4*e) + 15*a*sin(2*f*x + 2*e) - 6*a*sin(1/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) - 12*(a*sin(6*f*x + 6*e) + 15*a*sin(4*f*x + 4*e) +
15*a*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
 + a)*arctan2(sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 1) + 6*(15*(f*x + e)*a*cos(
4*f*x + 4*e) + 15*(f*x + e)*a*cos(2*f*x + 2*e) + (f*x + e)*a - 11*a*sin(4*f
*x + 4*e) - 11*a*sin(2*f*x + 2*e))*cos(6*f*x + 6*e) + 90*(15*(f*x + e)*a*co
s(2*f*x + 2*e) + (f*x + e)*a)*cos(4*f*x + 4*e) - 12*(3*(f*x + e)*a*cos(6*f*
x + 6*e) + 45*(f*x + e)*a*cos(4*f*x + 4*e) + 45*(f*x + e)*a*cos(2*f*x + 2*e
) - 60*(f*x + e)*a*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 1
8*(f*x + e)*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 3*(f*x
 + e)*a + 2*a*sin(6*f*x + 6*e) - 3*a*sin(4*f*x + 4*e) - 3*a*sin(2*f*x + 2*e
) + 10*a*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(5/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 20*(6*(f*x + e)*a*cos(6*f*x + 6*
e) + 90*(f*x + e)*a*cos(4*f*x + 4*e) + 90*(f*x + e)*a*cos(2*f*x + 2*e) - 36
```

```
*(f*x + e)*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 6*(f*x
+ e)*a + 5*a*sin(6*f*x + 6*e) + 9*a*sin(4*f*x + 4*e) + 9*a*sin(2*f*x + 2*e)
 - 6*a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 12*(3*(f*x + e)*a*cos(6*f*x + 6*e)
 + 45*(f*x + e)*a*cos(4*f*x + 4*e) + 45*(f*x + e)*a*cos(2*f*x + 2*e) + 3*(f
*x + e)*a + 2*a*sin(6*f*x + 6*e) - 3*a*sin(4*f*x + 4*e) - 3*a*sin(2*f*x + 2
*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 6*(15*(f*x + e)
*a*sin(4*f*x + 4*e) + 15*(f*x + e)*a*sin(2*f*x + 2*e) + 11*a*cos(4*f*x + 4*
e) + 11*a*cos(2*f*x + 2*e))*sin(6*f*x + 6*e) + 6*(225*(f*x + e)*a*sin(2*f*x
 + 2*e) - 11*a)*sin(4*f*x + 4*e) - 66*a*sin(2*f*x + 2*e) - 12*(3*(f*x + e)*
a*sin(6*f*x + 6*e) + 45*(f*x + e)*a*sin(4*f*x + 4*e) + 45*(f*x + e)*a*sin(2
*f*x + 2*e) - 60*(f*x + e)*a*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e))) - 18*(f*x + e)*a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
) - 2*a*cos(6*f*x + 6*e) + 3*a*cos(4*f*x + 4*e) + 3*a*cos(2*f*x + 2*e) - 10
*a*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*a)*sin(5/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 20*(6*(f*x + e)*a*sin(6*f*x + 6*
e) + 90*(f*x + e)*a*sin(4*f*x + 4*e) + 90*(f*x + e)*a*sin(2*f*x + 2*e) - 36
*(f*x + e)*a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 5*a*cos
(6*f*x + 6*e) - 9*a*cos(4*f*x + 4*e) - 9*a*cos(2*f*x + 2*e) + 6*a*cos(1/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 5*a)*sin(3/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e))) - 12*(3*(f*x + e)*a*sin(6*f*x + 6*e) + 45*(f*x
 + e)*a*sin(4*f*x + 4*e) + 45*(f*x + e)*a*sin(2*f*x + 2*e) - 2*a*cos(6*f*x
+ 6*e) + 3*a*cos(4*f*x + 4*e) + 3*a*cos(2*f*x + 2*e) - 2*a)*sin(1/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((c^4*cos(6*f*x + 6*
e)^2 + 225*c^4*cos(4*f*x + 4*e)^2 + 225*c^4*cos(2*f*x + 2*e)^2 + 36*c^4*cos
(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 400*c^4*cos(3/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 36*c^4*cos(1/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e)))^2 + c^4*sin(6*f*x + 6*e)^2 + 225*c^4*sin(4*f*x
 + 4*e)^2 + 450*c^4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 225*c^4*sin(2*f*x +
 2*e)^2 + 36*c^4*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4
00*c^4*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 36*c^4*sin(
1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 30*c^4*cos(2*f*x + 2*e
) + c^4 + 2*(15*c^4*cos(4*f*x + 4*e) + 15*c^4*cos(2*f*x + 2*e) + c^4)*cos(6
*f*x + 6*e) + 30*(15*c^4*cos(2*f*x + 2*e) + c^4)*cos(4*f*x + 4*e) - 12*(c^4
*cos(6*f*x + 6*e) + 15*c^4*cos(4*f*x + 4*e) + 15*c^4*cos(2*f*x + 2*e) - 20*
c^4*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 6*c^4*cos(1/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c^4)*cos(5/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e))) - 40*(c^4*cos(6*f*x + 6*e) + 15*c^4*cos(4*f*x +
 4*e) + 15*c^4*cos(2*f*x + 2*e) - 6*c^4*cos(1/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))) + c^4)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
)) - 12*(c^4*cos(6*f*x + 6*e) + 15*c^4*cos(4*f*x + 4*e) + 15*c^4*cos(2*f*x
+ 2*e) + c^4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 30*(c^
4*sin(4*f*x + 4*e) + c^4*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) - 12*(c^4*sin(6
*f*x + 6*e) + 15*c^4*sin(4*f*x + 4*e) + 15*c^4*sin(2*f*x + 2*e) - 20*c^4*si
n(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 6*c^4*sin(1/2*arctan2(
```

```
sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(5/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e))) - 40*(c^4*sin(6*f*x + 6*e) + 15*c^4*sin(4*f*x + 4*e) + 15*c
^4*sin(2*f*x + 2*e) - 6*c^4*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 12*(c^4*sin(6
*f*x + 6*e) + 15*c^4*sin(4*f*x + 4*e) + 15*c^4*sin(2*f*x + 2*e))*sin(1/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*f)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}\sqrt{-c\sec\left(fx+e\right)+c}}{c^4\sec\left(fx+e\right)^4-4\,c^4\sec\left(fx+e\right)^3+6\,c^4\sec\left(fx+e\right)^2-4\,c^4\sec\left(fx+e\right)+c^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="frica
s")
```

```
[Out] integral((a*sec(f*x + e) + a)^(3/2)*sqrt(-c*sec(f*x + e) + c)/(c^4*sec(f*x
+ e)^4 - 4*c^4*sec(f*x + e)^3 + 6*c^4*sec(f*x + e)^2 - 4*c^4*sec(f*x + e) +
 c^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

3.101 $\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}\,dx$

Optimal. Leaf size=153

$$-\frac{a^3c^3\tan^5(e+fx)}{4f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{a^3c^3\tan^3(e+fx)}{2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{a^3c^3\tan(e+fx)\log(\cos(e+}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec}$$

```
[Out] (a^3*c^3*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c
 - c*Sec[e + f*x]]) + (a^3*c^3*Tan[e + f*x]^3)/(2*f*Sqrt[a + a*Sec[e + f*x]
]*Sqrt[c - c*Sec[e + f*x]]) - (a^3*c^3*Tan[e + f*x]^5)/(4*f*Sqrt[a + a*Sec[
e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.120169, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3905, 3473, 3475}

$$-\frac{a^3c^3\tan^5(e+fx)}{4f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{a^3c^3\tan^3(e+fx)}{2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{a^3c^3\tan(e+fx)\log(\cos(e+}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (a^3*c^3*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c
 - c*Sec[e + f*x]]) + (a^3*c^3*Tan[e + f*x]^3)/(2*f*Sqrt[a + a*Sec[e + f*x]
]*Sqrt[c - c*Sec[e + f*x]]) - (a^3*c^3*Tan[e + f*x]^5)/(4*f*Sqrt[a + a*Sec[
e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3905

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Dist[((-(a*c))^(m + 1/2)*Cot[e + f*x])/(Sqrt
[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int[Cot[e + f*x]^(2*m), x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IntegerQ[m + 1/2]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}\,dx &= -\frac{\left(a^3c^3\tan(e+fx)\right)\int\tan^5(e+fx)\,dx}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\ &= -\frac{a^3c^3\tan^5(e+fx)}{4f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}+\frac{\left(a^3c^3\tan(e+fx)\right)\int\tan^3}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c}}\\ &= \frac{a^3c^3\tan^3(e+fx)}{2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}-\frac{a^3c^3\tan^5(e+fx}{4f\sqrt{a+a\sec(e+fx)}\sqrt{c-}}\\ &= \frac{a^3c^3\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}+\frac{a^3c^3\tan^3(e+fx)}{2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c}}\end{aligned}$$

Mathematica [C] time = 1.49186, size = 164, normalized size = 1.07

$$\frac{ia^2c^2\csc\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)\sec^3(e+fx)\sqrt{a(\sec(e+fx)+1)}\sqrt{c-c\sec(e+fx)}\left(3i\log\left(1+e^{2i(e+fx)}\right)+\left(fx+i\,\mathrm{lo}\right.\right.}{16f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] ((I/16)*a^2*c^2*Csc[(e + f*x)/2]*(2*I + 3*f*x + Cos[4*(e + f*x)]*(f*x + I*L
og[1 + E^((2*I)*(e + f*x))]) + 4*Cos[2*(e + f*x)]*(I + f*x + I*Log[1 + E^((
2*I)*(e + f*x))]) + (3*I)*Log[1 + E^((2*I)*(e + f*x))])*Sec[(e + f*x)/2]*Se
c[e + f*x]^3*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])/f
```

Maple [A] time = 0.299, size = 191, normalized size = 1.3

$$\frac{a^2}{4f\sin\left(fx+e\right)\left(-1+\cos\left(fx+e\right)\right)^2\cos\left(fx+e\right)}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{5}{2}}\left(4\left(\cos\left(fx+e\right)\right)^4\ln\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(5/2),x)
```

```
[Out] 1/4/f*a^2*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(c*(-1+cos(f*x+e))/cos(f*x+
e))^(5/2)*(4*cos(f*x+e)^4*ln(2/(1+cos(f*x+e)))-4*cos(f*x+e)^4*ln((1-cos(f*x
+e)+sin(f*x+e))/sin(f*x+e))-4*cos(f*x+e)^4*ln(-(-1+cos(f*x+e)+sin(f*x+e))/s
in(f*x+e))+3*cos(f*x+e)^4-4*cos(f*x+e)^2+1)/sin(f*x+e)/(-1+cos(f*x+e))^2/co
s(f*x+e)
```

Maxima [B] time = 2.43173, size = 2186, normalized size = 14.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] -((f*x + e)*a^2*c^2*cos(8*f*x + 8*e)^2 + 16*(f*x + e)*a^2*c^2*cos(6*f*x + 6
*e)^2 + 36*(f*x + e)*a^2*c^2*cos(4*f*x + 4*e)^2 + 16*(f*x + e)*a^2*c^2*cos(
2*f*x + 2*e)^2 + (f*x + e)*a^2*c^2*sin(8*f*x + 8*e)^2 + 16*(f*x + e)*a^2*c^
2*sin(6*f*x + 6*e)^2 + 36*(f*x + e)*a^2*c^2*sin(4*f*x + 4*e)^2 + 16*(f*x +
e)*a^2*c^2*sin(2*f*x + 2*e)^2 + 8*(f*x + e)*a^2*c^2*cos(2*f*x + 2*e) + (f*x
 + e)*a^2*c^2 - 4*a^2*c^2*sin(2*f*x + 2*e) - (a^2*c^2*cos(8*f*x + 8*e)^2 +
16*a^2*c^2*cos(6*f*x + 6*e)^2 + 36*a^2*c^2*cos(4*f*x + 4*e)^2 + 16*a^2*c^2*
cos(2*f*x + 2*e)^2 + a^2*c^2*sin(8*f*x + 8*e)^2 + 16*a^2*c^2*sin(6*f*x + 6*
e)^2 + 36*a^2*c^2*sin(4*f*x + 4*e)^2 + 48*a^2*c^2*sin(4*f*x + 4*e)*sin(2*f*
x + 2*e) + 16*a^2*c^2*sin(2*f*x + 2*e)^2 + 8*a^2*c^2*cos(2*f*x + 2*e) + a^2
*c^2 + 2*(4*a^2*c^2*cos(6*f*x + 6*e) + 6*a^2*c^2*cos(4*f*x + 4*e) + 4*a^2*c
^2*cos(2*f*x + 2*e) + a^2*c^2)*cos(8*f*x + 8*e) + 8*(6*a^2*c^2*cos(4*f*x +
4*e) + 4*a^2*c^2*cos(2*f*x + 2*e) + a^2*c^2)*cos(6*f*x + 6*e) + 12*(4*a^2*c
^2*cos(2*f*x + 2*e) + a^2*c^2)*cos(4*f*x + 4*e) + 4*(2*a^2*c^2*sin(6*f*x +
6*e) + 3*a^2*c^2*sin(4*f*x + 4*e) + 2*a^2*c^2*sin(2*f*x + 2*e))*sin(8*f*x +
 8*e) + 16*(3*a^2*c^2*sin(4*f*x + 4*e) + 2*a^2*c^2*sin(2*f*x + 2*e))*sin(6*
f*x + 6*e))*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) + 2*(4*(f*x + e
)*a^2*c^2*cos(6*f*x + 6*e) + 6*(f*x + e)*a^2*c^2*cos(4*f*x + 4*e) + 4*(f*x
+ e)*a^2*c^2*cos(2*f*x + 2*e) + (f*x + e)*a^2*c^2 - 2*a^2*c^2*sin(6*f*x + 6
*e) - 2*a^2*c^2*sin(4*f*x + 4*e) - 2*a^2*c^2*sin(2*f*x + 2*e))*cos(8*f*x +
8*e) + 8*(6*(f*x + e)*a^2*c^2*cos(4*f*x + 4*e) + 4*(f*x + e)*a^2*c^2*cos(2*
f*x + 2*e) + (f*x + e)*a^2*c^2 + a^2*c^2*sin(4*f*x + 4*e))*cos(6*f*x + 6*e)
 + 4*(12*(f*x + e)*a^2*c^2*cos(2*f*x + 2*e) + 3*(f*x + e)*a^2*c^2 - 2*a^2*c
^2*sin(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(2*(f*x + e)*a^2*c^2*sin(6*f*x +
6*e) + 3*(f*x + e)*a^2*c^2*sin(4*f*x + 4*e) + 2*(f*x + e)*a^2*c^2*sin(2*f*x
```

```
 + 2*e) + a^2*c^2*cos(6*f*x + 6*e) + a^2*c^2*cos(4*f*x + 4*e) + a^2*c^2*cos
(2*f*x + 2*e))*sin(8*f*x + 8*e) + 4*(12*(f*x + e)*a^2*c^2*sin(4*f*x + 4*e)
+ 8*(f*x + e)*a^2*c^2*sin(2*f*x + 2*e) - 2*a^2*c^2*cos(4*f*x + 4*e) - a^2*c
^2)*sin(6*f*x + 6*e) + 4*(12*(f*x + e)*a^2*c^2*sin(2*f*x + 2*e) + 2*a^2*c^2
*cos(2*f*x + 2*e) - a^2*c^2)*sin(4*f*x + 4*e))*sqrt(a)*sqrt(c)/((2*(4*cos(6
*f*x + 6*e) + 6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e) + 1)*cos(8*f*x + 8*e)
 + cos(8*f*x + 8*e)^2 + 8*(6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e) + 1)*cos
(6*f*x + 6*e) + 16*cos(6*f*x + 6*e)^2 + 12*(4*cos(2*f*x + 2*e) + 1)*cos(4*f
*x + 4*e) + 36*cos(4*f*x + 4*e)^2 + 16*cos(2*f*x + 2*e)^2 + 4*(2*sin(6*f*x
+ 6*e) + 3*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + sin(8*
f*x + 8*e)^2 + 16*(3*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*sin(6*f*x + 6*e
) + 16*sin(6*f*x + 6*e)^2 + 36*sin(4*f*x + 4*e)^2 + 48*sin(4*f*x + 4*e)*sin
(2*f*x + 2*e) + 16*sin(2*f*x + 2*e)^2 + 8*cos(2*f*x + 2*e) + 1)*f)
```

Fricas [A] time = 1.7419, size = 987, normalized size = 6.45

$$\left[\frac{2\sqrt{-ac}a^2c^2\cos\left(fx+e\right)^3\log\left(\frac{ac\cos\left(fx+e\right)^4-\left(\cos\left(fx+e\right)^3+\cos\left(fx+e\right)\right)\sqrt{-ac}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)+ac}{2\cos\left(fx+e\right)^2}\right)-\left(3a^2c^2\cos\left(fx\right.\right.}{4f\cos\left(fx+e\right)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] [1/4*(2*sqrt(-a*c)*a^2*c^2*cos(f*x + e)^3*log(1/2*(a*c*cos(f*x + e)^4 - (co
s(f*x + e)^3 + cos(f*x + e))*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x +
 e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) + a*c)/cos(f*x +
e)^2) - (3*a^2*c^2*cos(f*x + e)^2 - a^2*c^2)*sqrt((a*cos(f*x + e) + a)/cos(
f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x
+ e)^3), 1/4*(4*sqrt(a*c)*a^2*c^2*arctan(sqrt(a*c)*sqrt((a*cos(f*x + e) + a
)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)*sin(f*
x + e)/(a*c*cos(f*x + e)^2 + a*c))*cos(f*x + e)^3 - (3*a^2*c^2*cos(f*x + e)
^2 - a^2*c^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e)
- c)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

3.102 $\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=190

$$-\frac{a^2c^2\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}+\frac{a^3c^2\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac^2\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sec(e+fx)}}+$$

```
[Out] (a^3*c^2*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c
 - c*Sec[e + f*x]]) - (a^2*c^2*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(f*Sq
rt[c - c*Sec[e + f*x]]) - (a*c^2*(a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(
2*f*Sqrt[c - c*Sec[e + f*x]]) + (c^2*(a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x
])/(3*f*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.362838, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3909, 3906, 3905, 3475}

$$-\frac{a^2c^2\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}+\frac{a^3c^2\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac^2\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sec(e+fx)}}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (a^3*c^2*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c
 - c*Sec[e + f*x]]) - (a^2*c^2*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(f*Sq
rt[c - c*Sec[e + f*x]]) - (a*c^2*(a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(
2*f*Sqrt[c - c*Sec[e + f*x]]) + (c^2*(a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x
])/(3*f*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3909

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[(-2*a^2*Cot[e + f*x]*(c + d*Csc[e + f
*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[a, Int[Sqrt[a + b
*Csc[e + f*x]]*(c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LeQ[n, -2^(-1)]
```

Rule 3906

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e + f*x]*(c + d*Csc[e + f*x]
```

```
)^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[c, Int[Sqrt[a
+ b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && GtQ[n, 1/2]
```

Rule 3905

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Dist[((-(a*c))^(m + 1/2)*Cot[e + f*x])/(Sqrt
[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int[Cot[e + f*x]^(2*m), x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IntegerQ[m + 1/2]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\,dx &= \frac{c^2(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}}+c\int (a+a\sec(e+fx))^{5/2}\sqrt{c-\\
&= -\frac{ac^2(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{c-c\sec(e+fx)}}+\frac{c^2(a+a\sec(e+fx))^{5/2}\tan\\
&= -\frac{a^2c^2\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}-\frac{ac^2(a+a\sec(e+fx))^{3/2}\tan\\
&= -\frac{a^2c^2\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}-\frac{ac^2(a+a\sec(e+fx))^{3/2}\tan\\
&= \frac{a^3c^2\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}-\frac{a^2c^2\sqrt{a+a\sec(e+fx)}\tan\end{aligned}$$

Mathematica [C] time = 1.16559, size = 149, normalized size = 0.78

$$\frac{a^2c\csc\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\sqrt{a(\sec(e+fx)+1)}\sqrt{c-c\sec(e+fx)}\left(6\cos(2(e+fx))+3ifx\cos(3(e\right.}{24f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (a^2*c*Csc[(e + f*x)/2]*(2 + 6*Cos[2*(e + f*x)] + (3*I)*f*x*Cos[3*(e + f*x)
] + Cos[e + f*x]*(-6 + (9*I)*f*x - 9*Log[1 + E^((2*I)*(e + f*x))]) - 3*Cos[
3*(e + f*x)]*Log[1 + E^((2*I)*(e + f*x))])*Sec[(e + f*x)/2]*Sec[e + f*x]^2*
Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])/(24*f)
```

Maple [A] time = 0.281, size = 199, normalized size = 1.1

$$-\frac{a^2}{6f\sin(fx+e)\cos(fx+e)(-1+\cos(fx+e))}\sqrt{\frac{a(1+\cos(fx+e))}{\cos(fx+e)}}\left(\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}\right)^{\frac{3}{2}}\left(6\left(\cos(fx+e)\right)^3\ln\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(3/2),x)
```

```
[Out] -1/6/f*a^2*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(c*(-1+cos(f*x+e))/cos(f*x
+e))^(3/2)*(6*cos(f*x+e)^3*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+6*cos(f
*x+e)^3*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-6*cos(f*x+e)^3*ln(2/(1+c
os(f*x+e)))-7*cos(f*x+e)^3-6*cos(f*x+e)^2+3*cos(f*x+e)+2)/sin(f*x+e)/cos(f*
x+e)/(-1+cos(f*x+e))
```

Maxima [B] time = 2.19047, size = 1831, normalized size = 9.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] -1/3*(3*(f*x + e)*a^2*c*cos(6*f*x + 6*e)^2 + 27*(f*x + e)*a^2*c*cos(4*f*x +
 4*e)^2 + 27*(f*x + e)*a^2*c*cos(2*f*x + 2*e)^2 + 3*(f*x + e)*a^2*c*sin(6*f
*x + 6*e)^2 + 27*(f*x + e)*a^2*c*sin(4*f*x + 4*e)^2 + 27*(f*x + e)*a^2*c*si
n(2*f*x + 2*e)^2 + 18*(f*x + e)*a^2*c*cos(2*f*x + 2*e) + 3*(f*x + e)*a^2*c
- 6*a^2*c*sin(2*f*x + 2*e) - 3*(a^2*c*cos(6*f*x + 6*e)^2 + 9*a^2*c*cos(4*f*
x + 4*e)^2 + 9*a^2*c*cos(2*f*x + 2*e)^2 + a^2*c*sin(6*f*x + 6*e)^2 + 9*a^2*
c*sin(4*f*x + 4*e)^2 + 18*a^2*c*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 9*a^2*c
*sin(2*f*x + 2*e)^2 + 6*a^2*c*cos(2*f*x + 2*e) + a^2*c + 2*(3*a^2*c*cos(4*f
*x + 4*e) + 3*a^2*c*cos(2*f*x + 2*e) + a^2*c)*cos(6*f*x + 6*e) + 6*(3*a^2*c
*cos(2*f*x + 2*e) + a^2*c)*cos(4*f*x + 4*e) + 6*(a^2*c*sin(4*f*x + 4*e) + a
```

```
^2*c*sin(2*f*x + 2*e))*sin(6*f*x + 6*e))*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e) + 1) + 6*(3*(f*x + e)*a^2*c*cos(4*f*x + 4*e) + 3*(f*x + e)*a^2*c*c
os(2*f*x + 2*e) + (f*x + e)*a^2*c - a^2*c*sin(4*f*x + 4*e) - a^2*c*sin(2*f*
x + 2*e))*cos(6*f*x + 6*e) + 18*(3*(f*x + e)*a^2*c*cos(2*f*x + 2*e) + (f*x
+ e)*a^2*c)*cos(4*f*x + 4*e) - 6*(a^2*c*sin(6*f*x + 6*e) + 3*a^2*c*sin(4*f*
x + 4*e) + 3*a^2*c*sin(2*f*x + 2*e))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e))) - 4*(a^2*c*sin(6*f*x + 6*e) + 3*a^2*c*sin(4*f*x + 4*e) + 3*a
^2*c*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
 - 6*(a^2*c*sin(6*f*x + 6*e) + 3*a^2*c*sin(4*f*x + 4*e) + 3*a^2*c*sin(2*f*x
 + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 6*(3*(f*x +
 e)*a^2*c*sin(4*f*x + 4*e) + 3*(f*x + e)*a^2*c*sin(2*f*x + 2*e) + a^2*c*cos
(4*f*x + 4*e) + a^2*c*cos(2*f*x + 2*e))*sin(6*f*x + 6*e) + 6*(9*(f*x + e)*a
^2*c*sin(2*f*x + 2*e) - a^2*c)*sin(4*f*x + 4*e) + 6*(a^2*c*cos(6*f*x + 6*e)
 + 3*a^2*c*cos(4*f*x + 4*e) + 3*a^2*c*cos(2*f*x + 2*e) + a^2*c)*sin(5/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*(a^2*c*cos(6*f*x + 6*e) + 3*a
^2*c*cos(4*f*x + 4*e) + 3*a^2*c*cos(2*f*x + 2*e) + a^2*c)*sin(3/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e))) + 6*(a^2*c*cos(6*f*x + 6*e) + 3*a^2*c*c
os(4*f*x + 4*e) + 3*a^2*c*cos(2*f*x + 2*e) + a^2*c)*sin(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((2*(3*cos(4*f*x + 4*e) + 3*
cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + cos(6*f*x + 6*e)^2 + 6*(3*cos(2*f*
x + 2*e) + 1)*cos(4*f*x + 4*e) + 9*cos(4*f*x + 4*e)^2 + 9*cos(2*f*x + 2*e)^
2 + 6*(sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + sin(6*f*x +
6*e)^2 + 9*sin(4*f*x + 4*e)^2 + 18*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 9*si
n(2*f*x + 2*e)^2 + 6*cos(2*f*x + 2*e) + 1)*f)
```

Fricas [A] time = 1.72729, size = 1152, normalized size = 6.06

$$\left[\frac{\left(a^2c\cos\left(fx+e\right)^2-5\,a^2c\cos\left(fx+e\right)-2\,a^2c\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)+3\left(a^2c\cos\left(fx+e\right)^3+a^2c\,\mathrm{co}\right.}{6\left(f\cos\left(fx+e\right)^3+f\cos\left(fx\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] [1/6*((a^2*c*cos(f*x + e)^2 - 5*a^2*c*cos(f*x + e) - 2*a^2*c)*sqrt((a*cos(f
*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x
```

```
+ e) + 3*(a^2*c*cos(f*x + e)^3 + a^2*c*cos(f*x + e)^2)*sqrt(-a*c)*log(1/2*(
a*c*cos(f*x + e)^4 - (cos(f*x + e)^3 + cos(f*x + e))*sqrt(-a*c)*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*
x + e) + a*c)/cos(f*x + e)^2))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2), 1/6*(
(a^2*c*cos(f*x + e)^2 - 5*a^2*c*cos(f*x + e) - 2*a^2*c)*sqrt((a*cos(f*x + e
) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) +
 6*(a^2*c*cos(f*x + e)^3 + a^2*c*cos(f*x + e)^2)*sqrt(a*c)*arctan(sqrt(a*c)
*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x
+ e))*cos(f*x + e)*sin(f*x + e)/(a*c*cos(f*x + e)^2 + a*c)))/(f*cos(f*x + e
)^3 + f*cos(f*x + e)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e))**(3/2),x)

[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(3/2),x, algorithm="giac"
)

[Out] Timed out
```

3.103 $\int (a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\,dx$

Optimal. Leaf size=139

$$-\frac{a^2c\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}+\frac{a^3c\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sec(e+fx)}}$$

```
[Out] (a^3*c*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
 c*Sec[e + f*x]]) - (a^2*c*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[c
 - c*Sec[e + f*x]]) - (a*c*(a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(2*f*Sq
rt[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.262803, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3906, 3905, 3475}

$$-\frac{a^2c\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}+\frac{a^3c\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (a^3*c*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
 c*Sec[e + f*x]]) - (a^2*c*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[c
 - c*Sec[e + f*x]]) - (a*c*(a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(2*f*Sq
rt[c - c*Sec[e + f*x]])
```

Rule 3906

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e + f*x]*(c + d*Csc[e + f*x]
)^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[c, Int[Sqrt[a
+ b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && GtQ[n, 1/2]
```

Rule 3905

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Dist[((-(a*c))^(m + 1/2)*Cot[e + f*x])/(Sqrt
[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int[Cot[e + f*x]^(2*m), x],
```

```
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IntegerQ[m + 1/2]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\,dx = -\frac{ac(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{c-c\sec(e+fx)}} + a\int (a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}$$
$$= -\frac{a^2c\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} - \frac{ac(a+a\sec(e+fx))^{3/2}\tan(e+f}{2f\sqrt{c-c\sec(e+fx)}}$$
$$= -\frac{a^2c\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} - \frac{ac(a+a\sec(e+fx))^{3/2}\tan(e+f}{2f\sqrt{c-c\sec(e+fx)}}$$
$$= \frac{a^3c\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{a^2c\sqrt{a+a\sec(e+fx)}\tan(e+}{f\sqrt{c-c\sec(e+fx)}}$$

Mathematica [C] time = 1.25926, size = 164, normalized size = 1.18

$$\frac{a^2e^{-i(e+fx)}\left(1+e^{2i(e+fx)}\right)\left(\cot\left(\frac{1}{2}(e+fx)\right)+i\right)\sec^2(e+fx)\sqrt{a(\sec(e+fx)+1)}\sqrt{c-c\sec(e+fx)}\left(-\log\left(1+e^{2i(e+fx)}\right)+4\right.}{4f\left(1+e^{i(e+fx)}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (a^2*(1 + E^((2*I)*(e + f*x)))*(I + Cot[(e + f*x)/2])*(1 + I*f*x + 4*Cos[e
+ f*x] + Cos[2*(e + f*x)]*(I*f*x - Log[1 + E^((2*I)*(e + f*x))]) - Log[1 +
E^((2*I)*(e + f*x))])*Sec[e + f*x]^2*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*
Sec[e + f*x]])/(4*E^(I*(e + f*x))*(1 + E^(I*(e + f*x)))*f)
```

Maple [A] time = 0.307, size = 179, normalized size = 1.3

$$-\frac{a^2}{2f\cos(fx+e)\sin(fx+e)}\left(2\ln\left(\frac{1-\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)(\cos(fx+e))^2+2\ln\left(-\frac{-1+\cos(fx+e)+\sin(}{\sin(fx+e)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/2/f*a^2*(2*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2+2*ln(-(
-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-2*ln(2/(1+cos(f*x+e)))*c
os(f*x+e)^2-3*cos(f*x+e)^2-4*cos(f*x+e)-1)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(
1/2)*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)/cos(f*x+e)/sin(f*x+e)
```

Maxima [B] time = 1.84273, size = 959, normalized size = 6.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] -((f*x + e)*a^2*cos(4*f*x + 4*e)^2 + 4*(f*x + e)*a^2*cos(2*f*x + 2*e)^2 + (
f*x + e)*a^2*sin(4*f*x + 4*e)^2 + 4*(f*x + e)*a^2*sin(2*f*x + 2*e)^2 + 4*(f
*x + e)*a^2*cos(2*f*x + 2*e) + (f*x + e)*a^2 + 2*a^2*sin(2*f*x + 2*e) - (a^
2*cos(4*f*x + 4*e)^2 + 4*a^2*cos(2*f*x + 2*e)^2 + a^2*sin(4*f*x + 4*e)^2 +
4*a^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*a^2*sin(2*f*x + 2*e)^2 + 4*a^2*
cos(2*f*x + 2*e) + a^2 + 2*(2*a^2*cos(2*f*x + 2*e) + a^2)*cos(4*f*x + 4*e))
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) + 2*(2*(f*x + e)*a^2*cos(2
*f*x + 2*e) + (f*x + e)*a^2 + a^2*sin(2*f*x + 2*e))*cos(4*f*x + 4*e) - 4*(a
^2*sin(4*f*x + 4*e) + 2*a^2*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) - 4*(a^2*sin(4*f*x + 4*e) + 2*a^2*sin(2*f*x + 2*e))
*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*(2*(f*x + e)*a^2*
sin(2*f*x + 2*e) - a^2*cos(2*f*x + 2*e))*sin(4*f*x + 4*e) + 4*(a^2*cos(4*f*
x + 4*e) + 2*a^2*cos(2*f*x + 2*e) + a^2)*sin(3/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e))) + 4*(a^2*cos(4*f*x + 4*e) + 2*a^2*cos(2*f*x + 2*e) + a^2
)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((2
*(2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + cos(4*f*x + 4*e)^2 + 4*cos(2*f
*x + 2*e)^2 + sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*
sin(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*f)
```

Fricas [A] time = 1.64082, size = 1049, normalized size = 7.55

$$\left[\frac{\left(5\,a^2\cos\left(fx+e\right)+a^2\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)+\left(a^2\cos\left(fx+e\right)^2+a^2\cos\left(fx+e\right)\right)\sqrt{-ac}\log\left(\frac{ac\cos(f}{}\right.}{2\left(f\cos\left(fx+e\right)^2+f\cos\left(fx+e\right)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] [1/2*((5*a^2*cos(f*x + e) + a^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sq
rt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) + (a^2*cos(f*x + e)^2 +
a^2*cos(f*x + e))*sqrt(-a*c)*log(1/2*(a*c*cos(f*x + e)^4 - (cos(f*x + e)^3
+ cos(f*x + e))*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*
cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) + a*c)/cos(f*x + e)^2))/(f*cos
(f*x + e)^2 + f*cos(f*x + e)), 1/2*((5*a^2*cos(f*x + e) + a^2)*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x
 + e) + 2*(a^2*cos(f*x + e)^2 + a^2*cos(f*x + e))*sqrt(a*c)*arctan(sqrt(a*c
)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x
 + e))*cos(f*x + e)*sin(f*x + e)/(a*c*cos(f*x + e)^2 + a*c)))/(f*cos(f*x +
e)^2 + f*cos(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

3.104 $\int \frac{(a+a\sec(e+fx))^{5/2}}{\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal. Leaf size=152

$$\frac{a^3\tan(e+fx)\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{4a^3\tan(e+fx)\log(1-\sec(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{a^3\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] (a^3*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c
*Sec[e + f*x]]) + (4*a^3*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*
Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) + (a^3*Sec[e + f*x]*Tan[e + f*x])/(
f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.113735, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3912, 72}

$$\frac{a^3\tan(e+fx)\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{4a^3\tan(e+fx)\log(1-\sec(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{a^3\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)/Sqrt[c - c*Sec[e + f*x]],x]
```


```
[Out] (a^3*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c
*Sec[e + f*x]]) + (4*a^3*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*
Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) + (a^3*Sec[e + f*x]*Tan[e + f*x])/(
f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```


Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
x)^(n - 1/2))/x, x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
```

```
/; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{(ac\tan(e+fx))\operatorname{Subst}\left(\int \frac{(a+ax)^2}{x(c-cx)}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$
$$= -\frac{(ac\tan(e+fx))\operatorname{Subst}\left(\int \left(-\frac{a^2}{c}-\frac{4a^2}{c(-1+x)}+\frac{a^2}{cx}\right)dx,x,\sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$
$$= \frac{a^3\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}+\frac{4a^3\log(1-\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}+\frac{}{f\sqrt{a}}$$

Mathematica [C] time = 6.76663, size = 292, normalized size = 1.92

$$\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\sec(e+fx)(a(\sec(e+fx)+1))^{5/2}\sqrt{(\cos(e+fx)+1)\sec(e+fx)}}{f(\sec(e+fx)+1)^{5/2}\sqrt{c-c\sec(e+fx)}}+\frac{\sqrt{2}e^{\frac{1}{2}i(e+fx)}\sqrt{\frac{\left(1+e^{i(e+fx)}\right)^2}{1+e^{2i(e+fx)}}}\left(8\log\left(1-e^{i(}\right.\right.}{f\left(1+e^{i(e}\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)/Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (Sqrt[2]*E^((I/2)*(e + f*x))*Sqrt[(1 + E^(I*(e + f*x)))^2/(1 + E^((2*I)*(e
+ f*x)))]*((-I)*f*x + 8*Log[1 - E^(I*(e + f*x))] - 3*Log[1 + E^((2*I)*(e +
f*x))])*Sqrt[Sec[e + f*x]]*(a*(1 + Sec[e + f*x]))^(5/2)*Sin[e/2 + (f*x)/2])
/((1 + E^(I*(e + f*x)))*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)))]*f*(
1 + Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]]) + (Sec[e + f*x]*Sqrt[(1 +
 Cos[e + f*x])*Sec[e + f*x]]*(a*(1 + Sec[e + f*x]))^(5/2)*Tan[e/2 + (f*x)/2
])/(f*(1 + Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] time = 0.319, size = 183, normalized size = 1.2

$$\frac{a^2}{f\sin(fx+e)c}\left(3\cos(fx+e)\ln\left(\frac{1-\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)+3\cos(fx+e)\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx}{\sin(fx+e)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/f*a^2*(3*cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+3*cos(f*x+e)
*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-8*cos(f*x+e)*ln(-(-1+cos(f*x+e)
)/sin(f*x+e))+cos(f*x+e)*ln(2/(1+cos(f*x+e)))-cos(f*x+e)-1)*(1/cos(f*x+e)*a
*(1+cos(f*x+e)))^(1/2)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)/sin(f*x+e)/c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(a^2\sec\left(fx+e\right)^2+2\,a^2\sec\left(fx+e\right)+a^2\right)\sqrt{a\sec\left(fx+e\right)+a}\sqrt{-c\sec\left(fx+e\right)+c}}{c\sec\left(fx+e\right)-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-(a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*sqrt(a*sec(f*x +
e) + a)*sqrt(-c*sec(f*x + e) + c)/(c*sec(f*x + e) - c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

3.105 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=96

$$\frac{a^3\tan(e+fx)\log(\cos(e+fx))}{cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{4a^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}$$

```
[Out] (-4*a^3*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2
)) + (a^3*Log[Cos[e + f*x]]*Tan[e + f*x])/(c*f*Sqrt[a + a*Sec[e + f*x]]*Sqr
t[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.179263, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3910, 3905, 3475}

$$\frac{a^3\tan(e+fx)\log(\cos(e+fx))}{cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{4a^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-4*a^3*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2
)) + (a^3*Log[Cos[e + f*x]]*Tan[e + f*x])/(c*f*Sqrt[a + a*Sec[e + f*x]]*Sqr
t[c - c*Sec[e + f*x]])
```

Rule 3910

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(5/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[(-8*a^3*Cot[e + f*x]*(c + d*Csc[e + f
*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[a^2/c^2, Int[Sqrt
[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3905

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Dist[((-(a*c))^(m + 1/2)*Cot[e + f*x])/(Sqrt
[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int[Cot[e + f*x]^(2*m), x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
```

```
2, 0] && IntegerQ[m + 1/2]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{3/2}}\,dx &= -\frac{4a^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}+\frac{a^2\int\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx}{c^2}\\ &= -\frac{4a^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}-\frac{\left(a^3\tan(e+fx)\right)\int\tan(e+fx)\,dx}{c\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\ &= -\frac{4a^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}+\frac{a^3\log(\cos(e+fx))\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\end{aligned}$$

Mathematica [C] time = 1.2202, size = 111, normalized size = 1.16

$$\frac{a^2\tan\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}\left(-\log\left(1+e^{2i(e+fx)}\right)+\left(\log\left(1+e^{2i(e+fx)}\right)-ifx\right)\cos(e+fx)+ifx-4\right)}{cf(\cos(e+fx)-1)\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (a^2*(-4 + I*f*x - Log[1 + E^((2*I)*(e + f*x))] + Cos[e + f*x]*((-I)*f*x +
Log[1 + E^((2*I)*(e + f*x))]))*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])
/(c*f*(-1 + Cos[e + f*x])*Sqrt[c - c*Sec[e + f*x]])
```

Maple [B] time = 0.27, size = 237, normalized size = 2.5

$$-\frac{a^2\left(-1+\cos\left(fx+e\right)\right)}{f\cos\left(fx+e\right)\sin\left(fx+e\right)}\left(\cos\left(fx+e\right)\ln\left(\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)+\cos\left(fx+e\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)+}{\sin\left(fx+\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x)
```

```
[Out] -1/f*a^2*(-1+cos(f*x+e))*(cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e
))+cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-cos(f*x+e)*ln(2/(1
+cos(f*x+e)))-ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-ln(-(-1+cos(f*x+e)+s
in(f*x+e))/sin(f*x+e))-2*cos(f*x+e)+ln(2/(1+cos(f*x+e)))-2)*(1/cos(f*x+e)*a
*(1+cos(f*x+e)))^(1/2)/cos(f*x+e)/(c*(-1+cos(f*x+e))/cos(f*x+e))^(3/2)/sin(
f*x+e)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [B] time = 1.41231, size = 1062, normalized size = 11.06

$$\left[\frac{\left(a^2c\cos\left(fx+e\right)-a^2c\right)\sqrt{-\frac{a}{c}}\log\left(\frac{a\cos\left(fx+e\right)^4-\left(\cos\left(fx+e\right)^3+\cos\left(fx+e\right)\right)\sqrt{-\frac{a}{c}}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)+a}{2\cos\left(fx+e\right)^2}\right)\sin\left(fx+e\right)+4}{2\left(c^2f\cos\left(fx+e\right)-c^2f\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] [1/2*((a^2*c*cos(f*x + e) - a^2*c)*sqrt(-a/c)*log(1/2*(a*cos(f*x + e)^4 - (
cos(f*x + e)^3 + cos(f*x + e))*sqrt(-a/c)*sqrt((a*cos(f*x + e) + a)/cos(f*x
 + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) + a)/cos(f*x +
e)^2)*sin(f*x + e) + 4*(a^2*cos(f*x + e)^2 + a^2*cos(f*x + e))*sqrt((a*cos(
```

```
f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((c^2*
f*cos(f*x + e) - c^2*f)*sin(f*x + e)), ((a^2*c*cos(f*x + e) - a^2*c)*sqrt(a
/c)*arctan(sqrt(a/c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*
x + e) - c)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(a*cos(f*x + e)^2 + a))
*sin(f*x + e) + 2*(a^2*cos(f*x + e)^2 + a^2*cos(f*x + e))*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((c^2*f*cos
(f*x + e) - c^2*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

3.106 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=100

$$\frac{a^3\tan(e+fx)\log(1-\cos(e+fx))}{c^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{2a^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}$$

```
[Out] (-2*a^3*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2
)) + (a^3*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c^2*f*Sqrt[a + a*Sec[e + f*x
]]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.180779, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3910, 3911, 31}

$$\frac{a^3\tan(e+fx)\log(1-\cos(e+fx))}{c^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{2a^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-2*a^3*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2
)) + (a^3*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c^2*f*Sqrt[a + a*Sec[e + f*x
]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3910

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(5/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[(-8*a^3*Cot[e + f*x]*(c + d*Csc[e + f
*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[a^2/c^2, Int[Sqrt
[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3911

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> -Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((b + a*x)^(m - 1/2)*(d + c*x
)^(n - 1/2))/x^(m + n), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && EqQ
```

```
[m + n, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{2a^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}} + \frac{a^2\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx}{c^2}$$

$$= -\frac{2a^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}} + \frac{\left(a^3\tan(e+fx)\right)\operatorname{Subst}\left(\int \frac{1}{-c+cx}\,dx, x, \cos(\right.}{cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+f}}$$

$$= -\frac{2a^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}} + \frac{a^3\log(1-\cos(e+fx))\tan(e+fx)}{c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

Mathematica [C] time = 1.36979, size = 155, normalized size = 1.55

$$\frac{a^2\tan\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}\left(6\log\left(1-e^{i(e+fx)}\right)+\left(-8\log\left(1-e^{i(e+fx)}\right)+4ifx-8\right)\cos(e+fx)+\left(2\log\left(1-\right.\right.\right.}{2c^2f(\cos(e+fx)-1)^2\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (a^2*(4 - (3*I)*f*x + Cos[e + f*x]*(-8 + (4*I)*f*x - 8*Log[1 - E^(I*(e + f*
x))]) + 6*Log[1 - E^(I*(e + f*x))] + Cos[2*(e + f*x)]*((-I)*f*x + 2*Log[1 -
 E^(I*(e + f*x))]))*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(2*c^2*f*(
-1 + Cos[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]])
```

Maple [B] time = 0.266, size = 229, normalized size = 2.3

$$-\frac{a^2\left(-1+\cos\left(fx+e\right)\right)}{2f\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^2}\left(4\ln\left(-\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right)\left(\cos\left(fx+e\right)\right)^2-2\ln\left(2\left(1+\cos\left(fx+e\right)\right)^{-1}\right)\left(\cos\left(fx+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x)
```

```
[Out] -1/2/f*a^2*(-1+cos(f*x+e))*(4*ln(-(-1+cos(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-
2*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^2-8*cos(f*x+e)*ln(-(-1+cos(f*x+e))/sin(f*
x+e))-3*cos(f*x+e)^2+4*cos(f*x+e)*ln(2/(1+cos(f*x+e)))+4*ln(-(-1+cos(f*x+e)
)/sin(f*x+e))-2*cos(f*x+e)-2*ln(2/(1+cos(f*x+e)))+1)*(1/cos(f*x+e)*a*(1+cos
(f*x+e)))^(1/2)/sin(f*x+e)/cos(f*x+e)^2/(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2
)
```

Maxima [A] time = 1.51431, size = 188, normalized size = 1.88

$$\frac{\frac{4\sqrt{-a}a^2\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^{\frac{5}{2}}}-\frac{2\sqrt{-a}a^2\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{c^{\frac{5}{2}}}-\frac{\left(\sqrt{-a}a^2\sqrt{c}-\frac{2\sqrt{-a}a^2\sqrt{c}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)^4}{c^3\sin(fx+e)^4}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] 1/2*(4*sqrt(-a)*a^2*log(sin(f*x + e)/(cos(f*x + e) + 1))/c^(5/2) - 2*sqrt(-
a)*a^2*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(5/2) - (sqrt(-a)*a^2
*sqrt(c) - 2*sqrt(-a)*a^2*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos
(f*x + e) + 1)^4/(c^3*sin(f*x + e)^4))/f
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(a^2\sec\left(fx+e\right)^2+2\,a^2\sec\left(fx+e\right)+a^2\right)\sqrt{a\sec\left(fx+e\right)+a}\sqrt{-c\sec\left(fx+e\right)+c}}{c^3\sec\left(fx+e\right)^3-3\,c^3\sec\left(fx+e\right)^2+3\,c^3\sec\left(fx+e\right)-c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral(-(a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*sqrt(a*sec(f*x +
e) + a)*sqrt(-c*sec(f*x + e) + c)/(c^3*sec(f*x + e)^3 - 3*c^3*sec(f*x + e)^
2 + 3*c^3*sec(f*x + e) - c^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

3.107 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=148

$$-\frac{a^3\tan(e+fx)}{c^2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}+\frac{a^3\tan(e+fx)\log(1-\cos(e+fx))}{c^3f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{4a^3\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\,\mathrm{se}}$$

```
[Out] (-4*a^3*Tan[e + f*x])/(3*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(7
/2)) - (a^3*Tan[e + f*x])/(c^2*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*
x])^(3/2)) + (a^3*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c^3*f*Sqrt[a + a*Sec
[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.278129, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3910, 3907, 3911, 31}

$$-\frac{a^3\tan(e+fx)}{c^2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}+\frac{a^3\tan(e+fx)\log(1-\cos(e+fx))}{c^3f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{4a^3\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\,\mathrm{se}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(7/2),x]
```

```
[Out] (-4*a^3*Tan[e + f*x])/(3*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(7
/2)) - (a^3*Tan[e + f*x])/(c^2*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*
x])^(3/2)) + (a^3*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c^3*f*Sqrt[a + a*Sec
[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3910

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(5/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[(-8*a^3*Cot[e + f*x]*(c + d*Csc[e + f
*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[a^2/c^2, Int[Sqrt
[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3907

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_), x_Symbol] :> Simp[(-2*a*Cot[e + f*x]*(c + d*Csc[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[1/c, Int[Sqrt[a + b*Cs
```

```
c[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3911

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> -Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((b + a*x)^(m - 1/2)*(d + c*x
)^(n - 1/2))/x^(m + n), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && EqQ
[m + n, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = -\frac{4a^3\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}}+\frac{a^2\int\frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx}{c^2}$$

$$= -\frac{4a^3\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}}-\frac{a^3\tan(e+fx)}{c^2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/}}$$

$$= -\frac{4a^3\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}}-\frac{a^3\tan(e+fx)}{c^2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/}}$$

$$= -\frac{4a^3\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}}-\frac{a^3\tan(e+fx)}{c^2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/}}$$

Mathematica [C] time = 2.61524, size = 202, normalized size = 1.36

$$\frac{a^2\tan\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}\left(-60\log\left(1-e^{i(e+fx)}\right)-3ifx\cos(3(e+fx))+6i\left(6i\log\left(1-e^{i(e+fx)}\right)+3fx+5i\right.\right.}{12c^3f(\cos(e+fx)-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(7/2),x]
```

```
[Out] (a^2*(-58 + (30*I)*f*x - (3*I)*f*x*Cos[3*(e + f*x)] + (6*I)*Cos[2*(e + f*x)
]*(5*I + 3*f*x + (6*I)*Log[1 - E^(I*(e + f*x))]) - 60*Log[1 - E^(I*(e + f*x
))] + 6*Cos[3*(e + f*x)]*Log[1 - E^(I*(e + f*x))] + 9*Cos[e + f*x]*(8 - (5*
I)*f*x + 10*Log[1 - E^(I*(e + f*x))]))*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e +
f*x)/2])/(12*c^3*f*(-1 + Cos[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]])
```

Maple [B] time = 0.277, size = 281, normalized size = 1.9

$$-\frac{a^{2}\left(-1+\cos\left(fx+e\right)\right)}{3\,f\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^{3}}\left(6\,\left(\cos\left(fx+e\right)\right)^{3}\ln\left(-\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-3\,\left(\cos\left(fx+e\right)\right)^{3}\ln\left(2\,\left(1+\cos\left(fx+e\right)\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(7/2),x)
```

```
[Out] -1/3/f*a^2*(-1+cos(f*x+e))*(6*cos(f*x+e)^3*ln(-(-1+cos(f*x+e))/sin(f*x+e))-
3*cos(f*x+e)^3*ln(2/(1+cos(f*x+e)))-18*ln(-(-1+cos(f*x+e))/sin(f*x+e))*cos(
f*x+e)^2-5*cos(f*x+e)^3+9*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^2+18*cos(f*x+e)*l
n(-(-1+cos(f*x+e))/sin(f*x+e))-9*cos(f*x+e)*ln(2/(1+cos(f*x+e)))-6*ln(-(-1+
cos(f*x+e))/sin(f*x+e))+3*cos(f*x+e)+3*ln(2/(1+cos(f*x+e)))-2)*(1/cos(f*x+e
)*a*(1+cos(f*x+e)))^(1/2)/sin(f*x+e)/cos(f*x+e)^3/(c*(-1+cos(f*x+e))/cos(f*
x+e))^(7/2)
```

Maxima [B] time = 11.1486, size = 5046, normalized size = 34.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="maxim
a")
```

```
[Out] -1/3*(3*(f*x + e)*a^2*cos(6*f*x + 6*e)^2 + 675*(f*x + e)*a^2*cos(4*f*x + 4*
e)^2 + 675*(f*x + e)*a^2*cos(2*f*x + 2*e)^2 + 108*(f*x + e)*a^2*cos(5/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 1200*(f*x + e)*a^2*cos(3/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 108*(f*x + e)*a^2*cos(1/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 3*(f*x + e)*a^2*sin(6*f*x +
6*e)^2 + 675*(f*x + e)*a^2*sin(4*f*x + 4*e)^2 + 675*(f*x + e)*a^2*sin(2*f*x
```

```
 + 2*e)^2 + 108*(f*x + e)*a^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e)))^2 + 1200*(f*x + e)*a^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e)))^2 + 108*(f*x + e)*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e)))^2 + 90*(f*x + e)*a^2*cos(2*f*x + 2*e) + 3*(f*x + e)*a^2 - 72*a^2*s
in(2*f*x + 2*e) - 6*(a^2*cos(6*f*x + 6*e)^2 + 225*a^2*cos(4*f*x + 4*e)^2 +
225*a^2*cos(2*f*x + 2*e)^2 + 36*a^2*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e)))^2 + 400*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
)))^2 + 36*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + a^2
*sin(6*f*x + 6*e)^2 + 225*a^2*sin(4*f*x + 4*e)^2 + 450*a^2*sin(4*f*x + 4*e)
*sin(2*f*x + 2*e) + 225*a^2*sin(2*f*x + 2*e)^2 + 36*a^2*sin(5/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 400*a^2*sin(3/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e)))^2 + 36*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e)))^2 + 30*a^2*cos(2*f*x + 2*e) + a^2 + 2*(15*a^2*cos(4*f*x + 4*e)
 + 15*a^2*cos(2*f*x + 2*e) + a^2)*cos(6*f*x + 6*e) + 30*(15*a^2*cos(2*f*x +
 2*e) + a^2)*cos(4*f*x + 4*e) - 12*(a^2*cos(6*f*x + 6*e) + 15*a^2*cos(4*f*x
 + 4*e) + 15*a^2*cos(2*f*x + 2*e) - 20*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) - 6*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e))) + a^2)*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 40*(a^
2*cos(6*f*x + 6*e) + 15*a^2*cos(4*f*x + 4*e) + 15*a^2*cos(2*f*x + 2*e) - 6*
a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + a^2)*cos(3/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 12*(a^2*cos(6*f*x + 6*e) + 15*a
^2*cos(4*f*x + 4*e) + 15*a^2*cos(2*f*x + 2*e) + a^2)*cos(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))) + 30*(a^2*sin(4*f*x + 4*e) + a^2*sin(2*f*x +
 2*e))*sin(6*f*x + 6*e) - 12*(a^2*sin(6*f*x + 6*e) + 15*a^2*sin(4*f*x + 4*e
) + 15*a^2*sin(2*f*x + 2*e) - 20*a^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e))) - 6*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
)*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 40*(a^2*sin(6*f*x
+ 6*e) + 15*a^2*sin(4*f*x + 4*e) + 15*a^2*sin(2*f*x + 2*e) - 6*a^2*sin(1/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) - 12*(a^2*sin(6*f*x + 6*e) + 15*a^2*sin(4*f*x + 4*e
) + 15*a^2*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e))))*arctan2(sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 1) + 6*(15*(f*x + e)*a^2*
cos(4*f*x + 4*e) + 15*(f*x + e)*a^2*cos(2*f*x + 2*e) + (f*x + e)*a^2 - 12*a
^2*sin(4*f*x + 4*e) - 12*a^2*sin(2*f*x + 2*e))*cos(6*f*x + 6*e) + 90*(15*(f
*x + e)*a^2*cos(2*f*x + 2*e) + (f*x + e)*a^2)*cos(4*f*x + 4*e) - 6*(6*(f*x
+ e)*a^2*cos(6*f*x + 6*e) + 90*(f*x + e)*a^2*cos(4*f*x + 4*e) + 90*(f*x + e
)*a^2*cos(2*f*x + 2*e) - 120*(f*x + e)*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) - 36*(f*x + e)*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))) + 6*(f*x + e)*a^2 + 5*a^2*sin(6*f*x + 6*e) + 3*a^2*sin(4*
f*x + 4*e) + 3*a^2*sin(2*f*x + 2*e) + 16*a^2*sin(3/2*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e))))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
) - 4*(30*(f*x + e)*a^2*cos(6*f*x + 6*e) + 450*(f*x + e)*a^2*cos(4*f*x + 4*
e) + 450*(f*x + e)*a^2*cos(2*f*x + 2*e) - 180*(f*x + e)*a^2*cos(1/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 30*(f*x + e)*a^2 + 29*a^2*sin(6*f*x
```

+ 6*e) + 75*a^2*sin(4*f*x + 4*e) + 75*a^2*sin(2*f*x + 2*e) - 24*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 6*(6*(f*x + e)*a^2*cos(6*f*x + 6*e) + 90*(f*x + e)*a^2*cos(4*f*x + 4*e) + 90*(f*x + e)*a^2*cos(2*f*x + 2*e) + 6*(f*x + e)*a^2 + 5*a^2*sin(6*f*x + 6*e) + 3*a^2*sin(4*f*x + 4*e) + 3*a^2*sin(2*f*x + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 18*(5*(f*x + e)*a^2*sin(4*f*x + 4*e) + 5*(f*x + e)*a^2*sin(2*f*x + 2*e) + 4*a^2*cos(4*f*x + 4*e) + 4*a^2*cos(2*f*x + 2*e))*sin(6*f*x + 6*e) + 18*(75*(f*x + e)*a^2*sin(2*f*x + 2*e) - 4*a^2)*sin(4*f*x + 4*e) - 6*(6*(f*x + e)*a^2*sin(6*f*x + 6*e) + 90*(f*x + e)*a^2*sin(4*f*x + 4*e) + 90*(f*x + e)*a^2*sin(2*f*x + 2*e) - 120*(f*x + e)*a^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 36*(f*x + e)*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 5*a^2*cos(6*f*x + 6*e) - 3*a^2*cos(4*f*x + 4*e) - 3*a^2*cos(2*f*x + 2*e) - 16*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 5*a^2)*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*(30*(f*x + e)*a^2*sin(6*f*x + 6*e) + 450*(f*x + e)*a^2*sin(4*f*x + 4*e) + 450*(f*x + e)*a^2*sin(2*f*x + 2*e) - 180*(f*x + e)*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 29*a^2*cos(6*f*x + 6*e) - 75*a^2*cos(4*f*x + 4*e) - 75*a^2*cos(2*f*x + 2*e) + 24*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 29*a^2)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 6*(6*(f*x + e)*a^2*sin(6*f*x + 6*e) + 90*(f*x + e)*a^2*sin(4*f*x + 4*e) + 90*(f*x + e)*a^2*sin(2*f*x + 2*e) - 5*a^2*cos(6*f*x + 6*e) - 3*a^2*cos(4*f*x + 4*e) - 3*a^2*cos(2*f*x + 2*e) - 5*a^2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((c^4*cos(6*f*x + 6*e)^2 + 225*c^4*cos(4*f*x + 4*e)^2 + 225*c^4*cos(2*f*x + 2*e)^2 + 36*c^4*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 400*c^4*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 36*c^4*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + c^4*sin(6*f*x + 6*e)^2 + 225*c^4*sin(4*f*x + 4*e)^2 + 450*c^4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 225*c^4*sin(2*f*x + 2*e)^2 + 36*c^4*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 400*c^4*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 36*c^4*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 30*c^4*cos(2*f*x + 2*e) + c^4 + 2*(15*c^4*cos(4*f*x + 4*e) + 15*c^4*cos(2*f*x + 2*e) + c^4)*cos(6*f*x + 6*e) + 30*(15*c^4*cos(2*f*x + 2*e) + c^4)*cos(4*f*x + 4*e) - 12*(c^4*cos(6*f*x + 6*e) + 15*c^4*cos(4*f*x + 4*e) + 15*c^4*cos(2*f*x + 2*e) - 20*c^4*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 6*c^4*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c^4)*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 40*(c^4*cos(6*f*x + 6*e) + 15*c^4*cos(4*f*x + 4*e) + 15*c^4*cos(2*f*x + 2*e) - 6*c^4*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c^4)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 12*(c^4*cos(6*f*x + 6*e) + 15*c^4*cos(4*f*x + 4*e) + 15*c^4*cos(2*f*x + 2*e) + c^4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 30*(c^4*sin(4*f*x + 4*e) + c^4*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) - 12*(c^4*sin(6*f*x + 6*e) + 15*c^4*sin(4*f*x + 4*e) + 15*c^4*sin(2*f*x + 2*e) - 20*c^4*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 6*c^4*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*

```
e))))*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 40*(c^4*sin(6*
f*x + 6*e) + 15*c^4*sin(4*f*x + 4*e) + 15*c^4*sin(2*f*x + 2*e) - 6*c^4*sin(
1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e))) - 12*(c^4*sin(6*f*x + 6*e) + 15*c^4*sin(4*f*x +
 4*e) + 15*c^4*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))))*f)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2\sec\left(fx+e\right)^2+2\,a^2\sec\left(fx+e\right)+a^2\right)\sqrt{a\sec\left(fx+e\right)+a}\sqrt{-c\sec\left(fx+e\right)+c}}{c^4\sec\left(fx+e\right)^4-4\,c^4\sec\left(fx+e\right)^3+6\,c^4\sec\left(fx+e\right)^2-4\,c^4\sec\left(fx+e\right)+c^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="frica
s")
```

```
[Out] integral((a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*sqrt(a*sec(f*x + e
) + a)*sqrt(-c*sec(f*x + e) + c)/(c^4*sec(f*x + e)^4 - 4*c^4*sec(f*x + e)^3
 + 6*c^4*sec(f*x + e)^2 - 4*c^4*sec(f*x + e) + c^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

3.108 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=194

$$-\frac{a^3\tan(e+fx)}{c^3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}-\frac{a^3\tan(e+fx)}{2c^2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}+\frac{a^3\tan(e+fx)\log(1}{c^4f\sqrt{a\sec(e+fx)+a}\sqrt{}}$$

```
[Out] -((a^3*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(9/2)
)) - (a^3*Tan[e + f*x])/(2*c^2*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*
x])^(5/2)) - (a^3*Tan[e + f*x])/(c^3*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[
e + f*x])^(3/2)) + (a^3*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c^4*f*Sqrt[a +
 a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.375536, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3910, 3907, 3911, 31}

$$-\frac{a^3\tan(e+fx)}{c^3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}-\frac{a^3\tan(e+fx)}{2c^2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}+\frac{a^3\tan(e+fx)\log(1}{c^4f\sqrt{a\sec(e+fx)+a}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(9/2),x]
```

```
[Out] -((a^3*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(9/2)
)) - (a^3*Tan[e + f*x])/(2*c^2*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*
x])^(5/2)) - (a^3*Tan[e + f*x])/(c^3*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[
e + f*x])^(3/2)) + (a^3*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c^4*f*Sqrt[a +
 a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3910

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(5/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[(-8*a^3*Cot[e + f*x]*(c + d*Csc[e + f
*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[a^2/c^2, Int[Sqrt
[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3907

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_), x_Symbol] :> Simp[(-2*a*Cot[e + f*x]*(c + d*Csc[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[1/c, Int[Sqrt[a + b*Cs
c[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3911

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> -Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((b + a*x)^(m - 1/2)*(d + c*x
)^(n - 1/2))/x^(m + n), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && EqQ
[m + n, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}\,dx = -\frac{a^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{9/2}} + \frac{a^2\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx}{c^2}$$

$$= -\frac{a^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{9/2}} - \frac{a^3\tan(e+fx)}{2c^2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}$$

$$= -\frac{a^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{9/2}} - \frac{a^3\tan(e+fx)}{2c^2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}$$

$$= -\frac{a^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{9/2}} - \frac{a^3\tan(e+fx)}{2c^2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}$$

$$= -\frac{a^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{9/2}} - \frac{a^3\tan(e+fx)}{2c^2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}$$

Mathematica [C] time = 5.43484, size = 285, normalized size = 1.47

$$\frac{\sin^9\left(\frac{1}{2}(e+fx)\right)\sec^{\frac{9}{2}}(e+fx)(a(\sec(e+fx)+1))^{5/2}\left(\frac{(89\cos(e+fx)-60\cos(2(e+fx))+23\cos(3(e+fx))-6\cos(4(e+fx))-54)\csc^8\left(\frac{1}{2}(e+fx)\right)\mathrm{s}}{8f}\right.}{(\sec(e+fx)+1)^{5/2}(c-c\sec(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(9/2),x]
```

```
[Out] (Sec[e + f*x]^(9/2)*(a*(1 + Sec[e + f*x]))^(5/2)*((16*Sqrt[2]*E^((I/2)*(e +
 f*x))*Sqrt[(1 + E^(I*(e + f*x)))^2/(1 + E^((2*I)*(e + f*x)))]*((-I)*f*x +
2*Log[1 - E^(I*(e + f*x))]))/((1 + E^(I*(e + f*x)))*Sqrt[E^(I*(e + f*x))/(1
 + E^((2*I)*(e + f*x)))]*f) + ((-54 + 89*Cos[e + f*x] - 60*Cos[2*(e + f*x)]
 + 23*Cos[3*(e + f*x)] - 6*Cos[4*(e + f*x)])*Csc[(e + f*x)/2]^8*Sec[(e + f*
x)/2]*Sqrt[Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]])/(8*f))*Sin[(e + f*x)/2]^9)
/((1 + Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(9/2))
```

Maple [A] time = 0.282, size = 353, normalized size = 1.8

$$-\frac{a^2\left(-1+\cos\left(fx+e\right)\right)}{16\,f\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^4}\left(32\left(\cos\left(fx+e\right)\right)^4\ln\left(-\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-16\left(\cos\left(fx+e\right)\right)^4\ln\left(2\left(1+\cos\left(fx\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(9/2),x)
```

```
[Out] -1/16/f*a^2*(-1+cos(f*x+e))*(32*cos(f*x+e)^4*ln(-(-1+cos(f*x+e))/sin(f*x+e)
)-16*cos(f*x+e)^4*ln(2/(1+cos(f*x+e)))-29*cos(f*x+e)^4-128*cos(f*x+e)^3*ln(
-(-1+cos(f*x+e))/sin(f*x+e))+64*cos(f*x+e)^3*ln(2/(1+cos(f*x+e)))+20*cos(f*
x+e)^3+192*ln(-(-1+cos(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-96*ln(2/(1+cos(f*x+
e)))*cos(f*x+e)^2+10*cos(f*x+e)^2-128*cos(f*x+e)*ln(-(-1+cos(f*x+e))/sin(f*
x+e))+64*cos(f*x+e)*ln(2/(1+cos(f*x+e)))-28*cos(f*x+e)+32*ln(-(-1+cos(f*x+e
))/sin(f*x+e))-16*ln(2/(1+cos(f*x+e)))+11)*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^
(1/2)/sin(f*x+e)/cos(f*x+e)^4/(c*(-1+cos(f*x+e))/cos(f*x+e))^(9/2)
```

Maxima [B] time = 76.4326, size = 8281, normalized size = 42.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(9/2),x, algorithm="maxim
a")
```

```
[Out] -((f*x + e)*a^2*cos(8*f*x + 8*e)^2 + 784*(f*x + e)*a^2*cos(6*f*x + 6*e)^2 +
 4900*(f*x + e)*a^2*cos(4*f*x + 4*e)^2 + 784*(f*x + e)*a^2*cos(2*f*x + 2*e)
^2 + 64*(f*x + e)*a^2*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^
2 + 3136*(f*x + e)*a^2*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
^2 + 3136*(f*x + e)*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
)^2 + 64*(f*x + e)*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
^2 + (f*x + e)*a^2*sin(8*f*x + 8*e)^2 + 784*(f*x + e)*a^2*sin(6*f*x + 6*e)^
2 + 4900*(f*x + e)*a^2*sin(4*f*x + 4*e)^2 + 784*(f*x + e)*a^2*sin(2*f*x + 2
*e)^2 + 64*(f*x + e)*a^2*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
))^2 + 3136*(f*x + e)*a^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
)))^2 + 3136*(f*x + e)*a^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e)))^2 + 64*(f*x + e)*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
)))^2 + 56*(f*x + e)*a^2*cos(2*f*x + 2*e) + (f*x + e)*a^2 - 46*a^2*sin(2*f*
x + 2*e) - 2*(a^2*cos(8*f*x + 8*e)^2 + 784*a^2*cos(6*f*x + 6*e)^2 + 4900*a^
2*cos(4*f*x + 4*e)^2 + 784*a^2*cos(2*f*x + 2*e)^2 + 64*a^2*cos(7/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 3136*a^2*cos(5/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e)))^2 + 3136*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e)))^2 + 64*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e)))^2 + a^2*sin(8*f*x + 8*e)^2 + 784*a^2*sin(6*f*x + 6*e)^2 + 4900*a^2*
sin(4*f*x + 4*e)^2 + 3920*a^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 784*a^2*s
in(2*f*x + 2*e)^2 + 64*a^2*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e)))^2 + 3136*a^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 +
3136*a^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 64*a^2*si
n(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 56*a^2*cos(2*f*x + 2
*e) + a^2 + 2*(28*a^2*cos(6*f*x + 6*e) + 70*a^2*cos(4*f*x + 4*e) + 28*a^2*c
os(2*f*x + 2*e) + a^2)*cos(8*f*x + 8*e) + 56*(70*a^2*cos(4*f*x + 4*e) + 28*
a^2*cos(2*f*x + 2*e) + a^2)*cos(6*f*x + 6*e) + 140*(28*a^2*cos(2*f*x + 2*e)
 + a^2)*cos(4*f*x + 4*e) - 16*(a^2*cos(8*f*x + 8*e) + 28*a^2*cos(6*f*x + 6*
e) + 70*a^2*cos(4*f*x + 4*e) + 28*a^2*cos(2*f*x + 2*e) - 56*a^2*cos(5/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 56*a^2*cos(3/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e))) - 8*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e))) + a^2)*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
- 112*(a^2*cos(8*f*x + 8*e) + 28*a^2*cos(6*f*x + 6*e) + 70*a^2*cos(4*f*x +
4*e) + 28*a^2*cos(2*f*x + 2*e) - 56*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))) - 8*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
```

```
))) + a^2)*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 112*(a^2*
cos(8*f*x + 8*e) + 28*a^2*cos(6*f*x + 6*e) + 70*a^2*cos(4*f*x + 4*e) + 28*a
^2*cos(2*f*x + 2*e) - 8*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) + a^2)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 16*(a^2
*cos(8*f*x + 8*e) + 28*a^2*cos(6*f*x + 6*e) + 70*a^2*cos(4*f*x + 4*e) + 28*
a^2*cos(2*f*x + 2*e) + a^2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) + 28*(2*a^2*sin(6*f*x + 6*e) + 5*a^2*sin(4*f*x + 4*e) + 2*a^2*sin(2*f
*x + 2*e))*sin(8*f*x + 8*e) + 784*(5*a^2*sin(4*f*x + 4*e) + 2*a^2*sin(2*f*x
 + 2*e))*sin(6*f*x + 6*e) - 16*(a^2*sin(8*f*x + 8*e) + 28*a^2*sin(6*f*x + 6
*e) + 70*a^2*sin(4*f*x + 4*e) + 28*a^2*sin(2*f*x + 2*e) - 56*a^2*sin(5/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 56*a^2*sin(3/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e))) - 8*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e))))*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 112
*(a^2*sin(8*f*x + 8*e) + 28*a^2*sin(6*f*x + 6*e) + 70*a^2*sin(4*f*x + 4*e)
+ 28*a^2*sin(2*f*x + 2*e) - 56*a^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))) - 8*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*
sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 112*(a^2*sin(8*f*x +
 8*e) + 28*a^2*sin(6*f*x + 6*e) + 70*a^2*sin(4*f*x + 4*e) + 28*a^2*sin(2*f*
x + 2*e) - 8*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(
3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 16*(a^2*sin(8*f*x + 8*e)
 + 28*a^2*sin(6*f*x + 6*e) + 70*a^2*sin(4*f*x + 4*e) + 28*a^2*sin(2*f*x + 2
*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*arctan2(sin(1/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) - 1) + 2*(28*(f*x + e)*a^2*cos(6*f*x + 6*e) + 70*(f
*x + e)*a^2*cos(4*f*x + 4*e) + 28*(f*x + e)*a^2*cos(2*f*x + 2*e) + (f*x + e
)*a^2 - 23*a^2*sin(6*f*x + 6*e) - 66*a^2*sin(4*f*x + 4*e) - 23*a^2*sin(2*f*
x + 2*e))*cos(8*f*x + 8*e) + 28*(140*(f*x + e)*a^2*cos(4*f*x + 4*e) + 56*(f
*x + e)*a^2*cos(2*f*x + 2*e) + 2*(f*x + e)*a^2 - 17*a^2*sin(4*f*x + 4*e))*c
os(6*f*x + 6*e) + 28*(140*(f*x + e)*a^2*cos(2*f*x + 2*e) + 5*(f*x + e)*a^2
+ 17*a^2*sin(2*f*x + 2*e))*cos(4*f*x + 4*e) - 4*(4*(f*x + e)*a^2*cos(8*f*x
+ 8*e) + 112*(f*x + e)*a^2*cos(6*f*x + 6*e) + 280*(f*x + e)*a^2*cos(4*f*x +
 4*e) + 112*(f*x + e)*a^2*cos(2*f*x + 2*e) - 224*(f*x + e)*a^2*cos(5/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 224*(f*x + e)*a^2*cos(3/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 32*(f*x + e)*a^2*cos(1/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*(f*x + e)*a^2 + 3*a^2*sin(8*f*x + 8
*e) - 8*a^2*sin(6*f*x + 6*e) - 54*a^2*sin(4*f*x + 4*e) - 8*a^2*sin(2*f*x +
2*e) + 48*a^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 48*a^2
*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(7/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*(28*(f*x + e)*a^2*cos(8*f*x + 8*e) + 7
84*(f*x + e)*a^2*cos(6*f*x + 6*e) + 1960*(f*x + e)*a^2*cos(4*f*x + 4*e) + 7
84*(f*x + e)*a^2*cos(2*f*x + 2*e) - 1568*(f*x + e)*a^2*cos(3/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e))) - 224*(f*x + e)*a^2*cos(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))) + 28*(f*x + e)*a^2 + 27*a^2*sin(8*f*x + 8*e)
 + 112*a^2*sin(6*f*x + 6*e) + 42*a^2*sin(4*f*x + 4*e) + 112*a^2*sin(2*f*x +
 2*e) - 48*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(5/
```

```
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*(28*(f*x + e)*a^2*cos(8*
f*x + 8*e) + 784*(f*x + e)*a^2*cos(6*f*x + 6*e) + 1960*(f*x + e)*a^2*cos(4*
f*x + 4*e) + 784*(f*x + e)*a^2*cos(2*f*x + 2*e) - 224*(f*x + e)*a^2*cos(1/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 28*(f*x + e)*a^2 + 27*a^2*s
in(8*f*x + 8*e) + 112*a^2*sin(6*f*x + 6*e) + 42*a^2*sin(4*f*x + 4*e) + 112*
a^2*sin(2*f*x + 2*e) - 48*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e))))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*(4*(f*x +
 e)*a^2*cos(8*f*x + 8*e) + 112*(f*x + e)*a^2*cos(6*f*x + 6*e) + 280*(f*x +
e)*a^2*cos(4*f*x + 4*e) + 112*(f*x + e)*a^2*cos(2*f*x + 2*e) + 4*(f*x + e)*
a^2 + 3*a^2*sin(8*f*x + 8*e) - 8*a^2*sin(6*f*x + 6*e) - 54*a^2*sin(4*f*x +
4*e) - 8*a^2*sin(2*f*x + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e))) + 2*(28*(f*x + e)*a^2*sin(6*f*x + 6*e) + 70*(f*x + e)*a^2*sin(4*f*
x + 4*e) + 28*(f*x + e)*a^2*sin(2*f*x + 2*e) + 23*a^2*cos(6*f*x + 6*e) + 66
*a^2*cos(4*f*x + 4*e) + 23*a^2*cos(2*f*x + 2*e))*sin(8*f*x + 8*e) + 2*(1960
*(f*x + e)*a^2*sin(4*f*x + 4*e) + 784*(f*x + e)*a^2*sin(2*f*x + 2*e) + 238*
a^2*cos(4*f*x + 4*e) - 23*a^2)*sin(6*f*x + 6*e) + 4*(980*(f*x + e)*a^2*sin(
2*f*x + 2*e) - 119*a^2*cos(2*f*x + 2*e) - 33*a^2)*sin(4*f*x + 4*e) - 4*(4*(
f*x + e)*a^2*sin(8*f*x + 8*e) + 112*(f*x + e)*a^2*sin(6*f*x + 6*e) + 280*(f
*x + e)*a^2*sin(4*f*x + 4*e) + 112*(f*x + e)*a^2*sin(2*f*x + 2*e) - 224*(f*
x + e)*a^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 224*(f*x
+ e)*a^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 32*(f*x + e
)*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 3*a^2*cos(8*f*
x + 8*e) + 8*a^2*cos(6*f*x + 6*e) + 54*a^2*cos(4*f*x + 4*e) + 8*a^2*cos(2*f
*x + 2*e) - 48*a^2*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4
8*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 3*a^2)*sin(7/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*(28*(f*x + e)*a^2*sin(8*f
*x + 8*e) + 784*(f*x + e)*a^2*sin(6*f*x + 6*e) + 1960*(f*x + e)*a^2*sin(4*f
*x + 4*e) + 784*(f*x + e)*a^2*sin(2*f*x + 2*e) - 1568*(f*x + e)*a^2*sin(3/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 224*(f*x + e)*a^2*sin(1/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 27*a^2*cos(8*f*x + 8*e) - 112
*a^2*cos(6*f*x + 6*e) - 42*a^2*cos(4*f*x + 4*e) - 112*a^2*cos(2*f*x + 2*e)
+ 48*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 27*a^2)*sin
(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*(28*(f*x + e)*a^2*sin
(8*f*x + 8*e) + 784*(f*x + e)*a^2*sin(6*f*x + 6*e) + 1960*(f*x + e)*a^2*sin
(4*f*x + 4*e) + 784*(f*x + e)*a^2*sin(2*f*x + 2*e) - 224*(f*x + e)*a^2*sin(
1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 27*a^2*cos(8*f*x + 8*e)
- 112*a^2*cos(6*f*x + 6*e) - 42*a^2*cos(4*f*x + 4*e) - 112*a^2*cos(2*f*x +
2*e) + 48*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 27*a^2
)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*(4*(f*x + e)*a^2
*sin(8*f*x + 8*e) + 112*(f*x + e)*a^2*sin(6*f*x + 6*e) + 280*(f*x + e)*a^2*
sin(4*f*x + 4*e) + 112*(f*x + e)*a^2*sin(2*f*x + 2*e) - 3*a^2*cos(8*f*x + 8
*e) + 8*a^2*cos(6*f*x + 6*e) + 54*a^2*cos(4*f*x + 4*e) + 8*a^2*cos(2*f*x +
2*e) - 3*a^2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))))*sqrt(a)
*sqrt(c)/((c^5*cos(8*f*x + 8*e)^2 + 784*c^5*cos(6*f*x + 6*e)^2 + 4900*c^5*c
os(4*f*x + 4*e)^2 + 784*c^5*cos(2*f*x + 2*e)^2 + 64*c^5*cos(7/2*arctan2(sin
```

```
(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 3136*c^5*cos(5/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e)))^2 + 3136*c^5*cos(3/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e)))^2 + 64*c^5*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e)))^2 + c^5*sin(8*f*x + 8*e)^2 + 784*c^5*sin(6*f*x + 6*e)^2 + 4900*c^5*sin
(4*f*x + 4*e)^2 + 3920*c^5*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 784*c^5*sin(
2*f*x + 2*e)^2 + 64*c^5*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
)^2 + 3136*c^5*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 313
6*c^5*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 64*c^5*sin(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 56*c^5*cos(2*f*x + 2*e)
 + c^5 + 2*(28*c^5*cos(6*f*x + 6*e) + 70*c^5*cos(4*f*x + 4*e) + 28*c^5*cos(
2*f*x + 2*e) + c^5)*cos(8*f*x + 8*e) + 56*(70*c^5*cos(4*f*x + 4*e) + 28*c^5
*cos(2*f*x + 2*e) + c^5)*cos(6*f*x + 6*e) + 140*(28*c^5*cos(2*f*x + 2*e) +
c^5)*cos(4*f*x + 4*e) - 16*(c^5*cos(8*f*x + 8*e) + 28*c^5*cos(6*f*x + 6*e)
+ 70*c^5*cos(4*f*x + 4*e) + 28*c^5*cos(2*f*x + 2*e) - 56*c^5*cos(5/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 56*c^5*cos(3/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e))) - 8*c^5*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))) + c^5)*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 1
12*(c^5*cos(8*f*x + 8*e) + 28*c^5*cos(6*f*x + 6*e) + 70*c^5*cos(4*f*x + 4*e
) + 28*c^5*cos(2*f*x + 2*e) - 56*c^5*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e))) - 8*c^5*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
 + c^5)*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 112*(c^5*cos
(8*f*x + 8*e) + 28*c^5*cos(6*f*x + 6*e) + 70*c^5*cos(4*f*x + 4*e) + 28*c^5*
cos(2*f*x + 2*e) - 8*c^5*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
)) + c^5)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 16*(c^5*co
s(8*f*x + 8*e) + 28*c^5*cos(6*f*x + 6*e) + 70*c^5*cos(4*f*x + 4*e) + 28*c^5
*cos(2*f*x + 2*e) + c^5)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
)) + 28*(2*c^5*sin(6*f*x + 6*e) + 5*c^5*sin(4*f*x + 4*e) + 2*c^5*sin(2*f*x
+ 2*e))*sin(8*f*x + 8*e) + 784*(5*c^5*sin(4*f*x + 4*e) + 2*c^5*sin(2*f*x +
2*e))*sin(6*f*x + 6*e) - 16*(c^5*sin(8*f*x + 8*e) + 28*c^5*sin(6*f*x + 6*e)
 + 70*c^5*sin(4*f*x + 4*e) + 28*c^5*sin(2*f*x + 2*e) - 56*c^5*sin(5/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 56*c^5*sin(3/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e))) - 8*c^5*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))))*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 112*(c
^5*sin(8*f*x + 8*e) + 28*c^5*sin(6*f*x + 6*e) + 70*c^5*sin(4*f*x + 4*e) + 2
8*c^5*sin(2*f*x + 2*e) - 56*c^5*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e))) - 8*c^5*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin
(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 112*(c^5*sin(8*f*x + 8*
e) + 28*c^5*sin(6*f*x + 6*e) + 70*c^5*sin(4*f*x + 4*e) + 28*c^5*sin(2*f*x +
 2*e) - 8*c^5*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 16*(c^5*sin(8*f*x + 8*e) +
28*c^5*sin(6*f*x + 6*e) + 70*c^5*sin(4*f*x + 4*e) + 28*c^5*sin(2*f*x + 2*e)
)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*f)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(a^2\sec\left(fx+e\right)^2+2\,a^2\sec\left(fx+e\right)+a^2\right)\sqrt{a\sec\left(fx+e\right)+a}\sqrt{-c\sec\left(fx+e\right)+c}}{c^5\sec\left(fx+e\right)^5-5\,c^5\sec\left(fx+e\right)^4+10\,c^5\sec\left(fx+e\right)^3-10\,c^5\sec\left(fx+e\right)^2+5\,c^5\sec\left(fx+e\right)-c^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(9/2),x, algorithm="frica
s")
```

```
[Out] integral(-(a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*sqrt(a*sec(f*x +
e) + a)*sqrt(-c*sec(f*x + e) + c)/(c^5*sec(f*x + e)^5 - 5*c^5*sec(f*x + e)^
4 + 10*c^5*sec(f*x + e)^3 - 10*c^5*sec(f*x + e)^2 + 5*c^5*sec(f*x + e) - c^
5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(9/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

3.109 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx$

Optimal. Leaf size=244

$$-\frac{a^3\tan(e+fx)}{c^4 f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}-\frac{a^3\tan(e+fx)}{2c^3 f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}-\frac{a^3\tan(e\cdots}{3c^2 f\sqrt{a\sec(e+fx)+a(\cdots}}$$

```
[Out] (-4*a^3*Tan[e + f*x])/(5*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(1
1/2)) - (a^3*Tan[e + f*x])/(3*c^2*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e +
 f*x])^(7/2)) - (a^3*Tan[e + f*x])/(2*c^3*f*Sqrt[a + a*Sec[e + f*x]]*(c - c
*Sec[e + f*x])^(5/2)) - (a^3*Tan[e + f*x])/(c^4*f*Sqrt[a + a*Sec[e + f*x]]*
(c - c*Sec[e + f*x])^(3/2)) + (a^3*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c^5
*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.47412, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3910, 3907, 3911, 31}

$$-\frac{a^3\tan(e+fx)}{c^4 f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}-\frac{a^3\tan(e+fx)}{2c^3 f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}-\frac{a^3\tan(e\cdots}{3c^2 f\sqrt{a\sec(e+fx)+a(\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(11/2),x]
```


```
[Out] (-4*a^3*Tan[e + f*x])/(5*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(1
1/2)) - (a^3*Tan[e + f*x])/(3*c^2*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e +
 f*x])^(7/2)) - (a^3*Tan[e + f*x])/(2*c^3*f*Sqrt[a + a*Sec[e + f*x]]*(c - c
*Sec[e + f*x])^(5/2)) - (a^3*Tan[e + f*x])/(c^4*f*Sqrt[a + a*Sec[e + f*x]]*
(c - c*Sec[e + f*x])^(3/2)) + (a^3*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c^5
*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```


Rule 3910

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(5/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[(-8*a^3*Cot[e + f*x]*(c + d*Csc[e + f
*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[a^2/c^2, Int[Sqrt
[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3907

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_), x_Symbol] :> Simp[(-2*a*Cot[e + f*x]*(c + d*Csc[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[1/c, Int[Sqrt[a + b*Cs
c[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3911

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> -Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((b + a*x)^(m - 1/2)*(d + c*x
)^(n - 1/2))/x^(m + n), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && EqQ
[m + n, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx = -\frac{4a^3\tan(e+fx)}{5f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{11/2}} + \frac{a^2\int\frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{7/2}}\,dx}{c^2}$$

$$= -\frac{4a^3\tan(e+fx)}{5f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{11/2}} - \frac{a^3\tan(e+fx)}{3c^2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}$$

$$= -\frac{4a^3\tan(e+fx)}{5f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{11/2}} - \frac{a^3\tan(e+fx)}{3c^2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}$$

$$= -\frac{4a^3\tan(e+fx)}{5f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{11/2}} - \frac{a^3\tan(e+fx)}{3c^2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}$$

$$= -\frac{4a^3\tan(e+fx)}{5f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{11/2}} - \frac{a^3\tan(e+fx)}{3c^2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}$$

$$= -\frac{4a^3\tan(e+fx)}{5f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{11/2}} - \frac{a^3\tan(e+fx)}{3c^2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}$$

Mathematica [C] time = 5.94935, size = 299, normalized size = 1.23

$$\frac{\sin^{11}\left(\frac{1}{2}(e+fx)\right)\sec^{\frac{11}{2}}(e+fx)(a(\sec(e+fx)+1))^{5/2}\left(-\frac{(5612\cos(e+fx)-5(736\cos(2(e+fx))-367\cos(3(e+fx))+111\cos(4(e+fx))-21\cos(}{240f}\right.}{(\sec(e+fx)+1)^{5/2}(c-c\sec(}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(11/2),x]
```

```
[Out] (Sec[e + f*x]^(11/2)*(a*(1 + Sec[e + f*x]))^(5/2)*(((32*I)*Sqrt[2]*E^((I/2)
*(e + f*x))*Sqrt[(1 + E^(I*(e + f*x)))^2/(1 + E^((2*I)*(e + f*x)))]*(f*x +
(2*I)*Log[1 - E^(I*(e + f*x))]))/((1 + E^(I*(e + f*x)))*Sqrt[E^(I*(e + f*x)
)/(1 + E^((2*I)*(e + f*x)))]*f) - ((5612*Cos[e + f*x] - 5*(625 + 736*Cos[2*
(e + f*x)] - 367*Cos[3*(e + f*x)] + 111*Cos[4*(e + f*x)] - 21*Cos[5*(e + f*
x)]))*Csc[(e + f*x)/2]^10*Sec[(e + f*x)/2]*Sqrt[Sec[e + f*x]]*Sqrt[1 + Sec[
e + f*x]])/(240*f))*Sin[(e + f*x)/2]^11)/((1 + Sec[e + f*x])^(5/2)*(c - c*S
ec[e + f*x])^(11/2))
```

Maple [A] time = 0.292, size = 415, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(11/2),x)
```

```
[Out] -1/120/f*a^2*(-1+cos(f*x+e))*(240*ln(-(-1+cos(f*x+e))/sin(f*x+e))*cos(f*x+e
)^5-120*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^5-233*cos(f*x+e)^5-1200*cos(f*x+e)^
4*ln(-(-1+cos(f*x+e))/sin(f*x+e))+600*cos(f*x+e)^4*ln(2/(1+cos(f*x+e)))+325
*cos(f*x+e)^4+2400*cos(f*x+e)^3*ln(-(-1+cos(f*x+e))/sin(f*x+e))-1200*cos(f*
x+e)^3*ln(2/(1+cos(f*x+e)))-110*cos(f*x+e)^3-2400*ln(-(-1+cos(f*x+e))/sin(f
*x+e))*cos(f*x+e)^2+1200*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^2-290*cos(f*x+e)^2
+1200*cos(f*x+e)*ln(-(-1+cos(f*x+e))/sin(f*x+e))-600*cos(f*x+e)*ln(2/(1+cos
(f*x+e)))+295*cos(f*x+e)-240*ln(-(-1+cos(f*x+e))/sin(f*x+e))+120*ln(2/(1+co
s(f*x+e)))-83)*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)/(c*(-1+cos(f*x+e))/cos
(f*x+e))^(11/2)/sin(f*x+e)/cos(f*x+e)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(11/2),x, algorithm="maxi
ma")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2\sec\left(fx+e\right)^2+2\,a^2\sec\left(fx+e\right)+a^2\right)\sqrt{a\sec\left(fx+e\right)+a}\sqrt{-c\sec\left(fx+e\right)+c}}{c^6\sec\left(fx+e\right)^6-6\,c^6\sec\left(fx+e\right)^5+15\,c^6\sec\left(fx+e\right)^4-20\,c^6\sec\left(fx+e\right)^3+15\,c^6\sec\left(fx+e\right)^2-6\,c^6\sec\left(\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(11/2),x, algorithm="fric
as")
```

```
[Out] integral((a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*sqrt(a*sec(f*x + e
) + a)*sqrt(-c*sec(f*x + e) + c)/(c^6*sec(f*x + e)^6 - 6*c^6*sec(f*x + e)^5
 + 15*c^6*sec(f*x + e)^4 - 20*c^6*sec(f*x + e)^3 + 15*c^6*sec(f*x + e)^2 -
6*c^6*sec(f*x + e) + c^6), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(11/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(11/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

3.110 $\int \frac{(c-c\sec(e+fx))^{7/2}}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal. Leaf size=204

$$\frac{c^4\tan(e+fx)\sec^2(e+fx)}{2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{4c^4\tan(e+fx)\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{8c^4\tan(e+fx)\log(\sec(e+fx)+}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+}$$

```
[Out] (c^4*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c
*Sec[e + f*x]]) + (8*c^4*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*
Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) - (4*c^4*Sec[e + f*x]*Tan[e + f*x])
/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) + (c^4*Sec[e + f*x]^
2*Tan[e + f*x])/(2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.123592, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3912, 72}

$$\frac{c^4\tan(e+fx)\sec^2(e+fx)}{2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{4c^4\tan(e+fx)\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{8c^4\tan(e+fx)\log(\sec(e+fx)+}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^(7/2)/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (c^4*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c
*Sec[e + f*x]]) + (8*c^4*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*
Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) - (4*c^4*Sec[e + f*x]*Tan[e + f*x])
/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) + (c^4*Sec[e + f*x]^
2*Tan[e + f*x])/(2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
x)^(n - 1/2))/x, x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{(c-c\sec(e+fx))^{7/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = -\frac{(ac\tan(e+fx))\,\mathrm{Subst}\left(\int \frac{(c-cx)^3}{x(a+ax)}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$
$$= -\frac{(ac\tan(e+fx))\,\mathrm{Subst}\left(\int \left(\frac{4c^3}{a}+\frac{c^3}{ax}-\frac{c^3x}{a}-\frac{8c^3}{a(1+x)}\right)dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$
$$= \frac{c^4\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{8c^4\log(1+\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{\quad}{f\sqrt{a}}$$

Mathematica [C] time = 15.3406, size = 153, normalized size = 0.75

$$\frac{c^3\cot\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\sqrt{c-c\sec(e+fx)}\left(-16\log\left(1+e^{i(e+fx)}\right)+7\log\left(1+e^{2i(e+fx)}\right)+8\cos(e+fx)+\left(-16\log\right.\right.}{2f\sqrt{a(\sec(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^(7/2)/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (c^3*Cot[(e + f*x)/2]*(-1 + I*f*x + 8*Cos[e + f*x] - 16*Log[1 + E^(I*(e + f
*x))] + 7*Log[1 + E^((2*I)*(e + f*x))] + Cos[2*(e + f*x)]*(I*f*x - 16*Log[1
 + E^(I*(e + f*x))] + 7*Log[1 + E^((2*I)*(e + f*x))]))*Sec[e + f*x]^2*Sqrt[
c - c*Sec[e + f*x]])/(2*f*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [A] time = 0.309, size = 189, normalized size = 0.9

$$\frac{\left(\cos\left(fx+e\right)\right)^2}{2\,af\sin\left(fx+e\right)\left(-1+\cos\left(fx+e\right)\right)^3}\left(14\ln\left(\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)\left(\cos\left(fx+e\right)\right)^2+14\ln\left(-\frac{-1+\cos\left(fx\right.}{\sin}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/2/f/a*(14*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2+14*ln(-(-
1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2+2*ln(2/(1+cos(f*x+e)))*co
s(f*x+e)^2+9*cos(f*x+e)^2+8*cos(f*x+e)-1)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(7
/2)*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*cos(f*x+e)^2/sin(f*x+e)/(-1+cos(f
*x+e))^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(c^3\sec\left(fx+e\right)^3-3\,c^3\sec\left(fx+e\right)^2+3\,c^3\sec\left(fx+e\right)-c^3\right)\sqrt{-c\sec\left(fx+e\right)+c}}{\sqrt{a\sec\left(fx+e\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-(c^3*sec(f*x + e)^3 - 3*c^3*sec(f*x + e)^2 + 3*c^3*sec(f*x + e) -
 c^3)*sqrt(-c*sec(f*x + e) + c)/sqrt(a*sec(f*x + e) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**(7/2)/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError
```

3.111 $\int \frac{(c-c\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal. Leaf size=151

$$-\frac{c^3\tan(e+fx)\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{4c^3\tan(e+fx)\log(\sec(e+fx)+1)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{c^3\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] (c^3*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c
*Sec[e + f*x]]) + (4*c^3*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*
Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) - (c^3*Sec[e + f*x]*Tan[e + f*x])/(
f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.11346, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3912, 72}

$$-\frac{c^3\tan(e+fx)\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{4c^3\tan(e+fx)\log(\sec(e+fx)+1)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{c^3\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^(5/2)/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (c^3*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c
*Sec[e + f*x]]) + (4*c^3*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*
Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) - (c^3*Sec[e + f*x]*Tan[e + f*x])/(
f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
x)^(n - 1/2))/x, x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
```

```
/; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = -\frac{(ac\tan(e+fx))\,\text{Subst}\left(\int \frac{(c-cx)^2}{x(a+ax)}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$
$$= -\frac{(ac\tan(e+fx))\,\text{Subst}\left(\int\left(\frac{c^2}{a}+\frac{c^2}{ax}-\frac{4c^2}{a(1+x)}\right)dx,x,\sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$
$$= \frac{c^3\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}+\frac{4c^3\log(1+\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}-\frac{\quad}{f\sqrt{a}}$$

Mathematica [C] time = 3.88614, size = 181, normalized size = 1.2

$$\frac{c^2e^{-3i(e+fx)}\left(1+e^{2i(e+fx)}\right)^3\cos\left(\frac{1}{2}(e+fx)\right)\cot\left(\frac{1}{2}(e+fx)\right)\sec^4(e+fx)\sqrt{c-c\sec(e+fx)}\left(1+\left(-8\log\left(1+e^{i(e+fx)}\right)+3\right.\right.}{4f\left(1+e^{i(e+fx)}\right)\sqrt{a(\sec(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^(5/2)/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (c^2*(1 + E^((2*I)*(e + f*x)))^3*Cos[(e + f*x)/2]*Cot[(e + f*x)/2]*(1 + Cos
[e + f*x]*(I*f*x - 8*Log[1 + E^(I*(e + f*x))] + 3*Log[1 + E^((2*I)*(e + f*x
))]))*Sec[e + f*x]^4*Sqrt[c - c*Sec[e + f*x]]*(Cos[(e + f*x)/2] + I*Sin[(e
+ f*x)/2]))/(4*E^((3*I)*(e + f*x))*(1 + E^(I*(e + f*x)))*f*Sqrt[a*(1 + Sec[
e + f*x])])
```

Maple [A] time = 0.3, size = 169, normalized size = 1.1

$$\frac{\left(\cos\left(fx+e\right)\right)^2}{af\sin\left(fx+e\right)\left(-1+\cos\left(fx+e\right)\right)^2}\left(3\cos\left(fx+e\right)\ln\left(\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)+3\cos\left(fx+e\right)\ln\left(-\frac{-1+\cos}{\quad}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/f/a*(3*cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+3*cos(f*x+e)*l
n(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+cos(f*x+e)*ln(2/(1+cos(f*x+e)))+c
os(f*x+e)+1)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)*(1/cos(f*x+e)*a*(1+cos(f*
x+e)))^(1/2)*cos(f*x+e)^2/sin(f*x+e)/(-1+cos(f*x+e))^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c^2\sec\left(fx+e\right)^2-2\,c^2\sec\left(fx+e\right)+c^2\right)\sqrt{-c\sec\left(fx+e\right)+c}}{\sqrt{a\sec\left(fx+e\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral((c^2*sec(f*x + e)^2 - 2*c^2*sec(f*x + e) + c^2)*sqrt(-c*sec(f*x +
e) + c)/sqrt(a*sec(f*x + e) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError
```

3.112 $\int \frac{(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal. Leaf size=102

$$\frac{2c^2\tan(e+fx)\log(\sec(e+fx)+1)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{c^2\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] (c^2*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c
*Sec[e + f*x]]) + (2*c^2*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*
Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.106863, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3912, 72}

$$\frac{2c^2\tan(e+fx)\log(\sec(e+fx)+1)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{c^2\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^(3/2)/Sqrt[a + a*Sec[e + f*x]],x]
```


```
[Out] (c^2*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c
*Sec[e + f*x]]) + (2*c^2*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*
Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```


Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
x)^(n - 1/2))/x, x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = -\frac{(ac\tan(e+fx))\,\text{Subst}\left(\int \frac{c-cx}{x(a+ax)}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$
$$= -\frac{(ac\tan(e+fx))\,\text{Subst}\left(\int \left(\frac{c}{ax}-\frac{2c}{a(1+x)}\right)dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$
$$= \frac{c^2\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{2c^2\log(1+\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

Mathematica [C] time = 9.15407, size = 103, normalized size = 1.01

$$-\frac{c\left(1+e^{i(e+fx)}\right)\left(4i\log\left(1+e^{i(e+fx)}\right)-i\log\left(1+e^{2i(e+fx)}\right)+fx\right)\sqrt{c-c\sec(e+fx)}}{f\left(-1+e^{i(e+fx)}\right)\sqrt{a(\sec(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^(3/2)/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] -((c*(1 + E^(I*(e + f*x)))*(f*x + (4*I)*Log[1 + E^(I*(e + f*x))] - I*Log[1
+ E^((2*I)*(e + f*x))])*Sqrt[c - c*Sec[e + f*x]])/((-1 + E^(I*(e + f*x)))*f
*Sqrt[a*(1 + Sec[e + f*x])]))
```

Maple [A] time = 0.287, size = 93, normalized size = 0.9

$$\frac{\left(\cos\left(fx+e\right)\right)^2}{af\sin\left(fx+e\right)\left(-1+\cos\left(fx+e\right)\right)}\ln\left(-4\,\frac{\cos\left(fx+e\right)}{\left(1+\cos\left(fx+e\right)\right)^2}\right)\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/f/a*ln(-4*cos(f*x+e)/(1+cos(f*x+e))^2)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(3/
2)*cos(f*x+e)^2*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)/sin(f*x+e)/(-1+cos(f*
x+e))
```

Maxima [A] time = 1.81088, size = 81, normalized size = 0.79

$$-\frac{\left(\left(fx+e\right)c+c\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)+1\right)-4c\arctan\left(\sin\left(fx+e\right),\cos\left(fx+e\right)+1\right)\right)\sqrt{c}}{\sqrt{a}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] -((f*x + e)*c + c*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) - 4*c*arc
tan2(sin(f*x + e), cos(f*x + e) + 1))*sqrt(c)/(sqrt(a)*f)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-c\sec\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\sqrt{a\sec\left(fx+e\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral((-c*sec(f*x + e) + c)^(3/2)/sqrt(a*sec(f*x + e) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-c\left(\sec\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((-c*(sec(e + f*x) - 1))**(3/2)/sqrt(a*(sec(e + f*x) + 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError
```

3.113 $\int \frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal. Leaf size=49

$$\frac{c\tan(e+fx)\log(\cos(e+fx)+1)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] (c*Log[1 + Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
 c*Sec[e + f*x]])
```

Rubi [A] time = 0.084185, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3911, 31}

$$\frac{c\tan(e+fx)\log(\cos(e+fx)+1)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c*Sec[e + f*x]]/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (c*Log[1 + Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
 c*Sec[e + f*x]])
```

Rule 3911

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> -Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((b + a*x)^(m - 1/2)*(d + c*x
)^(n - 1/2))/x^(m + n), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && EqQ
[m + n, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{(ac\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{a+ax}\,dx, x, \cos(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$
$$= \frac{c\log(1+\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

Mathematica [C] time = 0.910697, size = 127, normalized size = 2.59

$$-\frac{\left(1+e^{i(e+fx)}\right)\sqrt{\frac{c\left(-1+e^{i(e+fx)}\right)^2}{1+e^{2i(e+fx)}}}\left(fx+2i\log\left(1+e^{i(e+fx)}\right)\right)}{f\left(-1+e^{i(e+fx)}\right)\sqrt{\frac{a\left(1+e^{i(e+fx)}\right)^2}{1+e^{2i(e+fx)}}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c*Sec[e + f*x]]/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] -(((1 + E^(I*(e + f*x)))*Sqrt[(c*(-1 + E^(I*(e + f*x)))^2)/(1 + E^((2*I)*(e
 + f*x)))]*(f*x + (2*I)*Log[1 + E^(I*(e + f*x))]))/((-1 + E^(I*(e + f*x)))*
Sqrt[(a*(1 + E^(I*(e + f*x)))^2)/(1 + E^((2*I)*(e + f*x)))]*f))
```

Maple [A] time = 0.302, size = 75, normalized size = 1.5

$$\frac{\cos\left(fx+e\right)}{af\sin\left(fx+e\right)}\sqrt{\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\ln\left(2\left(1+\cos\left(fx+e\right)\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/f/a*cos(f*x+e)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)*(1/cos(f*x+e)*a*(1+co
s(f*x+e)))^(1/2)*ln(2/(1+cos(f*x+e)))/sin(f*x+e)
```

Maxima [A] time = 1.57561, size = 46, normalized size = 0.94

$$\frac{\sqrt{c}\log\left(\frac{\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+1\right)}{\sqrt{-a}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] sqrt(c)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(sqrt(-a)*f)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-c\sec\left(fx+e\right)+c}}{\sqrt{a\sec\left(fx+e\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(-c*sec(f*x + e) + c)/sqrt(a*sec(f*x + e) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(sec(e + f*x) - 1))/sqrt(a*(sec(e + f*x) + 1)), x)
```

Giac [A] time = 1.79559, size = 92, normalized size = 1.88

$$\frac{\sqrt{-ac}c\log\left(\left|c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+c\right|\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)\operatorname{sgn}\left(\cos\left(fx+e\right)\right)}{af|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] sqrt(-a*c)*c*log(abs(c*tan(1/2*f*x + 1/2*e)^2 + c))*sgn(tan(1/2*f*x + 1/2*e
)^3 + tan(1/2*f*x + 1/2*e))*sgn(cos(f*x + e))/(a*f*abs(c))
```

3.114 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal. Leaf size=46

$$\frac{\tan(e+fx)\log(\sin(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] (Log[Sin[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec
[e + f*x]])
```

Rubi [A] time = 0.0896478, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3905, 3475}

$$\frac{\tan(e+fx)\log(\sin(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]),x]
```

```
[Out] (Log[Sin[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec
[e + f*x]])
```

Rule 3905

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Dist[((-(a*c))^(m + 1/2)*Cot[e + f*x])/(Sqrt
[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int[Cot[e + f*x]^(2*m), x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IntegerQ[m + 1/2]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx = \frac{\tan(e+fx)\int\cot(e+fx)\,dx}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$
$$= \frac{\log(\sin(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

Mathematica [C] time = 1.06858, size = 104, normalized size = 2.26

$$-\frac{2\left(-1+e^{i(e+fx)}\right)\left(fx+i\log\left(1-e^{2i(e+fx)}\right)\right)\cos^2\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)}{f\left(1+e^{i(e+fx)}\right)\sqrt{a(\sec(e+fx)+1)}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]),x]
```

```
[Out] (-2*(-1 + E^(I*(e + f*x)))*Cos[(e + f*x)/2]^2*(f*x + I*Log[1 - E^((2*I)*(e
+ f*x))])*Sec[e + f*x])/((1 + E^(I*(e + f*x)))*f*Sqrt[a*(1 + Sec[e + f*x])]
*Sqrt[c - c*Sec[e + f*x]])
```

Maple [B] time = 0.26, size = 100, normalized size = 2.2

$$\frac{\cos\left(fx+e\right)}{af\sin\left(fx+e\right)c}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\left(\ln\left(2\left(1+\cos\left(fx+e\right)\right)^{-1}\right)-\ln\left(-\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right)\right)\sqrt{\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/f/a*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(ln(2/(1+cos(f*x+e)))-ln(-(-1+c
os(f*x+e))/sin(f*x+e)))*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)*cos(f*x+e)/sin
(f*x+e)/c
```

Maxima [A] time = 1.77173, size = 53, normalized size = 1.15

$$-\frac{fx+e-\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)-1\right)}{\sqrt{a}\sqrt{c}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] -(f*x + e - arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) - 1))/(sqrt(a)*sqrt(
c)*f)
```

Fricas [B] time = 1.8759, size = 682, normalized size = 14.83

$$\left[-\frac{\sqrt{-ac}\log\left(-\frac{8\left(\left(256\cos\left(fx+e\right)^{5}-512\cos\left(fx+e\right)^{3}+175\cos\left(fx+e\right)\right)\sqrt{-ac}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}-\left(256\,ac\cos\left(fx+e\right)^{4}-512\,ac\cos\left(fx+e\right)^{2}+337\,ac\right.}{\left(\cos\left(fx+e\right)^{2}-1\right)\sin\left(fx+e\right)}\right.}{2\,acf}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] [-1/2*sqrt(-a*c)*log(-8*((256*cos(f*x + e)^5 - 512*cos(f*x + e)^3 + 175*cos
(f*x + e))*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f
*x + e) - c)/cos(f*x + e)) - (256*a*c*cos(f*x + e)^4 - 512*a*c*cos(f*x + e)
^2 + 337*a*c)*sin(f*x + e))/((cos(f*x + e)^2 - 1)*sin(f*x + e)))/(a*c*f), -
sqrt(a*c)*arctan((16*cos(f*x + e)^3 - 7*cos(f*x + e))*sqrt(a*c)*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((16*a
*c*cos(f*x + e)^2 - 25*a*c)*sin(f*x + e)))/(a*c*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-c*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sec(e + f*x) + 1))*sqrt(-c*(sec(e + f*x) - 1))), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] Timed out
```

3.115 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=168

$$\frac{\tan(e+fx)}{2cf(1-\cos(e+fx))\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{3\tan(e+fx)\log(1-\cos(e+fx))}{4cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{\tan(e+fx)\ldots}{4cf\sqrt{a\sec(e+\ldots}}$$

```
[Out] Tan[e + f*x]/(2*c*f*(1 - Cos[e + f*x])*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*
Sec[e + f*x]]) + (3*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(4*c*f*Sqrt[a + a*S
ec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) + (Log[1 + Cos[e + f*x]]*Tan[e + f*x
])/(4*c*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.136506, antiderivative size = 217, normalized size of antiderivative = 1.29, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3912, 72}

$$-\frac{\tan(e+fx)}{2cf(1-\sec(e+fx))\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{3\tan(e+fx)\log(1-\sec(e+fx))}{4cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{\tan(e+f\ldots}{4cf\sqrt{a\sec(e-\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2)),x]
```

```
[Out] (Log[Cos[e + f*x]]*Tan[e + f*x])/(c*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*S
ec[e + f*x]]) + (3*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(4*c*f*Sqrt[a + a*Se
c[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) + (Log[1 + Sec[e + f*x]]*Tan[e + f*x]
)/(4*c*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) - Tan[e + f*x]/
(2*c*f*(1 - Sec[e + f*x])*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]
)
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
x)^(n - 1/2))/x, x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
```

```
/; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{(ac\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{x(a+ax)(c-cx)^2}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{(ac\tan(e+fx))\,\text{Subst}\left(\int \left(\frac{1}{2ac^2(-1+x)^2}-\frac{3}{4ac^2(-1+x)}+\frac{1}{ac^2x}-\frac{1}{4ac^2(1+x)}\right)\right.}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= \frac{\log(\cos(e+fx))\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{3\log(1-\sec(e+fx))\,\text{t}}{4cf\sqrt{a+a\sec(e+fx)}\sqrt{c-}}$$

Mathematica [C] time = 8.14501, size = 143, normalized size = 0.85

$$\frac{\tan(e+fx)\left(-3\log\left(1-e^{i(e+fx)}\right)-\log\left(1+e^{i(e+fx)}\right)+\left(3\log\left(1-e^{i(e+fx)}\right)+\log\left(1+e^{i(e+fx)}\right)-2ifx\right)\cos(e+fx)+2if}{2cf(\cos(e+fx)-1)\sqrt{a(\sec(e+fx)+1)}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2)),x]
```

```
[Out] ((-1 + (2*I)*f*x - 3*Log[1 - E^(I*(e + f*x))] - Log[1 + E^(I*(e + f*x))] +
Cos[e + f*x]*((-2*I)*f*x + 3*Log[1 - E^(I*(e + f*x))] + Log[1 + E^(I*(e + f
*x))]))*Tan[e + f*x])/(2*c*f*(-1 + Cos[e + f*x])*Sqrt[a*(1 + Sec[e + f*x])]
*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] time = 0.294, size = 167, normalized size = 1.

$$-\frac{-1+\cos(fx+e)}{4af\cos(fx+e)\sin(fx+e)}\left(6\cos(fx+e)\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)-4\cos(fx+e)\ln\left(2\left(1+\cos(fx+e)\right)^{-1}\right)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/4/f/a*(-1+cos(f*x+e))*(6*cos(f*x+e)*ln(-(-1+cos(f*x+e))/sin(f*x+e))-4*co
s(f*x+e)*ln(2/(1+cos(f*x+e)))-cos(f*x+e)-6*ln(-(-1+cos(f*x+e))/sin(f*x+e))+
4*ln(2/(1+cos(f*x+e)))-1)*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)/(c*(-1+cos(
f*x+e))/cos(f*x+e))^(3/2)/cos(f*x+e)/sin(f*x+e)
```

Maxima [B] time = 1.9407, size = 1104, normalized size = 6.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] -1/2*(2*(f*x + e)*cos(2*f*x + 2*e)^2 + 8*(f*x + e)*cos(1/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e)))^2 + 2*(f*x + e)*sin(2*f*x + 2*e)^2 + 8*(f*x +
e)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*f*x - (cos(2*
f*x + 2*e)^2 - 4*(cos(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))) + 4*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^
2 + sin(2*f*x + 2*e)^2 - 4*sin(2*f*x + 2*e)*sin(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))) + 4*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
)))^2 + 2*cos(2*f*x + 2*e) + 1)*arctan2(sin(1/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1
) - 3*(cos(2*f*x + 2*e)^2 - 4*(cos(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))) + 4*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e)))^2 + sin(2*f*x + 2*e)^2 - 4*sin(2*f*x + 2*e)*sin(1/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*sin(1/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e)))^2 + 2*cos(2*f*x + 2*e) + 1)*arctan2(sin(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e))) - 1) + 4*(f*x + e)*cos(2*f*x + 2*e) - 2*(4*f*x + 4*(f*x + e)*cos(
2*f*x + 2*e) + 4*e + sin(2*f*x + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e))) - 2*(4*(f*x + e)*sin(2*f*x + 2*e) - cos(2*f*x + 2*e) - 1)*
sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*e)/((c*cos(2*f*x +
 2*e)^2 + 4*c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + c*si
n(2*f*x + 2*e)^2 - 4*c*sin(2*f*x + 2*e)*sin(1/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))) + 4*c*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
)^2 + 2*c*cos(2*f*x + 2*e) - 4*(c*cos(2*f*x + 2*e) + c)*cos(1/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))) + c)*sqrt(a)*sqrt(c)*f)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a\sec\left(fx+e\right)+a}\sqrt{-c\sec\left(fx+e\right)+c}}{ac^2\sec\left(fx+e\right)^3-ac^2\sec\left(fx+e\right)^2-ac^2\sec\left(fx+e\right)+ac^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a*c^2*sec(f*x
+ e)^3 - a*c^2*sec(f*x + e)^2 - a*c^2*sec(f*x + e) + a*c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\left(-c\left(\sec\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sec(e + f*x) + 1))*(-c*(sec(e + f*x) - 1))**(3/2)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] Timed out
```

3.116 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=274

$$-\frac{3\tan(e+fx)}{4c^2f(1-\sec(e+fx))\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)}{4c^2f(1-\sec(e+fx))^2\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+f}$$

```
[Out] (Log[Cos[e + f*x]]*Tan[e + f*x])/(c^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c
*Sec[e + f*x]]) + (7*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(8*c^2*f*Sqrt[a +
a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) + (Log[1 + Sec[e + f*x]]*Tan[e +
f*x])/(8*c^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) - Tan[e +
 f*x]/(4*c^2*f*(1 - Sec[e + f*x])^2*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec
[e + f*x]]) - (3*Tan[e + f*x])/(4*c^2*f*(1 - Sec[e + f*x])*Sqrt[a + a*Sec[e
 + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.160944, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3912, 72}

$$-\frac{3\tan(e+fx)}{4c^2f(1-\sec(e+fx))\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)}{4c^2f(1-\sec(e+fx))^2\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+f}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2)),x]
```


```
[Out] (Log[Cos[e + f*x]]*Tan[e + f*x])/(c^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c
*Sec[e + f*x]]) + (7*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(8*c^2*f*Sqrt[a +
a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) + (Log[1 + Sec[e + f*x]]*Tan[e +
f*x])/(8*c^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) - Tan[e +
 f*x]/(4*c^2*f*(1 - Sec[e + f*x])^2*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec
[e + f*x]]) - (3*Tan[e + f*x])/(4*c^2*f*(1 - Sec[e + f*x])*Sqrt[a + a*Sec[e
 + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```


Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
x)^(n - 1/2))/x, x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
```

```
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{(ac\tan(e+fx))\,\mathrm{Subst}\left(\int \frac{1}{x(a+ax)(c-cx)^3}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{(ac\tan(e+fx))\,\mathrm{Subst}\left(\int \left(-\frac{1}{2ac^3(-1+x)^3}+\frac{3}{4ac^3(-1+x)^2}-\frac{7}{8ac^3(-1+x)}+\frac{1}{ac}\right.\right.}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+f}}$$

$$= \frac{\log(\cos(e+fx))\tan(e+fx)}{c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{7\log(1-\sec(e+fx))}{8c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c}}$$

Mathematica [C] time = 1.87856, size = 194, normalized size = 0.71

$$\frac{\tan(e+fx)\left(21\log\left(1-e^{i(e+fx)}\right)+3\log\left(1+e^{i(e+fx)}\right)+\left(-28\log\left(1-e^{i(e+fx)}\right)-4\log\left(1+e^{i(e+fx)}\right)+16ifx-10\right)\cos(e}{8c^2f(\cos(e+fx)-1)^2\sqrt{a(\sec(e+fx)+1)}\sqrt{c-c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2)),x]
```

```
[Out] ((8 - (12*I)*f*x + 21*Log[1 - E^(I*(e + f*x))] + Cos[e + f*x]*(-10 + (16*I)
*f*x - 28*Log[1 - E^(I*(e + f*x))] - 4*Log[1 + E^(I*(e + f*x))]) + 3*Log[1
+ E^(I*(e + f*x))] + Cos[2*(e + f*x)]*((-4*I)*f*x + 7*Log[1 - E^(I*(e + f*x
))] + Log[1 + E^(I*(e + f*x))]))*Tan[e + f*x])/(8*c^2*f*(-1 + Cos[e + f*x])
^2*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] time = 0.303, size = 229, normalized size = 0.8

$$-\frac{-1+\cos(fx+e)}{16af\sin(fx+e)(\cos(fx+e))^2}\left(28\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)(\cos(fx+e))^2-16\ln\left(2(1+\cos(fx+e))^{-1}\right)(\cos\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/16/f/a*(-1+cos(f*x+e))*(28*ln(-(-1+cos(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-
16*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^2-9*cos(f*x+e)^2-56*cos(f*x+e)*ln(-(-1+c
os(f*x+e))/sin(f*x+e))+32*cos(f*x+e)*ln(2/(1+cos(f*x+e)))-2*cos(f*x+e)+28*l
n(-(-1+cos(f*x+e))/sin(f*x+e))-16*ln(2/(1+cos(f*x+e)))+7)*(1/cos(f*x+e)*a*(
1+cos(f*x+e)))^(1/2)/sin(f*x+e)/cos(f*x+e)^2/(c*(-1+cos(f*x+e))/cos(f*x+e))
^(5/2)
```

Maxima [B] time = 2.54955, size = 2978, normalized size = 10.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] -1/4*(4*(f*x + e)*cos(4*f*x + 4*e)^2 + 144*(f*x + e)*cos(2*f*x + 2*e)^2 + 6
4*(f*x + e)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 64*(f*
x + e)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*(f*x + e)
*sin(4*f*x + 4*e)^2 + 144*(f*x + e)*sin(2*f*x + 2*e)^2 + 64*(f*x + e)*sin(3
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 64*(f*x + e)*sin(1/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*f*x - (2*(6*cos(2*f*x + 2*
e) + 1)*cos(4*f*x + 4*e) + cos(4*f*x + 4*e)^2 + 36*cos(2*f*x + 2*e)^2 - 8*(
cos(4*f*x + 4*e) + 6*cos(2*f*x + 2*e) - 4*cos(1/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e))) + 1)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
)) + 16*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 8*(cos(4*f
*x + 4*e) + 6*cos(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e))) + 16*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 +
 sin(4*f*x + 4*e)^2 + 12*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 36*sin(2*f*x +
 2*e)^2 - 8*(sin(4*f*x + 4*e) + 6*sin(2*f*x + 2*e) - 4*sin(1/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))) + 16*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 8
*(sin(4*f*x + 4*e) + 6*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e))) + 16*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
)^2 + 12*cos(2*f*x + 2*e) + 1)*arctan2(sin(1/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1)
 - 7*(2*(6*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + cos(4*f*x + 4*e)^2 + 36
*cos(2*f*x + 2*e)^2 - 8*(cos(4*f*x + 4*e) + 6*cos(2*f*x + 2*e) - 4*cos(1/2*
```

```
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1)*cos(3/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e))) + 16*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e)))^2 - 8*(cos(4*f*x + 4*e) + 6*cos(2*f*x + 2*e) + 1)*cos(1/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 16*cos(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e)))^2 + sin(4*f*x + 4*e)^2 + 12*sin(4*f*x + 4*e)*sin(2*f*
x + 2*e) + 36*sin(2*f*x + 2*e)^2 - 8*(sin(4*f*x + 4*e) + 6*sin(2*f*x + 2*e)
 - 4*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 16*sin(3/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e)))^2 - 8*(sin(4*f*x + 4*e) + 6*sin(2*f*x + 2*e))*sin(1/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 16*sin(1/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e)))^2 + 12*cos(2*f*x + 2*e) + 1)*arctan2(sin(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) - 1) + 8*(f*x + 6*(f*x + e)*cos(2*f*x + 2*e) + e - 2*s
in(2*f*x + 2*e))*cos(4*f*x + 4*e) + 48*(f*x + e)*cos(2*f*x + 2*e) - 2*(16*f
*x + 16*(f*x + e)*cos(4*f*x + 4*e) + 96*(f*x + e)*cos(2*f*x + 2*e) - 64*(f*
x + e)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 16*e + 5*sin(
4*f*x + 4*e) - 2*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))) - 2*(16*f*x + 16*(f*x + e)*cos(4*f*x + 4*e) + 96*(f*x + e)*cos
(2*f*x + 2*e) + 16*e + 5*sin(4*f*x + 4*e) - 2*sin(2*f*x + 2*e))*cos(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 16*(3*(f*x + e)*sin(2*f*x + 2*e
) + cos(2*f*x + 2*e))*sin(4*f*x + 4*e) - 2*(16*(f*x + e)*sin(4*f*x + 4*e) +
 96*(f*x + e)*sin(2*f*x + 2*e) - 64*(f*x + e)*sin(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) - 5*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e) - 5)*sin(
3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*(16*(f*x + e)*sin(4*f*
x + 4*e) + 96*(f*x + e)*sin(2*f*x + 2*e) - 5*cos(4*f*x + 4*e) + 2*cos(2*f*x
 + 2*e) - 5)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*e - 1
6*sin(2*f*x + 2*e))/((c^2*cos(4*f*x + 4*e)^2 + 36*c^2*cos(2*f*x + 2*e)^2 +
16*c^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16*c^2*cos(
1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + c^2*sin(4*f*x + 4*e)^2
 + 12*c^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 36*c^2*sin(2*f*x + 2*e)^2 + 1
6*c^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16*c^2*sin(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 12*c^2*cos(2*f*x + 2*e)
 + c^2 + 2*(6*c^2*cos(2*f*x + 2*e) + c^2)*cos(4*f*x + 4*e) - 8*(c^2*cos(4*f
*x + 4*e) + 6*c^2*cos(2*f*x + 2*e) - 4*c^2*cos(1/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) + c^2)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) - 8*(c^2*cos(4*f*x + 4*e) + 6*c^2*cos(2*f*x + 2*e) + c^2)*cos(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 8*(c^2*sin(4*f*x + 4*e) + 6*c^2
*sin(2*f*x + 2*e) - 4*c^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 8*(c^2*sin(4*f*
x + 4*e) + 6*c^2*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))))*sqrt(a)*sqrt(c)*f)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{a\sec\left(fx+e\right)+a}\sqrt{-c\sec\left(fx+e\right)+c}}{ac^3\sec\left(fx+e\right)^4-2\,ac^3\sec\left(fx+e\right)^3+2\,ac^3\sec\left(fx+e\right)-ac^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a*c^3*sec(f*x
 + e)^4 - 2*a*c^3*sec(f*x + e)^3 + 2*a*c^3*sec(f*x + e) - a*c^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] Timed out
```

3.117 $\int \frac{(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=215

$$\frac{c^4\tan(e+fx)\sec(e+fx)}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{8c^4\tan(e+fx)}{af(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{4c^4\tan(e+f}{af\sqrt{a\sec(e+f}}$$

```
[Out] (c^4*Log[Cos[e + f*x]]*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
 c*Sec[e + f*x]]) - (4*c^4*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(a*f*Sqrt[a
+ a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) + (c^4*Sec[e + f*x]*Tan[e + f*x
])/(a*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) - (8*c^4*Tan[e +
 f*x])/(a*f*(1 + Sec[e + f*x])*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e +
f*x]])
```

Rubi [A] time = 0.144948, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3912, 88}

$$\frac{c^4\tan(e+fx)\sec(e+fx)}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{8c^4\tan(e+fx)}{af(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{4c^4\tan(e+f}{af\sqrt{a\sec(e+f}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^(7/2)/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (c^4*Log[Cos[e + f*x]]*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
 c*Sec[e + f*x]]) - (4*c^4*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(a*f*Sqrt[a
+ a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) + (c^4*Sec[e + f*x]*Tan[e + f*x
])/(a*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) - (8*c^4*Tan[e +
 f*x])/(a*f*(1 + Sec[e + f*x])*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e +
f*x]])
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
x)^(n - 1/2))/x, x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = -\frac{(ac\tan(e+fx))\,\text{Subst}\left(\int \frac{(c-cx)^3}{x(a+ax)^2}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$
$$= -\frac{(ac\tan(e+fx))\,\text{Subst}\left(\int \left(-\frac{c^3}{a^2}+\frac{c^3}{a^2x}-\frac{8c^3}{a^2(1+x)^2}+\frac{4c^3}{a^2(1+x)}\right)dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$
$$= \frac{c^4\log(\cos(e+fx))\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{4c^4\log(1+\sec(e+fx))\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{}{af\sqrt{}}$$

Mathematica [C] time = 2.08208, size = 204, normalized size = 0.95

$$\frac{c^3\cot\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\sqrt{c-c\sec(e+fx)}\left(8\log\left(1+e^{i(e+fx)}\right)-5\log\left(1+e^{2i(e+fx)}\right)+2\left(8\log\left(1+e^{i(e+fx)}\right)-5\log\right.\right.}{2af(\cos(e+fx)+1)\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^(7/2)/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (c^3*Cot[(e + f*x)/2]*(-2 + I*f*x + 8*Log[1 + E^(I*(e + f*x))] + 2*Cos[e +
f*x]*(-9 + I*f*x + 8*Log[1 + E^(I*(e + f*x))] - 5*Log[1 + E^((2*I)*(e + f*x
))]) + Cos[2*(e + f*x)]*(I*f*x + 8*Log[1 + E^(I*(e + f*x))] - 5*Log[1 + E^(
(2*I)*(e + f*x))]) - 5*Log[1 + E^((2*I)*(e + f*x))])*Sec[e + f*x]*Sqrt[c -
c*Sec[e + f*x]])/(2*a*f*(1 + Cos[e + f*x])*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [A] time = 0.276, size = 276, normalized size = 1.3

$$\frac{(\cos(fx+e))^3}{fa^2(\sin(fx+e))^3(-1+\cos(fx+e))^2}\left(5\ln\left(\frac{1-\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)(\cos(fx+e))^2+5\ln\left(-\frac{-1+\cos(fx+}{\sin(}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(3/2),x)
```

```
[Out] 1/f/a^2*(5*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2+5*ln(-(-1+
cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-ln(2/(1+cos(f*x+e)))*cos(f*
x+e)^2+5*cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+5*cos(f*x+e)*l
n(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-3*cos(f*x+e)^2-cos(f*x+e)*ln(2/(1
+cos(f*x+e)))+6*cos(f*x+e)+1)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(7/2)*cos(f*x+
e)^3*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)/sin(f*x+e)^3/(-1+cos(f*x+e))^2
```

Maxima [B] time = 2.71925, size = 3231, normalized size = 15.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] -((f*x + e)*c^3*cos(4*f*x + 4*e)^2 + 4*(f*x + e)*c^3*cos(2*f*x + 2*e)^2 + (
f*x + e)*c^3*sin(4*f*x + 4*e)^2 + 4*(f*x + e)*c^3*sin(2*f*x + 2*e)^2 + 4*(f
*x + e)*c^3*cos(2*f*x + 2*e) + (f*x + e)*c^3 - 4*c^3*sin(2*f*x + 2*e) + 4*(
(f*x + e)*c^3 - 5*c^3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*cos(
3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*((f*x + e)*c^3 - 5*c
^3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*cos(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*((f*x + e)*c^3 - 5*c^3*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e) + 1))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e)))^2 + 4*((f*x + e)*c^3 - 5*c^3*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 5*(c
^3*cos(4*f*x + 4*e)^2 + 4*c^3*cos(2*f*x + 2*e)^2 + c^3*sin(4*f*x + 4*e)^2 +
 4*c^3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*c^3*sin(2*f*x + 2*e)^2 + 4*c^3
*cos(2*f*x + 2*e) + c^3 + 2*(2*c^3*cos(2*f*x + 2*e) + c^3)*cos(4*f*x + 4*e)
)*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) + 8*(c^3*cos(4*f*x + 4*e)
^2 + 4*c^3*cos(2*f*x + 2*e)^2 + 4*c^3*cos(3/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e)))^2 + 4*c^3*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
)))^2 + c^3*sin(4*f*x + 4*e)^2 + 4*c^3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) +
4*c^3*sin(2*f*x + 2*e)^2 + 4*c^3*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e)))^2 + 4*c^3*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2
 + 4*c^3*cos(2*f*x + 2*e) + c^3 + 2*(2*c^3*cos(2*f*x + 2*e) + c^3)*cos(4*f*
x + 4*e) + 4*(c^3*cos(4*f*x + 4*e) + 2*c^3*cos(2*f*x + 2*e) + 2*c^3*cos(1/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c^3)*cos(3/2*arctan2(sin(2*
```

```
f*x + 2*e), cos(2*f*x + 2*e))) + 4*(c^3*cos(4*f*x + 4*e) + 2*c^3*cos(2*f*x
+ 2*e) + c^3)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*(c^3
*sin(4*f*x + 4*e) + 2*c^3*sin(2*f*x + 2*e) + 2*c^3*sin(1/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e))) + 4*(c^3*sin(4*f*x + 4*e) + 2*c^3*sin(2*f*x + 2*e))*sin(1/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*arctan2(sin(1/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
))) + 1) + 2*(2*(f*x + e)*c^3*cos(2*f*x + 2*e) + (f*x + e)*c^3 - 2*c^3*sin(
2*f*x + 2*e))*cos(4*f*x + 4*e) + 2*(2*(f*x + e)*c^3*cos(4*f*x + 4*e) + 4*(f
*x + e)*c^3*cos(2*f*x + 2*e) + 2*(f*x + e)*c^3 + 9*c^3*sin(4*f*x + 4*e) + 1
4*c^3*sin(2*f*x + 2*e) - 10*(c^3*cos(4*f*x + 4*e) + 2*c^3*cos(2*f*x + 2*e)
+ c^3)*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) + 4*((f*x + e)*c^3 -
 5*c^3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*cos(1/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))) + 2*(2*(f*x + e)*c^3*cos(4*f*x + 4*e) + 4*(f*x + e)*c^3*cos(2*
f*x + 2*e) + 2*(f*x + e)*c^3 + 9*c^3*sin(4*f*x + 4*e) + 14*c^3*sin(2*f*x +
2*e) - 10*(c^3*cos(4*f*x + 4*e) + 2*c^3*cos(2*f*x + 2*e) + c^3)*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e))) + 4*((f*x + e)*c^3*sin(2*f*x + 2*e) + c^3*cos(2*f*x + 2*e))
*sin(4*f*x + 4*e) + 2*(2*(f*x + e)*c^3*sin(4*f*x + 4*e) + 4*(f*x + e)*c^3*s
in(2*f*x + 2*e) - 9*c^3*cos(4*f*x + 4*e) - 14*c^3*cos(2*f*x + 2*e) - 9*c^3
- 10*(c^3*sin(4*f*x + 4*e) + 2*c^3*sin(2*f*x + 2*e))*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e) + 1) + 4*((f*x + e)*c^3 - 5*c^3*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*(2*(f*x + e)
*c^3*sin(4*f*x + 4*e) + 4*(f*x + e)*c^3*sin(2*f*x + 2*e) - 9*c^3*cos(4*f*x
+ 4*e) - 14*c^3*cos(2*f*x + 2*e) - 9*c^3 - 10*(c^3*sin(4*f*x + 4*e) + 2*c^3
*sin(2*f*x + 2*e))*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(c)/((a*cos(4*f*x + 4*e)
^2 + 4*a*cos(2*f*x + 2*e)^2 + 4*a*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e)))^2 + 4*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2
+ a*sin(4*f*x + 4*e)^2 + 4*a*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*a*sin(2*
f*x + 2*e)^2 + 4*a*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 +
 4*a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*(2*a*cos(2*
f*x + 2*e) + a)*cos(4*f*x + 4*e) + 4*a*cos(2*f*x + 2*e) + 4*(a*cos(4*f*x +
4*e) + 2*a*cos(2*f*x + 2*e) + 2*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))) + a)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*(
a*cos(4*f*x + 4*e) + 2*a*cos(2*f*x + 2*e) + a)*cos(1/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))) + 4*(a*sin(4*f*x + 4*e) + 2*a*sin(2*f*x + 2*e) + 2
*a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*(a*sin(4*f*x + 4*e) + 2*a*sin(2*f*x
+ 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + a)*sqrt(a)*f
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(c^3\sec\left(fx+e\right)^3-3\,c^3\sec\left(fx+e\right)^2+3\,c^3\sec\left(fx+e\right)-c^3\right)\sqrt{a\sec\left(fx+e\right)+a}\sqrt{-c\sec\left(fx+e\right)+c}}{a^2\sec\left(fx+e\right)^2+2\,a^2\sec\left(fx+e\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] integral(-(c^3*sec(f*x + e)^3 - 3*c^3*sec(f*x + e)^2 + 3*c^3*sec(f*x + e) -
 c^3)*sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^2*sec(f*x + e)^
2 + 2*a^2*sec(f*x + e) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**(7/2)/(a+a*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError
```

3.118 $\int \frac{(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=96

$$\frac{c^3\tan(e+fx)\log(\cos(e+fx))}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{4c^3\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] (-4*c^3*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]
]) + (c^3*Log[Cos[e + f*x]]*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e + f*x]]*Sqr
t[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.193182, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3910, 3905, 3475}

$$\frac{c^3\tan(e+fx)\log(\cos(e+fx))}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{4c^3\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^(5/2)/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-4*c^3*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]
]) + (c^3*Log[Cos[e + f*x]]*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e + f*x]]*Sqr
t[c - c*Sec[e + f*x]])
```

Rule 3910

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(5/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[(-8*a^3*Cot[e + f*x]*(c + d*Csc[e + f
*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[a^2/c^2, Int[Sqrt
[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3905

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Dist[((-(a*c))^(m + 1/2)*Cot[e + f*x])/(Sqrt
[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int[Cot[e + f*x]^(2*m), x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
```

```
2, 0] && IntegerQ[m + 1/2]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{3/2}}\,dx &= -\frac{4c^3\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}+\frac{c^2\int \sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,d}{a^2}\\ &= -\frac{4c^3\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}-\frac{\left(c^3\tan(e+fx)\right)\int \tan(e+fx)\,dx}{a\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\ &= -\frac{4c^3\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}+\frac{c^3\log(\cos(e+fx))\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\end{aligned}$$

Mathematica [C] time = 0.731135, size = 116, normalized size = 1.21

$$\frac{ic^2\cot\left(\frac{1}{2}(e+fx)\right)\sqrt{c-c\sec(e+fx)}\left(i\log\left(1+e^{2i(e+fx)}\right)+\left(fx+i\log\left(1+e^{2i(e+fx)}\right)\right)\cos(e+fx)+fx+4i\right)}{af(\cos(e+fx)+1)\sqrt{a(\sec(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^(5/2)/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (I*c^2*Cot[(e + f*x)/2]*(4*I + f*x + Cos[e + f*x]*(f*x + I*Log[1 + E^((2*I)
*(e + f*x))]) + I*Log[1 + E^((2*I)*(e + f*x))])*Sqrt[c - c*Sec[e + f*x]])/(
a*f*(1 + Cos[e + f*x])*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [B] time = 0.27, size = 236, normalized size = 2.5

$$\frac{\left(\cos\left(fx+e\right)\right)^3}{fa^2\left(\sin\left(fx+e\right)\right)^3\left(-1+\cos\left(fx+e\right)\right)}\left(\cos\left(fx+e\right)\ln\left(\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)+\cos\left(fx+e\right)\ln\left(-\frac{-1+\cos}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(3/2),x)
```

```
[Out] 1/f/a^2*(cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+cos(f*x+e)*ln(
-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-cos(f*x+e)*ln(2/(1+cos(f*x+e)))+ln(
(1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*
x+e))-2*cos(f*x+e)-ln(2/(1+cos(f*x+e)))+2)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(
5/2)*cos(f*x+e)^3*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)/sin(f*x+e)^3/(-1+co
s(f*x+e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [B] time = 1.35289, size = 1098, normalized size = 11.44

$$\left[-\frac{4\,c^2\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos\left(fx+e\right)\sin\left(fx+e\right)-\left(ac^2\cos\left(fx+e\right)^2+2\,ac^2\cos\left(fx+e\right)+ac^2\right)\sqrt{-\frac{c}{a}}\log\left(\frac{c\cos\cdots}{\cdots}\right.}{2\left(a^2f\cos\left(fx+e\right)^2+2\,a^2f\cos\left(fx+e\right)+a^2f\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] [-1/2*(4*c^2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) -
 c)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - (a*c^2*cos(f*x + e)^2 + 2*a*c
^2*cos(f*x + e) + a*c^2)*sqrt(-c/a)*log(1/2*(c*cos(f*x + e)^4 - (cos(f*x +
e)^3 + cos(f*x + e))*sqrt(-c/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqr
```

```
t((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) + c)/cos(f*x + e)^2))/(a^
2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f), -(2*c^2*sqrt((a*cos(f*x
 + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x +
e)*sin(f*x + e) - (a*c^2*cos(f*x + e)^2 + 2*a*c^2*cos(f*x + e) + a*c^2)*sqr
t(c/a)*arctan(sqrt(c/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos
(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(c*cos(f*x + e)^2 +
c)))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 3.05831, size = 185, normalized size = 1.93

$$-\frac{c\left(\frac{\sqrt{-ac}c^2\log\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)}{a^2|c|}-\frac{\sqrt{-ac}c^2\log\left(\left|c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+c\right|\right)}{a^2|c|}+\frac{2\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)\sqrt{-ac}c}{a^2|c|}\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+\tan\left(\frac{1}{2}\right.\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] -c*(sqrt(-a*c)*c^2*log(c*tan(1/2*f*x + 1/2*e)^2 - c)/(a^2*abs(c)) - sqrt(-a
*c)*c^2*log(abs(c*tan(1/2*f*x + 1/2*e)^2 + c))/(a^2*abs(c)) + 2*(c*tan(1/2*
f*x + 1/2*e)^2 - c)*sqrt(-a*c)*c/(a^2*abs(c)))*sgn(tan(1/2*f*x + 1/2*e)^3 +
 tan(1/2*f*x + 1/2*e))/f
```

3.119 $\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=98

$$\frac{c^2\tan(e+fx)\log(\cos(e+fx)+1)}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{2c^2\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] (-2*c^2*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]
]) + (c^2*Log[1 + Cos[e + f*x]]*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e + f*x]]
*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.195762, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3908, 3911, 31}

$$\frac{c^2\tan(e+fx)\log(\cos(e+fx)+1)}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{2c^2\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^(3/2)/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-2*c^2*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]
]) + (c^2*Log[1 + Cos[e + f*x]]*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e + f*x]]
*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3908

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[(-4*a^2*Cot[e + f*x]*(c + d*Csc[e + f
*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[a/c, Int[Sqrt[a +
 b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3911

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> -Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((b + a*x)^(m - 1/2)*(d + c*x
)^(n - 1/2))/x^(m + n), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && EqQ
```

```
[m + n, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = -\frac{2c^2\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}+\frac{c\int\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx}{a}$$

$$= -\frac{2c^2\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}+\frac{\left(c^2\tan(e+fx)\right)\text{Subst}\left(\int\frac{1}{a+ax}\,dx,x,\cos(e\right.}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+f}}$$

$$= -\frac{2c^2\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}+\frac{c^2\log(1+\cos(e+fx))\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

Mathematica [C] time = 1.11316, size = 114, normalized size = 1.16

$$\frac{ic\cot\left(\frac{1}{2}(e+fx)\right)\sqrt{c-c\sec(e+fx)}\left(2i\log\left(1+e^{i(e+fx)}\right)+\left(fx+2i\log\left(1+e^{i(e+fx)}\right)\right)\cos(e+fx)+fx+2i\right)}{af(\cos(e+fx)+1)\sqrt{a(\sec(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^(3/2)/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (I*c*Cot[(e + f*x)/2]*(2*I + f*x + Cos[e + f*x]*(f*x + (2*I)*Log[1 + E^(I*(
e + f*x))]) + (2*I)*Log[1 + E^(I*(e + f*x))])*Sqrt[c - c*Sec[e + f*x]])/(a*
f*(1 + Cos[e + f*x])*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [A] time = 0.256, size = 106, normalized size = 1.1

$$-\frac{\left(\cos\left(fx+e\right)\right)^2}{fa^2\left(\sin\left(fx+e\right)\right)^3}\left(\cos\left(fx+e\right)\ln\left(2\left(1+\cos\left(fx+e\right)\right)^{-1}\right)+\cos\left(fx+e\right)+\ln\left(2\left(1+\cos\left(fx+e\right)\right)^{-1}\right)-1\right)\left(\frac{c(-1}{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(3/2),x)
```

```
[Out] -1/f/a^2*(cos(f*x+e)*ln(2/(1+cos(f*x+e)))+cos(f*x+e)+ln(2/(1+cos(f*x+e)))-1
)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(3/2)*cos(f*x+e)^2*(1/cos(f*x+e)*a*(1+cos(
f*x+e)))^(1/2)/sin(f*x+e)^3
```

Maxima [A] time = 1.49343, size = 95, normalized size = 0.97

$$\frac{\frac{c^{\frac{3}{2}}\log\left(\frac{\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+1\right)}{\sqrt{-a}a}-\frac{c^{\frac{3}{2}}\sin\left(fx+e\right)^{2}}{\sqrt{-a}a\left(\cos\left(fx+e\right)+1\right)^{2}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] (c^(3/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(sqrt(-a)*a) - c^(3/2
)*sin(f*x + e)^2/(sqrt(-a)*a*(cos(f*x + e) + 1)^2))/f
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a\sec\left(fx+e\right)+a}\left(-c\sec\left(fx+e\right)+c\right)^{\frac{3}{2}}}{a^{2}\sec\left(fx+e\right)^{2}+2\,a^{2}\sec\left(fx+e\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^(3/2)/(a^2*sec(f*x
+ e)^2 + 2*a^2*sec(f*x + e) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.34846, size = 170, normalized size = 1.73

$$-\frac{\left(\frac{\sqrt{-ac}c^3\log(2|c|)}{a^2|c|}-\frac{\sqrt{-ac}c^3\log\left(\left|c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+c\right|\right)}{a^2|c|}+\frac{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)\sqrt{-ac}c^2}{a^2|c|}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] -(sqrt(-a*c)*c^3*log(2*abs(c))/(a^2*abs(c)) - sqrt(-a*c)*c^3*log(abs(c*tan(
1/2*f*x + 1/2*e)^2 + c))/(a^2*abs(c)) + (c*tan(1/2*f*x + 1/2*e)^2 - c)*sqrt
(-a*c)*c^2/(a^2*abs(c)))*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e))
/(c*f)
```

3.120 $\int \frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=94

$$\frac{c\tan(e+fx)\log(\cos(e+fx)+1)}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{c\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -((c*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]]))
 + (c*Log[1 + Cos[e + f*x]]*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e + f*x]]*Sqr
t[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.189419, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3907, 3911, 31}

$$\frac{c\tan(e+fx)\log(\cos(e+fx)+1)}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{c\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c*Sec[e + f*x]]/(a + a*Sec[e + f*x])^(3/2),x]
```


```
[Out] -((c*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]]))
 + (c*Log[1 + Cos[e + f*x]]*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e + f*x]]*Sqr
t[c - c*Sec[e + f*x]])
```


Rule 3907

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_), x_Symbol] :> Simp[(-2*a*Cot[e + f*x]*(c + d*Csc[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[1/c, Int[Sqrt[a + b*Cs
c[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3911

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> -Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((b + a*x)^(m - 1/2)*(d + c*x
)^(n - 1/2))/x^(m + n), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && EqQ
```

```
[m + n, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx = -\frac{c\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}} + \frac{\int \frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx}{a}$$

$$= -\frac{c\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}} + \frac{(c\tan(e+fx))\operatorname{Subst}\left(\int \frac{1}{a+ax}\,dx, x, \cos(e-$$

$$= -\frac{c\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}} + \frac{c\log(1+\cos(e+fx))\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

Mathematica [C] time = 0.561093, size = 106, normalized size = 1.13

$$\frac{i\cot\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\sqrt{c-c\sec(e+fx)}\left(2i\log\left(1+e^{i(e+fx)}\right)+\left(fx+2i\log\left(1+e^{i(e+fx)}\right)\right)\cos(e+fx)+fx+i\right)}{f(a(\sec(e+fx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c*Sec[e + f*x]]/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (I*Cot[(e + f*x)/2]*(I + f*x + Cos[e + f*x]*(f*x + (2*I)*Log[1 + E^(I*(e +
f*x))]) + (2*I)*Log[1 + E^(I*(e + f*x))])*Sec[e + f*x]*Sqrt[c - c*Sec[e + f
*x]])/(f*(a*(1 + Sec[e + f*x]))^(3/2))
```

Maple [A] time = 0.291, size = 119, normalized size = 1.3

$$-\frac{\cos\left(fx+e\right)}{2\,fa^{2}\left(\sin\left(fx+e\right)\right)^{3}}\left(2\,\ln\left(2\,\left(1+\cos\left(fx+e\right)\right)^{-1}\right)\left(\cos\left(fx+e\right)\right)^{2}+\left(\cos\left(fx+e\right)\right)^{2}-2\,\cos\left(fx+e\right)-2\,\ln\left(2\,\left(1+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(3/2),x)
```

```
[Out] -1/2/f/a^2*(2*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^2+cos(f*x+e)^2-2*cos(f*x+e)-2
*ln(2/(1+cos(f*x+e)))+1)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)*(1/cos(f*x+e)
*a*(1+cos(f*x+e)))^(1/2)*cos(f*x+e)/sin(f*x+e)^3
```

Maxima [B] time = 1.78872, size = 533, normalized size = 5.67

$$-\frac{\left(\left(fx+e\right)\cos\left(2\,fx+2\,e\right)^{2}+4\left(fx+e\right)\cos\left(fx+e\right)^{2}+\left(fx+e\right)\sin\left(2\,fx+2\,e\right)^{2}+4\left(fx+e\right)\sin\left(fx+e\right)^{2}+fx-2\left(2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] -((f*x + e)*cos(2*f*x + 2*e)^2 + 4*(f*x + e)*cos(f*x + e)^2 + (f*x + e)*sin
(2*f*x + 2*e)^2 + 4*(f*x + e)*sin(f*x + e)^2 + f*x - 2*(2*(2*cos(f*x + e) +
 1)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 4*cos(f*x + e)^2 + sin(2*f*x +
2*e)^2 + 4*sin(2*f*x + 2*e)*sin(f*x + e) + 4*sin(f*x + e)^2 + 4*cos(f*x + e
) + 1)*arctan2(sin(f*x + e), cos(f*x + e) + 1) + 2*(f*x + 2*(f*x + e)*cos(f
*x + e) + e - sin(f*x + e))*cos(2*f*x + 2*e) + 4*(f*x + e)*cos(f*x + e) + 2
*(2*(f*x + e)*sin(f*x + e) + cos(f*x + e))*sin(2*f*x + 2*e) + e - 2*sin(f*x
 + e))*sqrt(a)*sqrt(c)/((a^2*cos(2*f*x + 2*e)^2 + 4*a^2*cos(f*x + e)^2 + a^
2*sin(2*f*x + 2*e)^2 + 4*a^2*sin(2*f*x + 2*e)*sin(f*x + e) + 4*a^2*sin(f*x
+ e)^2 + 4*a^2*cos(f*x + e) + a^2 + 2*(2*a^2*cos(f*x + e) + a^2)*cos(2*f*x
+ 2*e))*f)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{a\sec\left(fx+e\right)+a}\sqrt{-c\sec\left(fx+e\right)+c}}{a^{2}\sec\left(fx+e\right)^{2}+2\,a^{2}\sec\left(fx+e\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^2*sec(f*x +
e)^2 + 2*a^2*sec(f*x + e) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}}{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(-c*(sec(e + f*x) - 1))/(a*(sec(e + f*x) + 1))**(3/2), x)
```

Giac [A] time = 1.89995, size = 158, normalized size = 1.68

$$\frac{\sqrt{2}c\left(\frac{2\sqrt{2}\sqrt{-ac}\log\left(\left|c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+c\right|\right)}{a^2|c|}-\frac{\sqrt{2}\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)\sqrt{-ac}}{a^2c|c|}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\cos\left(fx\right.\right.}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] 1/4*sqrt(2)*c*(2*sqrt(2)*sqrt(-a*c)*log(abs(c*tan(1/2*f*x + 1/2*e)^2 + c))/
(a^2*abs(c)) - sqrt(2)*(c*tan(1/2*f*x + 1/2*e)^2 - c)*sqrt(-a*c)/(a^2*c*abs
(c))*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e))*sgn(cos(f*x + e))/
f
```

3.121 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal. Leaf size=215

$$-\frac{\tan(e+fx)}{2af(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{\tan(e+fx)\log(1-\sec(e+fx))}{4af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{3\tan(e+}{4af\sqrt{a\sec(e}}$$

```
[Out] (Log[Cos[e + f*x]]*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*S
ec[e + f*x]]) + (Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(4*a*f*Sqrt[a + a*Sec[
e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) + (3*Log[1 + Sec[e + f*x]]*Tan[e + f*x]
)/(4*a*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) - Tan[e + f*x]/
(2*a*f*(1 + Sec[e + f*x])*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]
)
```

Rubi [A] time = 0.146146, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3912, 72}

$$-\frac{\tan(e+fx)}{2af(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{\tan(e+fx)\log(1-\sec(e+fx))}{4af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{3\tan(e+}{4af\sqrt{a\sec(e}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]]),x]
```

```
[Out] (Log[Cos[e + f*x]]*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*S
ec[e + f*x]]) + (Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(4*a*f*Sqrt[a + a*Sec[
e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) + (3*Log[1 + Sec[e + f*x]]*Tan[e + f*x]
)/(4*a*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) - Tan[e + f*x]/
(2*a*f*(1 + Sec[e + f*x])*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]
)
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
x)^(n - 1/2))/x, x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{(ac\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{x(a+ax)^2(c-cx)}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{(ac\tan(e+fx))\,\text{Subst}\left(\int \left(-\frac{1}{4a^2c(-1+x)}+\frac{1}{a^2cx}-\frac{1}{2a^2c(1+x)^2}-\frac{3}{4a^2c(1+x)}\right)\right.}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= \frac{\log(\cos(e+fx))\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{\log(1-\sec(e+fx))\,\text{ta}}{4af\sqrt{a+a\sec(e+fx)}\sqrt{c-}}$$

Mathematica [C] time = 1.38374, size = 141, normalized size = 0.66

$$\frac{\tan(e+fx)\left(\log\left(1-e^{i(e+fx)}\right)+3\log\left(1+e^{i(e+fx)}\right)+\left(\log\left(1-e^{i(e+fx)}\right)+3\log\left(1+e^{i(e+fx)}\right)-2ifx\right)\cos(e+fx)-2ifx}{2af(\cos(e+fx)+1)\sqrt{a(\sec(e+fx)+1)}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]]),x]
```

```
[Out] ((1 - (2*I)*f*x + Log[1 - E^(I*(e + f*x))] + 3*Log[1 + E^(I*(e + f*x))] + C
os[e + f*x]*((-2*I)*f*x + Log[1 - E^(I*(e + f*x))] + 3*Log[1 + E^(I*(e + f*
x))]))*Tan[e + f*x])/(2*a*f*(1 + Cos[e + f*x])*Sqrt[a*(1 + Sec[e + f*x])]*S
qrt[c - c*Sec[e + f*x]])
```

Maple [A] time = 0.3, size = 161, normalized size = 0.8

$$\frac{\left(-1+\cos\left(fx+e\right)\right)^2}{4\,fa^2\left(\sin\left(fx+e\right)\right)^3}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\left(2\,\cos\left(fx+e\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-4\,\cos\left(fx+e\right)\ln\left(2\,\left(1+\cos\left(f\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/4/f/a^2*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(-1+cos(f*x+e))^2*(2*cos(f*
x+e)*ln(-(-1+cos(f*x+e))/sin(f*x+e))-4*cos(f*x+e)*ln(2/(1+cos(f*x+e)))-cos(
f*x+e)+2*ln(-(-1+cos(f*x+e))/sin(f*x+e))-4*ln(2/(1+cos(f*x+e)))+1)/(c*(-1+c
os(f*x+e))/cos(f*x+e))^(1/2)/sin(f*x+e)^3
```

Maxima [B] time = 1.91558, size = 1104, normalized size = 5.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] -1/2*(2*(f*x + e)*cos(2*f*x + 2*e)^2 + 8*(f*x + e)*cos(1/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e)))^2 + 2*(f*x + e)*sin(2*f*x + 2*e)^2 + 8*(f*x +
e)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*f*x - 3*(cos(
2*f*x + 2*e)^2 + 4*(cos(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e))) + 4*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
)^2 + sin(2*f*x + 2*e)^2 + 4*sin(2*f*x + 2*e)*sin(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) + 4*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e)))^2 + 2*cos(2*f*x + 2*e) + 1)*arctan2(sin(1/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) +
 1) - (cos(2*f*x + 2*e)^2 + 4*(cos(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))) + 4*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e)))^2 + sin(2*f*x + 2*e)^2 + 4*sin(2*f*x + 2*e)*sin(1/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*sin(1/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e)))^2 + 2*cos(2*f*x + 2*e) + 1)*arctan2(sin(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e))) - 1) + 4*(f*x + e)*cos(2*f*x + 2*e) + 2*(4*f*x + 4*(f*x + e)*cos(
2*f*x + 2*e) + 4*e + sin(2*f*x + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e))) + 2*(4*(f*x + e)*sin(2*f*x + 2*e) - cos(2*f*x + 2*e) - 1)*
sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*e)/((a*cos(2*f*x +
 2*e)^2 + 4*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + a*si
n(2*f*x + 2*e)^2 + 4*a*sin(2*f*x + 2*e)*sin(1/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))) + 4*a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
)^2 + 2*a*cos(2*f*x + 2*e) + 4*(a*cos(2*f*x + 2*e) + a)*cos(1/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))) + a)*sqrt(a)*sqrt(c)*f)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{a\sec\left(fx+e\right)+a}\sqrt{-c\sec\left(fx+e\right)+c}}{a^2c\sec\left(fx+e\right)^3+a^2c\sec\left(fx+e\right)^2-a^2c\sec\left(fx+e\right)-a^2c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^2*c*sec(f*x
 + e)^3 + a^2*c*sec(f*x + e)^2 - a^2*c*sec(f*x + e) - a^2*c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/((a*(sec(e + f*x) + 1))**(3/2)*sqrt(-c*(sec(e + f*x) - 1))), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] Timed out
```

3.122 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=101

$$\frac{\cot(e+fx)}{2acf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{\tan(e+fx)\log(\sin(e+fx))}{acf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] Cot[e + f*x]/(2*a*c*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) +
(Log[Sin[e + f*x]]*Tan[e + f*x])/(a*c*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c
*Sec[e + f*x]])
```

Rubi [A] time = 0.121461, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3905, 3473, 3475}

$$\frac{\cot(e+fx)}{2acf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{\tan(e+fx)\log(\sin(e+fx))}{acf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(3/2)),x]
```

```
[Out] Cot[e + f*x]/(2*a*c*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) +
(Log[Sin[e + f*x]]*Tan[e + f*x])/(a*c*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c
*Sec[e + f*x]])
```

Rule 3905

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Dist[((-(a*c))^(m + 1/2)*Cot[e + f*x])/(Sqrt
[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int[Cot[e + f*x]^(2*m), x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IntegerQ[m + 1/2]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}}\,dx &= -\frac{\tan(e+fx)\int \cot^3(e+fx)\,dx}{ac\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} \\
&= \frac{\cot(e+fx)}{2acf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{\tan(e+fx)\int \mathrm{co}}{ac\sqrt{a+a\sec(e+fx)}\sqrt{}} \\
&= \frac{\cot(e+fx)}{2acf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{\log(\sin(e+fx))}{acf\sqrt{a+a\sec(e+fx)}}
\end{aligned}$$

Mathematica [C] time = 1.47835, size = 121, normalized size = 1.2

$$\frac{\tan(e+fx)\sec^2(e+fx)\left(\log\left(1-e^{2i(e+fx)}\right)+\left(ifx-\log\left(1-e^{2i(e+fx)}\right)\right)\cos(2(e+fx))-ifx+1\right)}{2cf(\sec(e+fx)-1)(a(\sec(e+fx)+1))^{3/2}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(3/2)),x]
```

```
[Out] ((1 - I*f*x + Cos[2*(e + f*x)]*(I*f*x - Log[1 - E^((2*I)*(e + f*x))]) + Log
[1 - E^((2*I)*(e + f*x))])*Sec[e + f*x]^2*Tan[e + f*x])/(2*c*f*(-1 + Sec[e
+ f*x])*(a*(1 + Sec[e + f*x]))^(3/2)*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] time = 0.264, size = 175, normalized size = 1.7

$$\frac{\left(-1+\cos\left(fx+e\right)\right)^2}{4fa^2\left(\sin\left(fx+e\right)\right)^3\cos\left(fx+e\right)}\left(4\ln\left(-\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right)\left(\cos\left(fx+e\right)\right)^2-4\ln\left(2\left(1+\cos\left(fx+e\right)\right)^{-1}\right)\left(\cos\left(fx\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x)
```

```
[Out] 1/4/f/a^2*(-1+cos(f*x+e))^2*(4*ln(-(-1+cos(f*x+e))/sin(f*x+e))*cos(f*x+e)^2
-4*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^2-cos(f*x+e)^2-4*ln(-(-1+cos(f*x+e))/sin
```

```
(f*x+e))+4*ln(2/(1+cos(f*x+e))-1)*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)/si
n(f*x+e)^3/(c*(-1+cos(f*x+e))/cos(f*x+e))^(3/2)/cos(f*x+e)
```

Maxima [B] time = 1.83737, size = 656, normalized size = 6.5

$$-\frac{\left(\left(fx+e\right)\cos\left(4\,fx+4\,e\right)^{2}+4\left(fx+e\right)\cos\left(2\,fx+2\,e\right)^{2}+\left(fx+e\right)\sin\left(4\,fx+4\,e\right)^{2}+4\left(fx+e\right)\sin\left(2\,fx+2\,e\right)^{2}+fx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="max
ima")
```

```
[Out] -((f*x + e)*cos(4*f*x + 4*e)^2 + 4*(f*x + e)*cos(2*f*x + 2*e)^2 + (f*x + e)
*sin(4*f*x + 4*e)^2 + 4*(f*x + e)*sin(2*f*x + 2*e)^2 + f*x + (2*(2*cos(2*f*
x + 2*e) - 1)*cos(4*f*x + 4*e) - cos(4*f*x + 4*e)^2 - 4*cos(2*f*x + 2*e)^2
- sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) - 4*sin(2*f*x +
2*e)^2 + 4*cos(2*f*x + 2*e) - 1)*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
 - 1) + 2*(f*x - 2*(f*x + e)*cos(2*f*x + 2*e) + e + sin(2*f*x + 2*e))*cos(4
*f*x + 4*e) - 4*(f*x + e)*cos(2*f*x + 2*e) - 2*(2*(f*x + e)*sin(2*f*x + 2*e
) + cos(2*f*x + 2*e))*sin(4*f*x + 4*e) + e + 2*sin(2*f*x + 2*e))*sqrt(a)*sq
rt(c)/((a^2*c^2*cos(4*f*x + 4*e)^2 + 4*a^2*c^2*cos(2*f*x + 2*e)^2 + a^2*c^2
*sin(4*f*x + 4*e)^2 - 4*a^2*c^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*a^2*c
^2*sin(2*f*x + 2*e)^2 - 4*a^2*c^2*cos(2*f*x + 2*e) + a^2*c^2 - 2*(2*a^2*c^2
*cos(2*f*x + 2*e) - a^2*c^2)*cos(4*f*x + 4*e))*f)
```

Fricas [B] time = 2.43781, size = 1242, normalized size = 12.3

$$\left[-\frac{9\sqrt{-ac}\left(\cos\left(fx+e\right)^{2}-1\right)\log\left(-\frac{8\left(\left(256\cos\left(fx+e\right)^{5}-512\cos\left(fx+e\right)^{3}+175\cos\left(fx+e\right)\right)\sqrt{-ac}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}-\left(256\,ac\cos\left(fx+e\right)^{4}\right.}{\left(\cos\left(fx+e\right)^{2}-1\right)\sin\left(fx+e\right)}\right.}{18\left(a^{2}c^{2}f\cos\left(fx+e\right)^{2}-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="fri
cas")
```

```
[Out] [-1/18*(9*sqrt(-a*c)*(cos(f*x + e)^2 - 1)*log(-8*((256*cos(f*x + e)^5 - 512
*cos(f*x + e)^3 + 175*cos(f*x + e))*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/co
s(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)) - (256*a*c*cos(f*x + e)
^4 - 512*a*c*cos(f*x + e)^2 + 337*a*c)*sin(f*x + e))/((cos(f*x + e)^2 - 1)*
sin(f*x + e)))*sin(f*x + e) + (16*cos(f*x + e)^3 - 25*cos(f*x + e))*sqrt((a
*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/(
(a^2*c^2*f*cos(f*x + e)^2 - a^2*c^2*f)*sin(f*x + e)), -1/18*(18*sqrt(a*c)*(
cos(f*x + e)^2 - 1)*arctan((16*cos(f*x + e)^3 - 7*cos(f*x + e))*sqrt(a*c)*s
qrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x +
e))/((16*a*c*cos(f*x + e)^2 - 25*a*c)*sin(f*x + e)))*sin(f*x + e) + (16*cos
(f*x + e)^3 - 25*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt
((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^2*c^2*f*cos(f*x + e)^2 - a^2*c^2*f
)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="gia
c")
```

```
[Out] Timed out
```

3.123 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=347

$$-\frac{\tan(e+fx)}{2ac^2f(1-\sec(e+fx))\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)}{8ac^2f(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+}}$$

```
[Out] (Log[Cos[e + f*x]]*Tan[e + f*x])/(a*c^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
 c*Sec[e + f*x]]) + (11*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(16*a*c^2*f*Sqr
t[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) + (5*Log[1 + Sec[e + f*x]]*
Tan[e + f*x])/(16*a*c^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]
) - Tan[e + f*x]/(8*a*c^2*f*(1 - Sec[e + f*x])^2*Sqrt[a + a*Sec[e + f*x]]*S
qrt[c - c*Sec[e + f*x]]) - Tan[e + f*x]/(2*a*c^2*f*(1 - Sec[e + f*x])*Sqrt[
a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) - Tan[e + f*x]/(8*a*c^2*f*(1
+ Sec[e + f*x])*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.183915, antiderivative size = 347, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3912, 88}

$$-\frac{\tan(e+fx)}{2ac^2f(1-\sec(e+fx))\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)}{8ac^2f(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(5/2)),x]
```


```
[Out] (Log[Cos[e + f*x]]*Tan[e + f*x])/(a*c^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
 c*Sec[e + f*x]]) + (11*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(16*a*c^2*f*Sqr
t[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) + (5*Log[1 + Sec[e + f*x]]*
Tan[e + f*x])/(16*a*c^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]
) - Tan[e + f*x]/(8*a*c^2*f*(1 - Sec[e + f*x])^2*Sqrt[a + a*Sec[e + f*x]]*S
qrt[c - c*Sec[e + f*x]]) - Tan[e + f*x]/(2*a*c^2*f*(1 - Sec[e + f*x])*Sqrt[
a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) - Tan[e + f*x]/(8*a*c^2*f*(1
+ Sec[e + f*x])*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```


Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
```

```
x)^(n - 1/2))/x, x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{(ac\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{x(a+ax)^2(c-cx)^3}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{(ac\tan(e+fx))\,\text{Subst}\left(\int \left(-\frac{1}{4a^2c^3(-1+x)^3}+\frac{1}{2a^2c^3(-1+x)^2}-\frac{11}{16a^2c^3(-1+x}\right.\right.}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-}}$$

$$= \frac{\log(\cos(e+fx))\tan(e+fx)}{ac^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}+\frac{11\log(1-\sec(e+}{16ac^2f\sqrt{a+a\sec(e+}}$$

Mathematica [C] time = 2.55283, size = 275, normalized size = 0.79

$$\frac{\tan(e+fx)\left(22\log\left(1-e^{i(e+fx)}\right)+10\log\left(1+e^{i(e+fx)}\right)-8ifx\cos(3(e+fx))+11\log\left(1-e^{i(e+fx)}\right)\cos(3(e+fx))+\left(-1\right.\right.}{32ac^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(5/2)),x]
```

```
[Out] ((14 - (16*I)*f*x - (8*I)*f*x*Cos[3*(e + f*x)] + 22*Log[1 - E^(I*(e + f*x))
] + 11*Cos[3*(e + f*x)]*Log[1 - E^(I*(e + f*x))] + Cos[e + f*x]*(-12 + (8*I
)*f*x - 11*Log[1 - E^(I*(e + f*x))] - 5*Log[1 + E^(I*(e + f*x))]) + 2*Cos[2
*(e + f*x)]*(-5 + (8*I)*f*x - 11*Log[1 - E^(I*(e + f*x))] - 5*Log[1 + E^(I*
(e + f*x))]) + 10*Log[1 + E^(I*(e + f*x))] + 5*Cos[3*(e + f*x)]*Log[1 + E^(
I*(e + f*x))])*Tan[e + f*x])/(32*a*c^2*f*(-1 + Cos[e + f*x])^2*(1 + Cos[e +
 f*x])*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] time = 0.273, size = 291, normalized size = 0.8

$$\frac{\left(-1+\cos\left(fx+e\right)\right)^{2}}{32\,f a^{2}\left(\sin\left(fx+e\right)\right)^{3}\left(\cos\left(fx+e\right)\right)^{2}}\left(44\left(\cos\left(fx+e\right)\right)^{3}\ln\left(-\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-32\left(\cos\left(fx+e\right)\right)^{3}\ln\left(2\left(1+\cos\left(\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x)
```

```
[Out] 1/32/f/a^2*(-1+cos(f*x+e))^2*(44*cos(f*x+e)^3*ln(-(-1+cos(f*x+e))/sin(f*x+e
))-32*cos(f*x+e)^3*ln(2/(1+cos(f*x+e)))-13*cos(f*x+e)^3-44*ln(-(-1+cos(f*x+
e))/sin(f*x+e))*cos(f*x+e)^2+32*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^2-7*cos(f*x
+e)^2-44*cos(f*x+e)*ln(-(-1+cos(f*x+e))/sin(f*x+e))+32*cos(f*x+e)*ln(2/(1+c
os(f*x+e)))+cos(f*x+e)+44*ln(-(-1+cos(f*x+e))/sin(f*x+e))-32*ln(2/(1+cos(f*
x+e)))+11)*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)/sin(f*x+e)^3/cos(f*x+e)^2/
(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)
```

Maxima [B] time = 11.2721, size = 5767, normalized size = 16.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="max
ima")
```

```
[Out] -1/8*(8*(f*x + e)*cos(6*f*x + 6*e)^2 + 8*(f*x + e)*cos(4*f*x + 4*e)^2 + 8*(
f*x + e)*cos(2*f*x + 2*e)^2 + 32*(f*x + e)*cos(5/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e)))^2 + 128*(f*x + e)*cos(3/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e)))^2 + 32*(f*x + e)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e)))^2 + 8*(f*x + e)*sin(6*f*x + 6*e)^2 + 8*(f*x + e)*sin(4*f*x + 4*
e)^2 + 8*(f*x + e)*sin(2*f*x + 2*e)^2 + 32*(f*x + e)*sin(5/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e)))^2 + 128*(f*x + e)*sin(3/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e)))^2 + 32*(f*x + e)*sin(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e)))^2 + 8*f*x + 5*(2*(cos(4*f*x + 4*e) + cos(2*f*x + 2*e)
 - 1)*cos(6*f*x + 6*e) - cos(6*f*x + 6*e)^2 - 2*(cos(2*f*x + 2*e) - 1)*cos(
4*f*x + 4*e) - cos(4*f*x + 4*e)^2 - cos(2*f*x + 2*e)^2 + 4*(cos(6*f*x + 6*e
) - cos(4*f*x + 4*e) - cos(2*f*x + 2*e) + 4*cos(3/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))) - 2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
))) + 1)*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*cos(5/2*a
```

```
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 8*(cos(6*f*x + 6*e) - cos(4
*f*x + 4*e) - cos(2*f*x + 2*e) - 2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))) + 1)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 16
*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*(cos(6*f*x + 6*
e) - cos(4*f*x + 4*e) - cos(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) - 4*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e)))^2 + 2*(sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) - sin(6*
f*x + 6*e)^2 - sin(4*f*x + 4*e)^2 - 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) - s
in(2*f*x + 2*e)^2 + 4*(sin(6*f*x + 6*e) - sin(4*f*x + 4*e) - sin(2*f*x + 2*
e) + 4*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*sin(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(5/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) - 4*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
))^2 - 8*(sin(6*f*x + 6*e) - sin(4*f*x + 4*e) - sin(2*f*x + 2*e) - 2*sin(1/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e))) - 16*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e)))^2 + 4*(sin(6*f*x + 6*e) - sin(4*f*x + 4*e) - sin(2*f*x + 2*e))*sin
(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*sin(1/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*cos(2*f*x + 2*e) - 1)*arctan2(sin(1/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))) + 1) + 11*(2*(cos(4*f*x + 4*e) + cos(2*f*x + 2*e)
- 1)*cos(6*f*x + 6*e) - cos(6*f*x + 6*e)^2 - 2*(cos(2*f*x + 2*e) - 1)*cos(4
*f*x + 4*e) - cos(4*f*x + 4*e)^2 - cos(2*f*x + 2*e)^2 + 4*(cos(6*f*x + 6*e)
 - cos(4*f*x + 4*e) - cos(2*f*x + 2*e) + 4*cos(3/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) - 2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
)) + 1)*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*cos(5/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 8*(cos(6*f*x + 6*e) - cos(4*
f*x + 4*e) - cos(2*f*x + 2*e) - 2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))) + 1)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 16*
cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*(cos(6*f*x + 6*e
) - cos(4*f*x + 4*e) - cos(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e))) - 4*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e)))^2 + 2*(sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) - sin(6*f
*x + 6*e)^2 - sin(4*f*x + 4*e)^2 - 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) - si
n(2*f*x + 2*e)^2 + 4*(sin(6*f*x + 6*e) - sin(4*f*x + 4*e) - sin(2*f*x + 2*e
) + 4*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*sin(1/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(5/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e))) - 4*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
)^2 - 8*(sin(6*f*x + 6*e) - sin(4*f*x + 4*e) - sin(2*f*x + 2*e) - 2*sin(1/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))) - 16*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e)))^2 + 4*(sin(6*f*x + 6*e) - sin(4*f*x + 4*e) - sin(2*f*x + 2*e))*sin(
1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*sin(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*cos(2*f*x + 2*e) - 1)*arctan2(sin(1/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) - 1) + 4*(4*f*x - 4*(f*x + e)*cos(4*f*x + 4*e) - 4*
```

```
(f*x + e)*cos(2*f*x + 2*e) + 4*e + 3*sin(4*f*x + 4*e) + 3*sin(2*f*x + 2*e))
*cos(6*f*x + 6*e) - 16*(f*x - (f*x + e)*cos(2*f*x + 2*e) + e)*cos(4*f*x + 4
*e) - 16*(f*x + e)*cos(2*f*x + 2*e) - 2*(16*f*x + 16*(f*x + e)*cos(6*f*x +
6*e) - 16*(f*x + e)*cos(4*f*x + 4*e) - 16*(f*x + e)*cos(2*f*x + 2*e) + 64*(
f*x + e)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 32*(f*x + e
)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 16*e + 5*sin(6*f*x
 + 6*e) + 7*sin(4*f*x + 4*e) + 7*sin(2*f*x + 2*e) - 8*sin(3/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))) + 4*(16*f*x + 16*(f*x + e)*cos(6*f*x + 6*e) - 16*(f*x + e)*cos(4
*f*x + 4*e) - 16*(f*x + e)*cos(2*f*x + 2*e) - 32*(f*x + e)*cos(1/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 16*e + 7*sin(6*f*x + 6*e) + 5*sin(4*
f*x + 4*e) + 5*sin(2*f*x + 2*e) - 4*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e))))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*(16
*f*x + 16*(f*x + e)*cos(6*f*x + 6*e) - 16*(f*x + e)*cos(4*f*x + 4*e) - 16*(
f*x + e)*cos(2*f*x + 2*e) + 16*e + 5*sin(6*f*x + 6*e) + 7*sin(4*f*x + 4*e)
+ 7*sin(2*f*x + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
- 4*(4*(f*x + e)*sin(4*f*x + 4*e) + 4*(f*x + e)*sin(2*f*x + 2*e) + 3*cos(4*
f*x + 4*e) + 3*cos(2*f*x + 2*e))*sin(6*f*x + 6*e) + 4*(4*(f*x + e)*sin(2*f*
x + 2*e) + 3)*sin(4*f*x + 4*e) - 2*(16*(f*x + e)*sin(6*f*x + 6*e) - 16*(f*x
 + e)*sin(4*f*x + 4*e) - 16*(f*x + e)*sin(2*f*x + 2*e) + 64*(f*x + e)*sin(3
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 32*(f*x + e)*sin(1/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 5*cos(6*f*x + 6*e) - 7*cos(4*f*x
 + 4*e) - 7*cos(2*f*x + 2*e) + 8*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))) - 5)*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*(1
6*(f*x + e)*sin(6*f*x + 6*e) - 16*(f*x + e)*sin(4*f*x + 4*e) - 16*(f*x + e)
*sin(2*f*x + 2*e) - 32*(f*x + e)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))) - 7*cos(6*f*x + 6*e) - 5*cos(4*f*x + 4*e) - 5*cos(2*f*x + 2*e) +
 4*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 7)*sin(3/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*(16*(f*x + e)*sin(6*f*x + 6*e) -
 16*(f*x + e)*sin(4*f*x + 4*e) - 16*(f*x + e)*sin(2*f*x + 2*e) - 5*cos(6*f*
x + 6*e) - 7*cos(4*f*x + 4*e) - 7*cos(2*f*x + 2*e) - 5)*sin(1/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))) + 8*e + 12*sin(2*f*x + 2*e))/((a*c^2*cos(
6*f*x + 6*e)^2 + a*c^2*cos(4*f*x + 4*e)^2 + a*c^2*cos(2*f*x + 2*e)^2 + 4*a*
c^2*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16*a*c^2*cos(3
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*a*c^2*cos(1/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + a*c^2*sin(6*f*x + 6*e)^2 + a*c^2*
sin(4*f*x + 4*e)^2 + 2*a*c^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + a*c^2*sin(
2*f*x + 2*e)^2 + 4*a*c^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
))^2 + 16*a*c^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*
a*c^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*a*c^2*cos(
2*f*x + 2*e) + a*c^2 - 2*(a*c^2*cos(4*f*x + 4*e) + a*c^2*cos(2*f*x + 2*e) -
 a*c^2)*cos(6*f*x + 6*e) + 2*(a*c^2*cos(2*f*x + 2*e) - a*c^2)*cos(4*f*x + 4
*e) - 4*(a*c^2*cos(6*f*x + 6*e) - a*c^2*cos(4*f*x + 4*e) - a*c^2*cos(2*f*x
+ 2*e) + 4*a*c^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*a
*c^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + a*c^2)*cos(5/2*
```

```
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 8*(a*c^2*cos(6*f*x + 6*e) -
a*c^2*cos(4*f*x + 4*e) - a*c^2*cos(2*f*x + 2*e) - 2*a*c^2*cos(1/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e))) + a*c^2)*cos(3/2*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e))) - 4*(a*c^2*cos(6*f*x + 6*e) - a*c^2*cos(4*f*x + 4*e)
 - a*c^2*cos(2*f*x + 2*e) + a*c^2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))) - 2*(a*c^2*sin(4*f*x + 4*e) + a*c^2*sin(2*f*x + 2*e))*sin(6*f*
x + 6*e) - 4*(a*c^2*sin(6*f*x + 6*e) - a*c^2*sin(4*f*x + 4*e) - a*c^2*sin(2
*f*x + 2*e) + 4*a*c^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
- 2*a*c^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(5/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 8*(a*c^2*sin(6*f*x + 6*e) - a*c
^2*sin(4*f*x + 4*e) - a*c^2*sin(2*f*x + 2*e) - 2*a*c^2*sin(1/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))) - 4*(a*c^2*sin(6*f*x + 6*e) - a*c^2*sin(4*f*x + 4*e) - a*c^2*si
n(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(
a)*sqrt(c)*f)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{a\sec\left(fx+e\right)+a}\sqrt{-c\sec\left(fx+e\right)+c}}{a^2c^3\sec\left(fx+e\right)^5-a^2c^3\sec\left(fx+e\right)^4-2\,a^2c^3\sec\left(fx+e\right)^3+2\,a^2c^3\sec\left(fx+e\right)^2+a^2c^3\sec\left(fx+e\right)-a^2c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^2*c^3*sec(f
*x + e)^5 - a^2*c^3*sec(f*x + e)^4 - 2*a^2*c^3*sec(f*x + e)^3 + 2*a^2*c^3*s
ec(f*x + e)^2 + a^2*c^3*sec(f*x + e) - a^2*c^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="gia
c")
```

```
[Out] Timed out
```

3.124 $\int \frac{(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=220

$$\frac{4c^4\tan(e+fx)}{a^2f(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{4c^4\tan(e+fx)}{a^2f(\sec(e+fx)+1)^2\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] (c^4*Log[Cos[e + f*x]]*Tan[e + f*x])/(a^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c
 - c*Sec[e + f*x]]) + (2*c^4*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(a^2*f*Sqr
t[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) - (4*c^4*Tan[e + f*x])/(a^2
*f*(1 + Sec[e + f*x])^2*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
+ (4*c^4*Tan[e + f*x])/(a^2*f*(1 + Sec[e + f*x])*Sqrt[a + a*Sec[e + f*x]]*S
qrt[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.143807, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3912, 88}

$$\frac{4c^4\tan(e+fx)}{a^2f(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{4c^4\tan(e+fx)}{a^2f(\sec(e+fx)+1)^2\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^(7/2)/(a + a*Sec[e + f*x])^(5/2),x]
```


```
[Out] (c^4*Log[Cos[e + f*x]]*Tan[e + f*x])/(a^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c
 - c*Sec[e + f*x]]) + (2*c^4*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(a^2*f*Sqr
t[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) - (4*c^4*Tan[e + f*x])/(a^2
*f*(1 + Sec[e + f*x])^2*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
+ (4*c^4*Tan[e + f*x])/(a^2*f*(1 + Sec[e + f*x])*Sqrt[a + a*Sec[e + f*x]]*S
qrt[c - c*Sec[e + f*x]])
```


Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
x)^(n - 1/2))/x, x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{(ac\tan(e+fx))\,\text{Subst}\left(\int \frac{(c-cx)^3}{x(a+ax)^3}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$
$$= -\frac{(ac\tan(e+fx))\,\text{Subst}\left(\int \left(\frac{c^3}{a^3x}-\frac{8c^3}{a^3(1+x)^3}+\frac{4c^3}{a^3(1+x)^2}-\frac{2c^3}{a^3(1+x)}\right)dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$
$$= \frac{c^4\log(\cos(e+fx))\tan(e+fx)}{a^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{2c^4\log(1+\sec(e+fx))\tan(e+fx)}{a^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{}{a^2}$$

Mathematica [C] time = 2.39511, size = 157, normalized size = 0.71

$$\frac{c^3\cot\left(\frac{1}{2}(e+fx)\right)\sqrt{c-c\sec(e+fx)}\left(4\left(-4\log\left(1+e^{i(e+fx)}\right)+\log\left(1+e^{2i(e+fx)}\right)+ifx-2\right)\cos(e+fx)+\left(-4\log\left(1+e^{i}\right.\right.\right.}{2a^2f(\cos(e+fx)+1)^2\sqrt{a(\sec(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^(7/2)/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] (c^3*Cot[(e + f*x)/2]*(4*Cos[e + f*x]*(-2 + I*f*x - 4*Log[1 + E^(I*(e + f*x
))] + Log[1 + E^((2*I)*(e + f*x))]) + (3 + Cos[2*(e + f*x)])*(I*f*x - 4*Log
[1 + E^(I*(e + f*x))] + Log[1 + E^((2*I)*(e + f*x))]))*Sqrt[c - c*Sec[e + f
*x]])/(2*a^2*f*(1 + Cos[e + f*x])^2*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [A] time = 0.26, size = 335, normalized size = 1.5

$$\frac{(\cos(fx+e))^4}{fa^3(\sin(fx+e))^5(-1+\cos(fx+e))}\left(\ln\left(\frac{1-\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)(\cos(fx+e))^2+\ln\left(-\frac{-1+\cos(fx+e)+}{\sin(fx+}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(5/2),x)
```

```
[Out] 1/f/a^3*(ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2+ln(-(-1+cos(
f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2+ln(2/(1+cos(f*x+e)))*cos(f*x+e)
^2+cos(f*x+e)^2+2*cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+2*cos
(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+2*cos(f*x+e)*ln(2/(1+cos
(f*x+e)))-2*cos(f*x+e)+ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+ln(-(-1+cos
(f*x+e)+sin(f*x+e))/sin(f*x+e))+ln(2/(1+cos(f*x+e)))+1)*(c*(-1+cos(f*x+e))/
cos(f*x+e))^(7/2)*cos(f*x+e)^4*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)/sin(f*
x+e)^5/(-1+cos(f*x+e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(c^3 \sec\left(fx+e\right)^3 - 3\,c^3 \sec\left(fx+e\right)^2 + 3\,c^3 \sec\left(fx+e\right) - c^3\right)\sqrt{a \sec\left(fx+e\right)+a}\sqrt{-c \sec\left(fx+e\right)+c}}{a^3 \sec\left(fx+e\right)^3 + 3\,a^3 \sec\left(fx+e\right)^2 + 3\,a^3 \sec\left(fx+e\right) + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral(-(c^3*sec(f*x + e)^3 - 3*c^3*sec(f*x + e)^2 + 3*c^3*sec(f*x + e) -
 c^3)*sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^3*sec(f*x + e)^
3 + 3*a^3*sec(f*x + e)^2 + 3*a^3*sec(f*x + e) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**(7/2)/(a+a*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError
```

3.125 $\int \frac{(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=98

$$\frac{c^3\tan(e+fx)\log(\cos(e+fx)+1)}{a^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{2c^3\tan(e+fx)}{f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] (-2*c^3*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]
]) + (c^3*Log[1 + Cos[e + f*x]]*Tan[e + f*x])/(a^2*f*Sqrt[a + a*Sec[e + f*x
]]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.187188, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3910, 3911, 31}

$$\frac{c^3\tan(e+fx)\log(\cos(e+fx)+1)}{a^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{2c^3\tan(e+fx)}{f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^(5/2)/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-2*c^3*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]
]) + (c^3*Log[1 + Cos[e + f*x]]*Tan[e + f*x])/(a^2*f*Sqrt[a + a*Sec[e + f*x
]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3910

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(5/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[(-8*a^3*Cot[e + f*x]*(c + d*Csc[e + f
*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[a^2/c^2, Int[Sqrt
[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3911

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> -Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((b + a*x)^(m - 1/2)*(d + c*x
)^(n - 1/2))/x^(m + n), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && EqQ
```

```
[m + n, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{2c^3\tan(e+fx)}{f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}} + \frac{c^2\int \frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx}{a^2}$$
$$= -\frac{2c^3\tan(e+fx)}{f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}} + \frac{\left(c^3\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{a+ax}\,dx, x, \cos(e+\right.}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx}}$$
$$= -\frac{2c^3\tan(e+fx)}{f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}} + \frac{c^3\log(1+\cos(e+fx))\tan(e+fx)}{a^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

Mathematica [C] time = 1.51971, size = 154, normalized size = 1.57

$$\frac{ic^2\cot\left(\frac{1}{2}(e+fx)\right)\sqrt{c-c\sec(e+fx)}\left(6i\log\left(1+e^{i(e+fx)}\right)+\left(fx+2i\log\left(1+e^{i(e+fx)}\right)\right)\cos(2(e+fx))+4\left(2i\log\left(1+e^{i(e\right.\right.}{2a^2f(\cos(e+fx)+1)^2\sqrt{a(\sec(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^(5/2)/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] ((I/2)*c^2*Cot[(e + f*x)/2]*(4*I + 3*f*x + Cos[2*(e + f*x)]*(f*x + (2*I)*Lo
g[1 + E^(I*(e + f*x))]) + 4*Cos[e + f*x]*(2*I + f*x + (2*I)*Log[1 + E^(I*(e
 + f*x))]) + (6*I)*Log[1 + E^(I*(e + f*x))])*Sqrt[c - c*Sec[e + f*x]])/(a^2
*f*(1 + Cos[e + f*x])^2*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [A] time = 0.252, size = 144, normalized size = 1.5

$$\frac{\left(\cos\left(fx+e\right)\right)^3}{2\,fa^3\left(\sin\left(fx+e\right)\right)^5}\left(2\,\ln\left(2\,\left(1+\cos\left(fx+e\right)\right)^{-1}\right)\left(\cos\left(fx+e\right)\right)^2+3\,\left(\cos\left(fx+e\right)\right)^2+4\,\cos\left(fx+e\right)\ln\left(2\,\left(1+\cos\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(5/2),x)
```

```
[Out] 1/2/f/a^3*(2*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^2+3*cos(f*x+e)^2+4*cos(f*x+e)*
ln(2/(1+cos(f*x+e)))-2*cos(f*x+e)+2*ln(2/(1+cos(f*x+e)))-1)*(c*(-1+cos(f*x+
e))/cos(f*x+e))^(5/2)*cos(f*x+e)^3*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)/si
n(f*x+e)^5
```

Maxima [A] time = 1.51093, size = 138, normalized size = 1.41

$$\frac{\frac{2\,c^{\frac{5}{2}}\log\left(\frac{\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+1\right)}{\sqrt{-a}a^{2}}+\frac{\frac{2\,\sqrt{-a}c^{\frac{5}{2}}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-\frac{\sqrt{-a}c^{\frac{5}{2}}\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}}{a^{3}}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] 1/2*(2*c^(5/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(sqrt(-a)*a^2)
+ (2*sqrt(-a)*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - sqrt(-a)*c^(5/2
)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)/a^3)/f
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c^{2}\sec\left(fx+e\right)^{2}-2\,c^{2}\sec\left(fx+e\right)+c^{2}\right)\sqrt{a\sec\left(fx+e\right)+a}\sqrt{-c\sec\left(fx+e\right)+c}}{a^{3}\sec\left(fx+e\right)^{3}+3\,a^{3}\sec\left(fx+e\right)^{2}+3\,a^{3}\sec\left(fx+e\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral((c^2*sec(f*x + e)^2 - 2*c^2*sec(f*x + e) + c^2)*sqrt(a*sec(f*x + e
) + a)*sqrt(-c*sec(f*x + e) + c)/(a^3*sec(f*x + e)^3 + 3*a^3*sec(f*x + e)^2
```

```
 + 3*a^3*sec(f*x + e) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 7.16082, size = 170, normalized size = 1.73

$$-\frac{c\left(\frac{2\sqrt{-ac}c^2\log(2|c|)}{a^3|c|}-\frac{2\sqrt{-ac}c^2\log\left(\left|c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+c\right|\right)}{a^3|c|}-\frac{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^2\sqrt{-ac}|c|}{a^3c^2}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] -1/2*c*(2*sqrt(-a*c)*c^2*log(2*abs(c))/(a^3*abs(c)) - 2*sqrt(-a*c)*c^2*log(
abs(c*tan(1/2*f*x + 1/2*e)^2 + c))/(a^3*abs(c)) - (c*tan(1/2*f*x + 1/2*e)^2
 - c)^2*sqrt(-a*c)*abs(c)/(a^3*c^2))*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f
*x + 1/2*e))/f
```

3.126 $\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=144

$$\frac{c^2\tan(e+fx)\log(\cos(e+fx)+1)}{a^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{c^2\tan(e+fx)}{af(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}-\frac{c^2\tan(e+fx)}{f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c}}$$

```
[Out] -((c^2*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]]
)) - (c^2*Tan[e + f*x])/(a*f*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e +
f*x]]) + (c^2*Log[1 + Cos[e + f*x]]*Tan[e + f*x])/(a^2*f*Sqrt[a + a*Sec[e +
 f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.287751, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3908, 3907, 3911, 31}

$$\frac{c^2\tan(e+fx)\log(\cos(e+fx)+1)}{a^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{c^2\tan(e+fx)}{af(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}-\frac{c^2\tan(e+fx)}{f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^(3/2)/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] -((c^2*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]]
)) - (c^2*Tan[e + f*x])/(a*f*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e +
f*x]]) + (c^2*Log[1 + Cos[e + f*x]]*Tan[e + f*x])/(a^2*f*Sqrt[a + a*Sec[e +
 f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3908

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[(-4*a^2*Cot[e + f*x]*(c + d*Csc[e + f
*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[a/c, Int[Sqrt[a +
 b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3907

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_), x_Symbol] :> Simp[(-2*a*Cot[e + f*x]*(c + d*Csc[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[1/c, Int[Sqrt[a + b*Cs
```

```
c[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3911

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> -Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((b + a*x)^(m - 1/2)*(d + c*x
)^(n - 1/2))/x^(m + n), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && EqQ
[m + n, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{c^2\tan(e+fx)}{f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}} + \frac{c\int \frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx}{a}$$

$$= -\frac{c^2\tan(e+fx)}{f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}} - \frac{c^2\tan(e+fx)}{af(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}} +$$

$$= -\frac{c^2\tan(e+fx)}{f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}} - \frac{c^2\tan(e+fx)}{af(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}} +$$

$$= -\frac{c^2\tan(e+fx)}{f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}} - \frac{c^2\tan(e+fx)}{af(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}} +$$

Mathematica [C] time = 0.838573, size = 152, normalized size = 1.06

$$\frac{ic\cot\left(\frac{1}{2}(e+fx)\right)\sqrt{c-c\sec(e+fx)}\left(6i\log\left(1+e^{i(e+fx)}\right)+\left(fx+2i\log\left(1+e^{i(e+fx)}\right)\right)\cos(2(e+fx))+\left(8i\log\left(1+e^{i(e+fx}\right.\right.\right.}{2a^2f(\cos(e+fx)+1)^2\sqrt{a(\sec(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^(3/2)/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] ((I/2)*c*Cot[(e + f*x)/2]*(4*I + 3*f*x + Cos[2*(e + f*x)]*(f*x + (2*I)*Log[
1 + E^(I*(e + f*x))]) + Cos[e + f*x]*(6*I + 4*f*x + (8*I)*Log[1 + E^(I*(e +
 f*x))]) + (6*I)*Log[1 + E^(I*(e + f*x))])*Sqrt[c - c*Sec[e + f*x]])/(a^2*f
*(1 + Cos[e + f*x])^2*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [A] time = 0.253, size = 152, normalized size = 1.1

$$\frac{\left(-1+\cos\left(fx+e\right)\right)\left(\cos\left(fx+e\right)\right)^{2}}{4\,fa^{3}\left(\sin\left(fx+e\right)\right)^{5}}\left(4\,\ln\left(2\,\left(1+\cos\left(fx+e\right)\right)^{-1}\right)\left(\cos\left(fx+e\right)\right)^{2}+5\,\left(\cos\left(fx+e\right)\right)^{2}+8\,\cos\left(fx+e\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(5/2),x)
```

```
[Out] 1/4/f/a^3*(-1+cos(f*x+e))*(4*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^2+5*cos(f*x+e)
^2+8*cos(f*x+e)*ln(2/(1+cos(f*x+e)))-2*cos(f*x+e)+4*ln(2/(1+cos(f*x+e)))-3)
*(c*(-1+cos(f*x+e))/cos(f*x+e))^(3/2)*cos(f*x+e)^2*(1/cos(f*x+e)*a*(1+cos(f
*x+e)))^(1/2)/sin(f*x+e)^5
```

Maxima [B] time = 2.54643, size = 2411, normalized size = 16.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] -((f*x + e)*c*cos(4*f*x + 4*e)^2 + 36*(f*x + e)*c*cos(2*f*x + 2*e)^2 + 16*(
f*x + e)*c*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16*(f*x
 + e)*c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + (f*x + e)*
c*sin(4*f*x + 4*e)^2 + 36*(f*x + e)*c*sin(2*f*x + 2*e)^2 + 16*(f*x + e)*c*s
in(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16*(f*x + e)*c*sin(
1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 12*(f*x + e)*c*cos(2*f
*x + 2*e) + (f*x + e)*c - 2*(c*cos(4*f*x + 4*e)^2 + 36*c*cos(2*f*x + 2*e)^2
 + 16*c*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16*c*cos(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + c*sin(4*f*x + 4*e)^2 +
12*c*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 36*c*sin(2*f*x + 2*e)^2 + 16*c*sin
```

```
(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16*c*sin(1/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*(6*c*cos(2*f*x + 2*e) + c)*cos(4
*f*x + 4*e) + 12*c*cos(2*f*x + 2*e) + 8*(c*cos(4*f*x + 4*e) + 6*c*cos(2*f*x
 + 2*e) + 4*c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c)*cos
(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 8*(c*cos(4*f*x + 4*e) +
 6*c*cos(2*f*x + 2*e) + c)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))) + 8*(c*sin(4*f*x + 4*e) + 6*c*sin(2*f*x + 2*e) + 4*c*sin(1/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e))) + 8*(c*sin(4*f*x + 4*e) + 6*c*sin(2*f*x + 2*e))*sin(1/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c)*arctan2(sin(1/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))) + 1) + 2*(6*(f*x + e)*c*cos(2*f*x + 2*e) + (f*x + e)*c - 4*c*sin
(2*f*x + 2*e))*cos(4*f*x + 4*e) + 2*(4*(f*x + e)*c*cos(4*f*x + 4*e) + 24*(f
*x + e)*c*cos(2*f*x + 2*e) + 16*(f*x + e)*c*cos(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))) + 4*(f*x + e)*c + 3*c*sin(4*f*x + 4*e) + 2*c*sin(2*f*
x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*(4*(f*x
+ e)*c*cos(4*f*x + 4*e) + 24*(f*x + e)*c*cos(2*f*x + 2*e) + 4*(f*x + e)*c +
 3*c*sin(4*f*x + 4*e) + 2*c*sin(2*f*x + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) + 4*(3*(f*x + e)*c*sin(2*f*x + 2*e) + 2*c*cos(2*f*x
 + 2*e))*sin(4*f*x + 4*e) - 8*c*sin(2*f*x + 2*e) + 2*(4*(f*x + e)*c*sin(4*f
*x + 4*e) + 24*(f*x + e)*c*sin(2*f*x + 2*e) + 16*(f*x + e)*c*sin(1/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 3*c*cos(4*f*x + 4*e) - 2*c*cos(2*f
*x + 2*e) - 3*c)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*(
4*(f*x + e)*c*sin(4*f*x + 4*e) + 24*(f*x + e)*c*sin(2*f*x + 2*e) - 3*c*cos(
4*f*x + 4*e) - 2*c*cos(2*f*x + 2*e) - 3*c)*sin(1/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((a^3*cos(4*f*x + 4*e)^2 + 36*a^3*cos
(2*f*x + 2*e)^2 + 16*a^3*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
))^2 + 16*a^3*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + a^3*
sin(4*f*x + 4*e)^2 + 12*a^3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 36*a^3*sin(
2*f*x + 2*e)^2 + 16*a^3*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
)^2 + 16*a^3*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 12*a^
3*cos(2*f*x + 2*e) + a^3 + 2*(6*a^3*cos(2*f*x + 2*e) + a^3)*cos(4*f*x + 4*e
) + 8*(a^3*cos(4*f*x + 4*e) + 6*a^3*cos(2*f*x + 2*e) + 4*a^3*cos(1/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + a^3)*cos(3/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) + 8*(a^3*cos(4*f*x + 4*e) + 6*a^3*cos(2*f*x + 2*e)
+ a^3)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 8*(a^3*sin(4*
f*x + 4*e) + 6*a^3*sin(2*f*x + 2*e) + 4*a^3*sin(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
 + 8*(a^3*sin(4*f*x + 4*e) + 6*a^3*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))))*f)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a\sec\left(fx+e\right)+a}\left(-c\sec\left(fx+e\right)+c\right)^{\frac{3}{2}}}{a^3\sec\left(fx+e\right)^3+3\,a^3\sec\left(fx+e\right)^2+3\,a^3\sec\left(fx+e\right)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^(3/2)/(a^3*sec(f*x
+ e)^3 + 3*a^3*sec(f*x + e)^2 + 3*a^3*sec(f*x + e) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 4.4283, size = 316, normalized size = 2.19

$$-\frac{\frac{4\sqrt{-ac}c^2\log(2|c|)\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{a^3|c|}-\frac{4\sqrt{-ac}c^2\log\left(\left|c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+c\right|\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{a^3|c|}-\frac{\left(c\tan\left(\frac{1}{2}fx+\right.\right.}{}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] -1/4*(4*sqrt(-a*c)*c^2*log(2*abs(c))*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f
*x + 1/2*e))/(a^3*abs(c)) - 4*sqrt(-a*c)*c^2*log(abs(c*tan(1/2*f*x + 1/2*e)
^2 + c))*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e))/(a^3*abs(c)) -
```

```
((c*tan(1/2*f*x + 1/2*e)^2 - c)^2*sqrt(-a*c)*a^3*abs(c)*sgn(tan(1/2*f*x + 1
/2*e)^3 + tan(1/2*f*x + 1/2*e)) - 2*(c*tan(1/2*f*x + 1/2*e)^2 - c)*sqrt(-a*
c)*a^3*c*abs(c)*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e)))/(a^6*c^
2))/f
```

3.127 $\int \frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=140

$$\frac{c\tan(e+fx)\log(\cos(e+fx)+1)}{a^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{c\tan(e+fx)}{af(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}-\frac{c\tan(e+fx)}{2f(a\sec(e+fx)+a)^{5/2}\sqrt{c-}}$$

```
[Out] -(c*Tan[e + f*x])/(2*f*(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]])
 - (c*Tan[e + f*x])/(a*f*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]
]) + (c*Log[1 + Cos[e + f*x]]*Tan[e + f*x])/(a^2*f*Sqrt[a + a*Sec[e + f*x]]
*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.282233, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3907, 3911, 31}

$$\frac{c\tan(e+fx)\log(\cos(e+fx)+1)}{a^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{c\tan(e+fx)}{af(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}-\frac{c\tan(e+fx)}{2f(a\sec(e+fx)+a)^{5/2}\sqrt{c-}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c*Sec[e + f*x]]/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] -(c*Tan[e + f*x])/(2*f*(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]])
 - (c*Tan[e + f*x])/(a*f*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]
]) + (c*Log[1 + Cos[e + f*x]]*Tan[e + f*x])/(a^2*f*Sqrt[a + a*Sec[e + f*x]]
*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3907

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_), x_Symbol] :> Simp[(-2*a*Cot[e + f*x]*(c + d*Csc[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[1/c, Int[Sqrt[a + b*Cs
c[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3911

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> -Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((b + a*x)^(m - 1/2)*(d + c*x
```

```
)^(n - 1/2))/x^(m + n), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && EqQ
[m + n, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{c\tan(e+fx)}{2f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}+\frac{\int \frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx}{a}$$

$$= -\frac{c\tan(e+fx)}{2f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}-\frac{c\tan(e+fx)}{af(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{c\tan(e+fx)}{2f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}-\frac{c\tan(e+fx)}{af(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{c\tan(e+fx)}{2f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}-\frac{c\tan(e+fx)}{af(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}$$

Mathematica [C] time = 0.641076, size = 151, normalized size = 1.08

$$\frac{i\cot\left(\frac{1}{2}(e+fx)\right)\sqrt{c-c\sec(e+fx)}\left(6i\log\left(1+e^{i(e+fx)}\right)+\left(fx+2i\log\left(1+e^{i(e+fx)}\right)\right)\cos(2(e+fx))+4\left(2i\log\left(1+e^{i(e+f}\right.\right.\right.}{2a^2f(\cos(e+fx)+1)^2\sqrt{a(\sec(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c*Sec[e + f*x]]/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] ((I/2)*Cot[(e + f*x)/2]*(3*I + 3*f*x + Cos[2*(e + f*x)]*(f*x + (2*I)*Log[1
+ E^(I*(e + f*x))]) + 4*Cos[e + f*x]*(I + f*x + (2*I)*Log[1 + E^(I*(e + f*x
))]) + (6*I)*Log[1 + E^(I*(e + f*x))])*Sqrt[c - c*Sec[e + f*x]])/(a^2*f*(1
+ Cos[e + f*x])^2*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [A] time = 0.291, size = 152, normalized size = 1.1

$$\frac{\left(-1+\cos\left(fx+e\right)\right)^{2}\cos\left(fx+e\right)}{8\,fa^{3}\left(\sin\left(fx+e\right)\right)^{5}}\left(8\,\ln\left(2\,\left(1+\cos\left(fx+e\right)\right)^{-1}\right)\left(\cos\left(fx+e\right)\right)^{2}+7\,\left(\cos\left(fx+e\right)\right)^{2}+16\,\cos\left(fx+e\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(5/2),x)
```

```
[Out] 1/8/f/a^3*(-1+cos(f*x+e))^2*(8*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^2+7*cos(f*x+
e)^2+16*cos(f*x+e)*ln(2/(1+cos(f*x+e)))-2*cos(f*x+e)+8*ln(2/(1+cos(f*x+e)))
-5)*cos(f*x+e)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)*(1/cos(f*x+e)*a*(1+cos(
f*x+e)))^(1/2)/sin(f*x+e)^5
```

Maxima [B] time = 2.43413, size = 1573, normalized size = 11.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] -((f*x + e)*cos(4*f*x + 4*e)^2 + 16*(f*x + e)*cos(3*f*x + 3*e)^2 + 36*(f*x
+ e)*cos(2*f*x + 2*e)^2 + 16*(f*x + e)*cos(f*x + e)^2 + (f*x + e)*sin(4*f*x
 + 4*e)^2 + 16*(f*x + e)*sin(3*f*x + 3*e)^2 + 36*(f*x + e)*sin(2*f*x + 2*e)
^2 + 16*(f*x + e)*sin(f*x + e)^2 + f*x - 2*(2*(4*cos(3*f*x + 3*e) + 6*cos(2
*f*x + 2*e) + 4*cos(f*x + e) + 1)*cos(4*f*x + 4*e) + cos(4*f*x + 4*e)^2 + 8
*(6*cos(2*f*x + 2*e) + 4*cos(f*x + e) + 1)*cos(3*f*x + 3*e) + 16*cos(3*f*x
+ 3*e)^2 + 12*(4*cos(f*x + e) + 1)*cos(2*f*x + 2*e) + 36*cos(2*f*x + 2*e)^2
 + 16*cos(f*x + e)^2 + 4*(2*sin(3*f*x + 3*e) + 3*sin(2*f*x + 2*e) + 2*sin(f
*x + e))*sin(4*f*x + 4*e) + sin(4*f*x + 4*e)^2 + 16*(3*sin(2*f*x + 2*e) + 2
*sin(f*x + e))*sin(3*f*x + 3*e) + 16*sin(3*f*x + 3*e)^2 + 36*sin(2*f*x + 2*
e)^2 + 48*sin(2*f*x + 2*e)*sin(f*x + e) + 16*sin(f*x + e)^2 + 8*cos(f*x + e
) + 1)*arctan2(sin(f*x + e), cos(f*x + e) + 1) + 2*(f*x + 4*(f*x + e)*cos(3
*f*x + 3*e) + 6*(f*x + e)*cos(2*f*x + 2*e) + 4*(f*x + e)*cos(f*x + e) + e -
 2*sin(3*f*x + 3*e) - 3*sin(2*f*x + 2*e) - 2*sin(f*x + e))*cos(4*f*x + 4*e)
 + 8*(f*x + 6*(f*x + e)*cos(2*f*x + 2*e) + 4*(f*x + e)*cos(f*x + e) + e)*co
s(3*f*x + 3*e) + 12*(f*x + 4*(f*x + e)*cos(f*x + e) + e)*cos(2*f*x + 2*e) +
 8*(f*x + e)*cos(f*x + e) + 2*(4*(f*x + e)*sin(3*f*x + 3*e) + 6*(f*x + e)*s
in(2*f*x + 2*e) + 4*(f*x + e)*sin(f*x + e) + 2*cos(3*f*x + 3*e) + 3*cos(2*f
```

```
*x + 2*e) + 2*cos(f*x + e))*sin(4*f*x + 4*e) + 4*(12*(f*x + e)*sin(2*f*x +
2*e) + 8*(f*x + e)*sin(f*x + e) - 1)*sin(3*f*x + 3*e) + 6*(8*(f*x + e)*sin(
f*x + e) - 1)*sin(2*f*x + 2*e) + e - 4*sin(f*x + e))*sqrt(a)*sqrt(c)/((a^3*
cos(4*f*x + 4*e)^2 + 16*a^3*cos(3*f*x + 3*e)^2 + 36*a^3*cos(2*f*x + 2*e)^2
+ 16*a^3*cos(f*x + e)^2 + a^3*sin(4*f*x + 4*e)^2 + 16*a^3*sin(3*f*x + 3*e)^
2 + 36*a^3*sin(2*f*x + 2*e)^2 + 48*a^3*sin(2*f*x + 2*e)*sin(f*x + e) + 16*a
^3*sin(f*x + e)^2 + 8*a^3*cos(f*x + e) + a^3 + 2*(4*a^3*cos(3*f*x + 3*e) +
6*a^3*cos(2*f*x + 2*e) + 4*a^3*cos(f*x + e) + a^3)*cos(4*f*x + 4*e) + 8*(6*
a^3*cos(2*f*x + 2*e) + 4*a^3*cos(f*x + e) + a^3)*cos(3*f*x + 3*e) + 12*(4*a
^3*cos(f*x + e) + a^3)*cos(2*f*x + 2*e) + 4*(2*a^3*sin(3*f*x + 3*e) + 3*a^3
*sin(2*f*x + 2*e) + 2*a^3*sin(f*x + e))*sin(4*f*x + 4*e) + 16*(3*a^3*sin(2*
f*x + 2*e) + 2*a^3*sin(f*x + e))*sin(3*f*x + 3*e))*f)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a\sec\left(fx+e\right)+a}\sqrt{-c\sec\left(fx+e\right)+c}}{a^3\sec\left(fx+e\right)^3+3\,a^3\sec\left(fx+e\right)^2+3\,a^3\sec\left(fx+e\right)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^3*sec(f*x +
e)^3 + 3*a^3*sec(f*x + e)^2 + 3*a^3*sec(f*x + e) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.40347, size = 215, normalized size = 1.54

$$\frac{\sqrt{2}\left(\frac{8\sqrt{2}\sqrt{-ac}c\log\left(\left|c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+c\right|\right)}{a^3|c|}+\frac{\sqrt{2}\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^2\sqrt{-ac}a^3c|c|-4\sqrt{2}\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)\sqrt{-ac}a^3c^2|c|}{a^6c^4}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3\right.}{16f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] 1/16*sqrt(2)*(8*sqrt(2)*sqrt(-a*c)*c*log(abs(c*tan(1/2*f*x + 1/2*e)^2 + c))
/(a^3*abs(c)) + (sqrt(2)*(c*tan(1/2*f*x + 1/2*e)^2 - c)^2*sqrt(-a*c)*a^3*c*
abs(c) - 4*sqrt(2)*(c*tan(1/2*f*x + 1/2*e)^2 - c)*sqrt(-a*c)*a^3*c^2*abs(c)
)/(a^6*c^4))*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e))*sgn(cos(f*x
 + e))/f
```

3.128 $\int \frac{1}{(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal. Leaf size=270

$$-\frac{3\tan(e+fx)}{4a^2f(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)}{4a^2f(\sec(e+fx)+1)^2\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+}$$

```
[Out] (Log[Cos[e + f*x]]*Tan[e + f*x])/(a^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c
*Sec[e + f*x]]) + (Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(8*a^2*f*Sqrt[a + a*
Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) + (7*Log[1 + Sec[e + f*x]]*Tan[e +
f*x])/(8*a^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) - Tan[e +
 f*x]/(4*a^2*f*(1 + Sec[e + f*x])^2*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec
[e + f*x]]) - (3*Tan[e + f*x])/(4*a^2*f*(1 + Sec[e + f*x])*Sqrt[a + a*Sec[e
 + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.153584, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3912, 72}

$$-\frac{3\tan(e+fx)}{4a^2f(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)}{4a^2f(\sec(e+fx)+1)^2\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]]),x]
```


```
[Out] (Log[Cos[e + f*x]]*Tan[e + f*x])/(a^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c
*Sec[e + f*x]]) + (Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(8*a^2*f*Sqrt[a + a*
Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) + (7*Log[1 + Sec[e + f*x]]*Tan[e +
f*x])/(8*a^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) - Tan[e +
 f*x]/(4*a^2*f*(1 + Sec[e + f*x])^2*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec
[e + f*x]]) - (3*Tan[e + f*x])/(4*a^2*f*(1 + Sec[e + f*x])*Sqrt[a + a*Sec[e
 + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```


Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
x)^(n - 1/2))/x, x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
```

```
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{(ac\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{x(a+ax)^3(c-cx)}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{(ac\tan(e+fx))\,\text{Subst}\left(\int \left(-\frac{1}{8a^3c(-1+x)}+\frac{1}{a^3cx}-\frac{1}{2a^3c(1+x)^3}-\frac{3}{4a^3c(1+x)^2}\right.\right.}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= \frac{\log(\cos(e+fx))\tan(e+fx)}{a^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{\log(1-\sec(e+fx))}{8a^2f\sqrt{a+a\sec(e+fx)}\sqrt{}}$$

Mathematica [C] time = 1.60522, size = 195, normalized size = 0.72

$$\frac{\tan(e+fx)\left(3\log\left(1-e^{i(e+fx)}\right)+21\log\left(1+e^{i(e+fx)}\right)+\left(\log\left(1-e^{i(e+fx)}\right)+7\log\left(1+e^{i(e+fx)}\right)-4ifx\right)\cos(2(e+fx))+\right.}{8a^2f(\cos(e+fx)+1)^2\sqrt{a(\sec(e+fx)+1)}\sqrt{c-c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]]),x]
```

```
[Out] ((8 - (12*I)*f*x + 3*Log[1 - E^(I*(e + f*x))] + 21*Log[1 + E^(I*(e + f*x))]
 + Cos[2*(e + f*x)]*((-4*I)*f*x + Log[1 - E^(I*(e + f*x))] + 7*Log[1 + E^(I
*(e + f*x))]) + 2*Cos[e + f*x]*(5 - (8*I)*f*x + 2*Log[1 - E^(I*(e + f*x))]
+ 14*Log[1 + E^(I*(e + f*x))]))*Tan[e + f*x])/(8*a^2*f*(1 + Cos[e + f*x])^2
*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] time = 0.298, size = 223, normalized size = 0.8

$$-\frac{\left(-1+\cos\left(fx+e\right)\right)^3}{16fa^3\left(\sin\left(fx+e\right)\right)^5}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\left(4\ln\left(-\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right)\left(\cos\left(fx+e\right)\right)^2-16\ln\left(2\left(1+\cos\left(fx+e\right)\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/16/f/a^3*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(-1+cos(f*x+e))^3*(4*ln(-
(-1+cos(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-16*ln(2/(1+cos(f*x+e)))*cos(f*x+e)
^2-9*cos(f*x+e)^2+8*cos(f*x+e)*ln(-(-1+cos(f*x+e))/sin(f*x+e))-32*cos(f*x+e
)*ln(2/(1+cos(f*x+e)))+2*cos(f*x+e)+4*ln(-(-1+cos(f*x+e))/sin(f*x+e))-16*ln
(2/(1+cos(f*x+e)))+7)/(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)/sin(f*x+e)^5
```

Maxima [B] time = 2.60178, size = 2978, normalized size = 11.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] -1/4*(4*(f*x + e)*cos(4*f*x + 4*e)^2 + 144*(f*x + e)*cos(2*f*x + 2*e)^2 + 6
4*(f*x + e)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 64*(f*
x + e)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*(f*x + e)
*sin(4*f*x + 4*e)^2 + 144*(f*x + e)*sin(2*f*x + 2*e)^2 + 64*(f*x + e)*sin(3
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 64*(f*x + e)*sin(1/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*f*x - 7*(2*(6*cos(2*f*x +
2*e) + 1)*cos(4*f*x + 4*e) + cos(4*f*x + 4*e)^2 + 36*cos(2*f*x + 2*e)^2 + 8
*(cos(4*f*x + 4*e) + 6*cos(2*f*x + 2*e) + 4*cos(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))) + 1)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))) + 16*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 8*(cos(4
*f*x + 4*e) + 6*cos(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e))) + 16*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2
 + sin(4*f*x + 4*e)^2 + 12*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 36*sin(2*f*x
 + 2*e)^2 + 8*(sin(4*f*x + 4*e) + 6*sin(2*f*x + 2*e) + 4*sin(1/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e))) + 16*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 +
 8*(sin(4*f*x + 4*e) + 6*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) + 16*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
)))^2 + 12*cos(2*f*x + 2*e) + 1)*arctan2(sin(1/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) +
1) - (2*(6*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + cos(4*f*x + 4*e)^2 + 36
*cos(2*f*x + 2*e)^2 + 8*(cos(4*f*x + 4*e) + 6*cos(2*f*x + 2*e) + 4*cos(1/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1)*cos(3/2*arctan2(sin(2*f*x
```

```
 + 2*e), cos(2*f*x + 2*e))) + 16*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e)))^2 + 8*(cos(4*f*x + 4*e) + 6*cos(2*f*x + 2*e) + 1)*cos(1/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 16*cos(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e)))^2 + sin(4*f*x + 4*e)^2 + 12*sin(4*f*x + 4*e)*sin(2*f*
x + 2*e) + 36*sin(2*f*x + 2*e)^2 + 8*(sin(4*f*x + 4*e) + 6*sin(2*f*x + 2*e)
 + 4*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 16*sin(3/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e)))^2 + 8*(sin(4*f*x + 4*e) + 6*sin(2*f*x + 2*e))*sin(1/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 16*sin(1/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e)))^2 + 12*cos(2*f*x + 2*e) + 1)*arctan2(sin(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) - 1) + 8*(f*x + 6*(f*x + e)*cos(2*f*x + 2*e) + e - 2*s
in(2*f*x + 2*e))*cos(4*f*x + 4*e) + 48*(f*x + e)*cos(2*f*x + 2*e) + 2*(16*f
*x + 16*(f*x + e)*cos(4*f*x + 4*e) + 96*(f*x + e)*cos(2*f*x + 2*e) + 64*(f*
x + e)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 16*e + 5*sin(
4*f*x + 4*e) - 2*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))) + 2*(16*f*x + 16*(f*x + e)*cos(4*f*x + 4*e) + 96*(f*x + e)*cos
(2*f*x + 2*e) + 16*e + 5*sin(4*f*x + 4*e) - 2*sin(2*f*x + 2*e))*cos(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 16*(3*(f*x + e)*sin(2*f*x + 2*e
) + cos(2*f*x + 2*e))*sin(4*f*x + 4*e) + 2*(16*(f*x + e)*sin(4*f*x + 4*e) +
 96*(f*x + e)*sin(2*f*x + 2*e) + 64*(f*x + e)*sin(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) - 5*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e) - 5)*sin(
3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*(16*(f*x + e)*sin(4*f*
x + 4*e) + 96*(f*x + e)*sin(2*f*x + 2*e) - 5*cos(4*f*x + 4*e) + 2*cos(2*f*x
 + 2*e) - 5)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*e - 1
6*sin(2*f*x + 2*e))/((a^2*cos(4*f*x + 4*e)^2 + 36*a^2*cos(2*f*x + 2*e)^2 +
16*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16*a^2*cos(
1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + a^2*sin(4*f*x + 4*e)^2
 + 12*a^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 36*a^2*sin(2*f*x + 2*e)^2 + 1
6*a^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16*a^2*sin(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 12*a^2*cos(2*f*x + 2*e)
 + a^2 + 2*(6*a^2*cos(2*f*x + 2*e) + a^2)*cos(4*f*x + 4*e) + 8*(a^2*cos(4*f
*x + 4*e) + 6*a^2*cos(2*f*x + 2*e) + 4*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) + a^2)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) + 8*(a^2*cos(4*f*x + 4*e) + 6*a^2*cos(2*f*x + 2*e) + a^2)*cos(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 8*(a^2*sin(4*f*x + 4*e) + 6*a^2
*sin(2*f*x + 2*e) + 4*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 8*(a^2*sin(4*f*
x + 4*e) + 6*a^2*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))))*sqrt(a)*sqrt(c)*f)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{a\sec\left(fx+e\right)+a}\sqrt{-c\sec\left(fx+e\right)+c}}{a^{3}c\sec\left(fx+e\right)^{4}+2\,a^{3}c\sec\left(fx+e\right)^{3}-2\,a^{3}c\sec\left(fx+e\right)-a^{3}c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^3*c*sec(f*x
 + e)^4 + 2*a^3*c*sec(f*x + e)^3 - 2*a^3*c*sec(f*x + e) - a^3*c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] Timed out
```

3.129 $\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=345

$$-\frac{\tan(e+fx)}{8a^2cf(1-\sec(e+fx))\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)}{2a^2cf(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e}}$$

```
[Out] (Log[Cos[e + f*x]]*Tan[e + f*x])/(a^2*c*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
 c*Sec[e + f*x]]) + (5*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(16*a^2*c*f*Sqrt
[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) + (11*Log[1 + Sec[e + f*x]]*
Tan[e + f*x])/(16*a^2*c*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]
) - Tan[e + f*x]/(8*a^2*c*f*(1 - Sec[e + f*x])*Sqrt[a + a*Sec[e + f*x]]*Sqr
t[c - c*Sec[e + f*x]]) - Tan[e + f*x]/(8*a^2*c*f*(1 + Sec[e + f*x])^2*Sqrt[
a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) - Tan[e + f*x]/(2*a^2*c*f*(1
+ Sec[e + f*x])*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.180822, antiderivative size = 345, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3912, 88}

$$-\frac{\tan(e+fx)}{8a^2cf(1-\sec(e+fx))\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)}{2a^2cf(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(3/2)),x]
```

```
[Out] (Log[Cos[e + f*x]]*Tan[e + f*x])/(a^2*c*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
 c*Sec[e + f*x]]) + (5*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(16*a^2*c*f*Sqrt
[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) + (11*Log[1 + Sec[e + f*x]]*
Tan[e + f*x])/(16*a^2*c*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]
) - Tan[e + f*x]/(8*a^2*c*f*(1 - Sec[e + f*x])*Sqrt[a + a*Sec[e + f*x]]*Sqr
t[c - c*Sec[e + f*x]]) - Tan[e + f*x]/(8*a^2*c*f*(1 + Sec[e + f*x])^2*Sqrt[
a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) - Tan[e + f*x]/(2*a^2*c*f*(1
+ Sec[e + f*x])*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
```

```
x)^(n - 1/2))/x, x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{(ac\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{x(a+ax)^3(c-cx)^2}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{(ac\tan(e+fx))\,\text{Subst}\left(\int \left(\frac{1}{8a^3c^2(-1+x)^2} - \frac{5}{16a^3c^2(-1+x)} + \frac{1}{a^3c^2x} - \frac{1}{4a^3c^2(1}\right.\right.}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\,\text{se}}}$$

$$= \frac{\log(\cos(e+fx))\tan(e+fx)}{a^2cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{5\log(1-\sec(e+f}{16a^2cf\sqrt{a+a\sec(e+f}}$$

Mathematica [C] time = 2.26347, size = 275, normalized size = 0.8

$$\frac{\tan(e+fx)\left(-10\log\left(1-e^{i(e+fx)}\right)-22\log\left(1+e^{i(e+fx)}\right)-8ifx\cos(3(e+fx))+5\log\left(1-e^{i(e+fx)}\right)\cos(3(e+fx))+\left(-5\,l\right.\right.}{32a^2cf(}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(3/2)),x]
```

```
[Out] ((-14 + (16*I)*f*x - (8*I)*f*x*Cos[3*(e + f*x)] - 10*Log[1 - E^(I*(e + f*x)
)] + 5*Cos[3*(e + f*x)]*Log[1 - E^(I*(e + f*x))] + Cos[e + f*x]*(-12 + (8*I
)*f*x - 5*Log[1 - E^(I*(e + f*x))] - 11*Log[1 + E^(I*(e + f*x))]) - 22*Log[
1 + E^(I*(e + f*x))] + 11*Cos[3*(e + f*x)]*Log[1 + E^(I*(e + f*x))] + 2*Cos
[2*(e + f*x)]*(5 - (8*I)*f*x + 5*Log[1 - E^(I*(e + f*x))] + 11*Log[1 + E^(I
*(e + f*x))]))*Tan[e + f*x])/(32*a^2*c*f*(-1 + Cos[e + f*x])*(1 + Cos[e + f
*x])^2*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] time = 0.268, size = 286, normalized size = 0.8

$$\frac{(\cos(fx+e))^2}{32\,fa^3c^3(\sin(fx+e))^5}\left(32\,(\cos(fx+e))^3\ln\left(2\,(1+\cos(fx+e))^{-1}\right)-20\,(\cos(fx+e))^3\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x)
```

```
[Out] 1/32/f/a^3*(32*cos(f*x+e)^3*ln(2/(1+cos(f*x+e)))-20*cos(f*x+e)^3*ln(-(-1+co
s(f*x+e))/sin(f*x+e))+13*cos(f*x+e)^3+32*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^2-
20*ln(-(-1+cos(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-7*cos(f*x+e)^2-32*cos(f*x+e
)*ln(2/(1+cos(f*x+e)))+20*cos(f*x+e)*ln(-(-1+cos(f*x+e))/sin(f*x+e))-cos(f*
x+e)-32*ln(2/(1+cos(f*x+e)))+20*ln(-(-1+cos(f*x+e))/sin(f*x+e))+11)*(1/cos(
f*x+e)*a*(1+cos(f*x+e)))^(1/2)*cos(f*x+e)^2*(c*(-1+cos(f*x+e))/cos(f*x+e))^
(3/2)/c^3/sin(f*x+e)^5
```

Maxima [B] time = 11.5139, size = 5767, normalized size = 16.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="max
ima")
```

```
[Out] -1/8*(8*(f*x + e)*cos(6*f*x + 6*e)^2 + 8*(f*x + e)*cos(4*f*x + 4*e)^2 + 8*(
f*x + e)*cos(2*f*x + 2*e)^2 + 32*(f*x + e)*cos(5/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e)))^2 + 128*(f*x + e)*cos(3/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e)))^2 + 32*(f*x + e)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e)))^2 + 8*(f*x + e)*sin(6*f*x + 6*e)^2 + 8*(f*x + e)*sin(4*f*x + 4*
e)^2 + 8*(f*x + e)*sin(2*f*x + 2*e)^2 + 32*(f*x + e)*sin(5/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e)))^2 + 128*(f*x + e)*sin(3/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e)))^2 + 32*(f*x + e)*sin(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e)))^2 + 8*f*x + 11*(2*(cos(4*f*x + 4*e) + cos(2*f*x + 2*e
) - 1)*cos(6*f*x + 6*e) - cos(6*f*x + 6*e)^2 - 2*(cos(2*f*x + 2*e) - 1)*cos
(4*f*x + 4*e) - cos(4*f*x + 4*e)^2 - cos(2*f*x + 2*e)^2 - 4*(cos(6*f*x + 6*
e) - cos(4*f*x + 4*e) - cos(2*f*x + 2*e) - 4*cos(3/2*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e))) + 2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))) + 1)*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*cos(5/2*
```

```
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 8*(cos(6*f*x + 6*e) - cos(
4*f*x + 4*e) - cos(2*f*x + 2*e) + 2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e))) + 1)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 1
6*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 4*(cos(6*f*x + 6
*e) - cos(4*f*x + 4*e) - cos(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))) - 4*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e)))^2 + 2*(sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) - sin(6
*f*x + 6*e)^2 - sin(4*f*x + 4*e)^2 - 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) -
sin(2*f*x + 2*e)^2 - 4*(sin(6*f*x + 6*e) - sin(4*f*x + 4*e) - sin(2*f*x + 2
*e) - 4*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*sin(1/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(5/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))) - 4*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
)))^2 + 8*(sin(6*f*x + 6*e) - sin(4*f*x + 4*e) - sin(2*f*x + 2*e) + 2*sin(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e))) - 16*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e)))^2 - 4*(sin(6*f*x + 6*e) - sin(4*f*x + 4*e) - sin(2*f*x + 2*e))*si
n(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*sin(1/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*cos(2*f*x + 2*e) - 1)*arctan2(sin(1/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e))) + 1) + 5*(2*(cos(4*f*x + 4*e) + cos(2*f*x + 2*e)
- 1)*cos(6*f*x + 6*e) - cos(6*f*x + 6*e)^2 - 2*(cos(2*f*x + 2*e) - 1)*cos(4
*f*x + 4*e) - cos(4*f*x + 4*e)^2 - cos(2*f*x + 2*e)^2 - 4*(cos(6*f*x + 6*e)
 - cos(4*f*x + 4*e) - cos(2*f*x + 2*e) - 4*cos(3/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) + 2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
)) + 1)*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*cos(5/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 8*(cos(6*f*x + 6*e) - cos(4*
f*x + 4*e) - cos(2*f*x + 2*e) + 2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))) + 1)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 16*
cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 4*(cos(6*f*x + 6*e
) - cos(4*f*x + 4*e) - cos(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e))) - 4*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e)))^2 + 2*(sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) - sin(6*f
*x + 6*e)^2 - sin(4*f*x + 4*e)^2 - 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) - si
n(2*f*x + 2*e)^2 - 4*(sin(6*f*x + 6*e) - sin(4*f*x + 4*e) - sin(2*f*x + 2*e
) - 4*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*sin(1/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(5/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e))) - 4*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
)^2 + 8*(sin(6*f*x + 6*e) - sin(4*f*x + 4*e) - sin(2*f*x + 2*e) + 2*sin(1/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))) - 16*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e)))^2 - 4*(sin(6*f*x + 6*e) - sin(4*f*x + 4*e) - sin(2*f*x + 2*e))*sin(
1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*sin(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*cos(2*f*x + 2*e) - 1)*arctan2(sin(1/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) - 1) + 4*(4*f*x - 4*(f*x + e)*cos(4*f*x + 4*e) - 4*
```

(f*x + e)*cos(2*f*x + 2*e) + 4*e + 3*sin(4*f*x + 4*e) + 3*sin(2*f*x + 2*e))
*cos(6*f*x + 6*e) - 16*(f*x - (f*x + e)*cos(2*f*x + 2*e) + e)*cos(4*f*x + 4
e) - 16(f*x + e)*cos(2*f*x + 2*e) + 2*(16*f*x + 16*(f*x + e)*cos(6*f*x +
6*e) - 16*(f*x + e)*cos(4*f*x + 4*e) - 16*(f*x + e)*cos(2*f*x + 2*e) - 64*(
f*x + e)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 32*(f*x + e
)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 16*e + 5*sin(6*f*x
 + 6*e) + 7*sin(4*f*x + 4*e) + 7*sin(2*f*x + 2*e) + 8*sin(3/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))) - 4*(16*f*x + 16*(f*x + e)*cos(6*f*x + 6*e) - 16*(f*x + e)*cos(4
*f*x + 4*e) - 16*(f*x + e)*cos(2*f*x + 2*e) + 32*(f*x + e)*cos(1/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 16*e + 7*sin(6*f*x + 6*e) + 5*sin(4*
f*x + 4*e) + 5*sin(2*f*x + 2*e) + 4*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e))))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*(16
*f*x + 16*(f*x + e)*cos(6*f*x + 6*e) - 16*(f*x + e)*cos(4*f*x + 4*e) - 16*(
f*x + e)*cos(2*f*x + 2*e) + 16*e + 5*sin(6*f*x + 6*e) + 7*sin(4*f*x + 4*e)
+ 7*sin(2*f*x + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
- 4*(4*(f*x + e)*sin(4*f*x + 4*e) + 4*(f*x + e)*sin(2*f*x + 2*e) + 3*cos(4*
f*x + 4*e) + 3*cos(2*f*x + 2*e))*sin(6*f*x + 6*e) + 4*(4*(f*x + e)*sin(2*f*
x + 2*e) + 3)*sin(4*f*x + 4*e) + 2*(16*(f*x + e)*sin(6*f*x + 6*e) - 16*(f*x
 + e)*sin(4*f*x + 4*e) - 16*(f*x + e)*sin(2*f*x + 2*e) - 64*(f*x + e)*sin(3
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 32*(f*x + e)*sin(1/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 5*cos(6*f*x + 6*e) - 7*cos(4*f*x
 + 4*e) - 7*cos(2*f*x + 2*e) - 8*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))) - 5)*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*(1
6*(f*x + e)*sin(6*f*x + 6*e) - 16*(f*x + e)*sin(4*f*x + 4*e) - 16*(f*x + e)
*sin(2*f*x + 2*e) + 32*(f*x + e)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))) - 7*cos(6*f*x + 6*e) - 5*cos(4*f*x + 4*e) - 5*cos(2*f*x + 2*e) -
 4*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 7)*sin(3/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*(16*(f*x + e)*sin(6*f*x + 6*e) -
 16*(f*x + e)*sin(4*f*x + 4*e) - 16*(f*x + e)*sin(2*f*x + 2*e) - 5*cos(6*f*
x + 6*e) - 7*cos(4*f*x + 4*e) - 7*cos(2*f*x + 2*e) - 5)*sin(1/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))) + 8*e + 12*sin(2*f*x + 2*e))/((a^2*c*cos(
6*f*x + 6*e)^2 + a^2*c*cos(4*f*x + 4*e)^2 + a^2*c*cos(2*f*x + 2*e)^2 + 4*a^
2*c*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16*a^2*c*cos(3
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*a^2*c*cos(1/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + a^2*c*sin(6*f*x + 6*e)^2 + a^2*c*
sin(4*f*x + 4*e)^2 + 2*a^2*c*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + a^2*c*sin(
2*f*x + 2*e)^2 + 4*a^2*c*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
))^2 + 16*a^2*c*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*
a^2*c*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*a^2*c*cos(
2*f*x + 2*e) + a^2*c - 2*(a^2*c*cos(4*f*x + 4*e) + a^2*c*cos(2*f*x + 2*e) -
 a^2*c)*cos(6*f*x + 6*e) + 2*(a^2*c*cos(2*f*x + 2*e) - a^2*c)*cos(4*f*x + 4
e) + 4(a^2*c*cos(6*f*x + 6*e) - a^2*c*cos(4*f*x + 4*e) - a^2*c*cos(2*f*x
+ 2*e) - 4*a^2*c*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*a
^2*c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + a^2*c)*cos(5/2*

```
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 8*(a^2*c*cos(6*f*x + 6*e) -
a^2*c*cos(4*f*x + 4*e) - a^2*c*cos(2*f*x + 2*e) + 2*a^2*c*cos(1/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e))) + a^2*c)*cos(3/2*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e))) + 4*(a^2*c*cos(6*f*x + 6*e) - a^2*c*cos(4*f*x + 4*e)
 - a^2*c*cos(2*f*x + 2*e) + a^2*c)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))) - 2*(a^2*c*sin(4*f*x + 4*e) + a^2*c*sin(2*f*x + 2*e))*sin(6*f*
x + 6*e) + 4*(a^2*c*sin(6*f*x + 6*e) - a^2*c*sin(4*f*x + 4*e) - a^2*c*sin(2
*f*x + 2*e) - 4*a^2*c*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
+ 2*a^2*c*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(5/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 8*(a^2*c*sin(6*f*x + 6*e) - a^2
*c*sin(4*f*x + 4*e) - a^2*c*sin(2*f*x + 2*e) + 2*a^2*c*sin(1/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))) + 4*(a^2*c*sin(6*f*x + 6*e) - a^2*c*sin(4*f*x + 4*e) - a^2*c*si
n(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(
a)*sqrt(c)*f)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a\sec\left(fx+e\right)+a}\sqrt{-c\sec\left(fx+e\right)+c}}{a^3c^2\sec\left(fx+e\right)^5+a^3c^2\sec\left(fx+e\right)^4-2\,a^3c^2\sec\left(fx+e\right)^3-2\,a^3c^2\sec\left(fx+e\right)^2+a^3c^2\sec\left(fx+e\right)+a^3c^2},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^3*c^2*sec(f*
x + e)^5 + a^3*c^2*sec(f*x + e)^4 - 2*a^3*c^2*sec(f*x + e)^3 - 2*a^3*c^2*se
c(f*x + e)^2 + a^3*c^2*sec(f*x + e) + a^3*c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="gia
c")
```

```
[Out] Timed out
```

3.130 $\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=151

$$-\frac{\cot^3(e+fx)}{4a^2c^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{\cot(e+fx)}{2a^2c^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{\tan(e+fx)\log(s}{a^2c^2f\sqrt{a\sec(e+fx)+a}}$$

```
[Out] Cot[e + f*x]/(2*a^2*c^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]
) - Cot[e + f*x]^3/(4*a^2*c^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e +
 f*x]]) + (Log[Sin[e + f*x]]*Tan[e + f*x])/(a^2*c^2*f*Sqrt[a + a*Sec[e + f*
x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.13111, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3905, 3473, 3475}

$$-\frac{\cot^3(e+fx)}{4a^2c^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{\cot(e+fx)}{2a^2c^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{\tan(e+fx)\log(s}{a^2c^2f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(5/2)),x]
```

```
[Out] Cot[e + f*x]/(2*a^2*c^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]
) - Cot[e + f*x]^3/(4*a^2*c^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e +
 f*x]]) + (Log[Sin[e + f*x]]*Tan[e + f*x])/(a^2*c^2*f*Sqrt[a + a*Sec[e + f*
x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3905

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Dist[((-(a*c))^(m + 1/2)*Cot[e + f*x])/(Sqrt
[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int[Cot[e + f*x]^(2*m), x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IntegerQ[m + 1/2]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\tan(e+fx)\int \cot^5(e+fx)\,dx}{a^2c^2\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$
$$= -\frac{\cot^3(e+fx)}{4a^2c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{\tan(e+fx)\int}{a^2c^2\sqrt{a+a\sec(e+}}$$
$$= \frac{\cot(e+fx)}{2a^2c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{\cot^3}{4a^2c^2f\sqrt{a+a\sec(e}}$$
$$= \frac{\cot(e+fx)}{2a^2c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{\cot^3}{4a^2c^2f\sqrt{a+a\sec(e}}$$

Mathematica [C] time = 2.10466, size = 149, normalized size = 0.99

$$\frac{\csc^3(e+fx)\sec(e+fx)\left(3\log\left(1-e^{2i(e+fx)}\right)+\left(-4\log\left(1-e^{2i(e+fx)}\right)+4ifx-4\right)\cos(2(e+fx))+\left(\log\left(1-e^{2i(e+fx)}\right)-\right.\right.}{8a^2c^2f\sqrt{a(\sec(e+fx)+1)}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(5/2)),x]
```

```
[Out] (Csc[e + f*x]^3*(2 - (3*I)*f*x + Cos[2*(e + f*x)]*(-4 + (4*I)*f*x - 4*Log[1
 - E^((2*I)*(e + f*x))]) + 3*Log[1 - E^((2*I)*(e + f*x))] + Cos[4*(e + f*x)
]*((-I)*f*x + Log[1 - E^((2*I)*(e + f*x))]))*Sec[e + f*x])/(8*a^2*c^2*f*Sqr
t[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] time = 0.276, size = 237, normalized size = 1.6

$$-\frac{\left(-1+\cos\left(fx+e\right)\right)^3}{32\,fa^3\left(\sin\left(fx+e\right)\right)^5\left(\cos\left(fx+e\right)\right)^2}\left(32\,\left(\cos\left(fx+e\right)\right)^4\ln\left(-\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-32\,\left(\cos\left(fx+e\right)\right)^4\ln\left(2\,\left(1+\mathrm{co}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x)
```

```
[Out] -1/32/f/a^3*(-1+cos(f*x+e))^3*(32*cos(f*x+e)^4*ln(-(-1+cos(f*x+e))/sin(f*x+
e))-32*cos(f*x+e)^4*ln(2/(1+cos(f*x+e)))-13*cos(f*x+e)^4-64*ln(-(-1+cos(f*x
+e))/sin(f*x+e))*cos(f*x+e)^2+64*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^2-6*cos(f*
x+e)^2+32*ln(-(-1+cos(f*x+e))/sin(f*x+e))-32*ln(2/(1+cos(f*x+e)))+11)*(1/co
s(f*x+e)*a*(1+cos(f*x+e)))^(1/2)/(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)/sin(f
*x+e)^5/cos(f*x+e)^2
```

Maxima [B] time = 2.48675, size = 1871, normalized size = 12.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="max
ima")
```

```
[Out] -((f*x + e)*cos(8*f*x + 8*e)^2 + 16*(f*x + e)*cos(6*f*x + 6*e)^2 + 36*(f*x
+ e)*cos(4*f*x + 4*e)^2 + 16*(f*x + e)*cos(2*f*x + 2*e)^2 + (f*x + e)*sin(8
*f*x + 8*e)^2 + 16*(f*x + e)*sin(6*f*x + 6*e)^2 + 36*(f*x + e)*sin(4*f*x +
4*e)^2 + 16*(f*x + e)*sin(2*f*x + 2*e)^2 + f*x + (2*(4*cos(6*f*x + 6*e) - 6
*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e) - 1)*cos(8*f*x + 8*e) - cos(8*f*x +
8*e)^2 + 8*(6*cos(4*f*x + 4*e) - 4*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) -
 16*cos(6*f*x + 6*e)^2 + 12*(4*cos(2*f*x + 2*e) - 1)*cos(4*f*x + 4*e) - 36*
cos(4*f*x + 4*e)^2 - 16*cos(2*f*x + 2*e)^2 + 4*(2*sin(6*f*x + 6*e) - 3*sin(
4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) - sin(8*f*x + 8*e)^2 +
16*(3*sin(4*f*x + 4*e) - 2*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) - 16*sin(6*f*
x + 6*e)^2 - 36*sin(4*f*x + 4*e)^2 + 48*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) -
 16*sin(2*f*x + 2*e)^2 + 8*cos(2*f*x + 2*e) - 1)*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e) - 1) + 2*(f*x - 4*(f*x + e)*cos(6*f*x + 6*e) + 6*(f*x + e)
*cos(4*f*x + 4*e) - 4*(f*x + e)*cos(2*f*x + 2*e) + e + 2*sin(6*f*x + 6*e) -
 2*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*cos(8*f*x + 8*e) - 8*(f*x + 6*(f*
x + e)*cos(4*f*x + 4*e) - 4*(f*x + e)*cos(2*f*x + 2*e) + e + sin(4*f*x + 4*
e))*cos(6*f*x + 6*e) + 4*(3*f*x - 12*(f*x + e)*cos(2*f*x + 2*e) + 3*e + 2*s
in(2*f*x + 2*e))*cos(4*f*x + 4*e) - 8*(f*x + e)*cos(2*f*x + 2*e) - 4*(2*(f*
x + e)*sin(6*f*x + 6*e) - 3*(f*x + e)*sin(4*f*x + 4*e) + 2*(f*x + e)*sin(2*
f*x + 2*e) + cos(6*f*x + 6*e) - cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*sin(8*
f*x + 8*e) - 4*(12*(f*x + e)*sin(4*f*x + 4*e) - 8*(f*x + e)*sin(2*f*x + 2*e
) - 2*cos(4*f*x + 4*e) - 1)*sin(6*f*x + 6*e) - 4*(12*(f*x + e)*sin(2*f*x +
```

```
2*e) + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + e + 4*sin(2*f*x + 2*e))*s
qrt(a)*sqrt(c)/((a^3*c^3*cos(8*f*x + 8*e)^2 + 16*a^3*c^3*cos(6*f*x + 6*e)^2
 + 36*a^3*c^3*cos(4*f*x + 4*e)^2 + 16*a^3*c^3*cos(2*f*x + 2*e)^2 + a^3*c^3*
sin(8*f*x + 8*e)^2 + 16*a^3*c^3*sin(6*f*x + 6*e)^2 + 36*a^3*c^3*sin(4*f*x +
 4*e)^2 - 48*a^3*c^3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 16*a^3*c^3*sin(2*f
*x + 2*e)^2 - 8*a^3*c^3*cos(2*f*x + 2*e) + a^3*c^3 - 2*(4*a^3*c^3*cos(6*f*x
 + 6*e) - 6*a^3*c^3*cos(4*f*x + 4*e) + 4*a^3*c^3*cos(2*f*x + 2*e) - a^3*c^3
)*cos(8*f*x + 8*e) - 8*(6*a^3*c^3*cos(4*f*x + 4*e) - 4*a^3*c^3*cos(2*f*x +
2*e) + a^3*c^3)*cos(6*f*x + 6*e) - 12*(4*a^3*c^3*cos(2*f*x + 2*e) - a^3*c^3
)*cos(4*f*x + 4*e) - 4*(2*a^3*c^3*sin(6*f*x + 6*e) - 3*a^3*c^3*sin(4*f*x +
4*e) + 2*a^3*c^3*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) - 16*(3*a^3*c^3*sin(4*f
*x + 4*e) - 2*a^3*c^3*sin(2*f*x + 2*e))*sin(6*f*x + 6*e))*f)
```

Fricas [A] time = 3.18785, size = 1446, normalized size = 9.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="fri
cas")
```

```
[Out] [-1/324*(162*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(-a*c)*log(-8*((25
6*cos(f*x + e)^5 - 512*cos(f*x + e)^3 + 175*cos(f*x + e))*sqrt(-a*c)*sqrt((
a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)) -
 (256*a*c*cos(f*x + e)^4 - 512*a*c*cos(f*x + e)^2 + 337*a*c)*sin(f*x + e))/
((cos(f*x + e)^2 - 1)*sin(f*x + e)))*sin(f*x + e) + (832*cos(f*x + e)^5 - 1
988*cos(f*x + e)^3 + 1075*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x +
 e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^3*c^3*f*cos(f*x + e)^4 -
2*a^3*c^3*f*cos(f*x + e)^2 + a^3*c^3*f)*sin(f*x + e)), -1/324*(324*(cos(f*x
 + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(a*c)*arctan((16*cos(f*x + e)^3 - 7*cos
(f*x + e))*sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*
x + e) - c)/cos(f*x + e))/((16*a*c*cos(f*x + e)^2 - 25*a*c)*sin(f*x + e)))*
sin(f*x + e) + (832*cos(f*x + e)^5 - 1988*cos(f*x + e)^3 + 1075*cos(f*x + e
))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*
x + e)))/((a^3*c^3*f*cos(f*x + e)^4 - 2*a^3*c^3*f*cos(f*x + e)^2 + a^3*c^3*
f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="gia
c")
```

```
[Out] Timed out
```

3.131 $\int (1+\sec(e+fx))^m (c-c\sec(e+fx))^n \, dx$

Optimal. Leaf size=92

$$\frac{2^{m+\frac{1}{2}}\tan(e+fx)(c-c\sec(e+fx))^n F_1\left(n+\frac{1}{2};\frac{1}{2}-m,1;n+\frac{3}{2};\frac{1}{2}(1-\sec(e+fx)),1-\sec(e+fx)\right)}{f(2n+1)\sqrt{\sec(e+fx)+1}}$$

```
[Out] (2^(1/2 + m)*AppellF1[1/2 + n, 1/2 - m, 1, 3/2 + n, (1 - Sec[e + f*x])/2, 1
 - Sec[e + f*x]]*(c - c*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[1 +
 Sec[e + f*x]])
```

Rubi [A] time = 0.0936537, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3912, 136}

$$\frac{2^{m+\frac{1}{2}}\tan(e+fx)(c-c\sec(e+fx))^n F_1\left(n+\frac{1}{2};\frac{1}{2}-m,1;n+\frac{3}{2};\frac{1}{2}(1-\sec(e+fx)),1-\sec(e+fx)\right)}{f(2n+1)\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sec[e + f*x])^m*(c - c*Sec[e + f*x])^n,x]
```

```
[Out] (2^(1/2 + m)*AppellF1[1/2 + n, 1/2 - m, 1, 3/2 + n, (1 - Sec[e + f*x])/2, 1
 - Sec[e + f*x]]*(c - c*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[1 +
 Sec[e + f*x]])
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
x)^(n - 1/2))/x, x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
```

```
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int (1+\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx = -\frac{(c\tan(e+fx))\operatorname{Subst}\left(\int \frac{(1+x)^{-\frac{1}{2}+m}(c-cx)^{-\frac{1}{2}+n}}{x}\,dx, x, \sec(e+fx)\right)}{f\sqrt{1+\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= \frac{2^{\frac{1}{2}+m}F_1\left(\frac{1}{2}+n;\frac{1}{2}-m,1;\frac{3}{2}+n;\frac{1}{2}(1-\sec(e+fx)),1-\sec(e+fx)\right)(c-c\,\mathrm{s}}{f(1+2n)\sqrt{1+\sec(e+fx)}}$$

Mathematica [F] time = 1.0421, size = 0, normalized size = 0.

$$\int (1+\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + Sec[e + f*x])^m*(c - c*Sec[e + f*x])^n,x]
```

```
[Out] Integrate[(1 + Sec[e + f*x])^m*(c - c*Sec[e + f*x])^n, x]
```

Maple [F] time = 0.407, size = 0, normalized size = 0.

$$\int \left(1+\sec\left(fx+e\right)\right)^{m}\left(c-c\sec\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x)
```

```
[Out] int((1+sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-c\sec\left(fx+e\right)+c\right)^{n}\left(\sec\left(fx+e\right)+1\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((-c*sec(f*x + e) + c)^n*(sec(f*x + e) + 1)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-c\sec\left(fx+e\right)+c\right)^{n}\left(\sec\left(fx+e\right)+1\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((-c*sec(f*x + e) + c)^n*(sec(f*x + e) + 1)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sec(f*x+e))**m*(c-c*sec(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-c\sec\left(fx+e\right)+c\right)^{n}\left(\sec\left(fx+e\right)+1\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((-c*sec(f*x + e) + c)^n*(sec(f*x + e) + 1)^m, x)
```

3.132 $\int (a + a\sec(e + fx))^m (c - c\sec(e + fx))^n \, dx$

Optimal. Leaf size=109

$$\frac{c2^{n+\frac{1}{2}}\tan(e+fx)(1-\sec(e+fx))^{\frac{1}{2}-n}(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{n-1}F_1\left(m+\frac{1}{2};\frac{1}{2}-n,1;m+\frac{3}{2};\frac{1}{2}(\sec(e+fx)\right.}{f(2m+1)}$$

```
[Out] (2^(1/2 + n)*c*AppellF1[1/2 + m, 1/2 - n, 1, 3/2 + m, (1 + Sec[e + f*x])/2,
 1 + Sec[e + f*x]]*(1 - Sec[e + f*x])^(1/2 - n)*(a + a*Sec[e + f*x])^m*(c -
 c*Sec[e + f*x])^(-1 + n)*Tan[e + f*x])/(f*(1 + 2*m))
```

Rubi [A] time = 0.121691, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3912, 137, 136}

$$\frac{c2^{n+\frac{1}{2}}\tan(e+fx)(1-\sec(e+fx))^{\frac{1}{2}-n}(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{n-1}F_1\left(m+\frac{1}{2};\frac{1}{2}-n,1;m+\frac{3}{2};\frac{1}{2}(\sec(e+fx)\right.}{f(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^n,x]
```

```
[Out] (2^(1/2 + n)*c*AppellF1[1/2 + m, 1/2 - n, 1, 3/2 + m, (1 + Sec[e + f*x])/2,
 1 + Sec[e + f*x]]*(1 - Sec[e + f*x])^(1/2 - n)*(a + a*Sec[e + f*x])^m*(c -
 c*Sec[e + f*x])^(-1 + n)*Tan[e + f*x])/(f*(1 + 2*m))
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
x)^(n - 1/2))/x, x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && !GtQ[b/(b*c
- a*d), 0] && !SimplerQ[c + d*x, a + b*x]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int (a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx=-\frac{(ac\tan(e+fx))\,\text{Subst}\left(\int\frac{(a+ax)^{-\frac{1}{2}+m}(c-cx)^{-\frac{1}{2}+n}}{x}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$=-\frac{\left(2^{-\frac{1}{2}+n}ac(c-c\sec(e+fx))^{-1+n}\left(\frac{c-c\sec(e+fx)}{c}\right)^{\frac{1}{2}-n}\tan(e+fx)\right)\text{Subst}}{f\sqrt{a+a\sec(e+fx)}}$$

$$=\frac{2^{\frac{1}{2}+n}cF_1\left(\frac{1}{2}+m;\frac{1}{2}-n,1;\frac{3}{2}+m;\frac{1}{2}(1+\sec(e+fx)),1+\sec(e+fx)\right)(1}{f(1}$$

Mathematica [F] time = 0.429061, size = 0, normalized size = 0.

$$\int (a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^n,x]
```

```
[Out] Integrate[(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^n, x]
```

Maple [F] time = 0.392, size = 0, normalized size = 0.

$$\int \left(a+a\sec\left(fx+e\right)\right)^m\left(c-c\sec\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x)
```

```
[Out] int((a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec\left(fx+e\right)+a\right)^{m}\left(-c\sec\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a\sec\left(fx+e\right)+a\right)^{m}\left(-c\sec\left(fx+e\right)+c\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\left(\sec\left(e+fx\right)+1\right)\right)^{m}\left(-c\left(\sec\left(e+fx\right)-1\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**m*(c-c*sec(f*x+e))**n,x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**m*(-c*(sec(e + f*x) - 1))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec \left(fx+e\right)+a\right)^m \left(-c\sec \left(fx+e\right)+c\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^n, x)
```

3.133 $\int (a + a\sec(e + fx))^3 (c - c\sec(e + fx))^n \, dx$

Optimal. Leaf size=101

$$\frac{c2^{n+\frac{1}{2}}\tan(e+fx)(a\sec(e+fx)+a)^3(1-\sec(e+fx))^{\frac{1}{2}-n}F_1\left(\frac{7}{2};\frac{1}{2}-n,1;\frac{9}{2};\frac{1}{2}(\sec(e+fx)+1),\sec(e+fx)+1\right)(c-c\sec}{7f}$$

```
[Out] (2^(1/2 + n)*c*AppellF1[7/2, 1/2 - n, 1, 9/2, (1 + Sec[e + f*x])/2, 1 + Sec
[e + f*x]]*(1 - Sec[e + f*x])^(1/2 - n)*(a + a*Sec[e + f*x])^3*(c - c*Sec[e
 + f*x])^(-1 + n)*Tan[e + f*x])/(7*f)
```

Rubi [A] time = 0.100126, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3912, 137, 136}

$$\frac{c2^{n+\frac{1}{2}}\tan(e+fx)(a\sec(e+fx)+a)^3(1-\sec(e+fx))^{\frac{1}{2}-n}F_1\left(\frac{7}{2};\frac{1}{2}-n,1;\frac{9}{2};\frac{1}{2}(\sec(e+fx)+1),\sec(e+fx)+1\right)(c-c\sec}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^n,x]
```

```
[Out] (2^(1/2 + n)*c*AppellF1[7/2, 1/2 - n, 1, 9/2, (1 + Sec[e + f*x])/2, 1 + Sec
[e + f*x]]*(1 - Sec[e + f*x])^(1/2 - n)*(a + a*Sec[e + f*x])^3*(c - c*Sec[e
 + f*x])^(-1 + n)*Tan[e + f*x])/(7*f)
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
x)^(n - 1/2))/x, x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && !GtQ[b/(b*c
- a*d), 0] && !SimplerQ[c + d*x, a + b*x]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^n\,dx = -\frac{(ac\tan(e+fx))\,\text{Subst}\left(\int \frac{(a+ax)^{5/2}(c-cx)^{-\frac{1}{2}+n}}{x}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{\left(2^{-\frac{1}{2}+n}ac(c-c\sec(e+fx))^{-1+n}\left(\frac{c-c\sec(e+fx)}{c}\right)^{\frac{1}{2}-n}\tan(e+fx)\right)\text{Subst}\Big(}{f\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2^{\frac{1}{2}+n}cF_1\left(\frac{7}{2};\frac{1}{2}-n,1;\frac{9}{2};\frac{1}{2}(1+\sec(e+fx)),1+\sec(e+fx)\right)(1-\sec(e+}{7f}$$

Mathematica [F] time = 3.21668, size = 0, normalized size = 0.

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^n,x]
```

```
[Out] Integrate[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^n, x]
```

Maple [F] time = 0.427, size = 0, normalized size = 0.

$$\int \left(a+a\sec\left(fx+e\right)\right)^3\left(c-c\sec\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^n,x)
```

```
[Out] int((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec\left(fx+e\right)+a\right)^{3}\left(-c\sec\left(fx+e\right)+c\right)^{n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^3*(-c*sec(f*x + e) + c)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^{3}\sec\left(fx+e\right)^{3}+3\,a^{3}\sec\left(fx+e\right)^{2}+3\,a^{3}\sec\left(fx+e\right)+a^{3}\right)\left(-c\sec\left(fx+e\right)+c\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((a^3*sec(f*x + e)^3 + 3*a^3*sec(f*x + e)^2 + 3*a^3*sec(f*x + e) +
a^3)*(-c*sec(f*x + e) + c)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^{3}\left(\int 3\left(-c\sec\left(e+fx\right)+c\right)^{n}\sec\left(e+fx\right)dx+\int 3\left(-c\sec\left(e+fx\right)+c\right)^{n}\sec^{2}\left(e+fx\right)dx+\int\left(-c\sec\left(e+fx\right)+c\right)^{n}\mathrm{s}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**n,x)
```

```
[Out] a**3*(Integral(3*(-c*sec(e + f*x) + c)**n*sec(e + f*x), x) + Integral(3*(-c
*sec(e + f*x) + c)**n*sec(e + f*x)**2, x) + Integral((-c*sec(e + f*x) + c)*
*n*sec(e + f*x)**3, x) + Integral((-c*sec(e + f*x) + c)**n, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec\left(fx+e\right)+a\right)^{3}\left(-c\sec\left(fx+e\right)+c\right)^{n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^3*(-c*sec(f*x + e) + c)^n, x)
```

3.134 $\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^n\,dx$

Optimal. Leaf size=101

$$\frac{c2^{n+\frac{1}{2}}\tan(e+fx)(a\sec(e+fx)+a)^2(1-\sec(e+fx))^{\frac{1}{2}-n}F_1\left(\frac{5}{2};\frac{1}{2}-n,1;\frac{7}{2};\frac{1}{2}(\sec(e+fx)+1),\sec(e+fx)+1\right)(c-c\sec}{5f}$$

```
[Out] (2^(1/2 + n)*c*AppellF1[5/2, 1/2 - n, 1, 7/2, (1 + Sec[e + f*x])/2, 1 + Sec
[e + f*x]]*(1 - Sec[e + f*x])^(1/2 - n)*(a + a*Sec[e + f*x])^2*(c - c*Sec[e
 + f*x])^(-1 + n)*Tan[e + f*x])/(5*f)
```

Rubi [A] time = 0.0990131, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3912, 137, 136}

$$\frac{c2^{n+\frac{1}{2}}\tan(e+fx)(a\sec(e+fx)+a)^2(1-\sec(e+fx))^{\frac{1}{2}-n}F_1\left(\frac{5}{2};\frac{1}{2}-n,1;\frac{7}{2};\frac{1}{2}(\sec(e+fx)+1),\sec(e+fx)+1\right)(c-c\sec}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^n,x]
```


```
[Out] (2^(1/2 + n)*c*AppellF1[5/2, 1/2 - n, 1, 7/2, (1 + Sec[e + f*x])/2, 1 + Sec
[e + f*x]]*(1 - Sec[e + f*x])^(1/2 - n)*(a + a*Sec[e + f*x])^2*(c - c*Sec[e
 + f*x])^(-1 + n)*Tan[e + f*x])/(5*f)
```


Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
x)^(n - 1/2))/x, x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && !GtQ[b/(b*c
- a*d), 0] && !SimplerQ[c + d*x, a + b*x]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^n\,dx = -\frac{(ac\tan(e+fx))\,\text{Subst}\left(\int\frac{(a+ax)^{3/2}(c-cx)^{-\frac{1}{2}+n}}{x}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{\left(2^{-\frac{1}{2}+n}ac(c-c\sec(e+fx))^{-1+n}\left(\frac{c-c\sec(e+fx)}{c}\right)^{\frac{1}{2}-n}\tan(e+fx)\right)\text{Subst}\left(\right.}{f\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2^{\frac{1}{2}+n}cF_1\left(\frac{5}{2};\frac{1}{2}-n,1;\frac{7}{2};\frac{1}{2}(1+\sec(e+fx)),1+\sec(e+fx)\right)(1-\sec(e+}{5f}$$

Mathematica [F] time = 1.44349, size = 0, normalized size = 0.

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^n,x]
```

```
[Out] Integrate[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^n, x]
```

Maple [F] time = 0.389, size = 0, normalized size = 0.

$$\int \left(a+a\sec\left(fx+e\right)\right)^2\left(c-c\sec\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^n,x)
```

```
[Out] int((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec\left(fx+e\right)+a\right)^{2}\left(-c\sec\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^2*(-c*sec(f*x + e) + c)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^{2}\sec\left(fx+e\right)^{2}+2\,a^{2}\sec\left(fx+e\right)+a^{2}\right)\left(-c\sec\left(fx+e\right)+c\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*(-c*sec(f*x + e) +
 c)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^{2}\left(\int 2\left(-c\sec\left(e+fx\right)+c\right)^{n}\sec\left(e+fx\right)dx+\int\left(-c\sec\left(e+fx\right)+c\right)^{n}\sec^{2}\left(e+fx\right)dx+\int\left(-c\sec\left(e+fx\right)+c\right)^{n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**n,x)
```

```
[Out] a**2*(Integral(2*(-c*sec(e + f*x) + c)**n*sec(e + f*x), x) + Integral((-c*s
ec(e + f*x) + c)**n*sec(e + f*x)**2, x) + Integral((-c*sec(e + f*x) + c)**n
, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec\left(fx+e\right)+a\right)^{2}\left(-c\sec\left(fx+e\right)+c\right)^{n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^2*(-c*sec(f*x + e) + c)^n, x)
```

3.135 $\int (a + a\sec(e + fx))(c - c\sec(e + fx))^n \, dx$

Optimal. Leaf size=99

$$\frac{c2^{n+\frac{1}{2}} \tan(e+fx)(a\sec(e+fx)+a)(1-\sec(e+fx))^{\frac{1}{2}-n} F_1\left(\frac{3}{2}; \frac{1}{2}-n, 1; \frac{5}{2}; \frac{1}{2}(\sec(e+fx)+1), \sec(e+fx)+1\right)(c-c\sec(}{3f}$$

```
[Out] (2^(1/2 + n)*c*AppellF1[3/2, 1/2 - n, 1, 5/2, (1 + Sec[e + f*x])/2, 1 + Sec
[e + f*x]]*(1 - Sec[e + f*x])^(1/2 - n)*(a + a*Sec[e + f*x])*(c - c*Sec[e +
 f*x])^(-1 + n)*Tan[e + f*x])/(3*f)
```

Rubi [A] time = 0.0754091, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3912, 137, 136}

$$\frac{c2^{n+\frac{1}{2}} \tan(e+fx)(a\sec(e+fx)+a)(1-\sec(e+fx))^{\frac{1}{2}-n} F_1\left(\frac{3}{2}; \frac{1}{2}-n, 1; \frac{5}{2}; \frac{1}{2}(\sec(e+fx)+1), \sec(e+fx)+1\right)(c-c\sec(}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^n,x]
```


```
[Out] (2^(1/2 + n)*c*AppellF1[3/2, 1/2 - n, 1, 5/2, (1 + Sec[e + f*x])/2, 1 + Sec
[e + f*x]]*(1 - Sec[e + f*x])^(1/2 - n)*(a + a*Sec[e + f*x])*(c - c*Sec[e +
 f*x])^(-1 + n)*Tan[e + f*x])/(3*f)
```


Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
x)^(n - 1/2))/x, x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int (a+a\sec(e+fx))(c-c\sec(e+fx))^n\,dx = -\frac{(ac\tan(e+fx))\,\mathrm{Subst}\left(\int\frac{\sqrt{a+ax}(c-cx)^{-\frac{1}{2}+n}}{x}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{\left(2^{-\frac{1}{2}+n}ac(c-c\sec(e+fx))^{-1+n}\left(\frac{c-c\sec(e+fx)}{c}\right)^{\frac{1}{2}-n}\tan(e+fx)\right)\mathrm{Subst}\left(\right.}{f\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2^{\frac{1}{2}+n}cF_1\left(\frac{3}{2};\frac{1}{2}-n,1;\frac{5}{2};\frac{1}{2}(1+\sec(e+fx)),1+\sec(e+fx)\right)(1-\sec(e+}{3f}$$

Mathematica [F] time = 1.57674, size = 0, normalized size = 0.

$$\int (a+a\sec(e+fx))(c-c\sec(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^n,x]
```

```
[Out] Integrate[(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^n, x]
```

Maple [F] time = 0.391, size = 0, normalized size = 0.

$$\int \left(a+a\sec\left(fx+e\right)\right)\left(c-c\sec\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))*(c-c*sec(f*x+e))^n,x)
```

```
[Out] int((a+a*sec(f*x+e))*(c-c*sec(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec\left(fx+e\right)+a\right)\left(-c\sec\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))*(c-c*sec(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a\sec\left(fx+e\right)+a\right)\left(-c\sec\left(fx+e\right)+c\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))*(c-c*sec(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int \left(-c\sec\left(e+fx\right)+c\right)^{n}\sec\left(e+fx\right)dx+\int \left(-c\sec\left(e+fx\right)+c\right)^{n}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))*(c-c*sec(f*x+e))**n,x)
```

```
[Out] a*(Integral((-c*sec(e + f*x) + c)**n*sec(e + f*x), x) + Integral((-c*sec(e
+ f*x) + c)**n, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec\left(fx+e\right)+a\right)\left(-c\sec\left(fx+e\right)+c\right)^{n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))*(c-c*sec(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^n, x)
```

3.136 $\int \frac{(c-c\sec(e+fx))^n}{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=99

$$-\frac{c2^{n+\frac{1}{2}}\tan(e+fx)(1-\sec(e+fx))^{\frac{1}{2}-n}F_1\left(-\frac{1}{2};\frac{1}{2}-n,1;\frac{1}{2};\frac{1}{2}(\sec(e+fx)+1),\sec(e+fx)+1\right)(c-c\sec(e+fx))^{n-1}}{f(a\sec(e+fx)+a)}$$

```
[Out] -((2^(1/2 + n)*c*AppellF1[-1/2, 1/2 - n, 1, 1/2, (1 + Sec[e + f*x])/2, 1 +
Sec[e + f*x]]*(1 - Sec[e + f*x])^(1/2 - n)*(c - c*Sec[e + f*x])^(-1 + n)*Ta
n[e + f*x])/(f*(a + a*Sec[e + f*x])))
```

Rubi [A] time = 0.100369, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3912, 137, 136}

$$-\frac{c2^{n+\frac{1}{2}}\tan(e+fx)(1-\sec(e+fx))^{\frac{1}{2}-n}F_1\left(-\frac{1}{2};\frac{1}{2}-n,1;\frac{1}{2};\frac{1}{2}(\sec(e+fx)+1),\sec(e+fx)+1\right)(c-c\sec(e+fx))^{n-1}}{f(a\sec(e+fx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^n/(a + a*Sec[e + f*x]),x]
```

```
[Out] -((2^(1/2 + n)*c*AppellF1[-1/2, 1/2 - n, 1, 1/2, (1 + Sec[e + f*x])/2, 1 +
Sec[e + f*x]]*(1 - Sec[e + f*x])^(1/2 - n)*(c - c*Sec[e + f*x])^(-1 + n)*Ta
n[e + f*x])/(f*(a + a*Sec[e + f*x])))
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
x)^(n - 1/2))/x, x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && !GtQ[b/(b*c
- a*d), 0] && !SimplerQ[c + d*x, a + b*x]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \frac{(c-c\sec(e+fx))^n}{a+a\sec(e+fx)}\,dx = -\frac{(ac\tan(e+fx))\,\text{Subst}\left(\int \frac{(c-cx)^{-\frac{1}{2}+n}}{x(a+ax)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{\left(2^{-\frac{1}{2}+n}ac(c-c\sec(e+fx))^{-1+n}\left(\frac{c-c\sec(e+fx)}{c}\right)^{\frac{1}{2}-n}\tan(e+fx)\right)\text{Subst}\left(\int \frac{\left(\frac{1}{2}-\frac{x}{2}\right)^{-\frac{1}{2}+n}}{x(a+ax)^{3/2}}\,dx, x,\right.}{f\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{2^{\frac{1}{2}+n}cF_1\left(-\frac{1}{2};\frac{1}{2}-n,1;\frac{1}{2};\frac{1}{2}(1+\sec(e+fx)),1+\sec(e+fx)\right)(1-\sec(e+fx))^{\frac{1}{2}-n}(c-c\,\text{se}}{f(a+a\sec(e+fx))}$$

Mathematica [F] time = 0.970258, size = 0, normalized size = 0.

$$\int \frac{(c-c\sec(e+fx))^n}{a+a\sec(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c - c*Sec[e + f*x])^n/(a + a*Sec[e + f*x]),x]
```

```
[Out] Integrate[(c - c*Sec[e + f*x])^n/(a + a*Sec[e + f*x]), x]
```

Maple [F] time = 0.396, size = 0, normalized size = 0.

$$\int \frac{\left(c-c\sec\left(fx+e\right)\right)^{n}}{a+a\sec\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e)),x)
```

```
[Out] int((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\sec\left(fx+e\right)+c\right)^{n}}{a\sec\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((-c*sec(f*x + e) + c)^n/(a*sec(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(-c\sec\left(fx+e\right)+c\right)^{n}}{a\sec\left(fx+e\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((-c*sec(f*x + e) + c)^n/(a*sec(f*x + e) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\left(-c\sec\left(e+fx\right)+c\right)^{n}}{\sec\left(e+fx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**n/(a+a*sec(f*x+e)),x)
```

```
[Out] Integral((-c*sec(e + f*x) + c)**n/(sec(e + f*x) + 1), x)/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\sec\left(fx+e\right)+c\right)^{n}}{a\sec\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((-c*sec(f*x + e) + c)^n/(a*sec(f*x + e) + a), x)
```

3.137 $\int \frac{(c-c\sec(e+fx))^n}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=101

$$-\frac{c2^{n+\frac{1}{2}}\tan(e+fx)(1-\sec(e+fx))^{\frac{1}{2}-n}F_1\left(-\frac{3}{2};\frac{1}{2}-n,1;-\frac{1}{2};\frac{1}{2}(\sec(e+fx)+1),\sec(e+fx)+1\right)(c-c\sec(e+fx))^{n-1}}{3f(a\sec(e+fx)+a)^2}$$

```
[Out] -(2^(1/2 + n)*c*AppellF1[-3/2, 1/2 - n, 1, -1/2, (1 + Sec[e + f*x])/2, 1 +
Sec[e + f*x]]*(1 - Sec[e + f*x])^(1/2 - n)*(c - c*Sec[e + f*x])^(-1 + n)*Ta
n[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2)
```

Rubi [A] time = 0.10022, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3912, 137, 136}

$$-\frac{c2^{n+\frac{1}{2}}\tan(e+fx)(1-\sec(e+fx))^{\frac{1}{2}-n}F_1\left(-\frac{3}{2};\frac{1}{2}-n,1;-\frac{1}{2};\frac{1}{2}(\sec(e+fx)+1),\sec(e+fx)+1\right)(c-c\sec(e+fx))^{n-1}}{3f(a\sec(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^n/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] -(2^(1/2 + n)*c*AppellF1[-3/2, 1/2 - n, 1, -1/2, (1 + Sec[e + f*x])/2, 1 +
Sec[e + f*x]]*(1 - Sec[e + f*x])^(1/2 - n)*(c - c*Sec[e + f*x])^(-1 + n)*Ta
n[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2)
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
x)^(n - 1/2))/x, x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && !GtQ[b/(b*c
- a*d), 0] && !SimplerQ[c + d*x, a + b*x]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \frac{(c-c\sec(e+fx))^n}{(a+a\sec(e+fx))^2}\,dx = -\frac{(ac\tan(e+fx))\operatorname{Subst}\left(\int \frac{(c-cx)^{-\frac{1}{2}+n}}{x(a+ax)^{5/2}}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{\left(2^{-\frac{1}{2}+n}ac(c-c\sec(e+fx))^{-1+n}\left(\frac{c-c\sec(e+fx)}{c}\right)^{\frac{1}{2}-n}\tan(e+fx)\right)\operatorname{Subst}\left(\int \frac{\left(\frac{1}{2}-\frac{x}{2}\right)^{-\frac{1}{2}+n}}{x(a+ax)^{5/2}}\,dx,x,\right.}{f\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{2^{\frac{1}{2}+n}cF_1\left(-\frac{3}{2};\frac{1}{2}-n,1;-\frac{1}{2};\frac{1}{2}(1+\sec(e+fx)),1+\sec(e+fx)\right)(1-\sec(e+fx))^{\frac{1}{2}-n}(c-c}{3f(a+a\sec(e+fx))^2}$$

Mathematica [F] time = 1.57377, size = 0, normalized size = 0.

$$\int \frac{(c-c\sec(e+fx))^n}{(a+a\sec(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c - c*Sec[e + f*x])^n/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] Integrate[(c - c*Sec[e + f*x])^n/(a + a*Sec[e + f*x])^2, x]
```

Maple [F] time = 0.287, size = 0, normalized size = 0.

$$\int \frac{\left(c-c\sec\left(fx+e\right)\right)^{n}}{\left(a+a\sec\left(fx+e\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^2,x)
```

```
[Out] int((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\sec\left(fx+e\right)+c\right)^{n}}{\left(a\sec\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((-c*sec(f*x + e) + c)^n/(a*sec(f*x + e) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-c\sec\left(fx+e\right)+c\right)^{n}}{a^{2}\sec\left(fx+e\right)^{2}+2\,a^{2}\sec\left(fx+e\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral((-c*sec(f*x + e) + c)^n/(a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) +
 a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\left(-c\sec\left(e+fx\right)+c\right)^{n}}{\sec^{2}\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**n/(a+a*sec(f*x+e))**2,x)
```

```
[Out] Integral((-c*sec(e + f*x) + c)**n/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x
)/a**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\sec\left(fx+e\right)+c\right)^{n}}{\left(a\sec\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((-c*sec(f*x + e) + c)^n/(a*sec(f*x + e) + a)^2, x)
```

3.138 $\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^n\,dx$

Optimal. Leaf size=172

$$\frac{2a^3\tan(e+fx)(c-c\sec(e+fx))^n\text{Hypergeometric2F1}\left(1,n+\frac{1}{2},n+\frac{3}{2},1-\sec(e+fx)\right)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}+\frac{6a^3\tan(e+fx)(c-c\sec(e}{f(2n+1)\sqrt{a\sec(e+fx}$$

```
[Out] (6*a^3*(c - c*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Sec[e +
 f*x]]) + (2*a^3*Hypergeometric2F1[1, 1/2 + n, 3/2 + n, 1 - Sec[e + f*x]]*(
c - c*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Sec[e + f*x]])
- (2*a^3*(c - c*Sec[e + f*x])^(1 + n)*Tan[e + f*x])/(c*f*(3 + 2*n)*Sqrt[a +
 a*Sec[e + f*x]])
```

Rubi [A] time = 0.145338, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3912, 88, 65}

$$\frac{2a^3\tan(e+fx)(c-c\sec(e+fx))^n\,{}_2F_1\left(1,n+\frac{1}{2};n+\frac{3}{2};1-\sec(e+fx)\right)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}+\frac{6a^3\tan(e+fx)(c-c\sec(e+fx))^n}{f(2n+1)\sqrt{a\sec(e+fx)+a}}-\frac{2a^3\tan}{cf(}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^n,x]
```

```
[Out] (6*a^3*(c - c*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Sec[e +
 f*x]]) + (2*a^3*Hypergeometric2F1[1, 1/2 + n, 3/2 + n, 1 - Sec[e + f*x]]*(
c - c*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Sec[e + f*x]])
- (2*a^3*(c - c*Sec[e + f*x])^(1 + n)*Tan[e + f*x])/(c*f*(3 + 2*n)*Sqrt[a +
 a*Sec[e + f*x]])
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
x)^(n - 1/2))/x, x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^n\,dx = -\frac{(ac\tan(e+fx))\,\text{Subst}\left(\int \frac{(a+ax)^2(c-cx)^{-\frac{1}{2}+n}}{x}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{(ac\tan(e+fx))\,\text{Subst}\left(\int \left(3a^2(c-cx)^{-\frac{1}{2}+n} + \frac{a^2(c-cx)^{-\frac{1}{2}+n}}{x} - \frac{a^2(c-cx)^{\frac{1}{2}+}}{c}\right.\right.}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= \frac{6a^3(c-c\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}} - \frac{2a^3(c-c\sec(e+fx))^{1+n}\tan(e}{cf(3+2n)\sqrt{a+a\sec(e+fx}}$$

$$= \frac{6a^3(c-c\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}} + \frac{2a^3\,{}_2F_1\left(1,\frac{1}{2}+n;\frac{3}{2}+n;1-\sec(\right.}{f(1+2n)}$$

Mathematica [F] time = 8.51105, size = 0, normalized size = 0.

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^n,x]
```

```
[Out] Integrate[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^n, x]
```

Maple [F] time = 0.282, size = 0, normalized size = 0.

$$\int \left(a+a\sec\left(fx+e\right)\right)^{\frac{5}{2}} \left(c-c\sec\left(fx+e\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^n,x)
```

```
[Out] int((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c\sec\left(fx+e\right)+c\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^(5/2)*(-c*sec(f*x + e) + c)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(a^{2}\sec\left(fx+e\right)^{2}+2\,a^{2}\sec\left(fx+e\right)+a^{2}\right)\sqrt{a\sec\left(fx+e\right)+a}\left(-c\sec\left(fx+e\right)+c\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*sqrt(a*sec(f*x + e
) + a)*(-c*sec(f*x + e) + c)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^n,x, algorithm="giac")
```

```
[Out] Timed out
```

3.139 $\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^n\,dx$

Optimal. Leaf size=119

$$\frac{2a^2\tan(e+fx)(c-c\sec(e+fx))^n\text{Hypergeometric2F1}\left(1,n+\frac{1}{2},n+\frac{3}{2},1-\sec(e+fx)\right)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}+\frac{2a^2\tan(e+fx)(c-c\sec(e+fx))^n}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

```
[Out] (2*a^2*(c - c*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Sec[e +
 f*x]]) + (2*a^2*Hypergeometric2F1[1, 1/2 + n, 3/2 + n, 1 - Sec[e + f*x]]*(
c - c*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Sec[e + f*x]])
```

Rubi [A] time = 0.162068, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3909, 3912, 65}

$$\frac{2a^2\tan(e+fx)(c-c\sec(e+fx))^n\,{}_2F_1\left(1,n+\frac{1}{2};n+\frac{3}{2};1-\sec(e+fx)\right)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}+\frac{2a^2\tan(e+fx)(c-c\sec(e+fx))^n}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^n,x]
```

```
[Out] (2*a^2*(c - c*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Sec[e +
 f*x]]) + (2*a^2*Hypergeometric2F1[1, 1/2 + n, 3/2 + n, 1 - Sec[e + f*x]]*(
c - c*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Sec[e + f*x]])
```

Rule 3909

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[(-2*a^2*Cot[e + f*x]*(c + d*Csc[e + f
*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[a, Int[Sqrt[a + b
*Csc[e + f*x]]*(c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LeQ[n, -2^(-1)]
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
x)^(n - 1/2))/x, x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
```

```
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^n\,dx=\frac{2a^2(c-c\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}}+a\int\sqrt{a+a\sec(e+fx)}(c-c\sec$$

$$=\frac{2a^2(c-c\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}}-\frac{\left(a^2c\tan(e+fx)\right)\text{Subst}\left(\int\frac{(c-cx}{}\right.}{f\sqrt{a+a\sec(e+fx)}\sqrt{c}}$$

$$=\frac{2a^2(c-c\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}}+\frac{2a^2\,{}_2F_1\left(1,\frac{1}{2}+n;\frac{3}{2}+n;1-\sec(\right.}{f(1+2n)}$$

Mathematica [F] time = 11.8065, size = 0, normalized size = 0.

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^n,x]
```

```
[Out] Integrate[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^n, x]
```

Maple [F] time = 0.302, size = 0, normalized size = 0.

$$\int \left(a+a\sec\left(fx+e\right)\right)^{\frac{3}{2}}\left(c-c\sec\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^n,x)
```

```
[Out] int((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sec\left(fx+e\right)+c\right)^{n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^(3/2)*(-c*sec(f*x + e) + c)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sec\left(fx+e\right)+c\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((a*sec(f*x + e) + a)^(3/2)*(-c*sec(f*x + e) + c)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^n,x, algorithm="giac")
```

```
[Out] Timed out
```

3.140 $\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^n\,dx$

Optimal. Leaf size=68

$$\frac{2a\tan(e+fx)(c-c\sec(e+fx))^n\text{Hypergeometric2F1}\left(1,n+\frac{1}{2},n+\frac{3}{2},1-\sec(e+fx)\right)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

```
[Out] (2*a*Hypergeometric2F1[1, 1/2 + n, 3/2 + n, 1 - Sec[e + f*x]]*(c - c*Sec[e
+ f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Sec[e + f*x]])
```

Rubi [A] time = 0.0827766, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3912, 65}

$$\frac{2a\tan(e+fx)(c-c\sec(e+fx))^n\,{}_2F_1\left(1,n+\frac{1}{2};n+\frac{3}{2};1-\sec(e+fx)\right)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^n,x]
```

```
[Out] (2*a*Hypergeometric2F1[1, 1/2 + n, 3/2 + n, 1 - Sec[e + f*x]]*(c - c*Sec[e
+ f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Sec[e + f*x]])
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
x)^(n - 1/2))/x, x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^n\,dx = -\frac{(ac\tan(e+fx))\,\mathrm{Subst}\left(\int \frac{(c-cx)^{-\frac{1}{2}+n}}{x}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$
$$= \frac{2a\,{}_2F_1\left(1,\frac{1}{2}+n;\frac{3}{2}+n;1-\sec(e+fx)\right)(c-c\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}}$$

Mathematica [F] time = 0.146506, size = 0, normalized size = 0.

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^n,x]
```

```
[Out] Integrate[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^n, x]
```

Maple [F] time = 0.302, size = 0, normalized size = 0.

$$\int \sqrt{a+a\sec\left(fx+e\right)}\left(c-c\sec\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^n,x)
```

```
[Out] int((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sec\left(fx+e\right)+a}\left(-c\sec\left(fx+e\right)+c\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^n,x, algorithm="maxima")
```

[Out] integrate(sqrt(a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^n, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{a\sec\left(fx+e\right)+a}\left(-c\sec\left(fx+e\right)+c\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^n,x, algorithm="fricas")

[Out] integral(sqrt(a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^n, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\left(-c\left(\sec\left(e+fx\right)-1\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))**(1/2)*(c-c*sec(f*x+e))**n,x)

[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*(-c*(sec(e + f*x) - 1))**n, x)

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^n,x, algorithm="giac")

[Out] Timed out

3.141 $\int \frac{(c-c\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal. Leaf size=139

$$\frac{2\tan(e+fx)(c-c\sec(e+fx))^n\text{Hypergeometric2F1}\left(1,n+\frac{1}{2},n+\frac{3}{2},1-\sec(e+fx)\right)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}-\frac{\tan(e+fx)(c-c\sec(e+f$$

```
[Out] -((Hypergeometric2F1[1, 1/2 + n, 3/2 + n, (1 - Sec[e + f*x])/2]*(c - c*Sec[
e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Sec[e + f*x]])) + (2*Hype
rgeometric2F1[1, 1/2 + n, 3/2 + n, 1 - Sec[e + f*x]]*(c - c*Sec[e + f*x])^n
*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Sec[e + f*x]])
```

Rubi [A] time = 0.112058, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3912, 86, 65, 68}

$$\frac{2\tan(e+fx)(c-c\sec(e+fx))^n\,{}_2F_1\left(1,n+\frac{1}{2};n+\frac{3}{2};1-\sec(e+fx)\right)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}-\frac{\tan(e+fx)(c-c\sec(e+fx))^n\,{}_2F_1\left(1,n+\frac{1}{2};n\right.}{f(2n+1)\sqrt{a\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^n/Sqrt[a + a*Sec[e + f*x]],x]
```



```
[Out] -((Hypergeometric2F1[1, 1/2 + n, 3/2 + n, (1 - Sec[e + f*x])/2]*(c - c*Sec[
e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Sec[e + f*x]])) + (2*Hype
rgeometric2F1[1, 1/2 + n, 3/2 + n, 1 - Sec[e + f*x]]*(c - c*Sec[e + f*x])^n
*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Sec[e + f*x]])
```


Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
x)^(n - 1/2))/x, x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
```

```
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(c-c\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx = -\frac{(ac\tan(e+fx))\,\text{Subst}\left(\int \frac{(c-cx)^{-\frac{1}{2}+n}}{x(a+ax)}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{(c\tan(e+fx))\,\text{Subst}\left(\int \frac{(c-cx)^{-\frac{1}{2}+n}}{x}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{(ac\tan(e+fx))\,\text{Subst}\left(\int \frac{(c-cx)^{-\frac{1}{2}}}{a+ax}\right.}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-}}$$

$$= -\frac{{}_2F_1\left(1,\frac{1}{2}+n;\frac{3}{2}+n;\frac{1}{2}(1-\sec(e+fx))\right)(c-c\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}} + \frac{2\,{}_2F_1\left(1,\frac{1}{2}+n;\right.}{}$$

Mathematica [F] time = 1.36948, size = 0, normalized size = 0.

$$\int \frac{(c-c\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c - c*Sec[e + f*x])^n/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] Integrate[(c - c*Sec[e + f*x])^n/Sqrt[a + a*Sec[e + f*x]], x]
```

Maple [F] time = 0.295, size = 0, normalized size = 0.

$$\int \left(c-c\sec\left(fx+e\right)\right)^{n} \frac{1}{\sqrt{a+a\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] int((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\sec\left(fx+e\right)+c\right)^{n}}{\sqrt{a\sec\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((-c*sec(f*x + e) + c)^n/sqrt(a*sec(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(-c\sec\left(fx+e\right)+c\right)^{n}}{\sqrt{a\sec\left(fx+e\right)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((-c*sec(f*x + e) + c)^n/sqrt(a*sec(f*x + e) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\left(\sec\left(e+fx\right)-1\right)\right)^{n}}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**n/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((-c*(sec(e + f*x) - 1))**n/sqrt(a*(sec(e + f*x) + 1)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.142 $\int \frac{(c-c\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=205

$$-\frac{(5-2n)\tan(e+fx)(c-c\sec(e+fx))^n\text{Hypergeometric2F1}\left(1,n+\frac{1}{2},n+\frac{3}{2},\frac{1}{2}(1-\sec(e+fx))\right)}{4af(2n+1)\sqrt{a\sec(e+fx)+a}}+\frac{2\tan(e+fx)(c}{}$$

```
[Out] -((5 - 2*n)*Hypergeometric2F1[1, 1/2 + n, 3/2 + n, (1 - Sec[e + f*x])/2]*(c
 - c*Sec[e + f*x])^n*Tan[e + f*x])/(4*a*f*(1 + 2*n)*Sqrt[a + a*Sec[e + f*x]
]) + (2*Hypergeometric2F1[1, 1/2 + n, 3/2 + n, 1 - Sec[e + f*x]]*(c - c*Sec
[e + f*x])^n*Tan[e + f*x])/(a*f*(1 + 2*n)*Sqrt[a + a*Sec[e + f*x]]) - ((c -
 c*Sec[e + f*x])^n*Tan[e + f*x])/(2*a*f*(1 + Sec[e + f*x])*Sqrt[a + a*Sec[e
 + f*x]])
```

Rubi [A] time = 0.167874, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3912, 103, 156, 65, 68}

$$-\frac{(5-2n)\tan(e+fx)(c-c\sec(e+fx))^n\,{}_2F_1\left(1,n+\frac{1}{2};n+\frac{3}{2};\frac{1}{2}(1-\sec(e+fx))\right)}{4af(2n+1)\sqrt{a\sec(e+fx)+a}}+\frac{2\tan(e+fx)(c-c\sec(e+fx))^n\,{}_2}{af(2n+1)\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^n/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] -((5 - 2*n)*Hypergeometric2F1[1, 1/2 + n, 3/2 + n, (1 - Sec[e + f*x])/2]*(c
 - c*Sec[e + f*x])^n*Tan[e + f*x])/(4*a*f*(1 + 2*n)*Sqrt[a + a*Sec[e + f*x]
]) + (2*Hypergeometric2F1[1, 1/2 + n, 3/2 + n, 1 - Sec[e + f*x]]*(c - c*Sec
[e + f*x])^n*Tan[e + f*x])/(a*f*(1 + 2*n)*Sqrt[a + a*Sec[e + f*x]]) - ((c -
 c*Sec[e + f*x])^n*Tan[e + f*x])/(2*a*f*(1 + Sec[e + f*x])*Sqrt[a + a*Sec[e
 + f*x]])
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
x)^(n - 1/2))/x, x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(c-c\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx = -\frac{(ac\tan(e+fx))\,\text{Subst}\left(\int \frac{(c-cx)^{-\frac{1}{2}+n}}{x(a+ax)^2}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{(c-c\sec(e+fx))^n\tan(e+fx)}{2af(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} - \frac{\tan(e+fx)\,\text{Subst}\left(\int \frac{(c-cx)^{-\frac{1}{2}+n}\left(2ac-\frac{1}{2}ac(1-2n)\right.}{x(a+ax)}\right.}{2af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec}}$$

$$= -\frac{(c-c\sec(e+fx))^n\tan(e+fx)}{2af(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} - \frac{(c\tan(e+fx))\,\text{Subst}\left(\int \frac{(c-cx)^{-\frac{1}{2}+n}}{x}\,dx, x, \sec\right.}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+f}}$$

$$= -\frac{(5-2n)\,{}_2F_1\left(1,\frac{1}{2}+n;\frac{3}{2}+n;\frac{1}{2}(1-\sec(e+fx))\right)(c-c\sec(e+fx))^n\tan(e+fx)}{4af(1+2n)\sqrt{a+a\sec(e+fx)}} + \frac{2\,{}_2F}{}$$

Mathematica [F] time = 1.6534, size = 0, normalized size = 0.

$$\int \frac{(c-c\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c - c*Sec[e + f*x])^n/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] Integrate[(c - c*Sec[e + f*x])^n/(a + a*Sec[e + f*x])^(3/2), x]
```

Maple [F] time = 0.264, size = 0, normalized size = 0.

$$\int \left(c-c\sec\left(fx+e\right)\right)^n\left(a+a\sec\left(fx+e\right)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x)
```

```
[Out] int((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\sec\left(fx+e\right)+c\right)^{n}}{\left(a\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-c*sec(f*x + e) + c)^n/(a*sec(f*x + e) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a\sec\left(fx+e\right)+a}\left(-c\sec\left(fx+e\right)+c\right)^{n}}{a^{2}\sec\left(fx+e\right)^{2}+2\,a^{2}\sec\left(fx+e\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^n/(a^2*sec(f*x + e)
^2 + 2*a^2*sec(f*x + e) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**n/(a+a*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

<u>**Giac**</u> **[F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.143 $\int \frac{\sqrt{a+a\sec(e+fx)}}{c+c\sec(e+fx)}\,dx$

Optimal. Leaf size=91

$$\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{cf}-\frac{\sqrt{2}\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{cf}$$

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c*f) -
 (Sqrt[2]*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e +
 f*x]])])/(c*f)
```

Rubi [A] time = 0.0824256, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {21, 3776, 3774, 203, 3795}

$$\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{cf}-\frac{\sqrt{2}\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{cf}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]/(c + c*Sec[e + f*x]),x]
```

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c*f) -
 (Sqrt[2]*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e +
 f*x]])])/(c*f)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3776

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[1/a, In
t[Sqrt[a + b*Csc[c + d*x]], x], x] - Dist[b/a, Int[Csc[c + d*x]/Sqrt[a + b*
Csc[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+a\sec(e+fx)}}{c+c\sec(e+fx)}\,dx &= \frac{a\int \frac{1}{\sqrt{a+a\sec(e+fx)}}\,dx}{c}\\ &= \frac{\int \sqrt{a+a\sec(e+fx)}\,dx}{c}-\frac{a\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx}{c}\\ &= -\frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{a\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf}+\frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{2a+x^2}\,dx, x, -\frac{a\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf}\\ &= \frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf}-\frac{\sqrt{2}\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{cf}\end{aligned}$$

Mathematica [C] time = 0.706105, size = 133, normalized size = 1.46

$$-\frac{i\sqrt{1+e^{2i(e+fx)}}\sqrt{a(\sec(e+fx)+1)}\left(\sinh^{-1}\left(e^{i(e+fx)}\right)-\sqrt{2}\tanh^{-1}\left(\frac{-1+e^{i(e+fx)}}{\sqrt{2}\sqrt{1+e^{2i(e+fx)}}}\right)-\tanh^{-1}\left(\sqrt{1+e^{2i(e+fx)}}\right)\right)}{cf\left(1+e^{i(e+fx)}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]/(c + c*Sec[e + f*x]),x]
```

```
[Out] ((-I)*Sqrt[1 + E^((2*I)*(e + f*x))]*(ArcSinh[E^(I*(e + f*x))] - Sqrt[2]*Arc
Tanh[(-1 + E^(I*(e + f*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(e + f*x))])] - ArcT
anh[Sqrt[1 + E^((2*I)*(e + f*x))]])*Sqrt[a*(1 + Sec[e + f*x]])/(c*(1 + E^(
I*(e + f*x)))*f)
```

Maple [A] time = 0.227, size = 141, normalized size = 1.6

$$-\frac{1}{fc}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\sqrt{-2\,\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\left(\sqrt{2}\mathrm{Artanh}\left(\frac{\sqrt{2}\sin\left(fx+e\right)}{2\,\cos\left(fx+e\right)}\sqrt{-2\,\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\right)+\ln\left(-\frac{1}{\sin\left(fx+\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x)
```

```
[Out] -1/c/f*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))
^(1/2)*(2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*si
n(f*x+e)/cos(f*x+e))+ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+
cos(f*x+e)-1)/sin(f*x+e)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\sec\left(fx+e\right)+a}}{c\sec\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sec(f*x + e) + a)/(c*sec(f*x + e) + c), x)
```

Fricas [A] time = 1.58395, size = 782, normalized size = 8.59

$$\left[\frac{\sqrt{2}\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+3a\cos(fx+e)^2+2a\cos(fx+e)-a}{\cos(fx+e)^2+2\cos(fx+e)+1}\right)+2\sqrt{-a}\log\left(\frac{2a\cos(fx+e)^2-2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cdots}{\cos(f\cdots}\right.}{2cf}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(2)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/co
s(f*x + e))*cos(f*x + e)*sin(f*x + e) + 3*a*cos(f*x + e)^2 + 2*a*cos(f*x +
e) - a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 2*sqrt(-a)*log((2*a*cos(f*
x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*
sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/(c*f), (sqrt(2)*sqr
t(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(s
qrt(a)*sin(f*x + e))) - 2*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x
+ e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))))/(c*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{a\sec(e+fx)+a}}{\sec(e+fx)+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(1/2)/(c+c*sec(f*x+e)),x)
```

```
[Out] Integral(sqrt(a*sec(e + f*x) + a)/(sec(e + f*x) + 1), x)/c
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.144 $\int \frac{(c+d\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=231

$$-\frac{2c\cot(e+fx)\sqrt{-\frac{d(1-\sec(e+fx))}{c+d\sec(e+fx)}}\sqrt{\frac{d(\sec(e+fx)+1)}{c+d\sec(e+fx)}}(c+d\sec(e+fx))\Pi\left(\frac{c}{c+d};\sin^{-1}\left(\frac{\sqrt{c+d}}{\sqrt{c+d\sec(e+fx)}}\right)|\frac{c-d}{c+d}\right)}{af\sqrt{c+d}}-\frac{(c-d)\sqrt{\frac{1}{\sec(e+fx}}}{}$$

```
[Out] (-2*c*Cot[e + f*x]*EllipticPi[c/(c + d), ArcSin[Sqrt[c + d]/Sqrt[c + d*Sec[
e + f*x]]], (c - d)/(c + d)]*Sqrt[-((d*(1 - Sec[e + f*x]))/(c + d*Sec[e + f
*x]))]*Sqrt[(d*(1 + Sec[e + f*x]))/(c + d*Sec[e + f*x])]*(c + d*Sec[e + f*x
]))/(a*Sqrt[c + d]*f) - ((c - d)*EllipticE[ArcSin[Tan[e + f*x]/(1 + Sec[e +
 f*x])], (c - d)/(c + d)]*Sqrt[(1 + Sec[e + f*x])^(-1)]*Sqrt[c + d*Sec[e +
f*x]])/(a*f*Sqrt[(c + d*Sec[e + f*x])/((c + d)*(1 + Sec[e + f*x]))])
```

Rubi [A] time = 0.257324, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3927, 3780, 3968}

$$-\frac{2c\cot(e+fx)\sqrt{-\frac{d(1-\sec(e+fx))}{c+d\sec(e+fx)}}\sqrt{\frac{d(\sec(e+fx)+1)}{c+d\sec(e+fx)}}(c+d\sec(e+fx))\Pi\left(\frac{c}{c+d};\sin^{-1}\left(\frac{\sqrt{c+d}}{\sqrt{c+d\sec(e+fx)}}\right)|\frac{c-d}{c+d}\right)}{af\sqrt{c+d}}-\frac{(c-d)\sqrt{\frac{1}{\sec(e+fx}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sec[e + f*x])^(3/2)/(a + a*Sec[e + f*x]),x]
```



```
[Out] (-2*c*Cot[e + f*x]*EllipticPi[c/(c + d), ArcSin[Sqrt[c + d]/Sqrt[c + d*Sec[
e + f*x]]], (c - d)/(c + d)]*Sqrt[-((d*(1 - Sec[e + f*x]))/(c + d*Sec[e + f
*x]))]*Sqrt[(d*(1 + Sec[e + f*x]))/(c + d*Sec[e + f*x])]*(c + d*Sec[e + f*x
]))/(a*Sqrt[c + d]*f) - ((c - d)*EllipticE[ArcSin[Tan[e + f*x]/(1 + Sec[e +
 f*x])], (c - d)/(c + d)]*Sqrt[(1 + Sec[e + f*x])^(-1)]*Sqrt[c + d*Sec[e +
f*x]])/(a*f*Sqrt[(c + d*Sec[e + f*x])/((c + d)*(1 + Sec[e + f*x]))])
```


Rule 3927

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_)), x_Symbol] :> Dist[a/c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] +
```

```
 Dist[(b*c - a*d)/c, Int[(Csc[e + f*x]*Sqrt[a + b*Csc[e + f*x]])/(c + d*Csc
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
(EqQ[a^2 - b^2, 0] || EqQ[c^2 - d^2, 0])
```

Rule 3780

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*(a + b
*Csc[c + d*x])*Sqrt[(b*(1 + Csc[c + d*x]))/(a + b*Csc[c + d*x])]*Sqrt[-((b*
(1 - Csc[c + d*x]))/(a + b*Csc[c + d*x]))]*EllipticPi[a/(a + b), ArcSin[Rt[
a + b, 2]/Sqrt[a + b*Csc[c + d*x]]], (a - b)/(a + b)])/(d*Rt[a + b, 2]*Cot[
c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3968

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> -Simp[(Sqrt[a + b*Csc[e
+ f*x]]*Sqrt[c/(c + d*Csc[e + f*x])]*EllipticE[ArcSin[(c*Cot[e + f*x])/(c +
 d*Csc[e + f*x])], -((b*c - a*d)/(b*c + a*d))])/(d*f*Sqrt[(c*d*(a + b*Csc[e
 + f*x]))/((b*c + a*d)*(c + d*Csc[e + f*x]))]), x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(c+d\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx = \frac{c\int\sqrt{c+d\sec(e+fx)}\,dx}{a} + (-c+d)\int\frac{\sec(e+fx)\sqrt{c+d\sec(e+fx)}}{a+a\sec(e+fx)}\,dx$$
$$= -\frac{2c\cot(e+fx)\Pi\left(\frac{c}{c+d};\sin^{-1}\left(\frac{\sqrt{c+d}}{\sqrt{c+d\sec(e+fx)}}\right)|\frac{c-d}{c+d}\right)\sqrt{-\frac{d(1-\sec(e+fx))}{c+d\sec(e+fx)}}\sqrt{\frac{d(1+\sec(e+fx))}{c+d\sec(e+fx)}}(c+d\,\mathrm{se}}{a\sqrt{c+d}f}$$

Mathematica [B] time = 17.9967, size = 814, normalized size = 3.52

$$\frac{(c+d\sec(e+fx))^{3/2}\left(2\sec\left(\frac{1}{2}(e+fx)\right)\left(d\sin\left(\frac{1}{2}(e+fx)\right)-c\sin\left(\frac{1}{2}(e+fx)\right)\right)-2(d-c)\sin(e+fx)\right)\cos^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{f(d+c\cos(e+fx))(\sec(e+fx)a+a)}+\frac{2c}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sec[e + f*x])^(3/2)/(a + a*Sec[e + f*x]),x]
```

```
[Out] (Cos[e/2 + (f*x)/2]^2*(c + d*Sec[e + f*x])^(3/2)*(2*Sec[(e + f*x)/2]*(-(c*S
in[(e + f*x)/2]) + d*Sin[(e + f*x)/2]) - 2*(-c + d)*Sin[e + f*x]))/(f*(d +
c*Cos[e + f*x])*(a + a*Sec[e + f*x])) + (2*Cos[e/2 + (f*x)/2]^2*(c + d*Sec[
e + f*x])^(3/2)*(c^2*Tan[(e + f*x)/2] - d^2*Tan[(e + f*x)/2] - 2*c^2*Tan[(e
 + f*x)/2]^3 + 2*c*d*Tan[(e + f*x)/2]^3 + c^2*Tan[(e + f*x)/2]^5 - 2*c*d*Ta
n[(e + f*x)/2]^5 + d^2*Tan[(e + f*x)/2]^5 + 4*c^2*EllipticPi[-1, -ArcSin[Ta
n[(e + f*x)/2]], (c - d)/(c + d)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[(c + d
- c*Tan[(e + f*x)/2]^2 + d*Tan[(e + f*x)/2]^2)/(c + d)] + 4*c^2*EllipticPi[
-1, -ArcSin[Tan[(e + f*x)/2]], (c - d)/(c + d)]*Tan[(e + f*x)/2]^2*Sqrt[1 -
 Tan[(e + f*x)/2]^2]*Sqrt[(c + d - c*Tan[(e + f*x)/2]^2 + d*Tan[(e + f*x)/2
]^2)/(c + d)] + (c^2 - d^2)*EllipticE[ArcSin[Tan[(e + f*x)/2]], (c - d)/(c
+ d)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*(1 + Tan[(e + f*x)/2]^2)*Sqrt[(c + d - c
*Tan[(e + f*x)/2]^2 + d*Tan[(e + f*x)/2]^2)/(c + d)] + 2*c*(c - d)*Elliptic
F[ArcSin[Tan[(e + f*x)/2]], (c - d)/(c + d)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*(
1 + Tan[(e + f*x)/2]^2)*Sqrt[(c + d - c*Tan[(e + f*x)/2]^2 + d*Tan[(e + f*x
)/2]^2)/(c + d)]))/(f*(d + c*Cos[e + f*x])^(3/2)*Sqrt[Sec[e + f*x]]*(a + a*
Sec[e + f*x])*Sqrt[(1 - Tan[(e + f*x)/2]^2)^(-1)]*(-1 + Tan[(e + f*x)/2]^2)
*(1 + Tan[(e + f*x)/2]^2)^(3/2)*Sqrt[(c + d - c*Tan[(e + f*x)/2]^2 + d*Tan[
(e + f*x)/2]^2)/(1 + Tan[(e + f*x)/2]^2)])
```

Maple [A] time = 0.398, size = 295, normalized size = 1.3

$$-\frac{\left(1+\cos\left(fx+e\right)\right)^{2}\left(-1+\cos\left(fx+e\right)\right)}{fa\left(d+c\cos\left(fx+e\right)\right)\left(\sin\left(fx+e\right)\right)^{2}}\sqrt{\frac{d+c\cos\left(fx+e\right)}{\cos\left(fx+e\right)}}\sqrt{\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\sqrt{\frac{d+c\cos\left(fx+e\right)}{\left(c+d\right)\left(1+\cos\left(fx+e\right)\right)}}\Big(2\,\mathrm{Ellipt}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e)),x)
```

```
[Out] -1/a/f*((d+c*cos(f*x+e))/cos(f*x+e))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2
)*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*(1+cos(f*x+e))^2*(2*Ellip
ticF((-1+cos(f*x+e))/sin(f*x+e),((c-d)/(c+d))^(1/2))*c^2-2*EllipticF((-1+co
s(f*x+e))/sin(f*x+e),((c-d)/(c+d))^(1/2))*c*d+EllipticE((-1+cos(f*x+e))/sin
(f*x+e),((c-d)/(c+d))^(1/2))*c^2-EllipticE((-1+cos(f*x+e))/sin(f*x+e),((c-d
)/(c+d))^(1/2))*d^2-4*c^2*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),-1,((c-d)/(
c+d))^(1/2)))*(-1+cos(f*x+e))/(d+c*cos(f*x+e))/sin(f*x+e)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(fx+e\right)+c\right)^{\frac{3}{2}}}{a\sec\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e) + c)^(3/2)/(a*sec(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(d\sec\left(fx+e\right)+c\right)^{\frac{3}{2}}}{a\sec\left(fx+e\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((d*sec(f*x + e) + c)^(3/2)/(a*sec(f*x + e) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{c\sqrt{c+d\sec\left(e+fx\right)}}{\sec\left(e+fx\right)+1}\,dx+\int \frac{d\sqrt{c+d\sec\left(e+fx\right)}\sec\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e)),x)
```

```
[Out] (Integral(c*sqrt(c + d*sec(e + f*x))/(sec(e + f*x) + 1), x) + Integral(d*sq
rt(c + d*sec(e + f*x))*sec(e + f*x)/(sec(e + f*x) + 1), x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(fx+e\right)+c\right)^{\frac{3}{2}}}{a\sec\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e) + c)^(3/2)/(a*sec(f*x + e) + a), x)
```

3.145 $\int \frac{\sqrt{c+d\sec(e+fx)}}{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=225

$$-\frac{2\cot(e+fx)\sqrt{-\frac{d(1-\sec(e+fx))}{c+d\sec(e+fx)}}\sqrt{\frac{d(\sec(e+fx)+1)}{c+d\sec(e+fx)}}(c+d\sec(e+fx))\Pi\left(\frac{c}{c+d};\sin^{-1}\left(\frac{\sqrt{c+d}}{\sqrt{c+d\sec(e+fx)}}\right)|\frac{c-d}{c+d}\right)}{af\sqrt{c+d}}-\frac{\sqrt{\frac{1}{\sec(e+fx)+1}}\sqrt{c+d\sec(e+fx)}\ldots}{\ldots}$$

```
[Out] (-2*Cot[e + f*x]*EllipticPi[c/(c + d), ArcSin[Sqrt[c + d]/Sqrt[c + d*Sec[e
+ f*x]]], (c - d)/(c + d)]*Sqrt[-((d*(1 - Sec[e + f*x]))/(c + d*Sec[e + f*x
]))]*Sqrt[(d*(1 + Sec[e + f*x]))/(c + d*Sec[e + f*x])]*(c + d*Sec[e + f*x])
)/(a*Sqrt[c + d]*f) - (EllipticE[ArcSin[Tan[e + f*x]/(1 + Sec[e + f*x])], (
c - d)/(c + d)]*Sqrt[(1 + Sec[e + f*x])^(-1)]*Sqrt[c + d*Sec[e + f*x]])/(a*
f*Sqrt[(c + d*Sec[e + f*x])/((c + d)*(1 + Sec[e + f*x]))])
```

Rubi [A] time = 0.21073, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3925, 3780, 3968}

$$-\frac{2\cot(e+fx)\sqrt{-\frac{d(1-\sec(e+fx))}{c+d\sec(e+fx)}}\sqrt{\frac{d(\sec(e+fx)+1)}{c+d\sec(e+fx)}}(c+d\sec(e+fx))\Pi\left(\frac{c}{c+d};\sin^{-1}\left(\frac{\sqrt{c+d}}{\sqrt{c+d\sec(e+fx)}}\right)|\frac{c-d}{c+d}\right)}{af\sqrt{c+d}}-\frac{\sqrt{\frac{1}{\sec(e+fx)+1}}\sqrt{c+d\sec(e+fx)}\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Sec[e + f*x]]/(a + a*Sec[e + f*x]),x]
```


```
[Out] (-2*Cot[e + f*x]*EllipticPi[c/(c + d), ArcSin[Sqrt[c + d]/Sqrt[c + d*Sec[e
+ f*x]]], (c - d)/(c + d)]*Sqrt[-((d*(1 - Sec[e + f*x]))/(c + d*Sec[e + f*x
]))]*Sqrt[(d*(1 + Sec[e + f*x]))/(c + d*Sec[e + f*x])]*(c + d*Sec[e + f*x])
)/(a*Sqrt[c + d]*f) - (EllipticE[ArcSin[Tan[e + f*x]/(1 + Sec[e + f*x])], (
c - d)/(c + d)]*Sqrt[(1 + Sec[e + f*x])^(-1)]*Sqrt[c + d*Sec[e + f*x]])/(a*
f*Sqrt[(c + d*Sec[e + f*x])/((c + d)*(1 + Sec[e + f*x]))])
```


Rule 3925

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[1/c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
```

```
ist[d/c, Int[(Csc[e + f*x]*Sqrt[a + b*Csc[e + f*x]])/(c + d*Csc[e + f*x]),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && (EqQ[a^2 - b
^2, 0] || EqQ[c^2 - d^2, 0])
```

Rule 3780

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*(a + b
*Csc[c + d*x])*Sqrt[(b*(1 + Csc[c + d*x]))/(a + b*Csc[c + d*x])]*Sqrt[-((b*
(1 - Csc[c + d*x]))/(a + b*Csc[c + d*x]))]*EllipticPi[a/(a + b), ArcSin[Rt[
a + b, 2]/Sqrt[a + b*Csc[c + d*x]]], (a - b)/(a + b)])/(d*Rt[a + b, 2]*Cot[
c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3968

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> -Simp[(Sqrt[a + b*Csc[e
+ f*x]]*Sqrt[c/(c + d*Csc[e + f*x])]*EllipticE[ArcSin[(c*Cot[e + f*x])/(c +
 d*Csc[e + f*x])], -((b*c - a*d)/(b*c + a*d))])/(d*f*Sqrt[(c*d*(a + b*Csc[e
 + f*x]))/((b*c + a*d)*(c + d*Csc[e + f*x]))]), x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{a+a\sec(e+fx)}\,dx = \frac{\int \sqrt{c+d\sec(e+fx)}\,dx}{a} - \int \frac{\sec(e+fx)\sqrt{c+d\sec(e+fx)}}{a+a\sec(e+fx)}\,dx$$

$$= -\frac{2\cot(e+fx)\Pi\left(\frac{c}{c+d};\sin^{-1}\left(\frac{\sqrt{c+d}}{\sqrt{c+d\sec(e+fx)}}\right)|\frac{c-d}{c+d}\right)\sqrt{-\frac{d(1-\sec(e+fx))}{c+d\sec(e+fx)}}\sqrt{\frac{d(1+\sec(e+fx))}{c+d\sec(e+fx)}}(c+d\sec(}{a\sqrt{c+d}f}$$

Mathematica [A] time = 8.51973, size = 180, normalized size = 0.8

$$-\frac{4\cos^4\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{1}{\sec(e+fx)+1}}\sqrt{c+d\sec(e+fx)}\left(2(c-d)\text{EllipticF}\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right),\frac{c-d}{c+d}\right)+(c+d)E\left(\sin^{-1}\right.\right.}{af(c+d)(\cos(e+fx)+1)^2\sqrt{\frac{c\cos(e+fx)+d}{(c+d)(\cos(e+fx)+1)}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*Sec[e + f*x]]/(a + a*Sec[e + f*x]),x]
```

```
[Out] (-4*Cos[(e + f*x)/2]^4*((c + d)*EllipticE[ArcSin[Tan[(e + f*x)/2]], (c - d)
/(c + d)] + 2*(c - d)*EllipticF[ArcSin[Tan[(e + f*x)/2]], (c - d)/(c + d)]
+ 4*c*EllipticPi[-1, -ArcSin[Tan[(e + f*x)/2]], (c - d)/(c + d)])*Sqrt[(1 +
 Sec[e + f*x])^(-1)]*Sqrt[c + d*Sec[e + f*x]])/(a*(c + d)*f*(1 + Cos[e + f*
x])^2*Sqrt[(d + c*Cos[e + f*x])/((c + d)*(1 + Cos[e + f*x]))])
```

Maple [A] time = 0.414, size = 285, normalized size = 1.3

$$-\frac{\left(1+\cos\left(fx+e\right)\right)^{2}\left(-1+\cos\left(fx+e\right)\right)}{fa\left(d+c\cos\left(fx+e\right)\right)\left(\sin\left(fx+e\right)\right)^{2}}\sqrt{\frac{d+c\cos\left(fx+e\right)}{\cos\left(fx+e\right)}}\sqrt{\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\sqrt{\frac{d+c\cos\left(fx+e\right)}{\left(c+d\right)\left(1+\cos\left(fx+e\right)\right)}}\left(2\,\mathrm{Elliptic}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e)),x)
```

```
[Out] -1/a/f*((d+c*cos(f*x+e))/cos(f*x+e))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2
)*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*(1+cos(f*x+e))^2*(-1+cos(
f*x+e))*(2*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((c-d)/(c+d))^(1/2))*c-2*El
lipticF((-1+cos(f*x+e))/sin(f*x+e),((c-d)/(c+d))^(1/2))*d+c*EllipticE((-1+c
os(f*x+e))/sin(f*x+e),((c-d)/(c+d))^(1/2))+d*EllipticE((-1+cos(f*x+e))/sin(
f*x+e),((c-d)/(c+d))^(1/2))-4*c*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),-1,((
c-d)/(c+d))^(1/2)))/(d+c*cos(f*x+e))/sin(f*x+e)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{d\sec\left(fx+e\right)+c}}{a\sec\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*sec(f*x + e) + c)/(a*sec(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\sec\left(fx+e\right)+c}}{a\sec\left(fx+e\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*sec(f*x + e) + c)/(a*sec(f*x + e) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\sqrt{c+d\sec\left(e+fx\right)}}{\sec\left(e+fx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e)),x)
```

```
[Out] Integral(sqrt(c + d*sec(e + f*x))/(sec(e + f*x) + 1), x)/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{d\sec\left(fx+e\right)+c}}{a\sec\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*sec(f*x + e) + c)/(a*sec(f*x + e) + a), x)
```

3.146 $\int \frac{1}{(a+a\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx$

Optimal. Leaf size=319

$$\frac{2\sqrt{c+d}\cot(e+fx)\sqrt{\frac{d(1-\sec(e+fx))}{c+d}}\sqrt{-\frac{d(\sec(e+fx)+1)}{c-d}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}}\right),\frac{c+d}{c-d}\right)}{af(c-d)}-\frac{2\sqrt{c+d}\cot(e+fx)\sqrt{\frac{d(1-\sec(e+fx))}{c+d}}}{}$$

```
[Out] (2*Sqrt[c + d]*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[c + d*Sec[e + f*x]]/Sqrt[
c + d]], (c + d)/(c - d)]*Sqrt[(d*(1 - Sec[e + f*x]))/(c + d)]*Sqrt[-((d*(1
 + Sec[e + f*x]))/(c - d))])/(a*(c - d)*f) - (2*Sqrt[c + d]*Cot[e + f*x]*El
lipticPi[(c + d)/c, ArcSin[Sqrt[c + d*Sec[e + f*x]]/Sqrt[c + d]], (c + d)/(
c - d)]*Sqrt[(d*(1 - Sec[e + f*x]))/(c + d)]*Sqrt[-((d*(1 + Sec[e + f*x]))/
(c - d))])/(a*c*f) - (EllipticE[ArcSin[Tan[e + f*x]/(1 + Sec[e + f*x])], (c
 - d)/(c + d)]*Sqrt[(1 + Sec[e + f*x])^(-1)]*Sqrt[c + d*Sec[e + f*x]])/(a*(
c - d)*f*Sqrt[(c + d*Sec[e + f*x])/((c + d)*(1 + Sec[e + f*x]))])
```

Rubi [A] time = 0.369961, antiderivative size = 319, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {3929, 3921, 3784, 3832, 3968}

$$\frac{2\sqrt{c+d}\cot(e+fx)\sqrt{\frac{d(1-\sec(e+fx))}{c+d}}\sqrt{-\frac{d(\sec(e+fx)+1)}{c-d}}F\left(\sin^{-1}\left(\frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}}\right)|\frac{c+d}{c-d}\right)}{af(c-d)}-\frac{2\sqrt{c+d}\cot(e+fx)\sqrt{\frac{d(1-\sec(e+fx))}{c+d}}\sqrt{-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])*Sqrt[c + d*Sec[e + f*x]]),x]
```


```
[Out] (2*Sqrt[c + d]*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[c + d*Sec[e + f*x]]/Sqrt[
c + d]], (c + d)/(c - d)]*Sqrt[(d*(1 - Sec[e + f*x]))/(c + d)]*Sqrt[-((d*(1
 + Sec[e + f*x]))/(c - d))])/(a*(c - d)*f) - (2*Sqrt[c + d]*Cot[e + f*x]*El
lipticPi[(c + d)/c, ArcSin[Sqrt[c + d*Sec[e + f*x]]/Sqrt[c + d]], (c + d)/(
c - d)]*Sqrt[(d*(1 - Sec[e + f*x]))/(c + d)]*Sqrt[-((d*(1 + Sec[e + f*x]))/
(c - d))])/(a*c*f) - (EllipticE[ArcSin[Tan[e + f*x]/(1 + Sec[e + f*x])], (c
 - d)/(c + d)]*Sqrt[(1 + Sec[e + f*x])^(-1)]*Sqrt[c + d*Sec[e + f*x]])/(a*(
c - d)*f*Sqrt[(c + d*Sec[e + f*x])/((c + d)*(1 + Sec[e + f*x]))])
```

Rule 3929

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))), x_Symbol] :> Dist[1/(c*(b*c - a*d)), Int[(b*c - a*d - b*d*C
sc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[d^2/(c*(b*c - a*d)), I
nt[(Csc[e + f*x]*Sqrt[a + b*Csc[e + f*x]])/(c + d*Csc[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && (EqQ[a^2 - b^2, 0] ||
EqQ[c^2 - d^2, 0])
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3968

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> -Simp[(Sqrt[a + b*Csc[e
+ f*x]]*Sqrt[c/(c + d*Csc[e + f*x])]*EllipticE[ArcSin[(c*Cot[e + f*x])/(c +
 d*Csc[e + f*x])], -((b*c - a*d)/(b*c + a*d))])/(d*f*Sqrt[(c*d*(a + b*Csc[e
 + f*x]))/((b*c + a*d)*(c + d*Csc[e + f*x]))]), x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx = -\frac{\int \frac{-ac+ad-ad\sec(e+fx)}{\sqrt{c+d\sec(e+fx)}}\,dx}{a^2(c-d)} + \frac{a\int \frac{\sec(e+fx)\sqrt{c+d\sec(e+fx)}}{a+a\sec(e+fx)}\,dx}{-ac+ad}$$

$$= -\frac{E\left(\sin^{-1}\left(\frac{\tan(e+fx)}{1+\sec(e+fx)}\right)|\frac{c-d}{c+d}\right)\sqrt{\frac{1}{1+\sec(e+fx)}}\sqrt{c+d\sec(e+fx)}}{a(c-d)f\sqrt{\frac{c+d\sec(e+fx)}{(c+d)(1+\sec(e+fx))}}} + \frac{\int \frac{1}{\sqrt{c+d\sec(}}}{a}$$

$$= \frac{2\sqrt{c+d}\cot(e+fx)F\left(\sin^{-1}\left(\frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}}\right)|\frac{c+d}{c-d}\right)\sqrt{\frac{d(1-\sec(e+fx))}{c+d}}\sqrt{-\frac{d(1+\sec}{c-}}}{a(c-d)f}$$

Mathematica [A] time = 5.37417, size = 193, normalized size = 0.61

$$\frac{4\cos^4\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{c\cos(e+fx)+d}{(c+d)(\cos(e+fx)+1)}}\left(2(c-2d)\text{EllipticF}\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right),\frac{c-d}{c+d}\right)+(c+d)E\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(e+f\right.\right.\right.\right.}{af(d-c)\sqrt{\frac{\cos(e+fx)}{\cos(e+fx)+1}}(\cos(e+fx)+1)^2\sqrt{c+d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[e + f*x])*Sqrt[c + d*Sec[e + f*x]]),x]
```

```
[Out] (4*Cos[(e + f*x)/2]^4*Sqrt[(d + c*Cos[e + f*x])/((c + d)*(1 + Cos[e + f*x])
)]*((c + d)*EllipticE[ArcSin[Tan[(e + f*x)/2]], (c - d)/(c + d)] + 2*(c - 2
*d)*EllipticF[ArcSin[Tan[(e + f*x)/2]], (c - d)/(c + d)] + 4*(c - d)*Ellipt
icPi[-1, -ArcSin[Tan[(e + f*x)/2]], (c - d)/(c + d)]))/(a*(-c + d)*f*Sqrt[C
os[e + f*x]/(1 + Cos[e + f*x])]*(1 + Cos[e + f*x])^2*Sqrt[c + d*Sec[e + f*x
]])
```

Maple [A] time = 0.351, size = 327, normalized size = 1.

$$-\frac{\left(1+\cos\left(fx+e\right)\right)^2\left(-1+\cos\left(fx+e\right)\right)}{fa\left(c-d\right)\left(d+c\cos\left(fx+e\right)\right)\left(\sin\left(fx+e\right)\right)^2}\sqrt{\frac{d+c\cos\left(fx+e\right)}{\cos\left(fx+e\right)}}\sqrt{\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\sqrt{\frac{d+c\cos\left(fx+e\right)}{\left(c+d\right)\left(1+\cos\left(fx+e\right)\right)}}\Big(2\,\mathrm{E}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/a/f/(c-d)*((d+c*cos(f*x+e))/cos(f*x+e))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e))
)^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*(1+cos(f*x+e))^2*(-
1+cos(f*x+e))*(2*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((c-d)/(c+d))^(1/2))*
c-4*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((c-d)/(c+d))^(1/2))*d+c*EllipticE
((-1+cos(f*x+e))/sin(f*x+e),((c-d)/(c+d))^(1/2))+d*EllipticE((-1+cos(f*x+e)
)/sin(f*x+e),((c-d)/(c+d))^(1/2))-4*c*EllipticPi((-1+cos(f*x+e))/sin(f*x+e)
,-1,((c-d)/(c+d))^(1/2))+4*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),-1,((c-d)/
(c+d))^(1/2))*d)/(d+c*cos(f*x+e))/sin(f*x+e)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sec\left(fx+e\right)+a\right)\sqrt{d\sec\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\sec\left(fx+e\right)+c}}{ad\sec\left(fx+e\right)^{2}+ac+(ac+ad)\sec\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*sec(f*x + e) + c)/(a*d*sec(f*x + e)^2 + a*c + (a*c + a*d)*s
ec(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{\sqrt{c+d\sec\left(e+fx\right)}\sec\left(e+fx\right)+\sqrt{c+d\sec\left(e+fx\right)}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(c + d*sec(e + f*x))*sec(e + f*x) + sqrt(c + d*sec(e + f*x)
)), x)/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sec\left(fx+e\right)+a\right)\sqrt{d\sec\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)), x)
```

3.147 $\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^4\,dx$

Optimal. Leaf size=271

$$\frac{2a^{3/2}c^4\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{2d^4\tan(e+fx)(a-a\sec(e+fx))^3}{7a^2f\sqrt{a\sec(e+fx)+a}}-\frac{2d^2\left(6c^2+8cd+3d^2\right)\tan(e+fx)(\cdots}{3f\sqrt{a\sec(e+fx)+\cdots}}$$

```
[Out] (2*a*d*(2*c + d)*(2*c^2 + 2*c*d + d^2)*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e +
f*x]]) + (2*a^(3/2)*c^4*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f
*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (2*d^2*(6*c^2
+ 8*c*d + 3*d^2)*(a - a*Sec[e + f*x])*Tan[e + f*x])/(3*f*Sqrt[a + a*Sec[e +
 f*x]]) + (2*d^3*(4*c + 3*d)*(a - a*Sec[e + f*x])^2*Tan[e + f*x])/(5*a*f*Sq
rt[a + a*Sec[e + f*x]]) - (2*d^4*(a - a*Sec[e + f*x])^3*Tan[e + f*x])/(7*a^
2*f*Sqrt[a + a*Sec[e + f*x]])
```

Rubi [A] time = 0.172894, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3940, 88, 63, 206}

$$\frac{2a^{3/2}c^4\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{2d^4\tan(e+fx)(a-a\sec(e+fx))^3}{7a^2f\sqrt{a\sec(e+fx)+a}}-\frac{2d^2\left(6c^2+8cd+3d^2\right)\tan(e+fx)(\cdots}{3f\sqrt{a\sec(e+fx)+\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^4,x]
```


```
[Out] (2*a*d*(2*c + d)*(2*c^2 + 2*c*d + d^2)*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e +
f*x]]) + (2*a^(3/2)*c^4*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f
*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (2*d^2*(6*c^2
+ 8*c*d + 3*d^2)*(a - a*Sec[e + f*x])*Tan[e + f*x])/(3*f*Sqrt[a + a*Sec[e +
 f*x]]) + (2*d^3*(4*c + 3*d)*(a - a*Sec[e + f*x])^2*Tan[e + f*x])/(5*a*f*Sq
rt[a + a*Sec[e + f*x]]) - (2*d^4*(a - a*Sec[e + f*x])^3*Tan[e + f*x])/(7*a^
2*f*Sqrt[a + a*Sec[e + f*x]])
```


Rule 3940

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(a^2*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
x)^n)/(x*Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
```

```
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^4\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int\frac{(c+dx)^4}{x\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int\left(\frac{d(2c+d)\left(2c^2+2cd+d^2\right)}{\sqrt{a-ax}}+\frac{c^4}{x\sqrt{a-ax}}-\frac{d^2\left(6c^2+8cd+3d^2\right)\sqrt{\cdots}}{a}\cdots\right.\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\,\text{s}\cdots}}$$

$$= \frac{2ad(2c+d)\left(2c^2+2cd+d^2\right)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{2d^2\left(6c^2+8cd+3d^2\right)(a-a\,\text{s}\cdots}{3f\sqrt{a+a\sec\cdots}}$$

$$= \frac{2ad(2c+d)\left(2c^2+2cd+d^2\right)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{2d^2\left(6c^2+8cd+3d^2\right)(a-a\,\text{s}\cdots}{3f\sqrt{a+a\sec\cdots}}$$

$$= \frac{2ad(2c+d)\left(2c^2+2cd+d^2\right)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2a^{3/2}c^4\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\cdots\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\cdots}}$$

Mathematica [C] time = 14.6181, size = 589, normalized size = 2.17

$$\frac{\cos^4(e+fx)\sec\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}(c+d\sec(e+fx))^4\left(\frac{8}{105}d\left(105c^2d+105c^3+56cd^2+12d^3\right)\sin\left(\frac{1}{2}(e+\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^4,x]
```

```
[Out] (Cos[e + f*x]^4*Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])]*(c + d*Sec[e +
f*x])^4*((8*d*(105*c^3 + 105*c^2*d + 56*c*d^2 + 12*d^3)*Sin[(e + f*x)/2])/1
05 + (2*d^4*Sec[e + f*x]^3*Sin[(e + f*x)/2])/7 + (4*Sec[e + f*x]^2*(14*c*d^
3*Sin[(e + f*x)/2] + 3*d^4*Sin[(e + f*x)/2]))/35 + (4*Sec[e + f*x]*(105*c^2
*d^2*Sin[(e + f*x)/2] + 56*c*d^3*Sin[(e + f*x)/2] + 12*d^4*Sin[(e + f*x)/2]
))/105))/(f*(d + c*Cos[e + f*x])^4) - (8*(-3 - 2*Sqrt[2])*c^4*Cos[(e + f*x)
/4]^4*Sqrt[(7 - 5*Sqrt[2] + (10 - 7*Sqrt[2])*Cos[(e + f*x)/2])/(1 + Cos[(e
+ f*x)/2])]*Sqrt[(-1 + Sqrt[2] - (-2 + Sqrt[2])*Cos[(e + f*x)/2])/(1 + Cos[
(e + f*x)/2])]*(1 - Sqrt[2] + (-2 + Sqrt[2])*Cos[(e + f*x)/2])*Cos[e + f*x]
^3*(EllipticF[ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]
] + 2*EllipticPi[-3 + 2*Sqrt[2], -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2
]]], 17 - 12*Sqrt[2]])*Sqrt[(-1 - Sqrt[2] + (2 + Sqrt[2])*Cos[(e + f*x)/2])
*Sec[(e + f*x)/4]^2]*Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])]*(c + d*Sec
[e + f*x])^4*Sqrt[3 - 2*Sqrt[2] - Tan[(e + f*x)/4]^2])/(f*(d + c*Cos[e + f*
x])^4)
```

Maple [B] time = 0.413, size = 546, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sec(f*x+e))^4*(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/840/f*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(-105*cos(f*x+e)^3*sin(f*x+e
)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(7/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(
1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*2^(1/2)*c^4-315*cos(f*x+e)^2*si
n(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(7/2)*arctanh(1/2*2^(1/2)*(-2*cos(f
*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*2^(1/2)*c^4-315*cos(f*x+
e)*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(7/2)*arctanh(1/2*2^(1/2)*(-2*
cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*2^(1/2)*c^4-105*(-2
```

```
*cos(f*x+e)/(1+cos(f*x+e)))^(7/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos
(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*2^(1/2)*c^4*sin(f*x+e)+6720*cos(f*x+
e)^4*c^3*d+6720*cos(f*x+e)^4*c^2*d^2+3584*cos(f*x+e)^4*c*d^3+768*cos(f*x+e)
^4*d^4-6720*cos(f*x+e)^3*c^3*d-3360*cos(f*x+e)^3*c^2*d^2-1792*cos(f*x+e)^3*
c*d^3-384*cos(f*x+e)^3*d^4-3360*cos(f*x+e)^2*c^2*d^2-448*cos(f*x+e)^2*c*d^3
-96*cos(f*x+e)^2*d^4-1344*cos(f*x+e)*c*d^3-48*cos(f*x+e)*d^4-240*d^4)/cos(f
*x+e)^3/sin(f*x+e)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^4*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.69881, size = 1181, normalized size = 4.36

$$\left[\frac{105\left(c^4\cos\left(fx+e\right)^4+c^4\cos\left(fx+e\right)^3\right)\sqrt{-a}\log\left(\frac{2\,a\cos\left(fx+e\right)^2-2\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)\sin\left(fx+e\right)+a\cos\left(fx+e\right)-a}{\cos\left(fx+e\right)+1}\right)+2\left(15\,d^4\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^4*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/105*(105*(c^4*cos(f*x + e)^4 + c^4*cos(f*x + e)^3)*sqrt(-a)*log((2*a*cos
(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x +
e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*(15*d^4 + 4*(
105*c^3*d + 105*c^2*d^2 + 56*c*d^3 + 12*d^4)*cos(f*x + e)^3 + 2*(105*c^2*d^
2 + 56*c*d^3 + 12*d^4)*cos(f*x + e)^2 + 6*(14*c*d^3 + 3*d^4)*cos(f*x + e))*
sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^4 + f
*cos(f*x + e)^3), -2/105*(105*(c^4*cos(f*x + e)^4 + c^4*cos(f*x + e)^3)*sqr
```

```
t(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*s
in(f*x + e))) - (15*d^4 + 4*(105*c^3*d + 105*c^2*d^2 + 56*c*d^3 + 12*d^4)*c
os(f*x + e)^3 + 2*(105*c^2*d^2 + 56*c*d^3 + 12*d^4)*cos(f*x + e)^2 + 6*(14*
c*d^3 + 3*d^4)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*
x + e))/(f*cos(f*x + e)^4 + f*cos(f*x + e)^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\left(\sec\left(e+fx\right)+1\right)}\left(c+d\sec\left(e+fx\right)\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))**4*(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*(c + d*sec(e + f*x))**4, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^4*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.148 $\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3\,dx$

Optimal. Leaf size=205

$$\frac{2a^{3/2}c^3\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2ad\left(3c^2+3cd+d^2\right)\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}-\frac{2d^2(3c+2d)\tan(e+fx)(a-a\sec(e+\ldots}{3f\sqrt{a\sec(e+fx)+a}}$$

```
[Out] (2*a*d*(3*c^2 + 3*c*d + d^2)*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) + (
2*a^(3/2)*c^3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(f*Sq
rt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (2*d^2*(3*c + 2*d)*(a -
a*Sec[e + f*x])*Tan[e + f*x])/(3*f*Sqrt[a + a*Sec[e + f*x]]) + (2*d^3*(a -
a*Sec[e + f*x])^2*Tan[e + f*x])/(5*a*f*Sqrt[a + a*Sec[e + f*x]])
```

Rubi [A] time = 0.140936, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3940, 88, 63, 206}

$$\frac{2a^{3/2}c^3\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2ad\left(3c^2+3cd+d^2\right)\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}-\frac{2d^2(3c+2d)\tan(e+fx)(a-a\sec(e+\ldots}{3f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (2*a*d*(3*c^2 + 3*c*d + d^2)*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) + (
2*a^(3/2)*c^3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(f*Sq
rt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (2*d^2*(3*c + 2*d)*(a -
a*Sec[e + f*x])*Tan[e + f*x])/(3*f*Sqrt[a + a*Sec[e + f*x]]) + (2*d^3*(a -
a*Sec[e + f*x])^2*Tan[e + f*x])/(5*a*f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 3940

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(a^2*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
x)^n)/(x*Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(c+dx)^3}{x\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int\left(\frac{d\left(3c^2+3cd+d^2\right)}{\sqrt{a-ax}}+\frac{c^3}{x\sqrt{a-ax}}-\frac{d^2(3c+2d)\sqrt{a-ax}}{a}+\frac{d^3\right.}{}}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2ad\left(3c^2+3cd+d^2\right)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{2d^2(3c+2d)(a-a\sec(e+fx))\tan}{3f\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2ad\left(3c^2+3cd+d^2\right)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{2d^2(3c+2d)(a-a\sec(e+fx))\tan}{3f\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2ad\left(3c^2+3cd+d^2\right)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2a^{3/2}c^3\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e}}$$

Mathematica [C] time = 14.5664, size = 519, normalized size = 2.53

$$\frac{\cos^3(e+fx)\sec\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}(c+d\sec(e+fx))^3\left(\frac{2}{15}d\left(45c^2+30cd+8d^2\right)\sin\left(\frac{1}{2}(e+fx)\right)+\frac{2}{15}\sec(e}{f(c\cos(e+fx)+d)^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (Cos[e + f*x]^3*Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])]*(c + d*Sec[e +
f*x])^3*((2*d*(45*c^2 + 30*c*d + 8*d^2)*Sin[(e + f*x)/2])/15 + (2*d^3*Sec[e
 + f*x]^2*Sin[(e + f*x)/2])/5 + (2*Sec[e + f*x]*(15*c*d^2*Sin[(e + f*x)/2]
+ 4*d^3*Sin[(e + f*x)/2]))/15))/(f*(d + c*Cos[e + f*x])^3) - (8*(-3 - 2*Sqr
t[2])*c^3*Cos[(e + f*x)/4]^4*Sqrt[(7 - 5*Sqrt[2] + (10 - 7*Sqrt[2])*Cos[(e
+ f*x)/2])/(1 + Cos[(e + f*x)/2])]*Sqrt[(-1 + Sqrt[2] - (-2 + Sqrt[2])*Cos[
(e + f*x)/2])/(1 + Cos[(e + f*x)/2])]*(1 - Sqrt[2] + (-2 + Sqrt[2])*Cos[(e
+ f*x)/2])*Cos[e + f*x]^2*(EllipticF[ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqr
t[2]]], 17 - 12*Sqrt[2]] + 2*EllipticPi[-3 + 2*Sqrt[2], -ArcSin[Tan[(e + f*
x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]])*Sqrt[(-1 - Sqrt[2] + (2 + Sqr
t[2])*Cos[(e + f*x)/2])*Sec[(e + f*x)/4]^2]*Sec[(e + f*x)/2]*Sqrt[a*(1 + Se
c[e + f*x])]*(c + d*Sec[e + f*x])^3*Sqrt[3 - 2*Sqrt[2] - Tan[(e + f*x)/4]^2
])/(f*(d + c*Cos[e + f*x])^3)
```

Maple [B] time = 0.307, size = 389, normalized size = 1.9

$$-\frac{1}{60f\left(\cos\left(fx+e\right)\right)^2\sin\left(fx+e\right)}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\left(15\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^2\sqrt{2}\left(-2\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}\right)^{5/2}\mathrm{Ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sec(f*x+e))^3*(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/60/f*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(15*sin(f*x+e)*cos(f*x+e)^2*2
^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x
+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*c^3+30*sin(f*x+e)*cos(f*x+
e)*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos
(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*c^3+15*(-2*cos(f*x+e)/
(1+cos(f*x+e)))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1
/2)*sin(f*x+e)/cos(f*x+e))*2^(1/2)*c^3*sin(f*x+e)+360*cos(f*x+e)^3*c^2*d+24
```

```
0*cos(f*x+e)^3*c*d^2+64*cos(f*x+e)^3*d^3-360*cos(f*x+e)^2*c^2*d-120*cos(f*x
+e)^2*c*d^2-32*cos(f*x+e)^2*d^3-120*cos(f*x+e)*c*d^2-8*cos(f*x+e)*d^3-24*d^
3)/cos(f*x+e)^2/sin(f*x+e)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^3*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.59197, size = 976, normalized size = 4.76

$$\left[\frac{15\left(c^3\cos\left(fx+e\right)^3+c^3\cos\left(fx+e\right)^2\right)\sqrt{-a}\log\left(\frac{2a\cos\left(fx+e\right)^2-2\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)\sin\left(fx+e\right)+a\cos\left(fx+e\right)-a}{\cos\left(fx+e\right)+1}\right)+2\left(3d^3 \cdots\right.}{15\left(f\cos\left(fx+e\right)^3+f\cos\left(\cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^3*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/15*(15*(c^3*cos(f*x + e)^3 + c^3*cos(f*x + e)^2)*sqrt(-a)*log((2*a*cos(f
*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)
*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*(3*d^3 + (45*c^
2*d + 30*c*d^2 + 8*d^3)*cos(f*x + e)^2 + (15*c*d^2 + 4*d^3)*cos(f*x + e))*s
qrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3 + f*
cos(f*x + e)^2), -2/15*(15*(c^3*cos(f*x + e)^3 + c^3*cos(f*x + e)^2)*sqrt(a
)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(
f*x + e))) - (3*d^3 + (45*c^2*d + 30*c*d^2 + 8*d^3)*cos(f*x + e)^2 + (15*c*
d^2 + 4*d^3)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x
+ e))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\left(\sec\left(e+fx\right)+1\right)}\left(c+d\sec\left(e+fx\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))**3*(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*(c + d*sec(e + f*x))**3, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^3*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.149 $\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2\,dx$

Optimal. Leaf size=144

$$\frac{2a^{3/2}c^2\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2ad(2c+d)\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}-\frac{2d^2\tan(e+fx)(a-a\sec(e+fx))}{3f\sqrt{a\sec(e+fx)+a}}$$

```
[Out] (2*a*d*(2*c + d)*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) + (2*a^(3/2)*c^
2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(f*Sqrt[a - a*Sec
[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (2*d^2*(a - a*Sec[e + f*x])*Tan[e +
f*x])/(3*f*Sqrt[a + a*Sec[e + f*x]])
```

Rubi [A] time = 0.11605, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3940, 88, 63, 206}

$$\frac{2a^{3/2}c^2\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2ad(2c+d)\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}-\frac{2d^2\tan(e+fx)(a-a\sec(e+fx))}{3f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (2*a*d*(2*c + d)*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) + (2*a^(3/2)*c^
2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(f*Sqrt[a - a*Sec
[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (2*d^2*(a - a*Sec[e + f*x])*Tan[e +
f*x])/(3*f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 3940

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(a^2*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
x)^n)/(x*Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
```

```
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int\frac{(c+dx)^2}{x\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int\left(\frac{d(2c+d)}{\sqrt{a-ax}}+\frac{c^2}{x\sqrt{a-ax}}-\frac{d^2\sqrt{a-ax}}{a}\right)dx,x,\sec(e+fx\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2ad(2c+d)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}-\frac{2d^2(a-a\sec(e+fx))\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}}-\frac{\left(a^2c^2\tan\right.}{f}$$

$$= \frac{2ad(2c+d)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}-\frac{2d^2(a-a\sec(e+fx))\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}}+\frac{\left(2ac^2\tan\right.}{}$$

$$= \frac{2ad(2c+d)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}+\frac{2a^{3/2}c^2\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}-$$

Mathematica [C] time = 6.59086, size = 444, normalized size = 3.08

$$\frac{\sqrt{\frac{1}{1-2\sin^2\left(\frac{1}{2}(e+fx)\right)}}\sqrt{1-2\sin^2\left(\frac{1}{2}(e+fx)\right)}\csc^3\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}(c+d\sec(e+fx))^2\left(256\,s\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (Csc[(e + f*x)/2]^3*Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])]*(c + d*Sec[
e + f*x])^2*Sqrt[(1 - 2*Sin[(e + f*x)/2]^2)^(-1)]*Sqrt[1 - 2*Sin[(e + f*x)/
2]^2]*(256*HypergeometricPFQ[{3/2, 2, 7/2}, {1, 9/2}, 2*Sin[(e + f*x)/2]^2]
*Sin[(e + f*x)/2]^6*(c + d - 2*c*Sin[(e + f*x)/2]^2)^2 + 1024*Hypergeometri
c2F1[3/2, 7/2, 9/2, 2*Sin[(e + f*x)/2]^2]*Sin[(e + f*x)/2]^6*(d^2 + c*d*(2
- 3*Sin[(e + f*x)/2]^2) + c^2*(1 - 3*Sin[(e + f*x)/2]^2 + 2*Sin[(e + f*x)/2
]^4)) - (7*Sqrt[2]*(-3*ArcSin[Sqrt[2]*Sqrt[Sin[(e + f*x)/2]^2]] + Sqrt[2]*S
qrt[Sin[(e + f*x)/2]^2]*Sqrt[1 - 2*Sin[(e + f*x)/2]^2]*(3 + 4*Sin[(e + f*x)
/2]^2))*(15*d^2 + 10*c*d*(3 - 2*Sin[(e + f*x)/2]^2) + c^2*(15 - 20*Sin[(e +
 f*x)/2]^2 + 12*Sin[(e + f*x)/2]^4)))/Sqrt[Sin[(e + f*x)/2]^2]))/(672*f*(d
+ c*Cos[e + f*x])^2*Sec[e + f*x]^(5/2))
```

Maple [A] time = 0.272, size = 248, normalized size = 1.7

$$\frac{1}{6 f \sin\left(fx+e\right)\cos\left(fx+e\right)}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\left(3\sqrt{2}\sin\left(fx+e\right)\cos\left(fx+e\right)\left(-2\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}\right)^{3/2}\mathrm{Artanh}\left(1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sec(f*x+e))^2*(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/6/f*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(3*2^(1/2)*sin(f*x+e)*cos(f*x+e
)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(
1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*c^2+3*2^(1/2)*(-2*cos(f*x+e)/(1
+cos(f*x+e)))^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2
)*sin(f*x+e)/cos(f*x+e))*c^2*sin(f*x+e)-24*cos(f*x+e)^2*c*d-8*cos(f*x+e)^2*
d^2+24*cos(f*x+e)*c*d+4*cos(f*x+e)*d^2+4*d^2)/sin(f*x+e)/cos(f*x+e)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^2*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.3009, size = 819, normalized size = 5.69

$$\left[\frac{3\left(c^2\cos\left(fx+e\right)^2+c^2\cos\left(fx+e\right)\right)\sqrt{-a}\log\left(\frac{2\,a\cos\left(fx+e\right)^2-2\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)\sin\left(fx+e\right)+a\cos\left(fx+e\right)-a}{\cos\left(fx+e\right)+1}\right)+2\left(d^2+2\left(3\right.\right.}{3\left(f\cos\left(fx+e\right)^2+f\cos\left(fx+e\right)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^2*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(c^2*cos(f*x + e)^2 + c^2*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x +
 e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin
(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*(d^2 + 2*(3*c*d + d
^2)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*
cos(f*x + e)^2 + f*cos(f*x + e)), -2/3*(3*(c^2*cos(f*x + e)^2 + c^2*cos(f*x
 + e))*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/
(sqrt(a)*sin(f*x + e))) - (d^2 + 2*(3*c*d + d^2)*cos(f*x + e))*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^2 + f*cos(f*x + e
))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\left(c+d\sec\left(e+fx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))**2*(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*(c + d*sec(e + f*x))**2, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^2*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.150 $\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))\,dx$

Optimal. Leaf size=66

$$\frac{2\sqrt{a}c\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}+\frac{2ad\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

```
[Out] (2*Sqrt[a]*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f + (
2*a*d*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]])
```

Rubi [A] time = 0.0868663, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3915, 3774, 203, 3792}

$$\frac{2\sqrt{a}c\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}+\frac{2ad\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x]),x]
```

```
[Out] (2*Sqrt[a]*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f + (
2*a*d*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 3915

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Dis
t[d, Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))\,dx &= c\int \sqrt{a+a\sec(e+fx)}\,dx + d\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}\,dx\\ &= \frac{2ad\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{(2ac)\,\mathrm{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{a\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}\\ &= \frac{2\sqrt{a}c\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} + \frac{2ad\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.316288, size = 76, normalized size = 1.15

$$\frac{\sec\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}\left(\sqrt{2}c\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{\cos(e+fx)}+2d\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x]),x]
```

```
[Out] (Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])]*(Sqrt[2]*c*ArcSin[Sqrt[2]*Sin[
(e + f*x)/2]]*Sqrt[Cos[e + f*x]] + 2*d*Sin[(e + f*x)/2]))/f
```

Maple [B] time = 0.214, size = 118, normalized size = 1.8

$$-\frac{1}{f\sin\left(fx+e\right)}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\left(\sqrt{-2\,\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\sqrt{2}\mathrm{Artanh}\left(\frac{\sqrt{2}\sin\left(fx+e\right)}{2\,\cos\left(fx+e\right)}\sqrt{-2\,\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\right)c\sin$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sec(f*x+e))*(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/f*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*((-2*cos(f*x+e)/(1+cos(f*x+e)))^
(1/2)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(
f*x+e)/cos(f*x+e))*c*sin(f*x+e)+2*d*cos(f*x+e)-2*d)/sin(f*x+e)
```

Maxima [B] time = 1.6785, size = 198, normalized size = 3.

$$\frac{\sqrt{a}c\arctan\left(\left(\cos\left(2\,fx+2\,e\right)^{2}+\sin\left(2\,fx+2\,e\right)^{2}+2\,\cos\left(2\,fx+2\,e\right)+1\right)^{\frac{1}{4}}\sin\left(\frac{1}{2}\,\arctan\left(\sin\left(2\,fx+2\,e\right),\cos\left(2\,fx+\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(a)*c*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x +
2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) +
sin(f*x + e), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
 + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + cos(
f*x + e))/f
```

Fricas [A] time = 1.15547, size = 620, normalized size = 9.39

$$\left[\frac{\left(c\cos\left(fx+e\right)+c\right)\sqrt{-a}\log\left(\frac{2\,a\cos\left(fx+e\right)^{2}-2\,\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)\sin\left(fx+e\right)+a\cos\left(fx+e\right)-a}{\cos\left(fx+e\right)+1}\right)+2\,d\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sin\left(fx+e\right.}{f\cos\left(fx+e\right)+f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [((c*cos(f*x + e) + c)*sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((
a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e
) - a)/(cos(f*x + e) + 1)) + 2*d*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*si
n(f*x + e))/(f*cos(f*x + e) + f), -2*((c*cos(f*x + e) + c)*sqrt(a)*arctan(s
qrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))
 - d*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)
+ f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\left(\sec\left(e+fx\right)+1\right)}\left(c+d\sec\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))*(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*(c + d*sec(e + f*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.151 $\int \frac{\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$

Optimal. Leaf size=105

$$\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{cf}-\frac{2\sqrt{a}\sqrt{d}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}\right)}{cf\sqrt{c+d}}$$

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c*f) -
 (2*Sqrt[a]*Sqrt[d]*ArcTan[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(Sqrt[c + d]*Sqrt
[a + a*Sec[e + f*x]])])/(c*Sqrt[c + d]*f)
```

Rubi [A] time = 0.22961, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {3925, 3774, 203, 3967, 205}

$$\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{cf}-\frac{2\sqrt{a}\sqrt{d}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}\right)}{cf\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]/(c + d*Sec[e + f*x]),x]
```

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c*f) -
 (2*Sqrt[a]*Sqrt[d]*ArcTan[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(Sqrt[c + d]*Sqrt
[a + a*Sec[e + f*x]])])/(c*Sqrt[c + d]*f)
```

Rule 3925

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[1/c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[d/c, Int[(Csc[e + f*x]*Sqrt[a + b*Csc[e + f*x]])/(c + d*Csc[e + f*x]),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && (EqQ[a^2 - b
^2, 0] || EqQ[c^2 - d^2, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3967

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[(-2*b)/f, Subst[Int
[1/(b*c + a*d + d*x^2), x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx &= \frac{\int \sqrt{a+a\sec(e+fx)}\,dx}{c} - \frac{d\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx}{c}\\
&= -\frac{(2a)\,\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{a\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf} + \frac{(2ad)\,\text{Subst}\left(\int \frac{1}{ac+ad+dx^2}\,dx, x, -\frac{a\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf}\\
&= \frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf} - \frac{2\sqrt{a}\sqrt{d}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}\right)}{c\sqrt{c+d}f}
\end{aligned}
$$

Mathematica [C] time = 24.9909, size = 2686, normalized size = 25.58

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]/(c + d*Sec[e + f*x]),x]
```

```
[Out] (-4*Sqrt[2]*Cos[(e + f*x)/4]^2*Sqrt[(-1 + Sqrt[2] - (-2 + Sqrt[2])*Cos[(e +
 f*x)/2])/(1 + Cos[(e + f*x)/2])]*(d + c*Cos[e + f*x])*(c*EllipticF[ArcSin[
Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + 2*(c + d)*Ellipti
cPi[-3 + 2*Sqrt[2], -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*
Sqrt[2]] - d*(EllipticPi[-(((-3 + 2*Sqrt[2])*(c + d))/(3*c + 2*Sqrt[2]*Sqrt
[c*(c - d)] - d)), -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*S
qrt[2]] + EllipticPi[((-3 + 2*Sqrt[2])*(c + d))/(-3*c + 2*Sqrt[2]*Sqrt[c*(c
 - d)] + d), -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]
]))*Sec[(e + f*x)/2]*((Cos[(e + f*x)/2]*Sqrt[Sec[e + f*x]])/(2*(d + c*Cos[e
 + f*x])) + (Cos[(3*(e + f*x))/2]*Sqrt[Sec[e + f*x]])/(2*(d + c*Cos[e + f*x
])))*Sec[e + f*x]*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[3 - 2*Sqrt[2] - Tan[(e +
f*x)/4]^2])/(c*(c + d)*f*(c + d*Sec[e + f*x])*((Sqrt[2]*Sqrt[(-1 + Sqrt[2]
- (-2 + Sqrt[2])*Cos[(e + f*x)/2])/(1 + Cos[(e + f*x)/2])]*(c*EllipticF[Arc
Sin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + 2*(c + d)*Ell
ipticPi[-3 + 2*Sqrt[2], -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 -
 12*Sqrt[2]] - d*(EllipticPi[-(((-3 + 2*Sqrt[2])*(c + d))/(3*c + 2*Sqrt[2]*
Sqrt[c*(c - d)] - d)), -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 -
12*Sqrt[2]] + EllipticPi[((-3 + 2*Sqrt[2])*(c + d))/(-3*c + 2*Sqrt[2]*Sqrt[
c*(c - d)] + d), -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqr
t[2]]))*Sqrt[Sec[e + f*x]]*Tan[(e + f*x)/4])/(c*(c + d)*Sqrt[3 - 2*Sqrt[2]
- Tan[(e + f*x)/4]^2]) + (2*Sqrt[2]*Cos[(e + f*x)/4]*Sqrt[(-1 + Sqrt[2] - (
-2 + Sqrt[2])*Cos[(e + f*x)/2])/(1 + Cos[(e + f*x)/2])]*(c*EllipticF[ArcSin
[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + 2*(c + d)*Ellipt
icPi[-3 + 2*Sqrt[2], -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12
*Sqrt[2]] - d*(EllipticPi[-(((-3 + 2*Sqrt[2])*(c + d))/(3*c + 2*Sqrt[2]*Sqr
t[c*(c - d)] - d)), -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*
Sqrt[2]] + EllipticPi[((-3 + 2*Sqrt[2])*(c + d))/(-3*c + 2*Sqrt[2]*Sqrt[c*(
c - d)] + d), -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2
]]))*Sqrt[Sec[e + f*x]]*Sin[(e + f*x)/4]*Sqrt[3 - 2*Sqrt[2] - Tan[(e + f*x)
/4]^2])/(c*(c + d)) - (2*Sqrt[2]*Cos[(e + f*x)/4]^2*(c*EllipticF[ArcSin[Tan
[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + 2*(c + d)*EllipticPi
[-3 + 2*Sqrt[2], -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqr
t[2]] - d*(EllipticPi[-(((-3 + 2*Sqrt[2])*(c + d))/(3*c + 2*Sqrt[2]*Sqrt[c*
(c - d)] - d)), -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt
[2]] + EllipticPi[((-3 + 2*Sqrt[2])*(c + d))/(-3*c + 2*Sqrt[2]*Sqrt[c*(c -
d)] + d), -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]]))
*Sqrt[Sec[e + f*x]]*(((-2 + Sqrt[2])*Sin[(e + f*x)/2])/(2*(1 + Cos[(e + f*x
)/2])) + ((-1 + Sqrt[2] - (-2 + Sqrt[2])*Cos[(e + f*x)/2])*Sin[(e + f*x)/2]
)/(2*(1 + Cos[(e + f*x)/2])^2))*Sqrt[3 - 2*Sqrt[2] - Tan[(e + f*x)/4]^2])/(
c*(c + d)*Sqrt[(-1 + Sqrt[2] - (-2 + Sqrt[2])*Cos[(e + f*x)/2])/(1 + Cos[(e
 + f*x)/2])]) - (2*Sqrt[2]*Cos[(e + f*x)/4]^2*Sqrt[(-1 + Sqrt[2] - (-2 + Sq
rt[2])*Cos[(e + f*x)/2])/(1 + Cos[(e + f*x)/2])]*(c*EllipticF[ArcSin[Tan[(e
 + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + 2*(c + d)*EllipticPi[-3
 + 2*Sqrt[2], -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2
]] - d*(EllipticPi[-(((-3 + 2*Sqrt[2])*(c + d))/(3*c + 2*Sqrt[2]*Sqrt[c*(c
```

```
- d)] - d)), -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]
] + EllipticPi[((-3 + 2*Sqrt[2])*(c + d))/(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)]
 + d), -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]]))*Se
c[e + f*x]^(3/2)*Sin[e + f*x]*Sqrt[3 - 2*Sqrt[2] - Tan[(e + f*x)/4]^2])/(c*
(c + d)) - (4*Sqrt[2]*Cos[(e + f*x)/4]^2*Sqrt[(-1 + Sqrt[2] - (-2 + Sqrt[2]
)*Cos[(e + f*x)/2])/(1 + Cos[(e + f*x)/2])]*Sqrt[Sec[e + f*x]]*Sqrt[3 - 2*S
qrt[2] - Tan[(e + f*x)/4]^2]*((c*Sec[(e + f*x)/4]^2)/(4*Sqrt[3 - 2*Sqrt[2]]
*Sqrt[1 - Tan[(e + f*x)/4]^2/(3 - 2*Sqrt[2])]*Sqrt[1 - ((17 - 12*Sqrt[2])*T
an[(e + f*x)/4]^2)/(3 - 2*Sqrt[2])]) - ((c + d)*Sec[(e + f*x)/4]^2)/(2*Sqrt
[3 - 2*Sqrt[2]]*Sqrt[1 - Tan[(e + f*x)/4]^2/(3 - 2*Sqrt[2])]*Sqrt[1 - ((17
- 12*Sqrt[2])*Tan[(e + f*x)/4]^2)/(3 - 2*Sqrt[2])]*(1 - ((-3 + 2*Sqrt[2])*T
an[(e + f*x)/4]^2)/(3 - 2*Sqrt[2]))) - d*(-Sec[(e + f*x)/4]^2/(4*Sqrt[3 - 2
*Sqrt[2]]*Sqrt[1 - Tan[(e + f*x)/4]^2/(3 - 2*Sqrt[2])]*Sqrt[1 - ((17 - 12*S
qrt[2])*Tan[(e + f*x)/4]^2)/(3 - 2*Sqrt[2])]*(1 + ((-3 + 2*Sqrt[2])*(c + d)
*Tan[(e + f*x)/4]^2)/((3 - 2*Sqrt[2])*(3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] - d)
))) - Sec[(e + f*x)/4]^2/(4*Sqrt[3 - 2*Sqrt[2]]*Sqrt[1 - Tan[(e + f*x)/4]^2
/(3 - 2*Sqrt[2])]*Sqrt[1 - ((17 - 12*Sqrt[2])*Tan[(e + f*x)/4]^2)/(3 - 2*Sq
rt[2])]*(1 - ((-3 + 2*Sqrt[2])*(c + d)*Tan[(e + f*x)/4]^2)/((3 - 2*Sqrt[2])
*(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] + d)))))))/(c*(c + d))))
```

Maple [B] time = 0.288, size = 501, normalized size = 4.8

$$-\frac{\sqrt{2}}{2cf}\left(2\sqrt{\frac{d}{c-d}}\sqrt{(c+d)(c-d)}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{2}\sin\left(fx+e\right)}{\cos\left(fx+e\right)}\sqrt{-2\,\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\right)+d\ln\left(-2\,\frac{}{\sqrt{(c+d)(c-d)}\sin\left(f\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x)
```

```
[Out] -1/2/f*2^(1/2)/(d/(c-d))^(1/2)/((c+d)*(c-d))^(1/2)/c*(2*(d/(c-d))^(1/2)*((c
+d)*(c-d))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*s
in(f*x+e)/cos(f*x+e))+d*ln(-2*((-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(d/(c-d
))^(1/2)*2^(1/2)*c*sin(f*x+e)-2^(1/2)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(1+cos
(f*x+e)))^(1/2)*d*sin(f*x+e)-((c+d)*(c-d))^(1/2)*cos(f*x+e)-c*sin(f*x+e)+d*
sin(f*x+e)+((c+d)*(c-d))^(1/2))/(((c+d)*(c-d))^(1/2)*sin(f*x+e)+c*cos(f*x+e
)-d*cos(f*x+e)-c+d))-d*ln(2*((-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(d/(c-d))
^(1/2)*2^(1/2)*c*sin(f*x+e)-2^(1/2)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(1+cos(f
*x+e)))^(1/2)*d*sin(f*x+e)+((c+d)*(c-d))^(1/2)*cos(f*x+e)-c*sin(f*x+e)+d*si
n(f*x+e)-((c+d)*(c-d))^(1/2))/(((c+d)*(c-d))^(1/2)*sin(f*x+e)-c*cos(f*x+e)+
d*cos(f*x+e)+c-d)))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/cos(f*x+e)*a*(1
+cos(f*x+e)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\sec\left(fx+e\right)+a}}{d\sec\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sec(f*x + e) + a)/(d*sec(f*x + e) + c), x)
```

Fricas [A] time = 2.95385, size = 1704, normalized size = 16.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] [(sqrt(-a*d/(c + d))*log((2*(c + d)*sqrt(-a*d/(c + d))*sqrt((a*cos(f*x + e)
 + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c + 2*a*d)*cos(f*x + e)^
2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2 + (c + d)*cos(f*x + e
) + d)) + sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e
) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f
*x + e) + 1)))/(c*f), -(2*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x
+ e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - sqrt(-a*d/(c + d))*log((2*(c +
 d)*sqrt(-a*d/(c + d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)
*sin(f*x + e) + (a*c + 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x +
e))/(c*cos(f*x + e)^2 + (c + d)*cos(f*x + e) + d)))/(c*f), (2*sqrt(a*d/(c +
 d))*arctan((c + d)*sqrt(a*d/(c + d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e
))*cos(f*x + e)/(a*d*sin(f*x + e))) + sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*
sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)
+ a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/(c*f), -2*(sqrt(a)*arctan(sqrt((
a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - sq
rt(a*d/(c + d))*arctan((c + d)*sqrt(a*d/(c + d))*sqrt((a*cos(f*x + e) + a)/
cos(f*x + e))*cos(f*x + e)/(a*d*sin(f*x + e))))/(c*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}}{c+d\sec\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e)),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))/(c + d*sec(e + f*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.152 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^2}\,dx$

Optimal. Leaf size=219

$$-\frac{a^{3/2}\sqrt{d}(3c+2d)\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{c^2f(c+d)^{3/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2a^{3/2}\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{ac}{cf(c+d)\sqrt{a\sec(e}}$$

```
[Out] (2*a^(3/2)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(c^2*f*S
qrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (a^(3/2)*Sqrt[d]*(3*c +
 2*d)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan
[e + f*x])/(c^2*(c + d)^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e +
 f*x]]) - (a*d*Tan[e + f*x])/(c*(c + d)*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*S
ec[e + f*x]))
```

Rubi [A] time = 0.222172, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3940, 103, 156, 63, 206, 208}

$$-\frac{a^{3/2}\sqrt{d}(3c+2d)\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{c^2f(c+d)^{3/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2a^{3/2}\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{ac}{cf(c+d)\sqrt{a\sec(e}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]/(c + d*Sec[e + f*x])^2,x]
```


```
[Out] (2*a^(3/2)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(c^2*f*S
qrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (a^(3/2)*Sqrt[d]*(3*c +
 2*d)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan
[e + f*x])/(c^2*(c + d)^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e +
 f*x]]) - (a*d*Tan[e + f*x])/(c*(c + d)*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*S
ec[e + f*x]))
```


Rule 3940

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(a^2*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
x)^n)/(x*Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
```

```
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^2}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}(c+dx)^2}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{ad\tan(e+fx)}{c(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))} - \frac{(a\tan(e+fx))\text{Subst}\left(\int \frac{a(c+d)-\frac{adx}{2}}{x\sqrt{a-ax}(c+dx)}\,dx,\right.}{c(c+d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\,\text{s}}}$$

$$= -\frac{ad\tan(e+fx)}{c(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))} - \frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}}\,dx, x, \text{s}\right.}{c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e}}$$

$$= -\frac{ad\tan(e+fx)}{c(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))} + \frac{(2a\tan(e+fx))\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx, x, \sqrt{a}\right.}{c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\,\text{se}}}$$

$$= \frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{a^{3/2}\sqrt{d}(3c+2d)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e}{c^2(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+}}$$

<u>**Mathematica**</u> **[C]** time = 28.4246, size = 2943, normalized size = 13.44

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]/(c + d*Sec[e + f*x])^2,x]
```

```
[Out] ((d + c*Cos[e + f*x])^2*Sec[(e + f*x)/2]*Sec[e + f*x]^2*Sqrt[a*(1 + Sec[e +
 f*x])]*(-((d*Sin[(e + f*x)/2])/(c^2*(c + d))) + (d^2*Sin[(e + f*x)/2])/(c^
2*(c + d)*(d + c*Cos[e + f*x]))))/(f*(c + d*Sec[e + f*x])^2) - (2*Sqrt[2]*C
os[(e + f*x)/4]^2*Sqrt[(-1 + Sqrt[2] - (-2 + Sqrt[2])*Cos[(e + f*x)/2])/(1
+ Cos[(e + f*x)/2])]*(d + c*Cos[e + f*x])^2*(c*(2*c + d)*EllipticF[ArcSin[T
an[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + 4*(c + d)^2*Ellipt
icPi[-3 + 2*Sqrt[2], -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12
*Sqrt[2]] - d*(3*c + 2*d)*(EllipticPi[-(((-3 + 2*Sqrt[2])*(c + d))/(3*c + 2
*Sqrt[2]*Sqrt[c*(c - d)] - d)), -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]
]], 17 - 12*Sqrt[2]] + EllipticPi[((-3 + 2*Sqrt[2])*(c + d))/(-3*c + 2*Sqrt
[2]*Sqrt[c*(c - d)] + d), -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17
 - 12*Sqrt[2]]))*Sec[(e + f*x)/2]*((Cos[(e + f*x)/2]*Sqrt[Sec[e + f*x]])/(2
*(c + d)*(d + c*Cos[e + f*x])) + (Cos[(3*(e + f*x))/2]*Sqrt[Sec[e + f*x]])/
(2*(c + d)*(d + c*Cos[e + f*x])) + (d*Cos[(3*(e + f*x))/2]*Sqrt[Sec[e + f*x
]])/(2*c*(c + d)*(d + c*Cos[e + f*x])))*Sec[e + f*x]^2*Sqrt[a*(1 + Sec[e +
```

```
f*x])]*Sqrt[3 - 2*Sqrt[2] - Tan[(e + f*x)/4]^2])/(c^2*(c + d)^2*f*(c + d*Se
c[e + f*x])^2*((Sqrt[(-1 + Sqrt[2] - (-2 + Sqrt[2])*Cos[(e + f*x)/2])/(1 +
Cos[(e + f*x)/2])]*(c*(2*c + d)*EllipticF[ArcSin[Tan[(e + f*x)/4]/Sqrt[3 -
2*Sqrt[2]]], 17 - 12*Sqrt[2]] + 4*(c + d)^2*EllipticPi[-3 + 2*Sqrt[2], -Arc
Sin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - d*(3*c + 2*d)
*(EllipticPi[-(((-3 + 2*Sqrt[2])*(c + d))/(3*c + 2*Sqrt[2]*Sqrt[c*(c - d)]
- d)), -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + El
lipticPi[((-3 + 2*Sqrt[2])*(c + d))/(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] + d),
 -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]]))*Sqrt[Sec
[e + f*x]]*Tan[(e + f*x)/4])/(Sqrt[2]*c^2*(c + d)^2*Sqrt[3 - 2*Sqrt[2] - Ta
n[(e + f*x)/4]^2]) + (Sqrt[2]*Cos[(e + f*x)/4]*Sqrt[(-1 + Sqrt[2] - (-2 + S
qrt[2])*Cos[(e + f*x)/2])/(1 + Cos[(e + f*x)/2])]*(c*(2*c + d)*EllipticF[Ar
cSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + 4*(c + d)^2*
EllipticPi[-3 + 2*Sqrt[2], -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 1
7 - 12*Sqrt[2]] - d*(3*c + 2*d)*(EllipticPi[-(((-3 + 2*Sqrt[2])*(c + d))/(3
*c + 2*Sqrt[2]*Sqrt[c*(c - d)] - d)), -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*S
qrt[2]]], 17 - 12*Sqrt[2]] + EllipticPi[((-3 + 2*Sqrt[2])*(c + d))/(-3*c +
2*Sqrt[2]*Sqrt[c*(c - d)] + d), -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]
]], 17 - 12*Sqrt[2]]))*Sqrt[Sec[e + f*x]]*Sin[(e + f*x)/4]*Sqrt[3 - 2*Sqrt[
2] - Tan[(e + f*x)/4]^2])/(c^2*(c + d)^2) - (Sqrt[2]*Cos[(e + f*x)/4]^2*(c*
(2*c + d)*EllipticF[ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*S
qrt[2]] + 4*(c + d)^2*EllipticPi[-3 + 2*Sqrt[2], -ArcSin[Tan[(e + f*x)/4]/S
qrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - d*(3*c + 2*d)*(EllipticPi[-(((-3 +
2*Sqrt[2])*(c + d))/(3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] - d)), -ArcSin[Tan[(e
+ f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + EllipticPi[((-3 + 2*Sqrt
[2])*(c + d))/(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] + d), -ArcSin[Tan[(e + f*x)
/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]]))*Sqrt[Sec[e + f*x]]*(((-2 + Sqr
t[2])*Sin[(e + f*x)/2])/(2*(1 + Cos[(e + f*x)/2])) + ((-1 + Sqrt[2] - (-2 +
 Sqrt[2])*Cos[(e + f*x)/2])*Sin[(e + f*x)/2])/(2*(1 + Cos[(e + f*x)/2])^2))
*Sqrt[3 - 2*Sqrt[2] - Tan[(e + f*x)/4]^2])/(c^2*(c + d)^2*Sqrt[(-1 + Sqrt[2
] - (-2 + Sqrt[2])*Cos[(e + f*x)/2])/(1 + Cos[(e + f*x)/2])]) - (Sqrt[2]*Co
s[(e + f*x)/4]^2*Sqrt[(-1 + Sqrt[2] - (-2 + Sqrt[2])*Cos[(e + f*x)/2])/(1 +
 Cos[(e + f*x)/2])]*(c*(2*c + d)*EllipticF[ArcSin[Tan[(e + f*x)/4]/Sqrt[3 -
 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + 4*(c + d)^2*EllipticPi[-3 + 2*Sqrt[2], -Ar
cSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - d*(3*c + 2*d
)*(EllipticPi[-(((-3 + 2*Sqrt[2])*(c + d))/(3*c + 2*Sqrt[2]*Sqrt[c*(c - d)]
 - d)), -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + E
llipticPi[((-3 + 2*Sqrt[2])*(c + d))/(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] + d)
, -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]]))*Sec[e +
 f*x]^(3/2)*Sin[e + f*x]*Sqrt[3 - 2*Sqrt[2] - Tan[(e + f*x)/4]^2])/(c^2*(c
+ d)^2) - (2*Sqrt[2]*Cos[(e + f*x)/4]^2*Sqrt[(-1 + Sqrt[2] - (-2 + Sqrt[2])
*Cos[(e + f*x)/2])/(1 + Cos[(e + f*x)/2])]*Sqrt[Sec[e + f*x]]*Sqrt[3 - 2*Sq
rt[2] - Tan[(e + f*x)/4]^2]*((c*(2*c + d)*Sec[(e + f*x)/4]^2)/(4*Sqrt[3 - 2
*Sqrt[2]]*Sqrt[1 - Tan[(e + f*x)/4]^2/(3 - 2*Sqrt[2])]*Sqrt[1 - ((17 - 12*S
qrt[2])*Tan[(e + f*x)/4]^2)/(3 - 2*Sqrt[2])]) - ((c + d)^2*Sec[(e + f*x)/4]
```

```
^2)/(Sqrt[3 - 2*Sqrt[2]]*Sqrt[1 - Tan[(e + f*x)/4]^2/(3 - 2*Sqrt[2])]*Sqrt[
1 - ((17 - 12*Sqrt[2])*Tan[(e + f*x)/4]^2)/(3 - 2*Sqrt[2])]*(1 - ((-3 + 2*S
qrt[2])*Tan[(e + f*x)/4]^2)/(3 - 2*Sqrt[2]))) - d*(3*c + 2*d)*(-Sec[(e + f*
x)/4]^2/(4*Sqrt[3 - 2*Sqrt[2]]*Sqrt[1 - Tan[(e + f*x)/4]^2/(3 - 2*Sqrt[2])]
*Sqrt[1 - ((17 - 12*Sqrt[2])*Tan[(e + f*x)/4]^2)/(3 - 2*Sqrt[2])]*(1 + ((-3
 + 2*Sqrt[2])*(c + d)*Tan[(e + f*x)/4]^2)/((3 - 2*Sqrt[2])*(3*c + 2*Sqrt[2]
*Sqrt[c*(c - d)] - d)))) - Sec[(e + f*x)/4]^2/(4*Sqrt[3 - 2*Sqrt[2]]*Sqrt[1
 - Tan[(e + f*x)/4]^2/(3 - 2*Sqrt[2])]*Sqrt[1 - ((17 - 12*Sqrt[2])*Tan[(e +
 f*x)/4]^2)/(3 - 2*Sqrt[2])]*(1 - ((-3 + 2*Sqrt[2])*(c + d)*Tan[(e + f*x)/4
]^2)/((3 - 2*Sqrt[2])*(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] + d)))))))/(c^2*(c
+ d)^2)))
```

Maple [B] time = 1.534, size = 97143, normalized size = 443.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^2,x)

[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^2,x, algorithm="maxima")

[Out] Timed out
```

Fricas [A] time = 13.7784, size = 3494, normalized size = 15.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*c*d*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x +
 e) - ((3*c^2 + 2*c*d)*cos(f*x + e)^2 + 3*c*d + 2*d^2 + (3*c^2 + 5*c*d + 2*
d^2)*cos(f*x + e))*sqrt(-a*d/(c + d))*log((2*(c + d)*sqrt(-a*d/(c + d))*sqr
t((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c + 2*a
*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2 + (c
 + d)*cos(f*x + e) + d)) - 2*((c^2 + c*d)*cos(f*x + e)^2 + c*d + d^2 + (c^2
 + 2*c*d + d^2)*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)
*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(
f*x + e) - a)/(cos(f*x + e) + 1)))/((c^4 + c^3*d)*f*cos(f*x + e)^2 + (c^4 +
 2*c^3*d + c^2*d^2)*f*cos(f*x + e) + (c^3*d + c^2*d^2)*f), -1/2*(2*c*d*sqrt
((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + 4*((c^2 + c
*d)*cos(f*x + e)^2 + c*d + d^2 + (c^2 + 2*c*d + d^2)*cos(f*x + e))*sqrt(a)*
arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*
x + e))) - ((3*c^2 + 2*c*d)*cos(f*x + e)^2 + 3*c*d + 2*d^2 + (3*c^2 + 5*c*d
 + 2*d^2)*cos(f*x + e))*sqrt(-a*d/(c + d))*log((2*(c + d)*sqrt(-a*d/(c + d)
)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c
+ 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2
 + (c + d)*cos(f*x + e) + d)))/((c^4 + c^3*d)*f*cos(f*x + e)^2 + (c^4 + 2*c
^3*d + c^2*d^2)*f*cos(f*x + e) + (c^3*d + c^2*d^2)*f), -(c*d*sqrt((a*cos(f*
x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - ((3*c^2 + 2*c*d)*cos(
f*x + e)^2 + 3*c*d + 2*d^2 + (3*c^2 + 5*c*d + 2*d^2)*cos(f*x + e))*sqrt(a*d
/(c + d))*arctan((c + d)*sqrt(a*d/(c + d))*sqrt((a*cos(f*x + e) + a)/cos(f*
x + e))*cos(f*x + e)/(a*d*sin(f*x + e))) - ((c^2 + c*d)*cos(f*x + e)^2 + c*
d + d^2 + (c^2 + 2*c*d + d^2)*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x + e)^
2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x
 + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/((c^4 + c^3*d)*f*cos(f*x +
 e)^2 + (c^4 + 2*c^3*d + c^2*d^2)*f*cos(f*x + e) + (c^3*d + c^2*d^2)*f), -(
c*d*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + 2*(
(c^2 + c*d)*cos(f*x + e)^2 + c*d + d^2 + (c^2 + 2*c*d + d^2)*cos(f*x + e))*
sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a
)*sin(f*x + e))) - ((3*c^2 + 2*c*d)*cos(f*x + e)^2 + 3*c*d + 2*d^2 + (3*c^2
 + 5*c*d + 2*d^2)*cos(f*x + e))*sqrt(a*d/(c + d))*arctan((c + d)*sqrt(a*d/(
c + d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(a*d*sin(f*x +
 e))))/((c^4 + c^3*d)*f*cos(f*x + e)^2 + (c^4 + 2*c^3*d + c^2*d^2)*f*cos(f*
x + e) + (c^3*d + c^2*d^2)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}}{\left(c+d\sec\left(e+fx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**2,x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))/(c + d*sec(e + f*x))**2, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.153 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^3}\,dx$

Optimal. Leaf size=287

$$-\frac{a^{3/2}\sqrt{d}\left(15c^2+20cd+8d^2\right)\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{4c^3f(c+d)^{5/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2a^{3/2}\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{}{4c^2f}$$

```
[Out] (2*a^(3/2)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(c^3*f*S
qrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (a^(3/2)*Sqrt[d]*(15*c^
2 + 20*c*d + 8*d^2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqr
t[c + d])]*Tan[e + f*x])/(4*c^3*(c + d)^(5/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sq
rt[a + a*Sec[e + f*x]]) - (a*d*Tan[e + f*x])/(2*c*(c + d)*f*Sqrt[a + a*Sec[
e + f*x]]*(c + d*Sec[e + f*x])^2) - (a*d*(7*c + 4*d)*Tan[e + f*x])/(4*c^2*(
c + d)^2*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x]))
```

Rubi [A] time = 0.306889, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {3940, 103, 151, 156, 63, 206, 208}

$$-\frac{a^{3/2}\sqrt{d}\left(15c^2+20cd+8d^2\right)\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{4c^3f(c+d)^{5/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2a^{3/2}\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{}{4c^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]/(c + d*Sec[e + f*x])^3,x]
```


```
[Out] (2*a^(3/2)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(c^3*f*S
qrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (a^(3/2)*Sqrt[d]*(15*c^
2 + 20*c*d + 8*d^2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqr
t[c + d])]*Tan[e + f*x])/(4*c^3*(c + d)^(5/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sq
rt[a + a*Sec[e + f*x]]) - (a*d*Tan[e + f*x])/(2*c*(c + d)*f*Sqrt[a + a*Sec[
e + f*x]]*(c + d*Sec[e + f*x])^2) - (a*d*(7*c + 4*d)*Tan[e + f*x])/(4*c^2*(
c + d)^2*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x]))
```


Rule 3940

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(a^2*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
```

```
e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
x)^n)/(x*Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^3}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a-ax}(c+dx)^3}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{ad\tan(e+fx)}{2c(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2} - \frac{(a\tan(e+fx))\,\mathrm{Subst}\left(\int \frac{2a(c+d)-\frac{3adx}{2}}{x\sqrt{a-ax}(c+dx)}\right.}{2c(c+d)f\sqrt{a-a\sec(e+fx)}\sqrt{a}}$$

$$= -\frac{ad\tan(e+fx)}{2c(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2} - \frac{ad(7c+4d)\tan(e+fx)}{4c^2(c+d)^2f\sqrt{a+a\sec(e+fx)}(c+d}$$

$$= -\frac{ad\tan(e+fx)}{2c(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2} - \frac{ad(7c+4d)\tan(e+fx)}{4c^2(c+d)^2f\sqrt{a+a\sec(e+fx)}(c+d}$$

$$= -\frac{ad\tan(e+fx)}{2c(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2} - \frac{ad(7c+4d)\tan(e+fx)}{4c^2(c+d)^2f\sqrt{a+a\sec(e+fx)}(c+d}$$

$$= \frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{a^{3/2}\sqrt{d}\left(15c^2+20cd+8d^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec}}{\sqrt{a}\sqrt{c-}}\right.}{4c^3(c+d)^{5/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a}}$$

Mathematica [C] time = 24.1367, size = 3106, normalized size = 10.82

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] ((d + c*Cos[e + f*x])^3*Sec[(e + f*x)/2]*Sec[e + f*x]^3*Sqrt[a*(1 + Sec[e +
 f*x])]*((-3*d*(3*c + 2*d)*Sin[(e + f*x)/2])/(4*c^3*(c + d)^2) - (d^3*Sin[(
e + f*x)/2])/(2*c^3*(c + d)*(d + c*Cos[e + f*x])^2) + (11*c*d^2*Sin[(e + f*
```

```
x)/2] + 8*d^3*Sin[(e + f*x)/2])/(4*c^3*(c + d)^2*(d + c*Cos[e + f*x]))))/(f
*(c + d*Sec[e + f*x])^3) - (Sqrt[3 - 2*Sqrt[2]]*Cos[(e + f*x)/4]^2*(d + c*C
os[e + f*x])^3*(c*(8*c^2 + 9*c*d + 4*d^2)*EllipticF[ArcSin[Tan[(e + f*x)/4]
/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + 16*(c + d)^3*EllipticPi[-3 + 2*Sq
rt[2], -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - d*
(15*c^2 + 20*c*d + 8*d^2)*(EllipticPi[-(((-3 + 2*Sqrt[2])*(c + d))/(3*c + 2
*Sqrt[2]*Sqrt[c*(c - d)] - d)), -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]
]], 17 - 12*Sqrt[2]] + EllipticPi[((-3 + 2*Sqrt[2])*(c + d))/(-3*c + 2*Sqrt
[2]*Sqrt[c*(c - d)] + d), -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17
 - 12*Sqrt[2]]))*Sec[(e + f*x)/2]*((Cos[(e + f*x)/2]*Sqrt[Sec[e + f*x]])/(2
*(c + d)^2*(d + c*Cos[e + f*x])) + (d*Cos[(e + f*x)/2]*Sqrt[Sec[e + f*x]])/
(8*c*(c + d)^2*(d + c*Cos[e + f*x])) + (Cos[(3*(e + f*x))/2]*Sqrt[Sec[e + f
*x]])/(2*(c + d)^2*(d + c*Cos[e + f*x])) + (d*Cos[(3*(e + f*x))/2]*Sqrt[Sec
[e + f*x]])/(c*(c + d)^2*(d + c*Cos[e + f*x])) + (d^2*Cos[(3*(e + f*x))/2]*
Sqrt[Sec[e + f*x]])/(2*c^2*(c + d)^2*(d + c*Cos[e + f*x])))*Sec[e + f*x]^3*
Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[1 + (-3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2]*Sq
rt[1 - (3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2])/(2*c^3*(c + d)^3*f*(c + d*Sec[e
 + f*x])^3*((Sqrt[3 - 2*Sqrt[2]]*(3 + 2*Sqrt[2])*(c*(8*c^2 + 9*c*d + 4*d^2)
*EllipticF[ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] +
 16*(c + d)^3*EllipticPi[-3 + 2*Sqrt[2], -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 -
2*Sqrt[2]]], 17 - 12*Sqrt[2]] - d*(15*c^2 + 20*c*d + 8*d^2)*(EllipticPi[-((
(-3 + 2*Sqrt[2])*(c + d))/(3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] - d)), -ArcSin[T
an[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + EllipticPi[((-3 +
2*Sqrt[2])*(c + d))/(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] + d), -ArcSin[Tan[(e
+ f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]]))*Sqrt[Sec[e + f*x]]*Tan[(
e + f*x)/4]*Sqrt[1 + (-3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2])/(8*c^3*(c + d)^3
*Sqrt[1 - (3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2]) - (Sqrt[3 - 2*Sqrt[2]]*(-3 +
 2*Sqrt[2])*(c*(8*c^2 + 9*c*d + 4*d^2)*EllipticF[ArcSin[Tan[(e + f*x)/4]/Sq
rt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + 16*(c + d)^3*EllipticPi[-3 + 2*Sqrt[
2], -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - d*(15
*c^2 + 20*c*d + 8*d^2)*(EllipticPi[-(((-3 + 2*Sqrt[2])*(c + d))/(3*c + 2*Sq
rt[2]*Sqrt[c*(c - d)] - d)), -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]],
 17 - 12*Sqrt[2]] + EllipticPi[((-3 + 2*Sqrt[2])*(c + d))/(-3*c + 2*Sqrt[2]
*Sqrt[c*(c - d)] + d), -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 -
12*Sqrt[2]]))*Sqrt[Sec[e + f*x]]*Tan[(e + f*x)/4]*Sqrt[1 - (3 + 2*Sqrt[2])*
Tan[(e + f*x)/4]^2])/(8*c^3*(c + d)^3*Sqrt[1 + (-3 + 2*Sqrt[2])*Tan[(e + f*
x)/4]^2]) + (Sqrt[3 - 2*Sqrt[2]]*Cos[(e + f*x)/4]*(c*(8*c^2 + 9*c*d + 4*d^2
)*EllipticF[ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]]
+ 16*(c + d)^3*EllipticPi[-3 + 2*Sqrt[2], -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 -
 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - d*(15*c^2 + 20*c*d + 8*d^2)*(EllipticPi[-(
((-3 + 2*Sqrt[2])*(c + d))/(3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] - d)), -ArcSin[
Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + EllipticPi[((-3 +
 2*Sqrt[2])*(c + d))/(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] + d), -ArcSin[Tan[(e
 + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]]))*Sqrt[Sec[e + f*x]]*Sin[
(e + f*x)/4]*Sqrt[1 + (-3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2]*Sqrt[1 - (3 + 2*
```

```
Sqrt[2])*Tan[(e + f*x)/4]^2])/(4*c^3*(c + d)^3) - (Sqrt[3 - 2*Sqrt[2]]*Cos[
(e + f*x)/4]^2*(c*(8*c^2 + 9*c*d + 4*d^2)*EllipticF[ArcSin[Tan[(e + f*x)/4]
/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + 16*(c + d)^3*EllipticPi[-3 + 2*Sq
rt[2], -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - d*
(15*c^2 + 20*c*d + 8*d^2)*(EllipticPi[-(((-3 + 2*Sqrt[2])*(c + d))/(3*c + 2
*Sqrt[2]*Sqrt[c*(c - d)] - d)), -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]
]], 17 - 12*Sqrt[2]] + EllipticPi[((-3 + 2*Sqrt[2])*(c + d))/(-3*c + 2*Sqrt
[2]*Sqrt[c*(c - d)] + d), -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17
 - 12*Sqrt[2]]))*Sec[e + f*x]^(3/2)*Sin[e + f*x]*Sqrt[1 + (-3 + 2*Sqrt[2])*
Tan[(e + f*x)/4]^2]*Sqrt[1 - (3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2])/(4*c^3*(c
 + d)^3) - (Sqrt[3 - 2*Sqrt[2]]*Cos[(e + f*x)/4]^2*Sqrt[Sec[e + f*x]]*Sqrt[
1 + (-3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2]*Sqrt[1 - (3 + 2*Sqrt[2])*Tan[(e +
f*x)/4]^2]*((c*(8*c^2 + 9*c*d + 4*d^2)*Sec[(e + f*x)/4]^2)/(4*Sqrt[3 - 2*Sq
rt[2]]*Sqrt[1 - Tan[(e + f*x)/4]^2/(3 - 2*Sqrt[2])]*Sqrt[1 - ((17 - 12*Sqrt
[2])*Tan[(e + f*x)/4]^2)/(3 - 2*Sqrt[2])]) - (4*(c + d)^3*Sec[(e + f*x)/4]^
2)/(Sqrt[3 - 2*Sqrt[2]]*Sqrt[1 - Tan[(e + f*x)/4]^2/(3 - 2*Sqrt[2])]*Sqrt[1
 - ((17 - 12*Sqrt[2])*Tan[(e + f*x)/4]^2)/(3 - 2*Sqrt[2])]*(1 - ((-3 + 2*Sq
rt[2])*Tan[(e + f*x)/4]^2)/(3 - 2*Sqrt[2]))) - d*(15*c^2 + 20*c*d + 8*d^2)*
(-Sec[(e + f*x)/4]^2/(4*Sqrt[3 - 2*Sqrt[2]]*Sqrt[1 - Tan[(e + f*x)/4]^2/(3
- 2*Sqrt[2])]*Sqrt[1 - ((17 - 12*Sqrt[2])*Tan[(e + f*x)/4]^2)/(3 - 2*Sqrt[2
])]*(1 + ((-3 + 2*Sqrt[2])*(c + d)*Tan[(e + f*x)/4]^2)/((3 - 2*Sqrt[2])*(3*
c + 2*Sqrt[2]*Sqrt[c*(c - d)] - d)))) - Sec[(e + f*x)/4]^2/(4*Sqrt[3 - 2*Sq
rt[2]]*Sqrt[1 - Tan[(e + f*x)/4]^2/(3 - 2*Sqrt[2])]*Sqrt[1 - ((17 - 12*Sqrt
[2])*Tan[(e + f*x)/4]^2)/(3 - 2*Sqrt[2])]*(1 - ((-3 + 2*Sqrt[2])*(c + d)*Ta
n[(e + f*x)/4]^2)/((3 - 2*Sqrt[2])*(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] + d)))
))))/(2*c^3*(c + d)^3)))
```

Maple [B] time = 14.995, size = 330372, normalized size = 1151.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^3,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 27.2375, size = 5542, normalized size = 19.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/8*((15*c^2*d^2 + 20*c*d^3 + 8*d^4 + (15*c^4 + 20*c^3*d + 8*c^2*d^2)*cos(
f*x + e)^3 + (15*c^4 + 50*c^3*d + 48*c^2*d^2 + 16*c*d^3)*cos(f*x + e)^2 + (
30*c^3*d + 55*c^2*d^2 + 36*c*d^3 + 8*d^4)*cos(f*x + e))*sqrt(-a*d/(c + d))*
log((2*(c + d)*sqrt(-a*d/(c + d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*c
os(f*x + e)*sin(f*x + e) + (a*c + 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)
*cos(f*x + e))/(c*cos(f*x + e)^2 + (c + d)*cos(f*x + e) + d)) + 8*(c^2*d^2
+ 2*c*d^3 + d^4 + (c^4 + 2*c^3*d + c^2*d^2)*cos(f*x + e)^3 + (c^4 + 4*c^3*d
 + 5*c^2*d^2 + 2*c*d^3)*cos(f*x + e)^2 + (2*c^3*d + 5*c^2*d^2 + 4*c*d^3 + d
^4)*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a
)/(cos(f*x + e) + 1)) - 2*(3*(3*c^3*d + 2*c^2*d^2)*cos(f*x + e)^2 + (7*c^2*
d^2 + 4*c*d^3)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*
x + e))/((c^7 + 2*c^6*d + c^5*d^2)*f*cos(f*x + e)^3 + (c^7 + 4*c^6*d + 5*c^
5*d^2 + 2*c^4*d^3)*f*cos(f*x + e)^2 + (2*c^6*d + 5*c^5*d^2 + 4*c^4*d^3 + c^
3*d^4)*f*cos(f*x + e) + (c^5*d^2 + 2*c^4*d^3 + c^3*d^4)*f), -1/8*(16*(c^2*d
^2 + 2*c*d^3 + d^4 + (c^4 + 2*c^3*d + c^2*d^2)*cos(f*x + e)^3 + (c^4 + 4*c^
3*d + 5*c^2*d^2 + 2*c*d^3)*cos(f*x + e)^2 + (2*c^3*d + 5*c^2*d^2 + 4*c*d^3
+ d^4)*cos(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))
*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - (15*c^2*d^2 + 20*c*d^3 + 8*d^4 + (1
5*c^4 + 20*c^3*d + 8*c^2*d^2)*cos(f*x + e)^3 + (15*c^4 + 50*c^3*d + 48*c^2*
d^2 + 16*c*d^3)*cos(f*x + e)^2 + (30*c^3*d + 55*c^2*d^2 + 36*c*d^3 + 8*d^4)
*cos(f*x + e))*sqrt(-a*d/(c + d))*log((2*(c + d)*sqrt(-a*d/(c + d))*sqrt((a
*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c + 2*a*d)*
cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2 + (c + d
)*cos(f*x + e) + d)) + 2*(3*(3*c^3*d + 2*c^2*d^2)*cos(f*x + e)^2 + (7*c^2*d
^2 + 4*c*d^3)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x
 + e))/((c^7 + 2*c^6*d + c^5*d^2)*f*cos(f*x + e)^3 + (c^7 + 4*c^6*d + 5*c^5
*d^2 + 2*c^4*d^3)*f*cos(f*x + e)^2 + (2*c^6*d + 5*c^5*d^2 + 4*c^4*d^3 + c^3
```

```
*d^4)*f*cos(f*x + e) + (c^5*d^2 + 2*c^4*d^3 + c^3*d^4)*f), 1/4*((15*c^2*d^2
 + 20*c*d^3 + 8*d^4 + (15*c^4 + 20*c^3*d + 8*c^2*d^2)*cos(f*x + e)^3 + (15*
c^4 + 50*c^3*d + 48*c^2*d^2 + 16*c*d^3)*cos(f*x + e)^2 + (30*c^3*d + 55*c^2
*d^2 + 36*c*d^3 + 8*d^4)*cos(f*x + e))*sqrt(a*d/(c + d))*arctan((c + d)*sqr
t(a*d/(c + d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(a*d*si
n(f*x + e))) + 4*(c^2*d^2 + 2*c*d^3 + d^4 + (c^4 + 2*c^3*d + c^2*d^2)*cos(f
*x + e)^3 + (c^4 + 4*c^3*d + 5*c^2*d^2 + 2*c*d^3)*cos(f*x + e)^2 + (2*c^3*d
 + 5*c^2*d^2 + 4*c*d^3 + d^4)*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x + e)^
2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x
 + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - (3*(3*c^3*d + 2*c^2*d^2)*
cos(f*x + e)^2 + (7*c^2*d^2 + 4*c*d^3)*cos(f*x + e))*sqrt((a*cos(f*x + e) +
 a)/cos(f*x + e))*sin(f*x + e))/((c^7 + 2*c^6*d + c^5*d^2)*f*cos(f*x + e)^3
 + (c^7 + 4*c^6*d + 5*c^5*d^2 + 2*c^4*d^3)*f*cos(f*x + e)^2 + (2*c^6*d + 5*
c^5*d^2 + 4*c^4*d^3 + c^3*d^4)*f*cos(f*x + e) + (c^5*d^2 + 2*c^4*d^3 + c^3*
d^4)*f), -1/4*(8*(c^2*d^2 + 2*c*d^3 + d^4 + (c^4 + 2*c^3*d + c^2*d^2)*cos(f
*x + e)^3 + (c^4 + 4*c^3*d + 5*c^2*d^2 + 2*c*d^3)*cos(f*x + e)^2 + (2*c^3*d
 + 5*c^2*d^2 + 4*c*d^3 + d^4)*cos(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(f*x
+ e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - (15*c^2*d^2
+ 20*c*d^3 + 8*d^4 + (15*c^4 + 20*c^3*d + 8*c^2*d^2)*cos(f*x + e)^3 + (15*c
^4 + 50*c^3*d + 48*c^2*d^2 + 16*c*d^3)*cos(f*x + e)^2 + (30*c^3*d + 55*c^2*
d^2 + 36*c*d^3 + 8*d^4)*cos(f*x + e))*sqrt(a*d/(c + d))*arctan((c + d)*sqrt
(a*d/(c + d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(a*d*sin
(f*x + e))) + (3*(3*c^3*d + 2*c^2*d^2)*cos(f*x + e)^2 + (7*c^2*d^2 + 4*c*d^
3)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/((c^
7 + 2*c^6*d + c^5*d^2)*f*cos(f*x + e)^3 + (c^7 + 4*c^6*d + 5*c^5*d^2 + 2*c^
4*d^3)*f*cos(f*x + e)^2 + (2*c^6*d + 5*c^5*d^2 + 4*c^4*d^3 + c^3*d^4)*f*cos
(f*x + e) + (c^5*d^2 + 2*c^4*d^3 + c^3*d^4)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}}{\left(c+d\sec\left(e+fx\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**3,x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))/(c + d*sec(e + f*x))**3, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.154 $\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^3\,dx$

Optimal. Leaf size=241

$$\frac{2a^2\tan(e+fx)\left(d\left(24c^2+111cd+52d^2\right)\sec(e+fx)+2\left(243c^2d+36c^3+189cd^2+52d^3\right)\right)}{105f\sqrt{a\sec(e+fx)+a}}+\frac{2a^{5/2}c^3\tan(e+fx)\tanh}{f\sqrt{a-a\sec(e+fx)}\sqrt{}}$$

```
[Out] (2*a^(5/2)*c^3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(f*S
qrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (2*a^2*(6*c + 13*d)*(c
+ d*Sec[e + f*x])^2*Tan[e + f*x])/(35*f*Sqrt[a + a*Sec[e + f*x]]) + (2*a^2*
(c + d*Sec[e + f*x])^3*Tan[e + f*x])/(7*f*Sqrt[a + a*Sec[e + f*x]]) + (2*a^
2*(2*(36*c^3 + 243*c^2*d + 189*c*d^2 + 52*d^3) + d*(24*c^2 + 111*c*d + 52*d
^2)*Sec[e + f*x])*Tan[e + f*x])/(105*f*Sqrt[a + a*Sec[e + f*x]])
```

Rubi [A] time = 0.199716, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {3940, 153, 147, 63, 206}

$$\frac{2a^2\tan(e+fx)\left(d\left(24c^2+111cd+52d^2\right)\sec(e+fx)+2\left(243c^2d+36c^3+189cd^2+52d^3\right)\right)}{105f\sqrt{a\sec(e+fx)+a}}+\frac{2a^{5/2}c^3\tan(e+fx)\tanh}{f\sqrt{a-a\sec(e+fx)}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])^3,x]
```


```
[Out] (2*a^(5/2)*c^3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(f*S
qrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (2*a^2*(6*c + 13*d)*(c
+ d*Sec[e + f*x])^2*Tan[e + f*x])/(35*f*Sqrt[a + a*Sec[e + f*x]]) + (2*a^2*
(c + d*Sec[e + f*x])^3*Tan[e + f*x])/(7*f*Sqrt[a + a*Sec[e + f*x]]) + (2*a^
2*(2*(36*c^3 + 243*c^2*d + 189*c*d^2 + 52*d^3) + d*(24*c^2 + 111*c*d + 52*d
^2)*Sec[e + f*x])*Tan[e + f*x])/(105*f*Sqrt[a + a*Sec[e + f*x]])
```


Rule 3940

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(a^2*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
x)^n)/(x*Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

```
 && IntegerQ[m - 1/2]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^3\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)(c+dx)^3}{x\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2a^2(c+d\sec(e+fx))^3\tan(e+fx)}{7f\sqrt{a+a\sec(e+fx)}} + \frac{(2a\tan(e+fx))\text{Subst}\left(\int \frac{(c+dx}{\cdots}\right.}{7f\sqrt{a-a\sec(e+\cdots}}$$

$$= \frac{2a^2(6c+13d)(c+d\sec(e+fx))^2\tan(e+fx)}{35f\sqrt{a+a\sec(e+fx)}} + \frac{2a^2(c+d\sec(e+fx)\cdots}{7f\sqrt{a+a\sec(\cdots}}$$

$$= \frac{2a^2(6c+13d)(c+d\sec(e+fx))^2\tan(e+fx)}{35f\sqrt{a+a\sec(e+fx)}} + \frac{2a^2(c+d\sec(e+fx)\cdots}{7f\sqrt{a+a\sec(\cdots}}$$

$$= \frac{2a^2(6c+13d)(c+d\sec(e+fx))^2\tan(e+fx)}{35f\sqrt{a+a\sec(e+fx)}} + \frac{2a^2(c+d\sec(e+fx)\cdots}{7f\sqrt{a+a\sec(\cdots}}$$

$$= \frac{2a^{5/2}c^3\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{2a^2(6c+13d)(c+d\sec(\cdots}{35f\sqrt{a+a\,\text{se}\cdots}}$$

Mathematica [A] time = 3.86925, size = 219, normalized size = 0.91

$$\frac{a\sec\left(\frac{1}{2}(e+fx)\right)\sec^3(e+fx)\sqrt{a(\sec(e+fx)+1)}\left(2\sin\left(\frac{1}{2}(e+fx)\right)\left(9\left(175c^2d+35c^3+154cd^2+52d^3\right)\cos(e+fx)+\cdots\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (a*Sec[(e + f*x)/2]*Sec[e + f*x]^3*Sqrt[a*(1 + Sec[e + f*x])]*(420*Sqrt[2]*
c^3*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]]*Cos[e + f*x]^(7/2) + 2*(210*c^2*d + 37
8*c*d^2 + 164*d^3 + 9*(35*c^3 + 175*c^2*d + 154*c*d^2 + 52*d^3)*Cos[e + f*x
] + 2*d*(105*c^2 + 189*c*d + 52*d^2)*Cos[2*(e + f*x)] + 105*c^3*Cos[3*(e +
f*x)] + 525*c^2*d*Cos[3*(e + f*x)] + 378*c*d^2*Cos[3*(e + f*x)] + 104*d^3*C
os[3*(e + f*x)])*Sin[(e + f*x)/2]))/(420*f)
```

Maple [B] time = 0.316, size = 539, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^3,x)
```

```
[Out] 1/840/f*a*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(105*2^(1/2)*sin(f*x+e)*cos
(f*x+e)^3*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+
e)/cos(f*x+e))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(7/2)*c^3+315*2^(1/2)*sin(f*x
+e)*cos(f*x+e)^2*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*s
in(f*x+e)/cos(f*x+e))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(7/2)*c^3+315*2^(1/2)*
sin(f*x+e)*cos(f*x+e)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1
/2)*sin(f*x+e)/cos(f*x+e))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(7/2)*c^3+105*(-2
*cos(f*x+e)/(1+cos(f*x+e)))^(7/2)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e
)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*c^3*sin(f*x+e)-1680*cos(f*x+
e)^4*c^3-8400*cos(f*x+e)^4*c^2*d-6048*cos(f*x+e)^4*c*d^2-1664*cos(f*x+e)^4*
d^3+1680*cos(f*x+e)^3*c^3+6720*cos(f*x+e)^3*c^2*d+3024*cos(f*x+e)^3*c*d^2+8
32*cos(f*x+e)^3*d^3+1680*cos(f*x+e)^2*c^2*d+2016*cos(f*x+e)^2*c*d^2+208*cos
(f*x+e)^2*d^3+1008*cos(f*x+e)*c*d^2+384*cos(f*x+e)*d^3+240*d^3)/cos(f*x+e)^
3/sin(f*x+e)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.34508, size = 1230, normalized size = 5.1

$$\left[\frac{105\left(ac^3\cos\left(fx+e\right)^4+ac^3\cos\left(fx+e\right)^3\right)\sqrt{-a}\log\left(\frac{2\,a\cos\left(fx+e\right)^2-2\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)\sin\left(fx+e\right)+a\cos\left(fx+e\right)-a}{\cos\left(fx+e\right)+1}\right)+2\left(1\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/105*(105*(a*c^3*cos(f*x + e)^4 + a*c^3*cos(f*x + e)^3)*sqrt(-a)*log((2*a
*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*
x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*(15*a*d^3
 + (105*a*c^3 + 525*a*c^2*d + 378*a*c*d^2 + 104*a*d^3)*cos(f*x + e)^3 + (10
5*a*c^2*d + 189*a*c*d^2 + 52*a*d^3)*cos(f*x + e)^2 + 3*(21*a*c*d^2 + 13*a*d
^3)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*
cos(f*x + e)^4 + f*cos(f*x + e)^3), -2/105*(105*(a*c^3*cos(f*x + e)^4 + a*c
^3*cos(f*x + e)^3)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*c
os(f*x + e)/(sqrt(a)*sin(f*x + e))) - (15*a*d^3 + (105*a*c^3 + 525*a*c^2*d
+ 378*a*c*d^2 + 104*a*d^3)*cos(f*x + e)^3 + (105*a*c^2*d + 189*a*c*d^2 + 52
*a*d^3)*cos(f*x + e)^2 + 3*(21*a*c*d^2 + 13*a*d^3)*cos(f*x + e))*sqrt((a*co
s(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^4 + f*cos(f*x +
 e)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(3/2)*(c+d*sec(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.155 $\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2\,dx$

Optimal. Leaf size=176

$$\frac{2a^2\tan(e+fx)\left(2\left(6c^2+25cd+9d^2\right)+d(4c+9d)\sec(e+fx)\right)}{15f\sqrt{a\sec(e+fx)+a}}+\frac{2a^{5/2}c^2\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2a^2\text{ta}}{}$$

```
[Out] (2*a^(5/2)*c^2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(f*S
qrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (2*a^2*(c + d*Sec[e + f
*x])^2*Tan[e + f*x])/(5*f*Sqrt[a + a*Sec[e + f*x]]) + (2*a^2*(2*(6*c^2 + 25
*c*d + 9*d^2) + d*(4*c + 9*d)*Sec[e + f*x])*Tan[e + f*x])/(15*f*Sqrt[a + a*
Sec[e + f*x]])
```

Rubi [A] time = 0.143539, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {3940, 153, 147, 63, 206}

$$\frac{2a^2\tan(e+fx)\left(2\left(6c^2+25cd+9d^2\right)+d(4c+9d)\sec(e+fx)\right)}{15f\sqrt{a\sec(e+fx)+a}}+\frac{2a^{5/2}c^2\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2a^2\text{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])^2,x]
```



```
[Out] (2*a^(5/2)*c^2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(f*S
qrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (2*a^2*(c + d*Sec[e + f
*x])^2*Tan[e + f*x])/(5*f*Sqrt[a + a*Sec[e + f*x]]) + (2*a^2*(2*(6*c^2 + 25
*c*d + 9*d^2) + d*(4*c + 9*d)*Sec[e + f*x])*Tan[e + f*x])/(15*f*Sqrt[a + a*
Sec[e + f*x]])
```


Rule 3940

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(a^2*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
x)^n)/(x*Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2\,dx = -\frac{\left(a^2\tan(e+fx)\right)\operatorname{Subst}\left(\int\frac{(a+ax)(c+dx)^2}{x\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2a^2(c+d\sec(e+fx))^2\tan(e+fx)}{5f\sqrt{a+a\sec(e+fx)}} + \frac{(2a\tan(e+fx))\operatorname{Subst}\left(\int\frac{(c+dx}{}\right.}{5f\sqrt{a-a\sec(e+}}$$

$$= \frac{2a^2(c+d\sec(e+fx))^2\tan(e+fx)}{5f\sqrt{a+a\sec(e+fx)}} + \frac{2a^2\left(2\left(6c^2+25cd+9d^2\right)+d(4}{15f\sqrt{a+a}}$$

$$= \frac{2a^2(c+d\sec(e+fx))^2\tan(e+fx)}{5f\sqrt{a+a\sec(e+fx)}} + \frac{2a^2\left(2\left(6c^2+25cd+9d^2\right)+d(4}{15f\sqrt{a+a}}$$

$$= \frac{2a^{5/2}c^2\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{2a^2(c+d\sec(e+fx))^2\,\mathrm{t}}{5f\sqrt{a+a\sec(e+}}$$

Mathematica [A] time = 1.33611, size = 145, normalized size = 0.82

$$\frac{a\sec\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\sqrt{a(\sec(e+fx)+1)}\left(2\sin\left(\frac{1}{2}(e+fx)\right)\left(\left(15c^2+50cd+18d^2\right)\cos(2(e+fx))+15c^2+2d(\right.\right.}{30f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (a*Sec[(e + f*x)/2]*Sec[e + f*x]^2*Sqrt[a*(1 + Sec[e + f*x])]*(30*Sqrt[2]*c
^2*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]]*Cos[e + f*x]^(5/2) + 2*(15*c^2 + 50*c*d
 + 24*d^2 + 2*d*(10*c + 9*d)*Cos[e + f*x] + (15*c^2 + 50*c*d + 18*d^2)*Cos[
2*(e + f*x)])*Sin[(e + f*x)/2]))/(30*f)
```

Maple [B] time = 0.271, size = 382, normalized size = 2.2

$$-\frac{a}{60f\left(\cos\left(fx+e\right)\right)^2\sin\left(fx+e\right)}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\left(15\sqrt{2}\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^2\left(-2\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}\right)^{5/2}A\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^2,x)
```

```
[Out] -1/60/f*a*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(15*2^(1/2)*sin(f*x+e)*cos(
f*x+e)^2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(f
*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*c^2+30*2^(1/2)*sin(f*x+e
)*cos(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)*arctanh(1/2*2^(1/2)*(-2*c
os(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*c^2+15*2^(1/2)*(-2*c
os(f*x+e)/(1+cos(f*x+e)))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f
*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*c^2*sin(f*x+e)+120*cos(f*x+e)^3*c^2+40
0*cos(f*x+e)^3*c*d+144*cos(f*x+e)^3*d^2-120*cos(f*x+e)^2*c^2-320*cos(f*x+e)
^2*c*d-72*cos(f*x+e)^2*d^2-80*cos(f*x+e)*c*d-48*cos(f*x+e)*d^2-24*d^2)/cos(
f*x+e)^2/sin(f*x+e)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.21746, size = 1006, normalized size = 5.72

$$\left[\frac{15\left(ac^2\cos\left(fx+e\right)^3 + ac^2\cos\left(fx+e\right)^2\right)\sqrt{-a}\log\left(\frac{2\,a\cos\left(fx+e\right)^2 - 2\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)\sin\left(fx+e\right) + a\cos\left(fx+e\right) - a}{\cos\left(fx+e\right)+1}\right) + 2\left(3\,ac\right.}{15\left(f\cos\left(fx+e\right)^3 + f\cos\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/15*(15*(a*c^2*cos(f*x + e)^3 + a*c^2*cos(f*x + e)^2)*sqrt(-a)*log((2*a*c
os(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x
+ e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*(3*a*d^2 +
(15*a*c^2 + 50*a*c*d + 18*a*d^2)*cos(f*x + e)^2 + (10*a*c*d + 9*a*d^2)*cos(
f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x
+ e)^3 + f*cos(f*x + e)^2), -2/15*(15*(a*c^2*cos(f*x + e)^3 + a*c^2*cos(f*x
 + e)^2)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e
)/(sqrt(a)*sin(f*x + e))) - (3*a*d^2 + (15*a*c^2 + 50*a*c*d + 18*a*d^2)*cos
(f*x + e)^2 + (10*a*c*d + 9*a*d^2)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/
cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(3/2)*(c+d*sec(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.156 $\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx$

Optimal. Leaf size=105

$$\frac{2a^2(3c+4d)\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}}+\frac{2a^{3/2}c\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}+\frac{2ad\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{3f}$$

```
[Out] (2*a^(3/2)*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f + (
2*a^2*(3*c + 4*d)*Tan[e + f*x])/(3*f*Sqrt[a + a*Sec[e + f*x]]) + (2*a*d*Sqr
t[a + a*Sec[e + f*x]]*Tan[e + f*x])/(3*f)
```

Rubi [A] time = 0.150098, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3917, 3915, 3774, 203, 3792}

$$\frac{2a^2(3c+4d)\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}}+\frac{2a^{3/2}c\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}+\frac{2ad\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x]),x]
```

```
[Out] (2*a^(3/2)*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f + (
2*a^2*(3*c + 4*d)*Tan[e + f*x])/(3*f*Sqrt[a + a*Sec[e + f*x]]) + (2*a*d*Sqr
t[a + a*Sec[e + f*x]]*Tan[e + f*x])/(3*f)
```

Rule 3917

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*c*m + (b
*c*m + a*d*(2*m - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3915

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Dis
t[d, Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d
```

```
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a + a\sec(e+fx))^{3/2}(c + d\sec(e+fx))\,dx = \frac{2ad\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{3f} + \frac{2}{3}\int \sqrt{a+a\sec(e+fx)}\left(\frac{3ac}{2} + \cdots\right.$$

$$= \frac{2ad\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{3f} + (ac)\int \sqrt{a+a\sec(e+fx)}\,dx + \cdots$$

$$= \frac{2a^2(3c+4d)\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}} + \frac{2ad\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{3f} - \cdots$$

$$= \frac{2a^{3/2}c\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} + \frac{2a^2(3c+4d)\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}} + \frac{2ad\sqrt{a+a\,\mathrm{s}\cdots}}{\cdots}$$

Mathematica [A] time = 0.561776, size = 102, normalized size = 0.97

$$\frac{a\sec\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\sqrt{a(\sec(e+fx)+1)}\left(2\sin\left(\frac{1}{2}(e+fx)\right)((3c+5d)\cos(e+fx)+d)+3\sqrt{2}c\sin^{-1}\left(\sqrt{2}\sin\left(\cdots\right.\right.\right.}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x]),x]
```

```
[Out] (a*Sec[(e + f*x)/2]*Sec[e + f*x]*Sqrt[a*(1 + Sec[e + f*x])]*(3*Sqrt[2]*c*Ar
cSin[Sqrt[2]*Sin[(e + f*x)/2]]*Cos[e + f*x]^(3/2) + 2*(d + (3*c + 5*d)*Cos[
e + f*x])*Sin[(e + f*x)/2]))/(3*f)
```

Maple [B] time = 0.229, size = 237, normalized size = 2.3

$$\frac{a}{6f\cos\left(fx+e\right)\sin\left(fx+e\right)}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\left(3\sqrt{2}\sin\left(fx+e\right)\cos\left(fx+e\right)\mathrm{Artanh}\left(1/2\,\frac{\sqrt{2}\sin\left(fx+e\right)}{\cos\left(fx+e\right)}\sqrt{-2\,\frac{}{1}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e)),x)
```

```
[Out] 1/6/f*a*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(3*2^(1/2)*sin(f*x+e)*cos(f*x
+e)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos
(f*x+e))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)*c+3*2^(1/2)*arctanh(1/2*2^(1/
2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*(-2*cos(f*x+
e)/(1+cos(f*x+e)))^(3/2)*c*sin(f*x+e)-12*cos(f*x+e)^2*c-20*cos(f*x+e)^2*d+1
2*c*cos(f*x+e)+16*d*cos(f*x+e)+4*d)/cos(f*x+e)/sin(f*x+e)
```

Maxima [B] time = 1.90324, size = 1347, normalized size = 12.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/2*((a*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*
e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(1/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - cos(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e)))), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
 + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*cos(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sin(1/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
```

```
 + 2*e)))) + 1) - a*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*co
s(2*f*x + 2*e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e)))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - cos(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e)))), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(
2*f*x + 2*e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
 + 1))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sin(1/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e)))) - 1) - a*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2
*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x
 + 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))
 + 1) + a*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x +
2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)), (
cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*cos
(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 1))*(cos(2*f*x + 2*
e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sqrt(a) + 4*(a*co
s(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e))) - (a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e))) - a)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1
)))*sqrt(a))*c/((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*
e) + 1)^(1/4)*f)
```

Fricas [A] time = 1.20415, size = 819, normalized size = 7.8

$$\left[\frac{3\left(ac\cos\left(fx+e\right)^2+ac\cos\left(fx+e\right)\right)\sqrt{-a}\log\left(\frac{2\,a\cos\left(fx+e\right)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)\sin\left(fx+e\right)+a\cos\left(fx+e\right)-a}{\cos\left(fx+e\right)+1}\right)+2\left(ad+(\cdots\right.}{3\left(f\cos\left(fx+e\right)^2+f\cos\left(fx+e\right)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(a*c*cos(f*x + e)^2 + a*c*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x +
 e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin
(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*(a*d + (3*a*c + 5*a
*d)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*
cos(f*x + e)^2 + f*cos(f*x + e)), -2/3*(3*(a*c*cos(f*x + e)^2 + a*c*cos(f*x
```

```
 + e))*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/
(sqrt(a)*sin(f*x + e))) - (a*d + (3*a*c + 5*a*d)*cos(f*x + e))*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^2 + f*cos(f*x + e
))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\left(c+d\sec\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(3/2)*(c+d*sec(f*x+e)),x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**(3/2)*(c + d*sec(e + f*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.157 $\int \frac{(a+a\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx$

Optimal. Leaf size=110

$$\frac{2a^{3/2}(c-d)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}\right)}{c\sqrt{d}f\sqrt{c+d}}+\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{cf}$$

```
[Out] (2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c*f) +
 (2*a^(3/2)*(c - d)*ArcTan[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(Sqrt[c + d]*Sqrt
[a + a*Sec[e + f*x]])])/(c*Sqrt[d]*Sqrt[c + d]*f)
```

Rubi [A] time = 0.240583, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {3927, 3774, 203, 3967, 205}

$$\frac{2a^{3/2}(c-d)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}\right)}{c\sqrt{d}f\sqrt{c+d}}+\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{cf}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x]),x]
```

```
[Out] (2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c*f) +
 (2*a^(3/2)*(c - d)*ArcTan[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(Sqrt[c + d]*Sqrt
[a + a*Sec[e + f*x]])])/(c*Sqrt[d]*Sqrt[c + d]*f)
```

Rule 3927

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_)), x_Symbol] :> Dist[a/c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] +
 Dist[(b*c - a*d)/c, Int[(Csc[e + f*x]*Sqrt[a + b*Csc[e + f*x]])/(c + d*Csc
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
(EqQ[a^2 - b^2, 0] || EqQ[c^2 - d^2, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3967

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[(-2*b)/f, Subst[Int
[1/(b*c + a*d + d*x^2), x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx = \frac{a\int \sqrt{a+a\sec(e+fx)}\,dx}{c} + \frac{(ac-ad)\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx}{c}$$

$$= -\frac{\left(2a^2\right)\operatorname{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{a\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf} - \frac{\left(2a^2(c-d)\right)\operatorname{Subst}\left(\int \frac{1}{ac+ad+dx^2}\,dx, x, -\frac{a\,\mathrm{ta}}{\sqrt{a+a}}\right.}{cf}$$

$$= \frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf} + \frac{2a^{3/2}(c-d)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}\right)}{c\sqrt{d}\sqrt{c+d}f}$$

Mathematica [A] time = 0.478421, size = 135, normalized size = 1.23

$$\frac{\sqrt{2}a\sqrt{\cos(e+fx)}\sec\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}\left(\sqrt{d}\sqrt{c+d}\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(e+fx)\right)\right)+(c-d)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sin\left(\frac{1}{2}(e\right.}{\sqrt{c+d}\sqrt{\cos(e}}\right.\right.}{c\sqrt{d}f\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x]),x]
```

```
[Out] (Sqrt[2]*a*(Sqrt[d]*Sqrt[c + d]*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]] + (c - d)*
ArcTan[(Sqrt[2]*Sqrt[d]*Sin[(e + f*x)/2])/(Sqrt[c + d]*Sqrt[Cos[e + f*x]])]
)*Sqrt[Cos[e + f*x]]*Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])])/(c*Sqrt[d
]*Sqrt[c + d]*f)
```

Maple [B] time = 0.217, size = 864, normalized size = 7.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x)
```

```
[Out] -1/2/f*2^(1/2)*a/((c+d)*(c-d))^(1/2)/c/(d/(c-d))^(1/2)*(2*(d/(c-d))^(1/2)*(
(c+d)*(c-d))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)
*sin(f*x+e)/cos(f*x+e))-ln(-2*((-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(d/(c-d
))^(1/2)*2^(1/2)*c*sin(f*x+e)-2^(1/2)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(1+cos
(f*x+e)))^(1/2)*d*sin(f*x+e)-((c+d)*(c-d))^(1/2)*cos(f*x+e)-c*sin(f*x+e)+d*
sin(f*x+e)+((c+d)*(c-d))^(1/2))/(((c+d)*(c-d))^(1/2)*sin(f*x+e)+c*cos(f*x+e
)-d*cos(f*x+e)-c+d))*c+d*ln(-2*((-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(d/(c-
d))^(1/2)*2^(1/2)*c*sin(f*x+e)-2^(1/2)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(1+co
s(f*x+e)))^(1/2)*d*sin(f*x+e)-((c+d)*(c-d))^(1/2)*cos(f*x+e)-c*sin(f*x+e)+d
*sin(f*x+e)+((c+d)*(c-d))^(1/2))/(((c+d)*(c-d))^(1/2)*sin(f*x+e)+c*cos(f*x+
e)-d*cos(f*x+e)-c+d))+ln(2*((-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(d/(c-d))^
(1/2)*2^(1/2)*c*sin(f*x+e)-2^(1/2)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(1+cos(f*
x+e)))^(1/2)*d*sin(f*x+e)+((c+d)*(c-d))^(1/2)*cos(f*x+e)-c*sin(f*x+e)+d*sin
(f*x+e)-((c+d)*(c-d))^(1/2))/(((c+d)*(c-d))^(1/2)*sin(f*x+e)-c*cos(f*x+e)+d
*cos(f*x+e)+c-d))*c-d*ln(2*((-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(d/(c-d))^
(1/2)*2^(1/2)*c*sin(f*x+e)-2^(1/2)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(1+cos(f*
x+e)))^(1/2)*d*sin(f*x+e)+((c+d)*(c-d))^(1/2)*cos(f*x+e)-c*sin(f*x+e)+d*sin
(f*x+e)-((c+d)*(c-d))^(1/2))/(((c+d)*(c-d))^(1/2)*sin(f*x+e)-c*cos(f*x+e)+d
*cos(f*x+e)+c-d)))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/cos(f*x+e)*a*(1+
cos(f*x+e)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}}{d\sec\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^(3/2)/(d*sec(f*x + e) + c), x)
```

Fricas [A] time = 8.84398, size = 1804, normalized size = 16.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] [-((a*c - a*d)*sqrt(-a/(c*d + d^2))*log((2*(c*d + d^2)*sqrt(-a/(c*d + d^2))
*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c +
 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2
+ (c + d)*cos(f*x + e) + d)) - sqrt(-a)*a*log((2*a*cos(f*x + e)^2 - 2*sqrt(
-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*c
os(f*x + e) - a)/(cos(f*x + e) + 1)))/(c*f), -(2*a^(3/2)*arctan(sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + (a*c -
a*d)*sqrt(-a/(c*d + d^2))*log((2*(c*d + d^2)*sqrt(-a/(c*d + d^2))*sqrt((a*c
os(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c + 2*a*d)*co
s(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2 + (c + d)*
cos(f*x + e) + d)))/(c*f), -(2*(a*c - a*d)*sqrt(a/(c*d + d^2))*arctan((c +
d)*sqrt(a/(c*d + d^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)
/(a*sin(f*x + e))) - sqrt(-a)*a*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((
a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e
) - a)/(cos(f*x + e) + 1)))/(c*f), -2*((a*c - a*d)*sqrt(a/(c*d + d^2))*arct
an((c + d)*sqrt(a/(c*d + d^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(
f*x + e)/(a*sin(f*x + e))) + a^(3/2)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f
*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))))/(c*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}{c+d\sec\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e)),x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**(3/2)/(c + d*sec(e + f*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.158 $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^2}\,dx$

Optimal. Leaf size=229

$$\frac{a^{5/2}\left(c^2-3cd-2d^2\right)\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{c^2\sqrt{d}f(c+d)^{3/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2a^{5/2}\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{a^2}{cf(c+d)\sqrt{a\,\mathrm{se}}}$$

```
[Out] (2*a^(5/2)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(c^2*f*S
qrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (a^(5/2)*(c^2 - 3*c*d -
 2*d^2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*T
an[e + f*x])/(c^2*Sqrt[d]*(c + d)^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a +
 a*Sec[e + f*x]]) + (a^2*(c - d)*Tan[e + f*x])/(c*(c + d)*f*Sqrt[a + a*Sec[
e + f*x]]*(c + d*Sec[e + f*x]))
```

Rubi [A] time = 0.24894, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3940, 151, 156, 63, 206, 208}

$$\frac{a^{5/2}\left(c^2-3cd-2d^2\right)\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{c^2\sqrt{d}f(c+d)^{3/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2a^{5/2}\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{a^2}{cf(c+d)\sqrt{a\,\mathrm{se}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (2*a^(5/2)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(c^2*f*S
qrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (a^(5/2)*(c^2 - 3*c*d -
 2*d^2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*T
an[e + f*x])/(c^2*Sqrt[d]*(c + d)^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a +
 a*Sec[e + f*x]]) + (a^2*(c - d)*Tan[e + f*x])/(c*(c + d)*f*Sqrt[a + a*Sec[
e + f*x]]*(c + d*Sec[e + f*x]))
```

Rule 3940

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(a^2*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
x)^n)/(x*Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
```

```
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^2}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\mathrm{Subst}\left(\int \frac{a+ax}{x\sqrt{a-ax}(c+dx)^2}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{a^2(c-d)\tan(e+fx)}{c(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))} - \frac{(a\tan(e+fx))\,\mathrm{Subst}\left(\int \frac{a^2(c+d)+\frac{1}{2}a^2(c-d)x}{x\sqrt{a-ax}(c+dx)}\right.}{c(c+d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+}}$$

$$= \frac{a^2(c-d)\tan(e+fx)}{c(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))} - \frac{\left(a^3\tan(e+fx)\right)\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a-ax}}\,dx,x,\mathrm{s}\right.}{c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e}}$$

$$= \frac{a^2(c-d)\tan(e+fx)}{c(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))} + \frac{\left(2a^2\tan(e+fx)\right)\mathrm{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx,x,\right.}{c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\,\mathrm{se}}}$$

$$= \frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{a^{5/2}\left(c^2-3cd-2d^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{c^2\sqrt{d}(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\,\mathrm{se}}}$$

Mathematica [C] time = 24.763, size = 2886, normalized size = 12.6

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^2,x]
```

```
[Out] ((d + c*Cos[e + f*x])^2*Sec[(e + f*x)/2]^3*Sec[e + f*x]*(a*(1 + Sec[e + f*x
]))^(3/2)*(((c - d)*Sin[(e + f*x)/2])/(2*c^2*(c + d)) + (-(c*d*Sin[(e + f*x
)/2]) + d^2*Sin[(e + f*x)/2])/(2*c^2*(c + d)*(d + c*Cos[e + f*x]))))/(f*(c
+ d*Sec[e + f*x])^2) - (Sqrt[3 - 2*Sqrt[2]]*Cos[(e + f*x)/4]^2*(d + c*Cos[e
 + f*x])^2*(c*(3*c + d)*EllipticF[ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2
]]], 17 - 12*Sqrt[2]] + 4*(c + d)^2*EllipticPi[-3 + 2*Sqrt[2], -ArcSin[Tan[
(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + (c^2 - 3*c*d - 2*d^2)
*(EllipticPi[-(((-3 + 2*Sqrt[2])*(c + d))/(3*c + 2*Sqrt[2]*Sqrt[c*(c - d)]
- d)), -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + El
lipticPi[((-3 + 2*Sqrt[2])*(c + d))/(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] + d),
 -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]]))*Sec[(e +
 f*x)/2]^3*((Cos[(e + f*x)/2]*Sqrt[Sec[e + f*x]])/(2*(c + d)*(d + c*Cos[e +
 f*x])) + (Cos[(3*(e + f*x))/2]*Sqrt[Sec[e + f*x]])/(4*(c + d)*(d + c*Cos[e
 + f*x])) + (d*Cos[(3*(e + f*x))/2]*Sqrt[Sec[e + f*x]])/(4*c*(c + d)*(d + c
*Cos[e + f*x])))*Sec[e + f*x]*(a*(1 + Sec[e + f*x]))^(3/2)*Sqrt[1 + (-3 + 2
```

```
*Sqrt[2])*Tan[(e + f*x)/4]^2]*Sqrt[1 - (3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2])
/(c^2*(c + d)^2*f*(c + d*Sec[e + f*x])^2*((Sqrt[3 - 2*Sqrt[2]]*(3 + 2*Sqrt[
2])*(c*(3*c + d)*EllipticF[ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17
 - 12*Sqrt[2]] + 4*(c + d)^2*EllipticPi[-3 + 2*Sqrt[2], -ArcSin[Tan[(e + f*
x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + (c^2 - 3*c*d - 2*d^2)*(Ellip
ticPi[-(((-3 + 2*Sqrt[2])*(c + d))/(3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] - d)),
-ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + EllipticP
i[((-3 + 2*Sqrt[2])*(c + d))/(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] + d), -ArcSi
n[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]]))*Sqrt[Sec[e + f*
x]]*Tan[(e + f*x)/4]*Sqrt[1 + (-3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2])/(4*c^2*
(c + d)^2*Sqrt[1 - (3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2]) - (Sqrt[3 - 2*Sqrt[
2]]*(-3 + 2*Sqrt[2])*(c*(3*c + d)*EllipticF[ArcSin[Tan[(e + f*x)/4]/Sqrt[3
- 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + 4*(c + d)^2*EllipticPi[-3 + 2*Sqrt[2], -A
rcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + (c^2 - 3*c*
d - 2*d^2)*(EllipticPi[-(((-3 + 2*Sqrt[2])*(c + d))/(3*c + 2*Sqrt[2]*Sqrt[c
*(c - d)] - d)), -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqr
t[2]] + EllipticPi[((-3 + 2*Sqrt[2])*(c + d))/(-3*c + 2*Sqrt[2]*Sqrt[c*(c -
 d)] + d), -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]])
)*Sqrt[Sec[e + f*x]]*Tan[(e + f*x)/4]*Sqrt[1 - (3 + 2*Sqrt[2])*Tan[(e + f*x
)/4]^2])/(4*c^2*(c + d)^2*Sqrt[1 + (-3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2]) +
(Sqrt[3 - 2*Sqrt[2]]*Cos[(e + f*x)/4]*(c*(3*c + d)*EllipticF[ArcSin[Tan[(e
+ f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + 4*(c + d)^2*EllipticPi[-
3 + 2*Sqrt[2], -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[
2]] + (c^2 - 3*c*d - 2*d^2)*(EllipticPi[-(((-3 + 2*Sqrt[2])*(c + d))/(3*c +
 2*Sqrt[2]*Sqrt[c*(c - d)] - d)), -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[
2]]], 17 - 12*Sqrt[2]] + EllipticPi[((-3 + 2*Sqrt[2])*(c + d))/(-3*c + 2*Sq
rt[2]*Sqrt[c*(c - d)] + d), -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]],
17 - 12*Sqrt[2]]))*Sqrt[Sec[e + f*x]]*Sin[(e + f*x)/4]*Sqrt[1 + (-3 + 2*Sqr
t[2])*Tan[(e + f*x)/4]^2]*Sqrt[1 - (3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2])/(2*
c^2*(c + d)^2) - (Sqrt[3 - 2*Sqrt[2]]*Cos[(e + f*x)/4]^2*(c*(3*c + d)*Ellip
ticF[ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + 4*(c
+ d)^2*EllipticPi[-3 + 2*Sqrt[2], -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[
2]]], 17 - 12*Sqrt[2]] + (c^2 - 3*c*d - 2*d^2)*(EllipticPi[-(((-3 + 2*Sqrt[
2])*(c + d))/(3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] - d)), -ArcSin[Tan[(e + f*x)/
4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + EllipticPi[((-3 + 2*Sqrt[2])*(c
 + d))/(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] + d), -ArcSin[Tan[(e + f*x)/4]/Sqr
t[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]]))*Sec[e + f*x]^(3/2)*Sin[e + f*x]*Sqrt[
1 + (-3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2]*Sqrt[1 - (3 + 2*Sqrt[2])*Tan[(e +
f*x)/4]^2])/(2*c^2*(c + d)^2) - (Sqrt[3 - 2*Sqrt[2]]*Cos[(e + f*x)/4]^2*Sqr
t[Sec[e + f*x]]*Sqrt[1 + (-3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2]*Sqrt[1 - (3 +
 2*Sqrt[2])*Tan[(e + f*x)/4]^2]*((c*(3*c + d)*Sec[(e + f*x)/4]^2)/(4*Sqrt[3
 - 2*Sqrt[2]]*Sqrt[1 - Tan[(e + f*x)/4]^2/(3 - 2*Sqrt[2])]*Sqrt[1 - ((17 -
12*Sqrt[2])*Tan[(e + f*x)/4]^2)/(3 - 2*Sqrt[2])]) - ((c + d)^2*Sec[(e + f*x
)/4]^2)/(Sqrt[3 - 2*Sqrt[2]]*Sqrt[1 - Tan[(e + f*x)/4]^2/(3 - 2*Sqrt[2])]*S
qrt[1 - ((17 - 12*Sqrt[2])*Tan[(e + f*x)/4]^2)/(3 - 2*Sqrt[2])]*(1 - ((-3 +
```

```
 2*Sqrt[2])*Tan[(e + f*x)/4]^2)/(3 - 2*Sqrt[2]))) + (c^2 - 3*c*d - 2*d^2)*(
-Sec[(e + f*x)/4]^2/(4*Sqrt[3 - 2*Sqrt[2]]*Sqrt[1 - Tan[(e + f*x)/4]^2/(3 -
 2*Sqrt[2])]*Sqrt[1 - ((17 - 12*Sqrt[2])*Tan[(e + f*x)/4]^2)/(3 - 2*Sqrt[2]
)]*(1 + ((-3 + 2*Sqrt[2])*(c + d)*Tan[(e + f*x)/4]^2)/((3 - 2*Sqrt[2])*(3*c
 + 2*Sqrt[2]*Sqrt[c*(c - d)] - d)))) - Sec[(e + f*x)/4]^2/(4*Sqrt[3 - 2*Sqr
t[2]]*Sqrt[1 - Tan[(e + f*x)/4]^2/(3 - 2*Sqrt[2])]*Sqrt[1 - ((17 - 12*Sqrt[
2])*Tan[(e + f*x)/4]^2)/(3 - 2*Sqrt[2])]*(1 - ((-3 + 2*Sqrt[2])*(c + d)*Tan
[(e + f*x)/4]^2)/((3 - 2*Sqrt[2])*(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] + d))))
)))/(c^2*(c + d)^2)))
```

Maple [B] time = 1.636, size = 62283, normalized size = 272.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^2,x)

[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^2,x, algorithm="maxima")

[Out] Timed out
```

Fricas [A] time = 30.3391, size = 4466, normalized size = 19.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a*c^2 - a*c*d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e
)*sin(f*x + e) - (a*c^2*d - 3*a*c*d^2 - 2*a*d^3 + (a*c^3 - 3*a*c^2*d - 2*a*
c*d^2)*cos(f*x + e)^2 + (a*c^3 - 2*a*c^2*d - 5*a*c*d^2 - 2*a*d^3)*cos(f*x +
 e))*sqrt(-a/(c*d + d^2))*log(((a*c^2 + 8*a*c*d + 8*a*d^2)*cos(f*x + e)^3 +
 a*d^2 + (a*c^2 + 2*a*c*d)*cos(f*x + e)^2 + 4*((c^2*d + 3*c*d^2 + 2*d^3)*co
s(f*x + e)^2 - (c*d^2 + d^3)*cos(f*x + e))*sqrt(-a/(c*d + d^2))*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - (6*a*c*d + 7*a*d^2)*cos(f*x + e
))/(c^2*cos(f*x + e)^3 + (c^2 + 2*c*d)*cos(f*x + e)^2 + d^2 + (2*c*d + d^2)
*cos(f*x + e))) + 2*(a*c*d + a*d^2 + (a*c^2 + a*c*d)*cos(f*x + e)^2 + (a*c^
2 + 2*a*c*d + a*d^2)*cos(f*x + e))*sqrt(-a)*log((8*a*cos(f*x + e)^3 - 4*(2*
cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x +
 e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1)))/((c^4 + c^3*
d)*f*cos(f*x + e)^2 + (c^4 + 2*c^3*d + c^2*d^2)*f*cos(f*x + e) + (c^3*d + c
^2*d^2)*f), 1/4*(4*(a*c^2 - a*c*d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*
cos(f*x + e)*sin(f*x + e) - 4*(a*c*d + a*d^2 + (a*c^2 + a*c*d)*cos(f*x + e)
^2 + (a*c^2 + 2*a*c*d + a*d^2)*cos(f*x + e))*sqrt(a)*arctan(1/2*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*(2*cos(f*x + e) - 1)/(sqrt(a)*sin(f*x + e))) -
 (a*c^2*d - 3*a*c*d^2 - 2*a*d^3 + (a*c^3 - 3*a*c^2*d - 2*a*c*d^2)*cos(f*x +
 e)^2 + (a*c^3 - 2*a*c^2*d - 5*a*c*d^2 - 2*a*d^3)*cos(f*x + e))*sqrt(-a/(c*
d + d^2))*log(((a*c^2 + 8*a*c*d + 8*a*d^2)*cos(f*x + e)^3 + a*d^2 + (a*c^2
+ 2*a*c*d)*cos(f*x + e)^2 + 4*((c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e)^2 - (
c*d^2 + d^3)*cos(f*x + e))*sqrt(-a/(c*d + d^2))*sqrt((a*cos(f*x + e) + a)/c
os(f*x + e))*sin(f*x + e) - (6*a*c*d + 7*a*d^2)*cos(f*x + e))/(c^2*cos(f*x
+ e)^3 + (c^2 + 2*c*d)*cos(f*x + e)^2 + d^2 + (2*c*d + d^2)*cos(f*x + e))))
/((c^4 + c^3*d)*f*cos(f*x + e)^2 + (c^4 + 2*c^3*d + c^2*d^2)*f*cos(f*x + e)
 + (c^3*d + c^2*d^2)*f), 1/2*(2*(a*c^2 - a*c*d)*sqrt((a*cos(f*x + e) + a)/c
os(f*x + e))*cos(f*x + e)*sin(f*x + e) - (a*c^2*d - 3*a*c*d^2 - 2*a*d^3 + (
a*c^3 - 3*a*c^2*d - 2*a*c*d^2)*cos(f*x + e)^2 + (a*c^3 - 2*a*c^2*d - 5*a*c*
d^2 - 2*a*d^3)*cos(f*x + e))*sqrt(a/(c*d + d^2))*arctan(1/2*((c + 2*d)*cos(
f*x + e) - d)*sqrt(a/(c*d + d^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))/(
a*sin(f*x + e))) + (a*c*d + a*d^2 + (a*c^2 + a*c*d)*cos(f*x + e)^2 + (a*c^2
 + 2*a*c*d + a*d^2)*cos(f*x + e))*sqrt(-a)*log((8*a*cos(f*x + e)^3 - 4*(2*c
os(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x +
e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1)))/((c^4 + c^3*d
)*f*cos(f*x + e)^2 + (c^4 + 2*c^3*d + c^2*d^2)*f*cos(f*x + e) + (c^3*d + c^
2*d^2)*f), 1/2*(2*(a*c^2 - a*c*d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*c
os(f*x + e)*sin(f*x + e) - (a*c^2*d - 3*a*c*d^2 - 2*a*d^3 + (a*c^3 - 3*a*c^
2*d - 2*a*c*d^2)*cos(f*x + e)^2 + (a*c^3 - 2*a*c^2*d - 5*a*c*d^2 - 2*a*d^3)
*cos(f*x + e))*sqrt(a/(c*d + d^2))*arctan(1/2*((c + 2*d)*cos(f*x + e) - d)*
sqrt(a/(c*d + d^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))/(a*sin(f*x + e)
)) - 2*(a*c*d + a*d^2 + (a*c^2 + a*c*d)*cos(f*x + e)^2 + (a*c^2 + 2*a*c*d +
 a*d^2)*cos(f*x + e))*sqrt(a)*arctan(1/2*sqrt((a*cos(f*x + e) + a)/cos(f*x
+ e))*(2*cos(f*x + e) - 1)/(sqrt(a)*sin(f*x + e))))/((c^4 + c^3*d)*f*cos(f*
x + e)^2 + (c^4 + 2*c^3*d + c^2*d^2)*f*cos(f*x + e) + (c^3*d + c^2*d^2)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}{\left(c+d\sec\left(e+fx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e))**2,x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**(3/2)/(c + d*sec(e + f*x))**2, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.159 $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^3}\,dx$

Optimal. Leaf size=310

$$\frac{a^2\left(3c^2-7cd-4d^2\right)\tan(e+fx)}{4c^2f(c+d)^2\sqrt{a\sec(e+fx)+a}\,(c+d\sec(e+fx))}+\frac{a^{5/2}\left(-15c^2d+3c^3-20cd^2-8d^3\right)\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{4c^3\sqrt{d}f(c+d)^{5/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

```
[Out] (2*a^(5/2)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(c^3*f*S
qrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (a^(5/2)*(3*c^3 - 15*c^
2*d - 20*c*d^2 - 8*d^3)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]
*Sqrt[c + d])]*Tan[e + f*x])/(4*c^3*Sqrt[d]*(c + d)^(5/2)*f*Sqrt[a - a*Sec[
e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (a^2*(c - d)*Tan[e + f*x])/(2*c*(c +
d)*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^2) + (a^2*(3*c^2 - 7*c*d
 - 4*d^2)*Tan[e + f*x])/(4*c^2*(c + d)^2*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*
Sec[e + f*x]))
```

Rubi [A] time = 0.341704, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3940, 151, 156, 63, 206, 208}

$$\frac{a^2\left(3c^2-7cd-4d^2\right)\tan(e+fx)}{4c^2f(c+d)^2\sqrt{a\sec(e+fx)+a}\,(c+d\sec(e+fx))}+\frac{a^{5/2}\left(-15c^2d+3c^3-20cd^2-8d^3\right)\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{4c^3\sqrt{d}f(c+d)^{5/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (2*a^(5/2)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(c^3*f*S
qrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (a^(5/2)*(3*c^3 - 15*c^
2*d - 20*c*d^2 - 8*d^3)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]
*Sqrt[c + d])]*Tan[e + f*x])/(4*c^3*Sqrt[d]*(c + d)^(5/2)*f*Sqrt[a - a*Sec[
e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (a^2*(c - d)*Tan[e + f*x])/(2*c*(c +
d)*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^2) + (a^2*(3*c^2 - 7*c*d
 - 4*d^2)*Tan[e + f*x])/(4*c^2*(c + d)^2*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*
Sec[e + f*x]))
```

Rule 3940

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(a^2*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
x)^n)/(x*Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^3}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{a+ax}{x\sqrt{a-ax}(c+dx)^3}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{a^2(c-d)\tan(e+fx)}{2c(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2} - \frac{(a\tan(e+fx))\,\text{Subst}\left(\int \frac{2a^2(c+d)+\frac{3}{2}a^2}{x\sqrt{a-ax}(c+}\right.}{2c(c+d)f\sqrt{a-a\sec(e+fx)}}$$

$$= \frac{a^2(c-d)\tan(e+fx)}{2c(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2} + \frac{a^2\left(3c^2-7cd-4d^2\right)\tan(e+}{4c^2(c+d)^2f\sqrt{a+a\sec(e+fx)}(c+}$$

$$= \frac{a^2(c-d)\tan(e+fx)}{2c(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2} + \frac{a^2\left(3c^2-7cd-4d^2\right)\tan(e+}{4c^2(c+d)^2f\sqrt{a+a\sec(e+fx)}(c+}$$

$$= \frac{a^2(c-d)\tan(e+fx)}{2c(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2} + \frac{a^2\left(3c^2-7cd-4d^2\right)\tan(e+}{4c^2(c+d)^2f\sqrt{a+a\sec(e+fx)}(c+}$$

$$= \frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{a^{5/2}\left(3c^3-15c^2d-20cd^2-8d^3\right)\tanh^{-1}\left(\frac{\sqrt{d}}{}\right.}{4c^3\sqrt{d}(c+d)^{5/2}f\sqrt{a-a\sec(e+fx)}}$$

Mathematica [C] time = 24.5808, size = 3190, normalized size = 10.29

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] ((d + c*Cos[e + f*x])^3*Sec[(e + f*x)/2]^3*Sec[e + f*x]^2*(a*(1 + Sec[e + f
*x]))^(3/2)*(-((-5*c^2 + 7*c*d + 6*d^2)*Sin[(e + f*x)/2])/(8*c^3*(c + d)^2)
 + (c*d^2*Sin[(e + f*x)/2] - d^3*Sin[(e + f*x)/2])/(4*c^3*(c + d)*(d + c*Co
s[e + f*x])^2) + (-7*c^2*d*Sin[(e + f*x)/2] + 7*c*d^2*Sin[(e + f*x)/2] + 8*
d^3*Sin[(e + f*x)/2])/(8*c^3*(c + d)^2*(d + c*Cos[e + f*x]))))/(f*(c + d*Se
c[e + f*x])^3) - (Sqrt[3 - 2*Sqrt[2]]*Cos[(e + f*x)/4]^2*(d + c*Cos[e + f*x
])^3*(c*(11*c^2 + 9*c*d + 4*d^2)*EllipticF[ArcSin[Tan[(e + f*x)/4]/Sqrt[3 -
 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + 16*(c + d)^3*EllipticPi[-3 + 2*Sqrt[2], -A
rcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + (3*c^3 - 15
*c^2*d - 20*c*d^2 - 8*d^3)*(EllipticPi[-(((-3 + 2*Sqrt[2])*(c + d))/(3*c +
```

2*Sqrt[2]*Sqrt[c*(c - d)] - d)), -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2
]]], 17 - 12*Sqrt[2]] + EllipticPi[((-3 + 2*Sqrt[2])*(c + d))/(-3*c + 2*Sqr
t[2]*Sqrt[c*(c - d)] + d), -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 1
7 - 12*Sqrt[2]]))*Sec[(e + f*x)/2]^3*((7*Cos[(e + f*x)/2]*Sqrt[Sec[e + f*x]
])/(16*(c + d)^2*(d + c*Cos[e + f*x])) + (d*Cos[(e + f*x)/2]*Sqrt[Sec[e + f
*x]])/(16*c*(c + d)^2*(d + c*Cos[e + f*x])) + (Cos[(3*(e + f*x))/2]*Sqrt[Se
c[e + f*x]])/(4*(c + d)^2*(d + c*Cos[e + f*x])) + (d*Cos[(3*(e + f*x))/2]*S
qrt[Sec[e + f*x]])/(2*c*(c + d)^2*(d + c*Cos[e + f*x])) + (d^2*Cos[(3*(e +
f*x))/2]*Sqrt[Sec[e + f*x]])/(4*c^2*(c + d)^2*(d + c*Cos[e + f*x])))*Sec[e
+ f*x]^2*(a*(1 + Sec[e + f*x]))^(3/2)*Sqrt[1 + (-3 + 2*Sqrt[2])*Tan[(e + f*
x)/4]^2]*Sqrt[1 - (3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2])/(4*c^3*(c + d)^3*f*(
c + d*Sec[e + f*x])^3*((Sqrt[3 - 2*Sqrt[2]]*(3 + 2*Sqrt[2])*(c*(11*c^2 + 9*
c*d + 4*d^2)*EllipticF[ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 1
2*Sqrt[2]] + 16*(c + d)^3*EllipticPi[-3 + 2*Sqrt[2], -ArcSin[Tan[(e + f*x)/
4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + (3*c^3 - 15*c^2*d - 20*c*d^2 -
8*d^3)*(EllipticPi[-(((-3 + 2*Sqrt[2])*(c + d))/(3*c + 2*Sqrt[2]*Sqrt[c*(c
- d)] - d)), -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]
] + EllipticPi[((-3 + 2*Sqrt[2])*(c + d))/(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)]
+ d), -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]]))*Sq
rt[Sec[e + f*x]]*Tan[(e + f*x)/4]*Sqrt[1 + (-3 + 2*Sqrt[2])*Tan[(e + f*x)/4
]^2])/(16*c^3*(c + d)^3*Sqrt[1 - (3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2]) - (Sq
rt[3 - 2*Sqrt[2]]*(-3 + 2*Sqrt[2])*(c*(11*c^2 + 9*c*d + 4*d^2)*EllipticF[Ar
cSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + 16*(c + d)^3
*EllipticPi[-3 + 2*Sqrt[2], -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]],
17 - 12*Sqrt[2]] + (3*c^3 - 15*c^2*d - 20*c*d^2 - 8*d^3)*(EllipticPi[-(((-3
+ 2*Sqrt[2])*(c + d))/(3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] - d)), -ArcSin[Tan[
(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + EllipticPi[((-3 + 2*S
qrt[2])*(c + d))/(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] + d), -ArcSin[Tan[(e + f
*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]]))*Sqrt[Sec[e + f*x]]*Tan[(e +
f*x)/4]*Sqrt[1 - (3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2])/(16*c^3*(c + d)^3*Sq
rt[1 + (-3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2]) + (Sqrt[3 - 2*Sqrt[2]]*Cos[(e
+ f*x)/4]*(c*(11*c^2 + 9*c*d + 4*d^2)*EllipticF[ArcSin[Tan[(e + f*x)/4]/Sqr
t[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + 16*(c + d)^3*EllipticPi[-3 + 2*Sqrt[2
], -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + (3*c^3
- 15*c^2*d - 20*c*d^2 - 8*d^3)*(EllipticPi[-(((-3 + 2*Sqrt[2])*(c + d))/(3
*c + 2*Sqrt[2]*Sqrt[c*(c - d)] - d)), -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*S
qrt[2]]], 17 - 12*Sqrt[2]] + EllipticPi[((-3 + 2*Sqrt[2])*(c + d))/(-3*c +
2*Sqrt[2]*Sqrt[c*(c - d)] + d), -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]
]], 17 - 12*Sqrt[2]]))*Sqrt[Sec[e + f*x]]*Sin[(e + f*x)/4]*Sqrt[1 + (-3 + 2
*Sqrt[2])*Tan[(e + f*x)/4]^2]*Sqrt[1 - (3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2])
/(8*c^3*(c + d)^3) - (Sqrt[3 - 2*Sqrt[2]]*Cos[(e + f*x)/4]^2*(c*(11*c^2 + 9
*c*d + 4*d^2)*EllipticF[ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 -
12*Sqrt[2]] + 16*(c + d)^3*EllipticPi[-3 + 2*Sqrt[2], -ArcSin[Tan[(e + f*x)
/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + (3*c^3 - 15*c^2*d - 20*c*d^2 -
8*d^3)*(EllipticPi[-(((-3 + 2*Sqrt[2])*(c + d))/(3*c + 2*Sqrt[2]*Sqrt[c*(c

```
 - d)] - d)), -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2
]] + EllipticPi[((-3 + 2*Sqrt[2])*(c + d))/(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)
] + d), -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]]))*S
ec[e + f*x]^(3/2)*Sin[e + f*x]*Sqrt[1 + (-3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2
]*Sqrt[1 - (3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2])/(8*c^3*(c + d)^3) - (Sqrt[3
 - 2*Sqrt[2]]*Cos[(e + f*x)/4]^2*Sqrt[Sec[e + f*x]]*Sqrt[1 + (-3 + 2*Sqrt[2
])*Tan[(e + f*x)/4]^2]*Sqrt[1 - (3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2]*((c*(11
*c^2 + 9*c*d + 4*d^2)*Sec[(e + f*x)/4]^2)/(4*Sqrt[3 - 2*Sqrt[2]]*Sqrt[1 - T
an[(e + f*x)/4]^2/(3 - 2*Sqrt[2])]*Sqrt[1 - ((17 - 12*Sqrt[2])*Tan[(e + f*x
)/4]^2)/(3 - 2*Sqrt[2])]) - (4*(c + d)^3*Sec[(e + f*x)/4]^2)/(Sqrt[3 - 2*Sq
rt[2]]*Sqrt[1 - Tan[(e + f*x)/4]^2/(3 - 2*Sqrt[2])]*Sqrt[1 - ((17 - 12*Sqrt
[2])*Tan[(e + f*x)/4]^2)/(3 - 2*Sqrt[2])]*(1 - ((-3 + 2*Sqrt[2])*Tan[(e + f
*x)/4]^2)/(3 - 2*Sqrt[2]))) + (3*c^3 - 15*c^2*d - 20*c*d^2 - 8*d^3)*(-Sec[(
e + f*x)/4]^2/(4*Sqrt[3 - 2*Sqrt[2]]*Sqrt[1 - Tan[(e + f*x)/4]^2/(3 - 2*Sqr
t[2])]*Sqrt[1 - ((17 - 12*Sqrt[2])*Tan[(e + f*x)/4]^2)/(3 - 2*Sqrt[2])]*(1
+ ((-3 + 2*Sqrt[2])*(c + d)*Tan[(e + f*x)/4]^2)/((3 - 2*Sqrt[2])*(3*c + 2*S
qrt[2]*Sqrt[c*(c - d)] - d)))) - Sec[(e + f*x)/4]^2/(4*Sqrt[3 - 2*Sqrt[2]]*
Sqrt[1 - Tan[(e + f*x)/4]^2/(3 - 2*Sqrt[2])]*Sqrt[1 - ((17 - 12*Sqrt[2])*Ta
n[(e + f*x)/4]^2)/(3 - 2*Sqrt[2])]*(1 - ((-3 + 2*Sqrt[2])*(c + d)*Tan[(e +
f*x)/4]^2)/((3 - 2*Sqrt[2])*(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] + d))))))/(4
*c^3*(c + d)^3)))
```

Maple [B] time = 10.07, size = 234091, normalized size = 755.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^3,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 79.1484, size = 6322, normalized size = 20.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [-1/8*((3*a*c^3*d^2 - 15*a*c^2*d^3 - 20*a*c*d^4 - 8*a*d^5 + (3*a*c^5 - 15*a
*c^4*d - 20*a*c^3*d^2 - 8*a*c^2*d^3)*cos(f*x + e)^3 + (3*a*c^5 - 9*a*c^4*d
- 50*a*c^3*d^2 - 48*a*c^2*d^3 - 16*a*c*d^4)*cos(f*x + e)^2 + (6*a*c^4*d - 2
7*a*c^3*d^2 - 55*a*c^2*d^3 - 36*a*c*d^4 - 8*a*d^5)*cos(f*x + e))*sqrt(-a/(c
*d + d^2))*log((2*(c*d + d^2)*sqrt(-a/(c*d + d^2))*sqrt((a*cos(f*x + e) + a
)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c + 2*a*d)*cos(f*x + e)^2 -
a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2 + (c + d)*cos(f*x + e) +
d)) - 8*(a*c^2*d^2 + 2*a*c*d^3 + a*d^4 + (a*c^4 + 2*a*c^3*d + a*c^2*d^2)*co
s(f*x + e)^3 + (a*c^4 + 4*a*c^3*d + 5*a*c^2*d^2 + 2*a*c*d^3)*cos(f*x + e)^2
 + (2*a*c^3*d + 5*a*c^2*d^2 + 4*a*c*d^3 + a*d^4)*cos(f*x + e))*sqrt(-a)*log
((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*c
os(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*((5*
a*c^4 - 7*a*c^3*d - 6*a*c^2*d^2)*cos(f*x + e)^2 + (3*a*c^3*d - 7*a*c^2*d^2
- 4*a*c*d^3)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x
+ e))/((c^7 + 2*c^6*d + c^5*d^2)*f*cos(f*x + e)^3 + (c^7 + 4*c^6*d + 5*c^5*
d^2 + 2*c^4*d^3)*f*cos(f*x + e)^2 + (2*c^6*d + 5*c^5*d^2 + 4*c^4*d^3 + c^3*
d^4)*f*cos(f*x + e) + (c^5*d^2 + 2*c^4*d^3 + c^3*d^4)*f), -1/8*(16*(a*c^2*d
^2 + 2*a*c*d^3 + a*d^4 + (a*c^4 + 2*a*c^3*d + a*c^2*d^2)*cos(f*x + e)^3 + (
a*c^4 + 4*a*c^3*d + 5*a*c^2*d^2 + 2*a*c*d^3)*cos(f*x + e)^2 + (2*a*c^3*d +
5*a*c^2*d^2 + 4*a*c*d^3 + a*d^4)*cos(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(f
*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + (3*a*c^3*
d^2 - 15*a*c^2*d^3 - 20*a*c*d^4 - 8*a*d^5 + (3*a*c^5 - 15*a*c^4*d - 20*a*c^
3*d^2 - 8*a*c^2*d^3)*cos(f*x + e)^3 + (3*a*c^5 - 9*a*c^4*d - 50*a*c^3*d^2 -
 48*a*c^2*d^3 - 16*a*c*d^4)*cos(f*x + e)^2 + (6*a*c^4*d - 27*a*c^3*d^2 - 55
*a*c^2*d^3 - 36*a*c*d^4 - 8*a*d^5)*cos(f*x + e))*sqrt(-a/(c*d + d^2))*log((
2*(c*d + d^2)*sqrt(-a/(c*d + d^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*
cos(f*x + e)*sin(f*x + e) + (a*c + 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d
)*cos(f*x + e))/(c*cos(f*x + e)^2 + (c + d)*cos(f*x + e) + d)) - 2*((5*a*c^
4 - 7*a*c^3*d - 6*a*c^2*d^2)*cos(f*x + e)^2 + (3*a*c^3*d - 7*a*c^2*d^2 - 4*
a*c*d^3)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e)
)/((c^7 + 2*c^6*d + c^5*d^2)*f*cos(f*x + e)^3 + (c^7 + 4*c^6*d + 5*c^5*d^2
+ 2*c^4*d^3)*f*cos(f*x + e)^2 + (2*c^6*d + 5*c^5*d^2 + 4*c^4*d^3 + c^3*d^4)
```

```
*f*cos(f*x + e) + (c^5*d^2 + 2*c^4*d^3 + c^3*d^4)*f), -1/4*((3*a*c^3*d^2 -
15*a*c^2*d^3 - 20*a*c*d^4 - 8*a*d^5 + (3*a*c^5 - 15*a*c^4*d - 20*a*c^3*d^2
- 8*a*c^2*d^3)*cos(f*x + e)^3 + (3*a*c^5 - 9*a*c^4*d - 50*a*c^3*d^2 - 48*a*
c^2*d^3 - 16*a*c*d^4)*cos(f*x + e)^2 + (6*a*c^4*d - 27*a*c^3*d^2 - 55*a*c^2
*d^3 - 36*a*c*d^4 - 8*a*d^5)*cos(f*x + e))*sqrt(a/(c*d + d^2))*arctan((c +
d)*sqrt(a/(c*d + d^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)
/(a*sin(f*x + e))) - 4*(a*c^2*d^2 + 2*a*c*d^3 + a*d^4 + (a*c^4 + 2*a*c^3*d
+ a*c^2*d^2)*cos(f*x + e)^3 + (a*c^4 + 4*a*c^3*d + 5*a*c^2*d^2 + 2*a*c*d^3)
*cos(f*x + e)^2 + (2*a*c^3*d + 5*a*c^2*d^2 + 4*a*c*d^3 + a*d^4)*cos(f*x + e
))*sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/
cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e)
 + 1)) - ((5*a*c^4 - 7*a*c^3*d - 6*a*c^2*d^2)*cos(f*x + e)^2 + (3*a*c^3*d -
 7*a*c^2*d^2 - 4*a*c*d^3)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x +
 e))*sin(f*x + e))/((c^7 + 2*c^6*d + c^5*d^2)*f*cos(f*x + e)^3 + (c^7 + 4*c
^6*d + 5*c^5*d^2 + 2*c^4*d^3)*f*cos(f*x + e)^2 + (2*c^6*d + 5*c^5*d^2 + 4*c
^4*d^3 + c^3*d^4)*f*cos(f*x + e) + (c^5*d^2 + 2*c^4*d^3 + c^3*d^4)*f), -1/4
*((3*a*c^3*d^2 - 15*a*c^2*d^3 - 20*a*c*d^4 - 8*a*d^5 + (3*a*c^5 - 15*a*c^4*
d - 20*a*c^3*d^2 - 8*a*c^2*d^3)*cos(f*x + e)^3 + (3*a*c^5 - 9*a*c^4*d - 50*
a*c^3*d^2 - 48*a*c^2*d^3 - 16*a*c*d^4)*cos(f*x + e)^2 + (6*a*c^4*d - 27*a*c
^3*d^2 - 55*a*c^2*d^3 - 36*a*c*d^4 - 8*a*d^5)*cos(f*x + e))*sqrt(a/(c*d + d
^2))*arctan((c + d)*sqrt(a/(c*d + d^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x +
 e))*cos(f*x + e)/(a*sin(f*x + e))) + 8*(a*c^2*d^2 + 2*a*c*d^3 + a*d^4 + (a
*c^4 + 2*a*c^3*d + a*c^2*d^2)*cos(f*x + e)^3 + (a*c^4 + 4*a*c^3*d + 5*a*c^2
*d^2 + 2*a*c*d^3)*cos(f*x + e)^2 + (2*a*c^3*d + 5*a*c^2*d^2 + 4*a*c*d^3 + a
*d^4)*cos(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*
cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - ((5*a*c^4 - 7*a*c^3*d - 6*a*c^2*d^2)
*cos(f*x + e)^2 + (3*a*c^3*d - 7*a*c^2*d^2 - 4*a*c*d^3)*cos(f*x + e))*sqrt(
(a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/((c^7 + 2*c^6*d + c^5*d^2)
*f*cos(f*x + e)^3 + (c^7 + 4*c^6*d + 5*c^5*d^2 + 2*c^4*d^3)*f*cos(f*x + e)^
2 + (2*c^6*d + 5*c^5*d^2 + 4*c^4*d^3 + c^3*d^4)*f*cos(f*x + e) + (c^5*d^2 +
 2*c^4*d^3 + c^3*d^4)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}{\left(c+d\sec\left(e+fx\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e))**3,x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**(3/2)/(c + d*sec(e + f*x))**3, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.160 $\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3\,dx$

Optimal. Leaf size=336

$$-\frac{2\left(12c^2d+c^3+24cd^2+12d^3\right)\tan(e+fx)\left(a^3-a^3\sec(e+fx)\right)}{3f\sqrt{a\sec(e+fx)+a}}+\frac{2a^3\left(12c^2d+3c^3+12cd^2+4d^3\right)\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}+\frac{2a^7}{f}$$

```
[Out] (2*a^3*(3*c^3 + 12*c^2*d + 12*c*d^2 + 4*d^3)*Tan[e + f*x])/(f*Sqrt[a + a*Se
c[e + f*x]]) + (2*a^(7/2)*c^3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan
[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (2*a*d*(
3*c^2 + 15*c*d + 13*d^2)*(a - a*Sec[e + f*x])^2*Tan[e + f*x])/(5*f*Sqrt[a +
 a*Sec[e + f*x]]) - (6*d^2*(c + 2*d)*(a - a*Sec[e + f*x])^3*Tan[e + f*x])/(
7*f*Sqrt[a + a*Sec[e + f*x]]) + (2*d^3*(a - a*Sec[e + f*x])^4*Tan[e + f*x])
/(9*a*f*Sqrt[a + a*Sec[e + f*x]]) - (2*(c^3 + 12*c^2*d + 24*c*d^2 + 12*d^3)
*(a^3 - a^3*Sec[e + f*x])*Tan[e + f*x])/(3*f*Sqrt[a + a*Sec[e + f*x]])
```

Rubi [A] time = 0.208889, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3940, 180, 63, 206}

$$-\frac{2\left(12c^2d+c^3+24cd^2+12d^3\right)\tan(e+fx)\left(a^3-a^3\sec(e+fx)\right)}{3f\sqrt{a\sec(e+fx)+a}}+\frac{2a^3\left(12c^2d+3c^3+12cd^2+4d^3\right)\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}+\frac{2a^7}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])^3,x]
```


```
[Out] (2*a^3*(3*c^3 + 12*c^2*d + 12*c*d^2 + 4*d^3)*Tan[e + f*x])/(f*Sqrt[a + a*Se
c[e + f*x]]) + (2*a^(7/2)*c^3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan
[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (2*a*d*(
3*c^2 + 15*c*d + 13*d^2)*(a - a*Sec[e + f*x])^2*Tan[e + f*x])/(5*f*Sqrt[a +
 a*Sec[e + f*x]]) - (6*d^2*(c + 2*d)*(a - a*Sec[e + f*x])^3*Tan[e + f*x])/(
7*f*Sqrt[a + a*Sec[e + f*x]]) + (2*d^3*(a - a*Sec[e + f*x])^4*Tan[e + f*x])
/(9*a*f*Sqrt[a + a*Sec[e + f*x]]) - (2*(c^3 + 12*c^2*d + 24*c*d^2 + 12*d^3)
*(a^3 - a^3*Sec[e + f*x])*Tan[e + f*x])/(3*f*Sqrt[a + a*Sec[e + f*x]])
```


Rule 3940

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(a^2*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
```

```
e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
x)^n)/(x*Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rule 180

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^2(c+dx)^3}{x\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int\left(\frac{a^2\left(3c^3+12c^2d+12cd^2+4d^3\right)}{\sqrt{a-ax}} + \frac{a^2c^3}{x\sqrt{a-ax}} - a\left(c^3 + \right.\right.\right.}{}$$

$$= \frac{2a^3\left(3c^3+12c^2d+12cd^2+4d^3\right)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2ad\left(3c^2+15cd+13d\right.}{5f\sqrt{}}$$

$$= \frac{2a^3\left(3c^3+12c^2d+12cd^2+4d^3\right)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2ad\left(3c^2+15cd+13d\right.}{5f\sqrt{}}$$

$$= \frac{2a^3\left(3c^3+12c^2d+12cd^2+4d^3\right)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2a^{7/2}c^3\tanh^{-1}\left(\frac{\sqrt{a-a\,\text{se}}}{}\right.}{f\sqrt{a-a\sec(e+fx)}}$$

Mathematica [A] time = 6.18927, size = 286, normalized size = 0.85

$$\frac{a^2\sec\left(\frac{1}{2}(e+fx)\right)\sec^4(e+fx)\sqrt{a(\sec(e+fx)+1)}\left(2\sin\left(\frac{1}{2}(e+fx)\right)\left(\left(5292c^2d+630c^3+7290cd^2+2792d^3\right)\cos(e+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (a^2*Sec[(e + f*x)/2]*Sec[e + f*x]^4*Sqrt[a*(1 + Sec[e + f*x])]*(2520*Sqrt[
2]*c^3*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]]*Cos[e + f*x]^(9/2) + 2*(2520*c^3 +
8883*c^2*d + 8370*c*d^2 + 2908*d^3 + (630*c^3 + 5292*c^2*d + 7290*c*d^2 + 2
792*d^3)*Cos[e + f*x] + 4*(840*c^3 + 2898*c^2*d + 2610*c*d^2 + 803*d^3)*Cos
[2*(e + f*x)] + 210*c^3*Cos[3*(e + f*x)] + 1764*c^2*d*Cos[3*(e + f*x)] + 20
70*c*d^2*Cos[3*(e + f*x)] + 584*d^3*Cos[3*(e + f*x)] + 840*c^3*Cos[4*(e + f
*x)] + 2709*c^2*d*Cos[4*(e + f*x)] + 2070*c*d^2*Cos[4*(e + f*x)] + 584*d^3*
Cos[4*(e + f*x)])*Sin[(e + f*x)/2]))/(2520*f)
```

Maple [B] time = 0.343, size = 677, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e))^3,x)
```

```
[Out] -1/5040/f*a^2*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(315*sin(f*x+e)*cos(f*x
+e)^4*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(9/2)*arctanh(1/2*2^(1/2)*(-2*
cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*c^3+1260*sin(f*x+e)
*cos(f*x+e)^3*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(9/2)*arctanh(1/2*2^(1
/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*c^3+1890*si
n(f*x+e)*cos(f*x+e)^2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(9/2)*arctanh(
1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*c^3
+1260*sin(f*x+e)*cos(f*x+e)*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(9/2)*ar
ctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e
))*c^3+315*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(9/2)*2^(1/2)*arctanh(1/2*2^(1/2)
*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*c^3*sin(f*x+e)
+26880*cos(f*x+e)^5*c^3+86688*cos(f*x+e)^5*c^2*d+66240*cos(f*x+e)^5*c*d^2+1
8688*cos(f*x+e)^5*d^3-23520*cos(f*x+e)^4*c^3-58464*cos(f*x+e)^4*c^2*d-33120
*cos(f*x+e)^4*c*d^2-9344*cos(f*x+e)^4*d^3-3360*cos(f*x+e)^3*c^3-22176*cos(f
*x+e)^3*c^2*d-15840*cos(f*x+e)^3*c*d^2-2336*cos(f*x+e)^3*d^3-6048*cos(f*x+e
)^2*c^2*d-12960*cos(f*x+e)^2*c*d^2-2848*cos(f*x+e)^2*d^3-4320*cos(f*x+e)*c*
d^2-3040*cos(f*x+e)*d^3-1120*d^3)/cos(f*x+e)^4/sin(f*x+e)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.09471, size = 1513, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/315*(315*(a^2*c^3*cos(f*x + e)^5 + a^2*c^3*cos(f*x + e)^4)*sqrt(-a)*log(
(2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*co
s(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*(35*a
^2*d^3 + (840*a^2*c^3 + 2709*a^2*c^2*d + 2070*a^2*c*d^2 + 584*a^2*d^3)*cos(
f*x + e)^4 + (105*a^2*c^3 + 882*a^2*c^2*d + 1035*a^2*c*d^2 + 292*a^2*d^3)*c
os(f*x + e)^3 + 3*(63*a^2*c^2*d + 180*a^2*c*d^2 + 73*a^2*d^3)*cos(f*x + e)^
2 + 5*(27*a^2*c*d^2 + 26*a^2*d^3)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/c
os(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^5 + f*cos(f*x + e)^4), -2/315*(3
15*(a^2*c^3*cos(f*x + e)^5 + a^2*c^3*cos(f*x + e)^4)*sqrt(a)*arctan(sqrt((a
*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - (35
*a^2*d^3 + (840*a^2*c^3 + 2709*a^2*c^2*d + 2070*a^2*c*d^2 + 584*a^2*d^3)*co
s(f*x + e)^4 + (105*a^2*c^3 + 882*a^2*c^2*d + 1035*a^2*c*d^2 + 292*a^2*d^3)
*cos(f*x + e)^3 + 3*(63*a^2*c^2*d + 180*a^2*c*d^2 + 73*a^2*d^3)*cos(f*x + e
)^2 + 5*(27*a^2*c*d^2 + 26*a^2*d^3)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)
/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^5 + f*cos(f*x + e)^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(5/2)*(c+d*sec(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.161 $\int (a + a\sec(e + fx))^{5/2}(c + d\sec(e + fx))^2\,dx$

Optimal. Leaf size=258

$$-\frac{2\left(c^2+8cd+8d^2\right)\tan(e+fx)\left(a^3-a^3\sec(e+fx)\right)}{3f\sqrt{a\sec(e+fx)+a}}+\frac{2a^{7/2}c^2\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2a^3(c+2d)(3c+2}{f\sqrt{a\sec(e+}}$$

```
[Out] (2*a^3*(c + 2*d)*(3*c + 2*d)*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) + (
2*a^(7/2)*c^2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(f*Sq
rt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (2*a*d*(2*c + 5*d)*(a -
a*Sec[e + f*x])^2*Tan[e + f*x])/(5*f*Sqrt[a + a*Sec[e + f*x]]) - (2*d^2*(a
- a*Sec[e + f*x])^3*Tan[e + f*x])/(7*f*Sqrt[a + a*Sec[e + f*x]]) - (2*(c^2
+ 8*c*d + 8*d^2)*(a^3 - a^3*Sec[e + f*x])*Tan[e + f*x])/(3*f*Sqrt[a + a*Sec
[e + f*x]])
```

Rubi [A] time = 0.17731, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3940, 180, 63, 206}

$$-\frac{2\left(c^2+8cd+8d^2\right)\tan(e+fx)\left(a^3-a^3\sec(e+fx)\right)}{3f\sqrt{a\sec(e+fx)+a}}+\frac{2a^{7/2}c^2\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2a^3(c+2d)(3c+2}{f\sqrt{a\sec(e+}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (2*a^3*(c + 2*d)*(3*c + 2*d)*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) + (
2*a^(7/2)*c^2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(f*Sq
rt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (2*a*d*(2*c + 5*d)*(a -
a*Sec[e + f*x])^2*Tan[e + f*x])/(5*f*Sqrt[a + a*Sec[e + f*x]]) - (2*d^2*(a
- a*Sec[e + f*x])^3*Tan[e + f*x])/(7*f*Sqrt[a + a*Sec[e + f*x]]) - (2*(c^2
+ 8*c*d + 8*d^2)*(a^3 - a^3*Sec[e + f*x])*Tan[e + f*x])/(3*f*Sqrt[a + a*Sec
[e + f*x]])
```

Rule 3940

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(a^2*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
x)^n)/(x*Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
```

```
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rule 180

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^2(c+dx)^2}{x\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int\left(\frac{a^2(c+2d)(3c+2d)}{\sqrt{a-ax}}+\frac{a^2c^2}{x\sqrt{a-ax}}-a\left(c^2+8cd+8\right.\right.\right.}{f\sqrt{a-a\sec(e+fx)}}$$

$$= \frac{2a^3(c+2d)(3c+2d)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}+\frac{2ad(2c+5d)(a-a\sec(e+fx))^2\,\mathrm{t}}{5f\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2a^3(c+2d)(3c+2d)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}+\frac{2ad(2c+5d)(a-a\sec(e+fx))^2\,\mathrm{t}}{5f\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2a^3(c+2d)(3c+2d)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}+\frac{2a^{7/2}c^2\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\mathrm{tan}}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(}$$

Mathematica [A] time = 2.68358, size = 191, normalized size = 0.74

$$\frac{a^2 \sec\left(\frac{1}{2}(e+fx)\right)\sec^3(e+fx)\sqrt{a(\sec(e+fx)+1)}\left(4\sin\left(\frac{1}{2}(e+fx)\right)\left(\left(420c^2+987cd+465d^2\right)\cos(e+fx)+\left(35c^2+19\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (a^2*Sec[(e + f*x)/2]*Sec[e + f*x]^3*Sqrt[a*(1 + Sec[e + f*x])]*(420*Sqrt[2
]*c^2*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]]*Cos[e + f*x]^(7/2) + 4*(35*c^2 + 196
*c*d + 145*d^2 + (420*c^2 + 987*c*d + 465*d^2)*Cos[e + f*x] + (35*c^2 + 196
*c*d + 115*d^2)*Cos[2*(e + f*x)] + 140*c^2*Cos[3*(e + f*x)] + 301*c*d*Cos[3
*(e + f*x)] + 115*d^2*Cos[3*(e + f*x)])*Sin[(e + f*x)/2]))/(420*f)
```

Maple [B] time = 0.283, size = 504, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e))^2,x)
```

```
[Out] 1/840/f*a^2*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(105*sin(f*x+e)*cos(f*x+e
)^3*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(7/2)*arctanh(1/2*2^(1/2)*(-2*co
s(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*c^2+315*sin(f*x+e)*co
s(f*x+e)^2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(7/2)*arctanh(1/2*2^(1/2)
*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*c^2+315*sin(f*
x+e)*cos(f*x+e)*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(7/2)*arctanh(1/2*2^
(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*c^2+105*(
-2*cos(f*x+e)/(1+cos(f*x+e)))^(7/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+c
os(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*2^(1/2)*c^2*sin(f*x+e)-4480*cos(f*
x+e)^4*c^2-9632*cos(f*x+e)^4*c*d-3680*cos(f*x+e)^4*d^2+3920*cos(f*x+e)^3*c^
2+6496*cos(f*x+e)^3*c*d+1840*cos(f*x+e)^3*d^2+560*cos(f*x+e)^2*c^2+2464*cos
(f*x+e)^2*c*d+880*cos(f*x+e)^2*d^2+672*cos(f*x+e)*c*d+720*cos(f*x+e)*d^2+24
0*d^2)/sin(f*x+e)/cos(f*x+e)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.10297, size = 1233, normalized size = 4.78

$$\left[\frac{105\left(a^{2}c^{2}\cos\left(fx+e\right)^{4}+a^{2}c^{2}\cos\left(fx+e\right)^{3}\right)\sqrt{-a}\log\left(\frac{2\,a\cos\left(fx+e\right)^{2}-2\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)\sin\left(fx+e\right)+a\cos\left(fx+e\right)-a}{\cos\left(fx+e\right)+1}\right)+2}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/105*(105*(a^2*c^2*cos(f*x + e)^4 + a^2*c^2*cos(f*x + e)^3)*sqrt(-a)*log(
(2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*co
s(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*(15*a
^2*d^2 + 2*(140*a^2*c^2 + 301*a^2*c*d + 115*a^2*d^2)*cos(f*x + e)^3 + (35*a
^2*c^2 + 196*a^2*c*d + 115*a^2*d^2)*cos(f*x + e)^2 + 6*(7*a^2*c*d + 10*a^2*
d^2)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f
*cos(f*x + e)^4 + f*cos(f*x + e)^3), -2/105*(105*(a^2*c^2*cos(f*x + e)^4 +
a^2*c^2*cos(f*x + e)^3)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x +
e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - (15*a^2*d^2 + 2*(140*a^2*c^2 + 3
01*a^2*c*d + 115*a^2*d^2)*cos(f*x + e)^3 + (35*a^2*c^2 + 196*a^2*c*d + 115*
a^2*d^2)*cos(f*x + e)^2 + 6*(7*a^2*c*d + 10*a^2*d^2)*cos(f*x + e))*sqrt((a*
cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^4 + f*cos(f*x
 + e)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(5/2)*(c+d*sec(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.162 $\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx$

Optimal. Leaf size=142

$$\frac{2a^3(35c+32d)\tan(e+fx)}{15f\sqrt{a\sec(e+fx)+a}}+\frac{2a^2(5c+8d)\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{15f}+\frac{2a^{5/2}c\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}+\frac{2ad\tan(e$$

```
[Out] (2*a^(5/2)*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f + (
2*a^3*(35*c + 32*d)*Tan[e + f*x])/(15*f*Sqrt[a + a*Sec[e + f*x]]) + (2*a^2*
(5*c + 8*d)*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(15*f) + (2*a*d*(a + a*S
ec[e + f*x])^(3/2)*Tan[e + f*x])/(5*f)
```

Rubi [A] time = 0.231934, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3917, 3915, 3774, 203, 3792}

$$\frac{2a^3(35c+32d)\tan(e+fx)}{15f\sqrt{a\sec(e+fx)+a}}+\frac{2a^2(5c+8d)\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{15f}+\frac{2a^{5/2}c\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}+\frac{2ad\tan(e$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x]),x]
```

```
[Out] (2*a^(5/2)*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f + (
2*a^3*(35*c + 32*d)*Tan[e + f*x])/(15*f*Sqrt[a + a*Sec[e + f*x]]) + (2*a^2*
(5*c + 8*d)*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(15*f) + (2*a*d*(a + a*S
ec[e + f*x])^(3/2)*Tan[e + f*x])/(5*f)
```

Rule 3917

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*c*m + (b
*c*m + a*d*(2*m - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3915

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Dis
```

```
t[d, Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx = \frac{2ad(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{5f}+\frac{2}{5}\int (a+a\sec(e+fx))^{3/2}\left(\frac{5ac}{2}\right.$$

$$= \frac{2a^2(5c+8d)\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{15f}+\frac{2ad(a+a\sec(e+fx))^{3/2}}{5f}$$

$$= \frac{2a^2(5c+8d)\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{15f}+\frac{2ad(a+a\sec(e+fx))^{3/2}}{5f}$$

$$= \frac{2a^3(35c+32d)\tan(e+fx)}{15f\sqrt{a+a\sec(e+fx)}}+\frac{2a^2(5c+8d)\sqrt{a+a\sec(e+fx)}\tan(e+f}{15f}$$

$$= \frac{2a^{5/2}c\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}+\frac{2a^3(35c+32d)\tan(e+fx)}{15f\sqrt{a+a\sec(e+fx)}}+\frac{2a^2(5c+8}{}$$

Mathematica [A] time = 0.921576, size = 128, normalized size = 0.9

$$\frac{a^2\sec\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\sqrt{a(\sec(e+fx)+1)}\left(2\sin\left(\frac{1}{2}(e+fx)\right)(2(5c+14d)\cos(e+fx)+(40c+43d)\cos(2(e+f\right.}{30f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x]),x]
```

```
[Out] (a^2*Sec[(e + f*x)/2]*Sec[e + f*x]^2*Sqrt[a*(1 + Sec[e + f*x])]*(30*Sqrt[2]
*c*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]]*Cos[e + f*x]^(5/2) + 2*(40*c + 49*d + 2
*(5*c + 14*d)*Cos[e + f*x] + (40*c + 43*d)*Cos[2*(e + f*x)])*Sin[(e + f*x)/
2]))/(30*f)
```

Maple [B] time = 0.241, size = 341, normalized size = 2.4

$$-\frac{a^2}{60 f \sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^2}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\left(15\sqrt{2}\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^2 \mathrm{Artanh}\left(1/2\,\frac{\sqrt{2}\sin\left(fx+\right.}{\cos\left(fx+e\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e)),x)
```

```
[Out] -1/60/f*a^2*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(15*2^(1/2)*sin(f*x+e)*co
s(f*x+e)^2*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x
+e)/cos(f*x+e))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)*c+30*2^(1/2)*sin(f*x+e
)*cos(f*x+e)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f
*x+e)/cos(f*x+e))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)*c+15*arctanh(1/2*2^(
1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*2^(1/2)*(-
2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)*c*sin(f*x+e)+320*cos(f*x+e)^3*c+344*cos(
f*x+e)^3*d-280*cos(f*x+e)^2*c-232*cos(f*x+e)^2*d-40*c*cos(f*x+e)-88*d*cos(f
*x+e)-24*d)/sin(f*x+e)/cos(f*x+e)^2
```

Maxima [B] time = 1.97029, size = 1885, normalized size = 13.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/6*(30*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^
(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 2*
```

```
(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*((
12*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(2*f*x + 2*e
) - 3*a^2*sin(2*f*x + 2*e) - 4*(3*a^2*cos(2*f*x + 2*e) + 4*a^2)*sin(3/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e) + 1)) + (12*a^2*sin(2*f*x + 2*e)*sin(3/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e))) + 3*a^2*cos(2*f*x + 2*e) - a^2 + 4*(3*a^2*cos
(2*f*x + 2*e) + 4*a^2)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))*sqrt(a) + 3*((a
^2*cos(2*f*x + 2*e)^2 + a^2*sin(2*f*x + 2*e)^2 + 2*a^2*cos(2*f*x + 2*e) + a
^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) +
 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - cos(1/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e)))), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1
)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*cos(1/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sin(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e)))) + 1) - (a^2*cos(2*f*x + 2*e)^2 + a^2*sin(2*f*x + 2*e)^2 + 2*a^2*cos(
2*f*x + 2*e) + a^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*co
s(2*f*x + 2*e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e)))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - cos(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e)))), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(
2*f*x + 2*e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
 + 1))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sin(1/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e)))) - 1) - (a^2*cos(2*f*x + 2*e)^2 + a^2*sin(2*f*x + 2*e
)^2 + 2*a^2*cos(2*f*x + 2*e) + a^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x
 + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2
*f*x + 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) +
 1)) + 1) + (a^2*cos(2*f*x + 2*e)^2 + a^2*sin(2*f*x + 2*e)^2 + 2*a^2*cos(2*
f*x + 2*e) + a^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(
2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
+ 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(
1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 1))*sqrt(a)
)*c/((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*f)
```

Fricas [A] time = 1.0254, size = 976, normalized size = 6.87

$$\left[\frac{15\left(a^2c\cos\left(fx+e\right)^3+a^2c\cos\left(fx+e\right)^2\right)\sqrt{-a}\log\left(\frac{2a\cos\left(fx+e\right)^2-2\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)\sin\left(fx+e\right)+a\cos\left(fx+e\right)-a}{\cos\left(fx+e\right)+1}\right)+2\left(3\right.}{15\left(f\cos\left(fx+e\right)^3+f\cos\left(f\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/15*(15*(a^2*c*cos(f*x + e)^3 + a^2*c*cos(f*x + e)^2)*sqrt(-a)*log((2*a*c
os(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x
+ e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*(3*a^2*d +
(40*a^2*c + 43*a^2*d)*cos(f*x + e)^2 + (5*a^2*c + 14*a^2*d)*cos(f*x + e))*s
qrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3 + f*
cos(f*x + e)^2), -2/15*(15*(a^2*c*cos(f*x + e)^3 + a^2*c*cos(f*x + e)^2)*sq
rt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*
sin(f*x + e))) - (3*a^2*d + (40*a^2*c + 43*a^2*d)*cos(f*x + e)^2 + (5*a^2*c
 + 14*a^2*d)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x
+ e))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(5/2)*(c+d*sec(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.163 $\int \frac{(a+a\sec(e+fx))^{5/2}}{c+d\sec(e+fx)}\,dx$

Optimal. Leaf size=203

$$-\frac{2a^{7/2}(c-d)^2\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{cd^{3/2}f\sqrt{c+d}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2a^{7/2}\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{cf\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2a^3\tan(e+fx)}{df\sqrt{a\sec(e+fx)+a}}$$

```
[Out] (2*a^3*Tan[e + f*x])/(d*f*Sqrt[a + a*Sec[e + f*x]]) + (2*a^(7/2)*ArcTanh[Sq
rt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(c*f*Sqrt[a - a*Sec[e + f*x]]
*Sqrt[a + a*Sec[e + f*x]]) - (2*a^(7/2)*(c - d)^2*ArcTanh[(Sqrt[d]*Sqrt[a -
 a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])/(c*d^(3/2)*Sqrt[c +
d]*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])
```

Rubi [A] time = 0.231801, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {3940, 180, 63, 206, 208}

$$-\frac{2a^{7/2}(c-d)^2\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{cd^{3/2}f\sqrt{c+d}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2a^{7/2}\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{cf\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2a^3\tan(e+fx)}{df\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x]),x]
```

```
[Out] (2*a^3*Tan[e + f*x])/(d*f*Sqrt[a + a*Sec[e + f*x]]) + (2*a^(7/2)*ArcTanh[Sq
rt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(c*f*Sqrt[a - a*Sec[e + f*x]]
*Sqrt[a + a*Sec[e + f*x]]) - (2*a^(7/2)*(c - d)^2*ArcTanh[(Sqrt[d]*Sqrt[a -
 a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])/(c*d^(3/2)*Sqrt[c +
d]*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])
```

Rule 3940

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(a^2*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
x)^n)/(x*Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rule 180

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{c+d\sec(e+fx)}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^2}{x\sqrt{a-ax}(c+dx)}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \left(\frac{a^2}{d\sqrt{a-ax}} + \frac{a^2}{cx\sqrt{a-ax}} - \frac{a^2(c-d)^2}{cd\sqrt{a-ax}(c+dx)}\right)dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2a^3\tan(e+fx)}{df\sqrt{a+a\sec(e+fx)}} - \frac{\left(a^4\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{cf\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{\left(a^4(c-\right.}{}$$

$$= \frac{2a^3\tan(e+fx)}{df\sqrt{a+a\sec(e+fx)}} + \frac{\left(2a^3\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx, x, \sqrt{a-a\sec(e+fx)}\right)}{cf\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{\left(2\right.}{}$$

$$= \frac{2a^3\tan(e+fx)}{df\sqrt{a+a\sec(e+fx)}} + \frac{2a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{cf\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{2a^{7/2}(c-d)^2\tanh^{-1}}{cd^{3/2}\sqrt{c+d}f\sqrt{a-a}}$$

Mathematica [C] time = 6.48915, size = 343, normalized size = 1.69

$$\frac{\cos^{\frac{3}{2}}(e+fx)\sec^5\left(\frac{1}{2}(e+fx)\right)(a(\sec(e+fx)+1))^{5/2}(c\cos(e+fx)+d)\left(-\frac{16d(c-d)^2\sin^3\left(\frac{1}{2}(e+fx)\right)(c\cos(e+fx)+d)\text{Hypergeometric}}{(c+d)^3\cos^{\frac{5}{2}}(e+fx)}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x]),x]
```

```
[Out] (Cos[e + f*x]^(3/2)*(d + c*Cos[e + f*x])*Sec[(e + f*x)/2]^5*(a*(1 + Sec[e +
 f*x]))^(5/2)*((10*(c - d)^2*(c + 3*d + 2*c*Cos[e + f*x])*Csc[(e + f*x)/2]*
(-ArcTanh[Sqrt[-((d*(-1 + Sec[e + f*x]))/(c + d))]] + Sqrt[-((d*(-1 + Sec[e
 + f*x]))/(c + d))]))/(d*(c + d)*Sqrt[Cos[e + f*x]]*Sqrt[-((d*(-1 + Sec[e +
 f*x]))/(c + d))]) + (20*(3*c - d)*Sin[(e + f*x)/2])/Sqrt[Cos[e + f*x]] - (
16*(c - d)^2*d*(d + c*Cos[e + f*x])*Hypergeometric2F1[2, 5/2, 7/2, (-2*d*Se
c[e + f*x]*Sin[(e + f*x)/2]^2)/(c + d)]*Sin[(e + f*x)/2]^3)/((c + d)^3*Cos[
e + f*x]^(5/2)) + 10*c*(Sqrt[2]*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]] - (2*Sin[(
e + f*x)/2])/Sqrt[Cos[e + f*x]])))/(40*c^2*f*(c + d*Sec[e + f*x]))
```

Maple [B] time = 0.237, size = 1487, normalized size = 7.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e)),x)
```

```
[Out] -1/2/f*a^2/c/((c+d)*(c-d))^(1/2)/d/(d/(c-d))^(1/2)*(2*2^(1/2)*((c+d)*(c-d))
^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x
+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*(d/(c-d))^(1/2)*d*sin(f*x+
e)+2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(-2*((-2*cos(f*x+e)/(1+co
s(f*x+e)))^(1/2)*(d/(c-d))^(1/2)*2^(1/2)*c*sin(f*x+e)-2^(1/2)*(d/(c-d))^(1/
2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*d*sin(f*x+e)-((c+d)*(c-d))^(1/2)*co
s(f*x+e)-c*sin(f*x+e)+d*sin(f*x+e)+((c+d)*(c-d))^(1/2))/(((c+d)*(c-d))^(1/2
)*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d))*c^2*sin(f*x+e)-2*2^(1/2)*(-2*c
os(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(-2*((-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)
*(d/(c-d))^(1/2)*2^(1/2)*c*sin(f*x+e)-2^(1/2)*(d/(c-d))^(1/2)*(-2*cos(f*x+e
)/(1+cos(f*x+e)))^(1/2)*d*sin(f*x+e)-((c+d)*(c-d))^(1/2)*cos(f*x+e)-c*sin(f
*x+e)+d*sin(f*x+e)+((c+d)*(c-d))^(1/2))/(((c+d)*(c-d))^(1/2)*sin(f*x+e)+c*c
os(f*x+e)-d*cos(f*x+e)-c+d))*c*d*sin(f*x+e)+2^(1/2)*(-2*cos(f*x+e)/(1+cos(f
*x+e)))^(1/2)*ln(-2*((-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(d/(c-d))^(1/2)*2
^(1/2)*c*sin(f*x+e)-2^(1/2)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^
(1/2)*d*sin(f*x+e)-((c+d)*(c-d))^(1/2)*cos(f*x+e)-c*sin(f*x+e)+d*sin(f*x+e)
+((c+d)*(c-d))^(1/2))/(((c+d)*(c-d))^(1/2)*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*
x+e)-c+d))*d^2*sin(f*x+e)-2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(2
*((-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(d/(c-d))^(1/2)*2^(1/2)*c*sin(f*x+e)
-2^(1/2)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*d*sin(f*x+e)+
((c+d)*(c-d))^(1/2)*cos(f*x+e)-c*sin(f*x+e)+d*sin(f*x+e)-((c+d)*(c-d))^(1/2
))/(((c+d)*(c-d))^(1/2)*sin(f*x+e)-c*cos(f*x+e)+d*cos(f*x+e)+c-d))*c^2*sin(
f*x+e)+2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(2*((-2*cos(f*x+e)/
(1+cos(f*x+e)))^(1/2)*(d/(c-d))^(1/2)*2^(1/2)*c*sin(f*x+e)-2^(1/2)*(d/(c-d)
)^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*d*sin(f*x+e)+((c+d)*(c-d))^(1/
2)*cos(f*x+e)-c*sin(f*x+e)+d*sin(f*x+e)-((c+d)*(c-d))^(1/2))/(((c+d)*(c-d))
^(1/2)*sin(f*x+e)-c*cos(f*x+e)+d*cos(f*x+e)+c-d))*c*d*sin(f*x+e)-2^(1/2)*(-
2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(2*((-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/
2)*(d/(c-d))^(1/2)*2^(1/2)*c*sin(f*x+e)-2^(1/2)*(d/(c-d))^(1/2)*(-2*cos(f*x
+e)/(1+cos(f*x+e)))^(1/2)*d*sin(f*x+e)+((c+d)*(c-d))^(1/2)*cos(f*x+e)-c*sin
(f*x+e)+d*sin(f*x+e)-((c+d)*(c-d))^(1/2))/(((c+d)*(c-d))^(1/2)*sin(f*x+e)-c
*cos(f*x+e)+d*cos(f*x+e)+c-d))*d^2*sin(f*x+e)+4*((c+d)*(c-d))^(1/2)*(d/(c-d
))^(1/2)*c*cos(f*x+e)-4*((c+d)*(c-d))^(1/2)*(d/(c-d))^(1/2)*c)*(1/cos(f*x+e
)*a*(1+cos(f*x+e)))^(1/2)/sin(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sec\left(fx+e\right)+a\right)^{\frac{5}{2}}}{d\sec\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^(5/2)/(d*sec(f*x + e) + c), x)
```

Fricas [A] time = 19.6106, size = 2773, normalized size = 13.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] [(2*a^2*c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) + (a^2*c^2 -
 2*a^2*c*d + a^2*d^2 + (a^2*c^2 - 2*a^2*c*d + a^2*d^2)*cos(f*x + e))*sqrt(-
a/(c*d + d^2))*log((2*(c*d + d^2)*sqrt(-a/(c*d + d^2))*sqrt((a*cos(f*x + e)
 + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c + 2*a*d)*cos(f*x + e)^
2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2 + (c + d)*cos(f*x + e
) + d)) + (a^2*d*cos(f*x + e) + a^2*d)*sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2
*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)
 + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/(c*d*f*cos(f*x + e) + c*d*f), (
2*a^2*c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 2*(a^2*d*cos
(f*x + e) + a^2*d)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*c
os(f*x + e)/(sqrt(a)*sin(f*x + e))) + (a^2*c^2 - 2*a^2*c*d + a^2*d^2 + (a^2
*c^2 - 2*a^2*c*d + a^2*d^2)*cos(f*x + e))*sqrt(-a/(c*d + d^2))*log((2*(c*d
+ d^2)*sqrt(-a/(c*d + d^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x
 + e)*sin(f*x + e) + (a*c + 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f
*x + e))/(c*cos(f*x + e)^2 + (c + d)*cos(f*x + e) + d)))/(c*d*f*cos(f*x + e
) + c*d*f), (2*a^2*c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) +
 2*(a^2*c^2 - 2*a^2*c*d + a^2*d^2 + (a^2*c^2 - 2*a^2*c*d + a^2*d^2)*cos(f*x
 + e))*sqrt(a/(c*d + d^2))*arctan((c + d)*sqrt(a/(c*d + d^2))*sqrt((a*cos(f
*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(a*sin(f*x + e))) + (a^2*d*cos(f*x
+ e) + a^2*d)*sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x
 + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(c
os(f*x + e) + 1)))/(c*d*f*cos(f*x + e) + c*d*f), 2*(a^2*c*sqrt((a*cos(f*x +
```

```
e) + a)/cos(f*x + e))*sin(f*x + e) + (a^2*c^2 - 2*a^2*c*d + a^2*d^2 + (a^2
*c^2 - 2*a^2*c*d + a^2*d^2)*cos(f*x + e))*sqrt(a/(c*d + d^2))*arctan((c + d
)*sqrt(a/(c*d + d^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/
(a*sin(f*x + e))) - (a^2*d*cos(f*x + e) + a^2*d)*sqrt(a)*arctan(sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))))/(c*d*f*c
os(f*x + e) + c*d*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.164 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^2}\,dx$

Optimal. Leaf size=329

$$\frac{2a^{7/2}(c-d)\sqrt{c+d}\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{c^2d^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2a^{7/2}\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{a^{7/2}(c-d)^2\,\mathrm{ta}}{cd^{3/2}f(c+d)^{3/2}}$$

```
[Out] (2*a^(7/2)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(c^2*f*S
qrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (a^(7/2)*(c - d)^2*ArcT
anh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])
/(c*d^(3/2)*(c + d)^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x
]]) + (2*a^(7/2)*(c - d)*Sqrt[c + d]*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*
x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])/(c^2*d^(3/2)*f*Sqrt[a - a*Sec[e +
 f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (a^3*(c - d)^2*Tan[e + f*x])/(c*d*(c + d
)*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x]))
```

Rubi [A] time = 0.337278, antiderivative size = 329, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3940, 180, 63, 206, 51, 208}

$$\frac{2a^{7/2}(c-d)\sqrt{c+d}\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{c^2d^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2a^{7/2}\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{a^{7/2}(c-d)^2\,\mathrm{ta}}{cd^{3/2}f(c+d)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (2*a^(7/2)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(c^2*f*S
qrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (a^(7/2)*(c - d)^2*ArcT
anh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])
/(c*d^(3/2)*(c + d)^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x
]]) + (2*a^(7/2)*(c - d)*Sqrt[c + d]*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*
x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])/(c^2*d^(3/2)*f*Sqrt[a - a*Sec[e +
 f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (a^3*(c - d)^2*Tan[e + f*x])/(c*d*(c + d
)*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x]))
```

Rule 3940

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(a^2*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
x)^n)/(x*Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rule 180

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^2}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^2}{x\sqrt{a-ax}(c+dx)^2}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \left(\frac{a^2}{c^2x\sqrt{a-ax}} - \frac{a^2(c-d)^2}{cd\sqrt{a-ax}(c+dx)^2} + \frac{a^2\left(c^2-d^2\right)}{c^2d\sqrt{a-ax}(c+dx)}\right)dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\left(a^4\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{\left(a^4(c-d)^2\tan(e+fx)\right)\text{Subst}}{cdf\sqrt{a-a\sec(e+f}}$$

$$= -\frac{a^3(c-d)^2\tan(e+fx)}{cd(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))} + \frac{\left(2a^3\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx\right.}{c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+}}$$

$$= \frac{2a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{2a^{7/2}(c-d)\sqrt{c+d}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{c^2d^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e}}$$

$$= \frac{2a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{a^{7/2}(c-d)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e}{cd^{3/2}(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec}}$$

Mathematica [A] time = 3.7247, size = 280, normalized size = 0.85

$$\frac{\sqrt{\cos(e+fx)}\sec^5\left(\frac{1}{2}(e+fx)\right)(a(\sec(e+fx)+1))^{5/2}(c\cos(e+fx)+d)^2\left(\frac{4\sqrt{2}(c-d)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}\sqrt{\cos(e+fx)}}\right)}{\sqrt{d}\sqrt{c+d}} - \frac{(c-d)^2\sin\left(\frac{1}{2}(e+fx\right.}{}\right.}{8c^2f(c+d\sec(e+fx))^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (Sqrt[Cos[e + f*x]]*(d + c*Cos[e + f*x])^2*Sec[(e + f*x)/2]^5*(a*(1 + Sec[e
 + f*x]))^(5/2)*(2*Sqrt[2]*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]] + (4*Sqrt[2]*(c
 - d)*ArcTan[(Sqrt[2]*Sqrt[d]*Sin[(e + f*x)/2])/(Sqrt[c + d]*Sqrt[Cos[e + f
*x]])])/(Sqrt[d]*Sqrt[c + d]) - ((c - d)^2*(2*c*Cos[e + f*x] - (2*(c + 2*d)
*ArcTanh[Sqrt[-((d*(-1 + Sec[e + f*x]))/(c + d))]]*(d + c*Cos[e + f*x]))/((
c + d)*Sqrt[-((d*(-1 + Sec[e + f*x]))/(c + d))]))*Sin[(e + f*x)/2])/(d*(c +
```

```
 d)*Sqrt[Cos[e + f*x]]*(d + c*Cos[e + f*x]))))/(8*c^2*f*(c + d*Sec[e + f*x]
)^2)
```

Maple [B] time = 0.848, size = 46082, normalized size = 140.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^2,x)

[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^2,x, algorithm="maxima")

[Out] Timed out
```

Fricas [A] time = 89.0912, size = 4579, normalized size = 13.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^2,x, algorithm="fricas")

[Out] [-1/2*(2*(a^2*c^3 - 2*a^2*c^2*d + a^2*c*d^2)*sqrt((a*cos(f*x + e) + a)/cos(
f*x + e))*cos(f*x + e)*sin(f*x + e) + (a^2*c^3*d + 4*a^2*c^2*d^2 - 3*a^2*c*
d^3 - 2*a^2*d^4 + (a^2*c^4 + 4*a^2*c^3*d - 3*a^2*c^2*d^2 - 2*a^2*c*d^3)*cos
(f*x + e)^2 + (a^2*c^4 + 5*a^2*c^3*d + a^2*c^2*d^2 - 5*a^2*c*d^3 - 2*a^2*d^
4)*cos(f*x + e))*sqrt(-a/(c*d + d^2))*log((2*(c*d + d^2)*sqrt(-a/(c*d + d^2
))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c
```

```
 + 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^
2 + (c + d)*cos(f*x + e) + d)) - 2*(a^2*c*d^2 + a^2*d^3 + (a^2*c^2*d + a^2*
c*d^2)*cos(f*x + e)^2 + (a^2*c^2*d + 2*a^2*c*d^2 + a^2*d^3)*cos(f*x + e))*s
qrt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(
f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1
)))/((c^4*d + c^3*d^2)*f*cos(f*x + e)^2 + (c^4*d + 2*c^3*d^2 + c^2*d^3)*f*c
os(f*x + e) + (c^3*d^2 + c^2*d^3)*f), -1/2*(2*(a^2*c^3 - 2*a^2*c^2*d + a^2*
c*d^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) +
4*(a^2*c*d^2 + a^2*d^3 + (a^2*c^2*d + a^2*c*d^2)*cos(f*x + e)^2 + (a^2*c^2*
d + 2*a^2*c*d^2 + a^2*d^3)*cos(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(f*x + e
) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + (a^2*c^3*d + 4*
a^2*c^2*d^2 - 3*a^2*c*d^3 - 2*a^2*d^4 + (a^2*c^4 + 4*a^2*c^3*d - 3*a^2*c^2*
d^2 - 2*a^2*c*d^3)*cos(f*x + e)^2 + (a^2*c^4 + 5*a^2*c^3*d + a^2*c^2*d^2 -
5*a^2*c*d^3 - 2*a^2*d^4)*cos(f*x + e))*sqrt(-a/(c*d + d^2))*log((2*(c*d + d
^2)*sqrt(-a/(c*d + d^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x +
e)*sin(f*x + e) + (a*c + 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x
+ e))/(c*cos(f*x + e)^2 + (c + d)*cos(f*x + e) + d)))/((c^4*d + c^3*d^2)*f*
cos(f*x + e)^2 + (c^4*d + 2*c^3*d^2 + c^2*d^3)*f*cos(f*x + e) + (c^3*d^2 +
c^2*d^3)*f), -((a^2*c^3 - 2*a^2*c^2*d + a^2*c*d^2)*sqrt((a*cos(f*x + e) + a
)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a^2*c^3*d + 4*a^2*c^2*d^2 - 3*
a^2*c*d^3 - 2*a^2*d^4 + (a^2*c^4 + 4*a^2*c^3*d - 3*a^2*c^2*d^2 - 2*a^2*c*d^
3)*cos(f*x + e)^2 + (a^2*c^4 + 5*a^2*c^3*d + a^2*c^2*d^2 - 5*a^2*c*d^3 - 2*
a^2*d^4)*cos(f*x + e))*sqrt(a/(c*d + d^2))*arctan((c + d)*sqrt(a/(c*d + d^2
))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(a*sin(f*x + e))) -
 (a^2*c*d^2 + a^2*d^3 + (a^2*c^2*d + a^2*c*d^2)*cos(f*x + e)^2 + (a^2*c^2*d
 + 2*a^2*c*d^2 + a^2*d^3)*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x + e)^2 -
2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e
) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/((c^4*d + c^3*d^2)*f*cos(f*x +
 e)^2 + (c^4*d + 2*c^3*d^2 + c^2*d^3)*f*cos(f*x + e) + (c^3*d^2 + c^2*d^3)*
f), -((a^2*c^3 - 2*a^2*c^2*d + a^2*c*d^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x
 + e))*cos(f*x + e)*sin(f*x + e) + (a^2*c^3*d + 4*a^2*c^2*d^2 - 3*a^2*c*d^3
 - 2*a^2*d^4 + (a^2*c^4 + 4*a^2*c^3*d - 3*a^2*c^2*d^2 - 2*a^2*c*d^3)*cos(f*
x + e)^2 + (a^2*c^4 + 5*a^2*c^3*d + a^2*c^2*d^2 - 5*a^2*c*d^3 - 2*a^2*d^4)*
cos(f*x + e))*sqrt(a/(c*d + d^2))*arctan((c + d)*sqrt(a/(c*d + d^2))*sqrt((
a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(a*sin(f*x + e))) + 2*(a^2*c
*d^2 + a^2*d^3 + (a^2*c^2*d + a^2*c*d^2)*cos(f*x + e)^2 + (a^2*c^2*d + 2*a^
2*c*d^2 + a^2*d^3)*cos(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/c
os(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))))/((c^4*d + c^3*d^2)*f*cos
(f*x + e)^2 + (c^4*d + 2*c^3*d^2 + c^2*d^3)*f*cos(f*x + e) + (c^3*d^2 + c^2
*d^3)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.165 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^3}\,dx$

Optimal. Leaf size=536

$$\frac{a^{7/2}(c-d)\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{c^2d^{3/2}f\sqrt{c+d}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{a^3(c-d)\tan(e+fx)}{c^2df\sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))}-\frac{2a^{7/2}\sqrt{d}\tan(e+f}{c^3f\sqrt{c+d}\sqrt{a-a\,\mathrm{s}}}$$

```
[Out] (2*a^(7/2)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(c^3*f*S
qrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (3*a^(7/2)*(c - d)^2*Ar
cTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x
])/(4*c*d^(3/2)*(c + d)^(5/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e +
 f*x]]) + (a^(7/2)*(c - d)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt
[a]*Sqrt[c + d])]*Tan[e + f*x])/(c^2*d^(3/2)*Sqrt[c + d]*f*Sqrt[a - a*Sec[e
 + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (2*a^(7/2)*Sqrt[d]*ArcTanh[(Sqrt[d]*Sq
rt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])/(c^3*Sqrt[c +
d]*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (a^3*(c - d)^2*Ta
n[e + f*x])/(2*c*d*(c + d)*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^
2) + (a^3*(c - d)*Tan[e + f*x])/(c^2*d*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*Se
c[e + f*x])) - (3*a^3*(c - d)^2*Tan[e + f*x])/(4*c*d*(c + d)^2*f*Sqrt[a + a
*Sec[e + f*x]]*(c + d*Sec[e + f*x]))
```

Rubi [A] time = 0.492023, antiderivative size = 536, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3940, 180, 63, 206, 51, 208}

$$\frac{a^{7/2}(c-d)\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{c^2d^{3/2}f\sqrt{c+d}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{a^3(c-d)\tan(e+fx)}{c^2df\sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))}-\frac{2a^{7/2}\sqrt{d}\tan(e+f}{c^3f\sqrt{c+d}\sqrt{a-a\,\mathrm{s}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (2*a^(7/2)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(c^3*f*S
qrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (3*a^(7/2)*(c - d)^2*Ar
cTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x
])/(4*c*d^(3/2)*(c + d)^(5/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e +
 f*x]]) + (a^(7/2)*(c - d)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt
[a]*Sqrt[c + d])]*Tan[e + f*x])/(c^2*d^(3/2)*Sqrt[c + d]*f*Sqrt[a - a*Sec[e
```

```
 + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (2*a^(7/2)*Sqrt[d]*ArcTanh[(Sqrt[d]*Sq
rt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])/(c^3*Sqrt[c +
d]*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (a^3*(c - d)^2*Ta
n[e + f*x])/(2*c*d*(c + d)*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^
2) + (a^3*(c - d)*Tan[e + f*x])/(c^2*d*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*Se
c[e + f*x])) - (3*a^3*(c - d)^2*Tan[e + f*x])/(4*c*d*(c + d)^2*f*Sqrt[a + a
*Sec[e + f*x]]*(c + d*Sec[e + f*x]))
```

Rule 3940

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(a^2*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
x)^n)/(x*Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rule 180

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
```

```
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^3}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^2}{x\sqrt{a-ax}(c+dx)^3}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \left(\frac{a^2}{c^3x\sqrt{a-ax}} - \frac{a^2(c-d)^2}{cd\sqrt{a-ax}(c+dx)^3} + \frac{a^2\left(c^2-d^2\right)}{c^2d\sqrt{a-ax}(c+dx)^2} - \frac{a^2d}{c^3\sqrt{a-ax}(c+dx)}\right) dx\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\left(a^4\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{\left(a^4(c-d)^2\tan(e+fx)\right)\text{Subst}}{cdf\sqrt{a-a\sec(e+f}}$$

$$= -\frac{a^3(c-d)^2\tan(e+fx)}{2cd(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2} + \frac{a^3(c-d)\tan(e+fx)}{c^2df\sqrt{a+a\sec(e+fx)}(c+d\sec}$$

$$= \frac{2a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{2a^{7/2}\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+f}{c^3\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+}$$

$$= \frac{2a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{a^{7/2}(c-d)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e}{c^2d^{3/2}\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(}$$

$$= \frac{2a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{3a^{7/2}(c-d)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan}{4cd^{3/2}(c+d)^{5/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec}}$$

Mathematica [C] time = 25.668, size = 3368, normalized size = 6.28

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] ((d + c*Cos[e + f*x])^3*Sec[(e + f*x)/2]^5*Sec[e + f*x]*(a*(1 + Sec[e + f*x
]))^(5/2)*(-((c^3 - 12*c^2*d + 5*c*d^2 + 6*d^3)*Sin[(e + f*x)/2])/(16*c^3*d
*(c + d)^2) + (-(c^2*d*Sin[(e + f*x)/2]) + 2*c*d^2*Sin[(e + f*x)/2] - d^3*S
in[(e + f*x)/2])/(8*c^3*(c + d)*(d + c*Cos[e + f*x])^2) + (3*c^3*Sin[(e + f
*x)/2] - 14*c^2*d*Sin[(e + f*x)/2] + 3*c*d^2*Sin[(e + f*x)/2] + 8*d^3*Sin[(
e + f*x)/2])/(16*c^3*(c + d)^2*(d + c*Cos[e + f*x]))))/(f*(c + d*Sec[e + f*
x])^3) - (Sqrt[3 - 2*Sqrt[2]]*Cos[(e + f*x)/4]^2*(d + c*Cos[e + f*x])^3*(c*
(c^3 + 18*c^2*d + 9*c*d^2 + 4*d^3)*EllipticF[ArcSin[Tan[(e + f*x)/4]/Sqrt[3
 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + 16*d*(c + d)^3*EllipticPi[-3 + 2*Sqrt[2]
, -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + (c^4 +
10*c^3*d - 15*c^2*d^2 - 20*c*d^3 - 8*d^4)*(EllipticPi[-(((-3 + 2*Sqrt[2])*(
c + d))/(3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] - d)), -ArcSin[Tan[(e + f*x)/4]/Sq
rt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + EllipticPi[((-3 + 2*Sqrt[2])*(c + d)
)/(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] + d), -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 -
 2*Sqrt[2]]], 17 - 12*Sqrt[2]]))*Sec[(e + f*x)/2]^5*((7*Cos[(e + f*x)/2]*Sq
rt[Sec[e + f*x]])/(16*(c + d)^2*(d + c*Cos[e + f*x])) + (c*Cos[(e + f*x)/2]
*Sqrt[Sec[e + f*x]])/(32*d*(c + d)^2*(d + c*Cos[e + f*x])) + (d*Cos[(e + f*
x)/2]*Sqrt[Sec[e + f*x]])/(32*c*(c + d)^2*(d + c*Cos[e + f*x])) + (Cos[(3*(
e + f*x))/2]*Sqrt[Sec[e + f*x]])/(8*(c + d)^2*(d + c*Cos[e + f*x])) + (d*Co
s[(3*(e + f*x))/2]*Sqrt[Sec[e + f*x]])/(4*c*(c + d)^2*(d + c*Cos[e + f*x]))
 + (d^2*Cos[(3*(e + f*x))/2]*Sqrt[Sec[e + f*x]])/(8*c^2*(c + d)^2*(d + c*Co
s[e + f*x])))*Sec[e + f*x]*(a*(1 + Sec[e + f*x]))^(5/2)*Sqrt[1 + (-3 + 2*Sq
rt[2])*Tan[(e + f*x)/4]^2]*Sqrt[1 - (3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2])/(8
*c^3*d*(c + d)^3*f*(c + d*Sec[e + f*x])^3*((Sqrt[3 - 2*Sqrt[2]]*(3 + 2*Sqrt
[2])*(c*(c^3 + 18*c^2*d + 9*c*d^2 + 4*d^3)*EllipticF[ArcSin[Tan[(e + f*x)/4
]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + 16*d*(c + d)^3*EllipticPi[-3 + 2
*Sqrt[2], -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] +
 (c^4 + 10*c^3*d - 15*c^2*d^2 - 20*c*d^3 - 8*d^4)*(EllipticPi[-(((-3 + 2*Sq
rt[2])*(c + d))/(3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] - d)), -ArcSin[Tan[(e + f*
x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + EllipticPi[((-3 + 2*Sqrt[2])
*(c + d))/(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] + d), -ArcSin[Tan[(e + f*x)/4]/
Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]]))*Sqrt[Sec[e + f*x]]*Tan[(e + f*x)/4
]*Sqrt[1 + (-3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2])/(32*c^3*d*(c + d)^3*Sqrt[1
 - (3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2]) - (Sqrt[3 - 2*Sqrt[2]]*(-3 + 2*Sqrt
[2])*(c*(c^3 + 18*c^2*d + 9*c*d^2 + 4*d^3)*EllipticF[ArcSin[Tan[(e + f*x)/4
]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + 16*d*(c + d)^3*EllipticPi[-3 + 2
*Sqrt[2], -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] +
 (c^4 + 10*c^3*d - 15*c^2*d^2 - 20*c*d^3 - 8*d^4)*(EllipticPi[-(((-3 + 2*Sq
rt[2])*(c + d))/(3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] - d)), -ArcSin[Tan[(e + f*
x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + EllipticPi[((-3 + 2*Sqrt[2])
*(c + d))/(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] + d), -ArcSin[Tan[(e + f*x)/4]/
Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]]))*Sqrt[Sec[e + f*x]]*Tan[(e + f*x)/4
]*Sqrt[1 - (3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2])/(32*c^3*d*(c + d)^3*Sqrt[1
+ (-3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2]) + (Sqrt[3 - 2*Sqrt[2]]*Cos[(e + f*x
)/4]*(c*(c^3 + 18*c^2*d + 9*c*d^2 + 4*d^3)*EllipticF[ArcSin[Tan[(e + f*x)/4
```

```
]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + 16*d*(c + d)^3*EllipticPi[-3 + 2
*Sqrt[2], -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] +
 (c^4 + 10*c^3*d - 15*c^2*d^2 - 20*c*d^3 - 8*d^4)*(EllipticPi[-(((-3 + 2*Sq
rt[2])*(c + d))/(3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] - d)), -ArcSin[Tan[(e + f*
x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + EllipticPi[((-3 + 2*Sqrt[2])
*(c + d))/(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] + d), -ArcSin[Tan[(e + f*x)/4]/
Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]]))*Sqrt[Sec[e + f*x]]*Sin[(e + f*x)/4
]*Sqrt[1 + (-3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2]*Sqrt[1 - (3 + 2*Sqrt[2])*Ta
n[(e + f*x)/4]^2])/(16*c^3*d*(c + d)^3) - (Sqrt[3 - 2*Sqrt[2]]*Cos[(e + f*x
)/4]^2*(c*(c^3 + 18*c^2*d + 9*c*d^2 + 4*d^3)*EllipticF[ArcSin[Tan[(e + f*x)
/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + 16*d*(c + d)^3*EllipticPi[-3 +
 2*Sqrt[2], -ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]]
 + (c^4 + 10*c^3*d - 15*c^2*d^2 - 20*c*d^3 - 8*d^4)*(EllipticPi[-(((-3 + 2*
Sqrt[2])*(c + d))/(3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] - d)), -ArcSin[Tan[(e +
f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + EllipticPi[((-3 + 2*Sqrt[2
])*(c + d))/(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] + d), -ArcSin[Tan[(e + f*x)/4
]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]]))*Sec[e + f*x]^(3/2)*Sin[e + f*x]*
Sqrt[1 + (-3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2]*Sqrt[1 - (3 + 2*Sqrt[2])*Tan[
(e + f*x)/4]^2])/(16*c^3*d*(c + d)^3) - (Sqrt[3 - 2*Sqrt[2]]*Cos[(e + f*x)/
4]^2*Sqrt[Sec[e + f*x]]*Sqrt[1 + (-3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2]*Sqrt[
1 - (3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2]*((c*(c^3 + 18*c^2*d + 9*c*d^2 + 4*d
^3)*Sec[(e + f*x)/4]^2)/(4*Sqrt[3 - 2*Sqrt[2]]*Sqrt[1 - Tan[(e + f*x)/4]^2/
(3 - 2*Sqrt[2])]*Sqrt[1 - ((17 - 12*Sqrt[2])*Tan[(e + f*x)/4]^2)/(3 - 2*Sqr
t[2])]) - (4*d*(c + d)^3*Sec[(e + f*x)/4]^2)/(Sqrt[3 - 2*Sqrt[2]]*Sqrt[1 -
Tan[(e + f*x)/4]^2/(3 - 2*Sqrt[2])]*Sqrt[1 - ((17 - 12*Sqrt[2])*Tan[(e + f*
x)/4]^2)/(3 - 2*Sqrt[2])]*(1 - ((-3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2)/(3 - 2
*Sqrt[2]))) + (c^4 + 10*c^3*d - 15*c^2*d^2 - 20*c*d^3 - 8*d^4)*(-Sec[(e + f
*x)/4]^2/(4*Sqrt[3 - 2*Sqrt[2]]*Sqrt[1 - Tan[(e + f*x)/4]^2/(3 - 2*Sqrt[2])
]*Sqrt[1 - ((17 - 12*Sqrt[2])*Tan[(e + f*x)/4]^2)/(3 - 2*Sqrt[2])]*(1 + ((-
3 + 2*Sqrt[2])*(c + d)*Tan[(e + f*x)/4]^2)/((3 - 2*Sqrt[2])*(3*c + 2*Sqrt[2
]*Sqrt[c*(c - d)] - d)))) - Sec[(e + f*x)/4]^2/(4*Sqrt[3 - 2*Sqrt[2]]*Sqrt[
1 - Tan[(e + f*x)/4]^2/(3 - 2*Sqrt[2])]*Sqrt[1 - ((17 - 12*Sqrt[2])*Tan[(e
+ f*x)/4]^2)/(3 - 2*Sqrt[2])]*(1 - ((-3 + 2*Sqrt[2])*(c + d)*Tan[(e + f*x)/
4]^2)/((3 - 2*Sqrt[2])*(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] + d)))))))/(8*c^3*
d*(c + d)^3)))
```

Maple [B] time = 6.465, size = 209489, normalized size = 390.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^3,x)
```

```
[Out] result too large to display
```

Maximo [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 172.515, size = 7337, normalized size = 13.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [-1/8*((a^2*c^4*d^2 + 10*a^2*c^3*d^3 - 15*a^2*c^2*d^4 - 20*a^2*c*d^5 - 8*a^
2*d^6 + (a^2*c^6 + 10*a^2*c^5*d - 15*a^2*c^4*d^2 - 20*a^2*c^3*d^3 - 8*a^2*c
^2*d^4)*cos(f*x + e)^3 + (a^2*c^6 + 12*a^2*c^5*d + 5*a^2*c^4*d^2 - 50*a^2*c
^3*d^3 - 48*a^2*c^2*d^4 - 16*a^2*c*d^5)*cos(f*x + e)^2 + (2*a^2*c^5*d + 21*
a^2*c^4*d^2 - 20*a^2*c^3*d^3 - 55*a^2*c^2*d^4 - 36*a^2*c*d^5 - 8*a^2*d^6)*c
os(f*x + e))*sqrt(-a/(c*d + d^2))*log((2*(c*d + d^2)*sqrt(-a/(c*d + d^2))*s
qrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c + 2
*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2 +
(c + d)*cos(f*x + e) + d)) - 8*(a^2*c^2*d^3 + 2*a^2*c*d^4 + a^2*d^5 + (a^2*
c^4*d + 2*a^2*c^3*d^2 + a^2*c^2*d^3)*cos(f*x + e)^3 + (a^2*c^4*d + 4*a^2*c^
3*d^2 + 5*a^2*c^2*d^3 + 2*a^2*c*d^4)*cos(f*x + e)^2 + (2*a^2*c^3*d^2 + 5*a^
2*c^2*d^3 + 4*a^2*c*d^4 + a^2*d^5)*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x
+ e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*si
n(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*((a^2*c^5 - 12*a^2
*c^4*d + 5*a^2*c^3*d^2 + 6*a^2*c^2*d^3)*cos(f*x + e)^2 - (a^2*c^4*d + 10*a^
2*c^3*d^2 - 7*a^2*c^2*d^3 - 4*a^2*c*d^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)
 + a)/cos(f*x + e))*sin(f*x + e))/((c^7*d + 2*c^6*d^2 + c^5*d^3)*f*cos(f*x
+ e)^3 + (c^7*d + 4*c^6*d^2 + 5*c^5*d^3 + 2*c^4*d^4)*f*cos(f*x + e)^2 + (2*
```

c^6*d^2 + 5*c^5*d^3 + 4*c^4*d^4 + c^3*d^5)*f*cos(f*x + e) + (c^5*d^3 + 2*c^
4*d^4 + c^3*d^5)*f), -1/8*(16*(a^2*c^2*d^3 + 2*a^2*c*d^4 + a^2*d^5 + (a^2*c
^4*d + 2*a^2*c^3*d^2 + a^2*c^2*d^3)*cos(f*x + e)^3 + (a^2*c^4*d + 4*a^2*c^3
*d^2 + 5*a^2*c^2*d^3 + 2*a^2*c*d^4)*cos(f*x + e)^2 + (2*a^2*c^3*d^2 + 5*a^2
*c^2*d^3 + 4*a^2*c*d^4 + a^2*d^5)*cos(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + (a^2*c^4
*d^2 + 10*a^2*c^3*d^3 - 15*a^2*c^2*d^4 - 20*a^2*c*d^5 - 8*a^2*d^6 + (a^2*c^
6 + 10*a^2*c^5*d - 15*a^2*c^4*d^2 - 20*a^2*c^3*d^3 - 8*a^2*c^2*d^4)*cos(f*x
 + e)^3 + (a^2*c^6 + 12*a^2*c^5*d + 5*a^2*c^4*d^2 - 50*a^2*c^3*d^3 - 48*a^2
*c^2*d^4 - 16*a^2*c*d^5)*cos(f*x + e)^2 + (2*a^2*c^5*d + 21*a^2*c^4*d^2 - 2
0*a^2*c^3*d^3 - 55*a^2*c^2*d^4 - 36*a^2*c*d^5 - 8*a^2*d^6)*cos(f*x + e))*sq
rt(-a/(c*d + d^2))*log((2*(c*d + d^2)*sqrt(-a/(c*d + d^2))*sqrt((a*cos(f*x
+ e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c + 2*a*d)*cos(f*x +
 e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2 + (c + d)*cos(f*x
 + e) + d)) + 2*((a^2*c^5 - 12*a^2*c^4*d + 5*a^2*c^3*d^2 + 6*a^2*c^2*d^3)*c
os(f*x + e)^2 - (a^2*c^4*d + 10*a^2*c^3*d^2 - 7*a^2*c^2*d^3 - 4*a^2*c*d^4)*
cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/((c^7*d
 + 2*c^6*d^2 + c^5*d^3)*f*cos(f*x + e)^3 + (c^7*d + 4*c^6*d^2 + 5*c^5*d^3 +
 2*c^4*d^4)*f*cos(f*x + e)^2 + (2*c^6*d^2 + 5*c^5*d^3 + 4*c^4*d^4 + c^3*d^5
)*f*cos(f*x + e) + (c^5*d^3 + 2*c^4*d^4 + c^3*d^5)*f), -1/4*((a^2*c^4*d^2 +
 10*a^2*c^3*d^3 - 15*a^2*c^2*d^4 - 20*a^2*c*d^5 - 8*a^2*d^6 + (a^2*c^6 + 10
*a^2*c^5*d - 15*a^2*c^4*d^2 - 20*a^2*c^3*d^3 - 8*a^2*c^2*d^4)*cos(f*x + e)^
3 + (a^2*c^6 + 12*a^2*c^5*d + 5*a^2*c^4*d^2 - 50*a^2*c^3*d^3 - 48*a^2*c^2*d
^4 - 16*a^2*c*d^5)*cos(f*x + e)^2 + (2*a^2*c^5*d + 21*a^2*c^4*d^2 - 20*a^2*
c^3*d^3 - 55*a^2*c^2*d^4 - 36*a^2*c*d^5 - 8*a^2*d^6)*cos(f*x + e))*sqrt(a/(
c*d + d^2))*arctan((c + d)*sqrt(a/(c*d + d^2))*sqrt((a*cos(f*x + e) + a)/co
s(f*x + e))*cos(f*x + e)/(a*sin(f*x + e))) - 4*(a^2*c^2*d^3 + 2*a^2*c*d^4 +
 a^2*d^5 + (a^2*c^4*d + 2*a^2*c^3*d^2 + a^2*c^2*d^3)*cos(f*x + e)^3 + (a^2*
c^4*d + 4*a^2*c^3*d^2 + 5*a^2*c^2*d^3 + 2*a^2*c*d^4)*cos(f*x + e)^2 + (2*a^
2*c^3*d^2 + 5*a^2*c^2*d^3 + 4*a^2*c*d^4 + a^2*d^5)*cos(f*x + e))*sqrt(-a)*l
og((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))
*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + ((a^
2*c^5 - 12*a^2*c^4*d + 5*a^2*c^3*d^2 + 6*a^2*c^2*d^3)*cos(f*x + e)^2 - (a^2
*c^4*d + 10*a^2*c^3*d^2 - 7*a^2*c^2*d^3 - 4*a^2*c*d^4)*cos(f*x + e))*sqrt((
a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/((c^7*d + 2*c^6*d^2 + c^5*d
^3)*f*cos(f*x + e)^3 + (c^7*d + 4*c^6*d^2 + 5*c^5*d^3 + 2*c^4*d^4)*f*cos(f*
x + e)^2 + (2*c^6*d^2 + 5*c^5*d^3 + 4*c^4*d^4 + c^3*d^5)*f*cos(f*x + e) + (
c^5*d^3 + 2*c^4*d^4 + c^3*d^5)*f), -1/4*((a^2*c^4*d^2 + 10*a^2*c^3*d^3 - 15
*a^2*c^2*d^4 - 20*a^2*c*d^5 - 8*a^2*d^6 + (a^2*c^6 + 10*a^2*c^5*d - 15*a^2*
c^4*d^2 - 20*a^2*c^3*d^3 - 8*a^2*c^2*d^4)*cos(f*x + e)^3 + (a^2*c^6 + 12*a^
2*c^5*d + 5*a^2*c^4*d^2 - 50*a^2*c^3*d^3 - 48*a^2*c^2*d^4 - 16*a^2*c*d^5)*c
os(f*x + e)^2 + (2*a^2*c^5*d + 21*a^2*c^4*d^2 - 20*a^2*c^3*d^3 - 55*a^2*c^2
*d^4 - 36*a^2*c*d^5 - 8*a^2*d^6)*cos(f*x + e))*sqrt(a/(c*d + d^2))*arctan((
c + d)*sqrt(a/(c*d + d^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x
+ e)/(a*sin(f*x + e))) + 8*(a^2*c^2*d^3 + 2*a^2*c*d^4 + a^2*d^5 + (a^2*c^4*

```
d + 2*a^2*c^3*d^2 + a^2*c^2*d^3)*cos(f*x + e)^3 + (a^2*c^4*d + 4*a^2*c^3*d^
2 + 5*a^2*c^2*d^3 + 2*a^2*c*d^4)*cos(f*x + e)^2 + (2*a^2*c^3*d^2 + 5*a^2*c^
2*d^3 + 4*a^2*c*d^4 + a^2*d^5)*cos(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(f*x
 + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + ((a^2*c^5 -
 12*a^2*c^4*d + 5*a^2*c^3*d^2 + 6*a^2*c^2*d^3)*cos(f*x + e)^2 - (a^2*c^4*d
+ 10*a^2*c^3*d^2 - 7*a^2*c^2*d^3 - 4*a^2*c*d^4)*cos(f*x + e))*sqrt((a*cos(f
*x + e) + a)/cos(f*x + e))*sin(f*x + e))/((c^7*d + 2*c^6*d^2 + c^5*d^3)*f*c
os(f*x + e)^3 + (c^7*d + 4*c^6*d^2 + 5*c^5*d^3 + 2*c^4*d^4)*f*cos(f*x + e)^
2 + (2*c^6*d^2 + 5*c^5*d^3 + 4*c^4*d^4 + c^3*d^5)*f*cos(f*x + e) + (c^5*d^3
 + 2*c^4*d^4 + c^3*d^5)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.166 $\int \frac{(c+d\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal. Leaf size=258

$$\frac{2\sqrt{a}c^3\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2d^2(3c-d)\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}-\frac{\sqrt{2}\sqrt{a}(c-d)^3\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

```
[Out] (2*(3*c - d)*d^2*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) + (2*d^3*Tan[e
+ f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) - (2*d^3*(1 - Sec[e + f*x])*Tan[e + f*
x])/(3*f*Sqrt[a + a*Sec[e + f*x]]) + (2*Sqrt[a]*c^3*ArcTanh[Sqrt[a - a*Sec[
e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec
[e + f*x]]) - (Sqrt[2]*Sqrt[a]*(c - d)^3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(
Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[
e + f*x]])
```

Rubi [A] time = 0.207306, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {3940, 180, 63, 206, 43}

$$\frac{2\sqrt{a}c^3\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2d^2(3c-d)\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}-\frac{\sqrt{2}\sqrt{a}(c-d)^3\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sec[e + f*x])^3/Sqrt[a + a*Sec[e + f*x]],x]
```


```
[Out] (2*(3*c - d)*d^2*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) + (2*d^3*Tan[e
+ f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) - (2*d^3*(1 - Sec[e + f*x])*Tan[e + f*
x])/(3*f*Sqrt[a + a*Sec[e + f*x]]) + (2*Sqrt[a]*c^3*ArcTanh[Sqrt[a - a*Sec[
e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec
[e + f*x]]) - (Sqrt[2]*Sqrt[a]*(c - d)^3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(
Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[
e + f*x]])
```


Rule 3940

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(a^2*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
```

```
x)^n)/(x*Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rule 180

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(c+d\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(c+dx)^3}{x\sqrt{a-ax}(a+ax)}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \left(\frac{(3c-d)d^2}{a\sqrt{a-ax}} + \frac{c^3}{ax\sqrt{a-ax}} + \frac{d^3x}{a\sqrt{a-ax}} - \frac{(c-d)^3}{a(1+x)\sqrt{a-ax}}\right)dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2(3c-d)d^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{\left(ac^3\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{\left(a(c-\right.}{}$$

$$= \frac{2(3c-d)d^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{\left(2c^3\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx, x, \sqrt{a-a\sec(e+fx)}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{(2}{}$$

$$= \frac{2(3c-d)d^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2d^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{2d^3(1-\sec(e+fx))\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}} + \frac{2\sqrt{a}}{f\sqrt{}}$$

Mathematica [C] time = 8.11463, size = 787, normalized size = 3.05

$$\frac{2\sqrt{\frac{1}{1-2\sin^2\left(\frac{1}{2}(e+fx)\right)}}\sqrt{1-2\sin^2\left(\frac{1}{2}(e+fx)\right)}\cos\left(\frac{1}{2}(e+fx)\right)(c+d\sec(e+fx))^3\left(-\frac{(c-d)^3\csc^5\left(\frac{1}{2}(e+fx)\right)\left(-12\sin^8\left(\frac{1}{2}(e+fx)\right)\cos^4\left(\frac{1}{2}\right.\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sec[e + f*x])^3/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*Cos[(e + f*x)/2]*(c + d*Sec[e + f*x])^3*Sqrt[(1 - 2*Sin[(e + f*x)/2]^2)^
(-1)]*Sqrt[1 - 2*Sin[(e + f*x)/2]^2]*((2*c*(c^2 + 3*d^2)*Sin[(e + f*x)/2])/
(3*(1 - 2*Sin[(e + f*x)/2]^2)^(3/2)) - (4*c^2*(c + 3*d)*Sin[(e + f*x)/2]^3)
/(3*(1 - 2*Sin[(e + f*x)/2]^2)^(3/2)) + (4*c*(c^2 + 3*d^2)*Sin[(e + f*x)/2]
)/(3*Sqrt[1 - 2*Sin[(e + f*x)/2]^2]) + (c^3*Csc[(e + f*x)/2]*Sqrt[1 - 2*Sin
[(e + f*x)/2]^2]*((4*Sin[(e + f*x)/2]^4)/(1 - 2*Sin[(e + f*x)/2]^2)^2 - (6*
Sin[(e + f*x)/2]^2)/(1 - 2*Sin[(e + f*x)/2]^2) + (3*Sqrt[2]*ArcSin[Sqrt[2]*
Sin[(e + f*x)/2]]*Sin[(e + f*x)/2])/Sqrt[1 - 2*Sin[(e + f*x)/2]^2]))/3 - ((
c - d)^3*Csc[(e + f*x)/2]^5*(-12*Cos[(e + f*x)/2]^4*HypergeometricPFQ[{2, 2
, 7/2}, {1, 9/2}, -(Sin[(e + f*x)/2]^2/(1 - 2*Sin[(e + f*x)/2]^2))]*Sin[(e
+ f*x)/2]^8 - 12*Hypergeometric2F1[2, 7/2, 9/2, -(Sin[(e + f*x)/2]^2/(1 - 2
```

```
*Sin[(e + f*x)/2]^2))]*Sin[(e + f*x)/2]^8*(4 - 7*Sin[(e + f*x)/2]^2 + 3*Sin
[(e + f*x)/2]^4) + 7*Sqrt[-(Sin[(e + f*x)/2]^2/(1 - 2*Sin[(e + f*x)/2]^2))]
*(1 - 2*Sin[(e + f*x)/2]^2)^3*(15 - 20*Sin[(e + f*x)/2]^2 + 8*Sin[(e + f*x)
/2]^4)*((3 - 7*Sin[(e + f*x)/2]^2)*Sqrt[-(Sin[(e + f*x)/2]^2/(1 - 2*Sin[(e
+ f*x)/2]^2))] - 3*ArcTanh[Sqrt[-(Sin[(e + f*x)/2]^2/(1 - 2*Sin[(e + f*x)/2
]^2))]]*(1 - 2*Sin[(e + f*x)/2]^2))))/(63*(1 - 2*Sin[(e + f*x)/2]^2)^(7/2))
))/(f*(d + c*Cos[e + f*x])^3*Sec[e + f*x]^(5/2)*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [B] time = 0.307, size = 907, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/6/f/a*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(3*sin(f*x+e)*cos(f*x+e)*2^(1
/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos
(f*x+e))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)*c^3+3*sin(f*x+e)*cos(f*x+e)*l
n(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+
e))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)*c^3-9*sin(f*x+e)*cos(f*x+e)*ln(-(-
(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))*(
-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)*c^2*d+9*sin(f*x+e)*cos(f*x+e)*ln(-(-(-2
*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))*(-2*
cos(f*x+e)/(1+cos(f*x+e)))^(3/2)*c*d^2-3*sin(f*x+e)*cos(f*x+e)*ln(-(-(-2*co
s(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))*(-2*cos
(f*x+e)/(1+cos(f*x+e)))^(3/2)*d^3+3*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^
(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/c
os(f*x+e))*c^3*sin(f*x+e)+3*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)*ln(-(-(-2*
cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))*c^3*s
in(f*x+e)-9*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)*ln(-(-(-2*cos(f*x+e)/(1+co
s(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))*c^2*d*sin(f*x+e)+9*(-
2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/
2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))*c*d^2*sin(f*x+e)-3*(-2*cos(f*x+e)/(
1+cos(f*x+e)))^(3/2)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+
cos(f*x+e)-1)/sin(f*x+e))*d^3*sin(f*x+e)-36*cos(f*x+e)^2*c*d^2+4*cos(f*x+e)
^2*d^3+36*cos(f*x+e)*c*d^2-8*cos(f*x+e)*d^3+4*d^3)/sin(f*x+e)/cos(f*x+e)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 72.6639, size = 1553, normalized size = 6.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*sqrt(2)*((a*c^3 - 3*a*c^2*d + 3*a*c*d^2 - a*d^3)*cos(f*x + e)^2 +
(a*c^3 - 3*a*c^2*d + 3*a*c*d^2 - a*d^3)*cos(f*x + e))*sqrt(-1/a)*log(-(2*sq
rt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x + e)*sin(f
*x + e) - 3*cos(f*x + e)^2 - 2*cos(f*x + e) + 1)/(cos(f*x + e)^2 + 2*cos(f*
x + e) + 1)) + 6*(c^3*cos(f*x + e)^2 + c^3*cos(f*x + e))*sqrt(-a)*log((2*a*
cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x
 + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 4*(d^3 + (9*
c*d^2 - d^3)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x
+ e))/(a*f*cos(f*x + e)^2 + a*f*cos(f*x + e)), -1/3*(6*(c^3*cos(f*x + e)^2
+ c^3*cos(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*
cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 2*(d^3 + (9*c*d^2 - d^3)*cos(f*x + e
))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 3*sqrt(2)*((a*c^3
 - 3*a*c^2*d + 3*a*c*d^2 - a*d^3)*cos(f*x + e)^2 + (a*c^3 - 3*a*c^2*d + 3*a
*c*d^2 - a*d^3)*cos(f*x + e))*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(
f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))/sqrt(a))/(a*f*cos(f*x + e)^2
 + a*f*cos(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c + d \sec \left(e + fx\right)\right)^{3}}{\sqrt{a \left(\sec \left(e + fx\right) + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))**3/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((c + d*sec(e + f*x))**3/sqrt(a*(sec(e + f*x) + 1)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.167 $\int \frac{(c+d\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal. Leaf size=183

$$\frac{2\sqrt{a}c^2\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{\sqrt{2}\sqrt{a}(c-d)^2\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2d^2\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

```
[Out] (2*d^2*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) + (2*Sqrt[a]*c^2*ArcTanh[
Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]
*Sqrt[a + a*Sec[e + f*x]]) - (Sqrt[2]*Sqrt[a]*(c - d)^2*ArcTanh[Sqrt[a - a*
Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*
Sqrt[a + a*Sec[e + f*x]])
```

Rubi [A] time = 0.158058, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3940, 180, 63, 206}

$$\frac{2\sqrt{a}c^2\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{\sqrt{2}\sqrt{a}(c-d)^2\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2d^2\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sec[e + f*x])^2/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*d^2*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) + (2*Sqrt[a]*c^2*ArcTanh[
Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]
*Sqrt[a + a*Sec[e + f*x]]) - (Sqrt[2]*Sqrt[a]*(c - d)^2*ArcTanh[Sqrt[a - a*
Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*
Sqrt[a + a*Sec[e + f*x]])
```

Rule 3940

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(a^2*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
x)^n)/(x*Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rule 180

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(c+d\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\operatorname{Subst}\left(\int \frac{(c+dx)^2}{x\sqrt{a-ax}(a+ax)}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\left(a^2\tan(e+fx)\right)\operatorname{Subst}\left(\int \left(\frac{d^2}{a\sqrt{a-ax}}+\frac{c^2}{ax\sqrt{a-ax}}-\frac{(c-d)^2}{a(1+x)\sqrt{a-ax}}\right)dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2d^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{\left(ac^2\tan(e+fx)\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{\left(a(c-d)^2\,\mathrm{ta}\cdots\right.}{\cdots}$$

$$= \frac{2d^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{\left(2c^2\tan(e+fx)\right)\operatorname{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx, x, \sqrt{a-a\sec(e+fx)}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{\left(2(c-\cdots\right.}{\cdots}$$

$$= \frac{2d^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2\sqrt{a}c^2\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{\sqrt{2}\sqrt{a}(c-d)^2\tanh^{-1}\left(\cdots\right.}{f\sqrt{a-a\sec(e+f\cdots}}$$

Mathematica [C] time = 2.46796, size = 295, normalized size = 1.61

$$\frac{2\cos\left(\frac{1}{2}(e+fx)\right)\cos^{\frac{3}{2}}(e+fx)(c+d\sec(e+fx))^2\left(-\frac{(c-d)^2\sin\left(\frac{1}{2}(e+fx)\right)\sin^2(e+fx)\text{Hypergeometric2F1}\left(2,\frac{5}{2},\frac{7}{2},\sin^2\left(\frac{1}{2}(e+fx)\right)(-\sec(e+f}{10\cos^{\frac{5}{2}}(e+fx)}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sec[e + f*x])^2/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*Cos[(e + f*x)/2]*Cos[e + f*x]^(3/2)*(c + d*Sec[e + f*x])^2*(-((c - d)^2*
Sqrt[-1 + Cos[e + f*x]]*(2 + Cos[e + f*x])*Csc[(e + f*x)/2]^3*(-2*ArcTanh[S
qrt[-(Sec[e + f*x]*Sin[(e + f*x)/2]^2)]] + Sqrt[2 - 2*Sec[e + f*x]]))/(2*Sq
rt[2]) + (4*c*d*Sin[(e + f*x)/2])/Sqrt[Cos[e + f*x]] + c^2*(Sqrt[2]*ArcSin[
Sqrt[2]*Sin[(e + f*x)/2]] - (2*Sin[(e + f*x)/2])/Sqrt[Cos[e + f*x]]) - ((c
- d)^2*Hypergeometric2F1[2, 5/2, 7/2, -(Sec[e + f*x]*Sin[(e + f*x)/2]^2)]*S
in[(e + f*x)/2]*Sin[e + f*x]^2)/(10*Cos[e + f*x]^(5/2))))/(f*(d + c*Cos[e +
 f*x])^2*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [B] time = 0.24, size = 358, normalized size = 2.

$$-\frac{1}{af\sin\left(fx+e\right)}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\left(\sqrt{-2\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\sqrt{2}\text{Artanh}\left(\frac{\sqrt{2}\sin\left(fx+e\right)}{2\cos\left(fx+e\right)}\sqrt{-2\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\right)c^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/f/a*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*((-2*cos(f*x+e)/(1+cos(f*x+e))
)^(1/2)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*si
n(f*x+e)/cos(f*x+e))*c^2*sin(f*x+e)+(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln
(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e
))*c^2*sin(f*x+e)-2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(-(-(-2*cos(f*x+
e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))*c*d*sin(f*x+e
)+(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))
^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))*d^2*sin(f*x+e)+2*cos(f*x+e)*d^2
-2*d^2)/sin(f*x+e)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 16.9463, size = 1246, normalized size = 6.81

$$\left[\frac{4\,d^2\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin\left(fx+e\right)+\sqrt{2}\left(ac^2-2\,acd+ad^2+\left(ac^2-2\,acd+ad^2\right)\cos\left(fx+e\right)\right)\sqrt{-\frac{1}{a}}\log\left(\frac{2\,\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{-\frac{1}{a}}\cdots}{\cdots}\right.}{2\left(\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(4*d^2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) + sqrt(2)*
(a*c^2 - 2*a*c*d + a*d^2 + (a*c^2 - 2*a*c*d + a*d^2)*cos(f*x + e))*sqrt(-1/
a)*log((2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*
x + e)*sin(f*x + e) + 3*cos(f*x + e)^2 + 2*cos(f*x + e) - 1)/(cos(f*x + e)^
2 + 2*cos(f*x + e) + 1)) - 2*(c^2*cos(f*x + e) + c^2)*sqrt(-a)*log((2*a*cos
(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x +
e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/(a*f*cos(f*x + e
) + a*f), (2*d^2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 2*(
c^2*cos(f*x + e) + c^2)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x +
e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + sqrt(2)*(a*c^2 - 2*a*c*d + a*d^2
 + (a*c^2 - 2*a*c*d + a*d^2)*cos(f*x + e))*arctan(sqrt(2)*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))/sqrt(a))/(a*f*c
os(f*x + e) + a*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c+d\sec\left(e+fx\right)\right)^2}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))**2/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((c + d*sec(e + f*x))**2/sqrt(a*(sec(e + f*x) + 1)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.168 $\int \frac{c+d\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal. Leaf size=91

$$\frac{2c\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f}-\frac{\sqrt{2}(c-d)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f}$$

```
[Out] (2*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(Sqrt[a]*f) -
 (Sqrt[2]*(c - d)*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e +
 f*x]])])/(Sqrt[a]*f)
```

Rubi [A] time = 0.109533, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3920, 3774, 203, 3795}

$$\frac{2c\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f}-\frac{\sqrt{2}(c-d)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sec[e + f*x])/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(Sqrt[a]*f) -
 (Sqrt[2]*(c - d)*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e +
 f*x]])])/(Sqrt[a]*f)
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{c+d\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx &= \frac{c\int\sqrt{a+a\sec(e+fx)}\,dx}{a}-(c-d)\int\frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx\\
&= -\frac{(2c)\operatorname{Subst}\left(\int\frac{1}{a+x^2}\,dx,x,-\frac{a\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}+\frac{(2(c-d))\operatorname{Subst}\left(\int\frac{1}{2a+x^2}\,dx,x,-\frac{a\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}\\
&= \frac{2c\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}f}-\frac{\sqrt{2}(c-d)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}f}
\end{aligned}$$

Mathematica [A] time = 0.28695, size = 92, normalized size = 1.01

$$\frac{2\cos\left(\frac{1}{2}(e+fx)\right)\left((d-c)\tan^{-1}\left(\frac{\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{\cos(e+fx)}}\right)+\sqrt{2}c\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)}{f\sqrt{\cos(e+fx)}\sqrt{a(\sec(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sec[e + f*x])/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*(Sqrt[2]*c*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]] + (-c + d)*ArcTan[Sin[(e + f
*x)/2]/Sqrt[Cos[e + f*x]]])*Cos[(e + f*x)/2])/(f*Sqrt[Cos[e + f*x]]*Sqrt[a*
(1 + Sec[e + f*x])])
```

Maple [B] time = 0.217, size = 194, normalized size = 2.1

$$-\frac{1}{af}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\sqrt{-2\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\left(c\sqrt{2}\mathrm{Artanh}\left(\frac{\sqrt{2}\sin\left(fx+e\right)}{2\cos\left(fx+e\right)}\sqrt{-2\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\right)+c\ln\left(-\frac{1}{\sin\left(f\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/f/a*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))
^(1/2)*(c*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*
sin(f*x+e)/cos(f*x+e))+c*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x
+e)+cos(f*x+e)-1)/sin(f*x+e))-d*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*
sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 5.14631, size = 824, normalized size = 9.05

$$\left[-\frac{\sqrt{2}(ac-ad)\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{-\frac{1}{a}}\cos(fx+e)\sin(fx+e)-3\cos(fx+e)^2-2\cos(fx+e)+1}{\cos(fx+e)^2+2\cos(fx+e)+1}\right)+2\sqrt{-a}c\log\left(\frac{2a\cos(fx+e)^2+2\sqrt{-a}\dots}{\dots}\right.}{2af}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(2)*(a*c - a*d)*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(f*x + e)
+ a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) - 3*cos(f*x + e)^2
- 2*cos(f*x + e) + 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 2*sqrt(-a)*c
*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e
))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/(a*
f), -(2*sqrt(a)*c*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x +
e)/(sqrt(a)*sin(f*x + e))) - sqrt(2)*(a*c - a*d)*arctan(sqrt(2)*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))/sqrt(a))/
(a*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{c + d\sec\left(e + fx\right)}{\sqrt{a\left(\sec\left(e + fx\right) + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((c + d*sec(e + f*x))/sqrt(a*(sec(e + f*x) + 1)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.169 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx$

Optimal. Leaf size=166

$$\frac{2d^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}cf(c-d)\sqrt{c+d}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f(c-d)}+\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}cf}$$

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(Sqrt[a]*c*f) -
 (Sqrt[2]*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])]
)/(Sqrt[a]*(c - d)*f) + (2*d^(3/2)*ArcTan[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(S
qrt[c + d]*Sqrt[a + a*Sec[e + f*x]])])/(Sqrt[a]*c*(c - d)*Sqrt[c + d]*f)
```

Rubi [A] time = 0.372448, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {3929, 3920, 3774, 203, 3795, 3967, 205}

$$\frac{2d^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}cf(c-d)\sqrt{c+d}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f(c-d)}+\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}cf}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(Sqrt[a]*c*f) -
 (Sqrt[2]*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])]
)/(Sqrt[a]*(c - d)*f) + (2*d^(3/2)*ArcTan[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(S
qrt[c + d]*Sqrt[a + a*Sec[e + f*x]])])/(Sqrt[a]*c*(c - d)*Sqrt[c + d]*f)
```

Rule 3929

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))), x_Symbol] :> Dist[1/(c*(b*c - a*d)), Int[(b*c - a*d - b*d*C
sc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[d^2/(c*(b*c - a*d)), I
nt[(Csc[e + f*x]*Sqrt[a + b*Csc[e + f*x]])/(c + d*Csc[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && (EqQ[a^2 - b^2, 0] ||
EqQ[c^2 - d^2, 0])
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3967

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[(-2*b)/f, Subst[Int
[1/(b*c + a*d + d*x^2), x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \frac{\int \frac{ac-ad-ad\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx}{ac(c-d)} + \frac{d^2\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx}{ac(c-d)}$$

$$= \frac{\int \sqrt{a+a\sec(e+fx)}\,dx}{ac} - \frac{\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx}{c-d} - \frac{(2d^2)\,\text{Subst}\left(\int \frac{1}{ac+ad+dx^2}\right.}{c(c-d}$$

$$= \frac{2d^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}c(c-d)\sqrt{c+d}f} - \frac{2\,\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{a\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf} +$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}cf} - \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}(c-d)f} + \frac{2d^{3/2}\tan^{-1}\left(\frac{\sqrt{a}}{\sqrt{c+a}}\right.}{\sqrt{a}c(c-d)}$$

Mathematica [C] time = 38.7076, size = 431238, normalized size = 2597.82

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.249, size = 662, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/2/f/(d/(c-d))^(1/2)/(c-d)/c/((c+d)*(c-d))^(1/2)/a*(2*((c+d)*(c-d))^(1/2)
*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e
)/cos(f*x+e))*(d/(c-d))^(1/2)*c-2*((c+d)*(c-d))^(1/2)*2^(1/2)*arctanh(1/2*2
^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*(d/(c-d)
)^(1/2)*d+2*((c+d)*(c-d))^(1/2)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*
sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))*(d/(c-d))^(1/2)*c-2^(1/2)*ln(-2*((-2*c
```

```
os(f*x+e)/(1+cos(f*x+e)))^(1/2)*(d/(c-d))^(1/2)*2^(1/2)*c*sin(f*x+e)-2^(1/2
)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*d*sin(f*x+e)-((c+d)*
(c-d))^(1/2)*cos(f*x+e)-c*sin(f*x+e)+d*sin(f*x+e)+((c+d)*(c-d))^(1/2))/(((c
+d)*(c-d))^(1/2)*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d))*d^2+2^(1/2)*ln(
2*((-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(d/(c-d))^(1/2)*2^(1/2)*c*sin(f*x+e
)-2^(1/2)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*d*sin(f*x+e)
+((c+d)*(c-d))^(1/2)*cos(f*x+e)-c*sin(f*x+e)+d*sin(f*x+e)-((c+d)*(c-d))^(1/
2))/(((c+d)*(c-d))^(1/2)*sin(f*x+e)-c*cos(f*x+e)+d*cos(f*x+e)+c-d))*d^2)*(-
2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\sec\left(fx+e\right)+a}\left(d\sec\left(fx+e\right)+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*sec(f*x + e) + a)*(d*sec(f*x + e) + c)), x)
```

Fricas [A] time = 100.191, size = 2695, normalized size = 16.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(2)*a*c*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos
(f*x + e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) - 3*cos(f*x + e)^2 - 2*cos(
f*x + e) + 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 2*a*d*sqrt(-d/(a*c +
 a*d))*log((2*(c + d)*sqrt(-d/(a*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*
x + e))*cos(f*x + e)*sin(f*x + e) + (c + 2*d)*cos(f*x + e)^2 + (c + d)*cos(
f*x + e) - d)/(c*cos(f*x + e)^2 + (c + d)*cos(f*x + e) + d)) + 2*sqrt(-a)*(
c - d)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f
*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)
))/((a*c^2 - a*c*d)*f), -1/2*(sqrt(2)*a*c*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((
a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) - 3*
```

```
cos(f*x + e)^2 - 2*cos(f*x + e) + 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1))
 + 4*a*d*sqrt(d/(a*c + a*d))*arctan((c + d)*sqrt(d/(a*c + a*d))*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(d*sin(f*x + e))) + 2*sqrt(-a)*(c
 - d)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*
x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1))
)/((a*c^2 - a*c*d)*f), -(a*d*sqrt(-d/(a*c + a*d))*log((2*(c + d)*sqrt(-d/(a
*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e
) + (c + 2*d)*cos(f*x + e)^2 + (c + d)*cos(f*x + e) - d)/(c*cos(f*x + e)^2
+ (c + d)*cos(f*x + e) + d)) - sqrt(2)*sqrt(a)*c*arctan(sqrt(2)*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + 2*sqrt(
a)*(c - d)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqr
t(a)*sin(f*x + e))))/((a*c^2 - a*c*d)*f), -(2*a*d*sqrt(d/(a*c + a*d))*arcta
n((c + d)*sqrt(d/(a*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f
*x + e)/(d*sin(f*x + e))) - sqrt(2)*sqrt(a)*c*arctan(sqrt(2)*sqrt((a*cos(f*
x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + 2*sqrt(a)*
(c - d)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a
)*sin(f*x + e))))/((a*c^2 - a*c*d)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\left(c+d\sec\left(e+fx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sec(e + f*x) + 1))*(c + d*sec(e + f*x))), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.170 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2}\,dx$

Optimal. Leaf size=416

$$\frac{d^2\tan(e+fx)}{cf\left(c^2-d^2\right)\sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))}+\frac{2\sqrt{a}d^{3/2}(2c-d)\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{c^2f(c-d)^2\sqrt{c+d}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2\sqrt{a}\,\mathrm{ta}}{c^2f\sqrt{a}}$$

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(c^2*f*S
qrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (Sqrt[2]*Sqrt[a]*ArcTan
h[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/((c - d)^2*f*Sq
rt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (Sqrt[a]*d^(3/2)*ArcTanh
[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])/(c
*(c - d)*(c + d)^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])
 + (2*Sqrt[a]*(2*c - d)*d^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/
(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])/(c^2*(c - d)^2*Sqrt[c + d]*f*Sqrt[a -
a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (d^2*Tan[e + f*x])/(c*(c^2 - d^
2)*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x]))
```

Rubi [A] time = 0.422065, antiderivative size = 416, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3940, 180, 63, 206, 51, 208}

$$\frac{d^2\tan(e+fx)}{cf\left(c^2-d^2\right)\sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))}+\frac{2\sqrt{a}d^{3/2}(2c-d)\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{c^2f(c-d)^2\sqrt{c+d}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2\sqrt{a}\,\mathrm{ta}}{c^2f\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^2),x]
```


```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(c^2*f*S
qrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (Sqrt[2]*Sqrt[a]*ArcTan
h[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/((c - d)^2*f*Sq
rt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (Sqrt[a]*d^(3/2)*ArcTanh
[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])/(c
*(c - d)*(c + d)^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])
 + (2*Sqrt[a]*(2*c - d)*d^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/
(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])/(c^2*(c - d)^2*Sqrt[c + d]*f*Sqrt[a -
a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (d^2*Tan[e + f*x])/(c*(c^2 - d^
```

```
2)*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x]))
```

Rule 3940

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(a^2*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
x)^n)/(x*Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rule 180

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}(a+ax)(c+dx)^2}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \left(\frac{1}{ac^2x\sqrt{a-ax}} - \frac{1}{a(c-d)^2(1+x)\sqrt{a-ax}} + \frac{d^2}{ac(c-d)\sqrt{a-ax}(}\right.\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec}}$$

$$= -\frac{(a\tan(e+fx))\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{(a\tan(e+fx))}{(c-d)^2f\sqrt{a}}$$

$$= \frac{d^2\tan(e+fx)}{c\left(c^2-d^2\right)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))} + \frac{(2\tan(e+fx))\,\text{Su}}{c^2f\sqrt{a-a}}$$

$$= \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{\sqrt{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e}}{\sqrt{2}\sqrt{a}}\right.}{(c-d)^2f\sqrt{a-a\sec(e+fx}}$$

$$= \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{\sqrt{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e}}{\sqrt{2}\sqrt{a}}\right.}{(c-d)^2f\sqrt{a-a\sec(e+fx}}$$

Mathematica [C] time = 35.8607, size = 472069, normalized size = 1134.78

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 2.335, size = 117715, normalized size = 283.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\sec\left(fx+e\right)+a}\left(d\sec\left(fx+e\right)+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/(sqrt(a*sec(f*x + e) + a)*(d*sec(f*x + e) + c)^2), x)
```

Fricas [A] time = 96.0913, size = 6564, normalized size = 15.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/4*(4*(c^2*d^2 - c*d^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x +
 e)*sin(f*x + e) + sqrt(2)*(a*c^3*d + a*c^2*d^2 + (a*c^4 + a*c^3*d)*cos(f*x
 + e)^2 + (a*c^4 + 2*a*c^3*d + a*c^2*d^2)*cos(f*x + e))*sqrt(-1/a)*log((4*s
qrt(2)*(3*cos(f*x + e)^2 - cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x
+ e))*sqrt(-1/a)*sin(f*x + e) + 17*cos(f*x + e)^3 + 3*cos(f*x + e)^2 - 13*c
os(f*x + e) + 1)/(cos(f*x + e)^3 + 3*cos(f*x + e)^2 + 3*cos(f*x + e) + 1))
- (5*a*c^2*d^2 + a*c*d^3 - 2*a*d^4 + (5*a*c^3*d + a*c^2*d^2 - 2*a*c*d^3)*co
s(f*x + e)^2 + (5*a*c^3*d + 6*a*c^2*d^2 - a*c*d^3 - 2*a*d^4)*cos(f*x + e))*
sqrt(-d/(a*c + a*d))*log(((c^2 + 8*c*d + 8*d^2)*cos(f*x + e)^3 + (c^2 + 2*c
*d)*cos(f*x + e)^2 + 4*((c^2 + 3*c*d + 2*d^2)*cos(f*x + e)^2 - (c*d + d^2)*
cos(f*x + e))*sqrt(-d/(a*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*
sin(f*x + e) + d^2 - (6*c*d + 7*d^2)*cos(f*x + e))/(c^2*cos(f*x + e)^3 + (c
^2 + 2*c*d)*cos(f*x + e)^2 + d^2 + (2*c*d + d^2)*cos(f*x + e))) - 2*(c^3*d
- c^2*d^2 - c*d^3 + d^4 + (c^4 - c^3*d - c^2*d^2 + c*d^3)*cos(f*x + e)^2 +
```

```
(c^4 - 2*c^2*d^2 + d^4)*cos(f*x + e))*sqrt(-a)*log((8*a*cos(f*x + e)^3 + 4*
(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*
x + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1)))/((a*c^6 -
 a*c^5*d - a*c^4*d^2 + a*c^3*d^3)*f*cos(f*x + e)^2 + (a*c^6 - 2*a*c^4*d^2 +
 a*c^2*d^4)*f*cos(f*x + e) + (a*c^5*d - a*c^4*d^2 - a*c^3*d^3 + a*c^2*d^4)*
f), 1/4*(4*(c^2*d^2 - c*d^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*
x + e)*sin(f*x + e) + sqrt(2)*(a*c^3*d + a*c^2*d^2 + (a*c^4 + a*c^3*d)*cos(
f*x + e)^2 + (a*c^4 + 2*a*c^3*d + a*c^2*d^2)*cos(f*x + e))*sqrt(-1/a)*log((
4*sqrt(2)*(3*cos(f*x + e)^2 - cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f
*x + e))*sqrt(-1/a)*sin(f*x + e) + 17*cos(f*x + e)^3 + 3*cos(f*x + e)^2 - 1
3*cos(f*x + e) + 1)/(cos(f*x + e)^3 + 3*cos(f*x + e)^2 + 3*cos(f*x + e) + 1
)) - 2*(5*a*c^2*d^2 + a*c*d^3 - 2*a*d^4 + (5*a*c^3*d + a*c^2*d^2 - 2*a*c*d^
3)*cos(f*x + e)^2 + (5*a*c^3*d + 6*a*c^2*d^2 - a*c*d^3 - 2*a*d^4)*cos(f*x +
 e))*sqrt(d/(a*c + a*d))*arctan(1/2*((c + 2*d)*cos(f*x + e) - d)*sqrt(d/(a*
c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))/(d*sin(f*x + e))) - 2*(c^
3*d - c^2*d^2 - c*d^3 + d^4 + (c^4 - c^3*d - c^2*d^2 + c*d^3)*cos(f*x + e)^
2 + (c^4 - 2*c^2*d^2 + d^4)*cos(f*x + e))*sqrt(-a)*log((8*a*cos(f*x + e)^3
+ 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/co
s(f*x + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1)))/((a*c
^6 - a*c^5*d - a*c^4*d^2 + a*c^3*d^3)*f*cos(f*x + e)^2 + (a*c^6 - 2*a*c^4*d
^2 + a*c^2*d^4)*f*cos(f*x + e) + (a*c^5*d - a*c^4*d^2 - a*c^3*d^3 + a*c^2*d
^4)*f), 1/4*(4*(c^2*d^2 - c*d^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*co
s(f*x + e)*sin(f*x + e) - 4*(c^3*d - c^2*d^2 - c*d^3 + d^4 + (c^4 - c^3*d -
 c^2*d^2 + c*d^3)*cos(f*x + e)^2 + (c^4 - 2*c^2*d^2 + d^4)*cos(f*x + e))*sq
rt(a)*arctan(1/2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*(2*cos(f*x + e) -
1)/(sqrt(a)*sin(f*x + e))) - (5*a*c^2*d^2 + a*c*d^3 - 2*a*d^4 + (5*a*c^3*d
+ a*c^2*d^2 - 2*a*c*d^3)*cos(f*x + e)^2 + (5*a*c^3*d + 6*a*c^2*d^2 - a*c*d^
3 - 2*a*d^4)*cos(f*x + e))*sqrt(-d/(a*c + a*d))*log(((c^2 + 8*c*d + 8*d^2)*
cos(f*x + e)^3 + (c^2 + 2*c*d)*cos(f*x + e)^2 + 4*((c^2 + 3*c*d + 2*d^2)*co
s(f*x + e)^2 - (c*d + d^2)*cos(f*x + e))*sqrt(-d/(a*c + a*d))*sqrt((a*cos(f
*x + e) + a)/cos(f*x + e))*sin(f*x + e) + d^2 - (6*c*d + 7*d^2)*cos(f*x + e
))/(c^2*cos(f*x + e)^3 + (c^2 + 2*c*d)*cos(f*x + e)^2 + d^2 + (2*c*d + d^2)
*cos(f*x + e))) + 2*sqrt(2)*(a*c^3*d + a*c^2*d^2 + (a*c^4 + a*c^3*d)*cos(f*
x + e)^2 + (a*c^4 + 2*a*c^3*d + a*c^2*d^2)*cos(f*x + e))*arctan(1/4*sqrt(2)
*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*(3*cos(f*x + e) - 1)/(sqrt(a)*sin(
f*x + e)))/sqrt(a))/((a*c^6 - a*c^5*d - a*c^4*d^2 + a*c^3*d^3)*f*cos(f*x +
e)^2 + (a*c^6 - 2*a*c^4*d^2 + a*c^2*d^4)*f*cos(f*x + e) + (a*c^5*d - a*c^4*
d^2 - a*c^3*d^3 + a*c^2*d^4)*f), 1/2*(2*(c^2*d^2 - c*d^3)*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - (5*a*c^2*d^2 + a*c*d^3 -
 2*a*d^4 + (5*a*c^3*d + a*c^2*d^2 - 2*a*c*d^3)*cos(f*x + e)^2 + (5*a*c^3*d
+ 6*a*c^2*d^2 - a*c*d^3 - 2*a*d^4)*cos(f*x + e))*sqrt(d/(a*c + a*d))*arctan
(1/2*((c + 2*d)*cos(f*x + e) - d)*sqrt(d/(a*c + a*d))*sqrt((a*cos(f*x + e)
+ a)/cos(f*x + e))/(d*sin(f*x + e))) - 2*(c^3*d - c^2*d^2 - c*d^3 + d^4 + (
c^4 - c^3*d - c^2*d^2 + c*d^3)*cos(f*x + e)^2 + (c^4 - 2*c^2*d^2 + d^4)*cos
(f*x + e))*sqrt(a)*arctan(1/2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*(2*co
```

```
s(f*x + e) - 1)/(sqrt(a)*sin(f*x + e))) + sqrt(2)*(a*c^3*d + a*c^2*d^2 + (a
*c^4 + a*c^3*d)*cos(f*x + e)^2 + (a*c^4 + 2*a*c^3*d + a*c^2*d^2)*cos(f*x +
e))*arctan(1/4*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*(3*cos(f*x +
 e) - 1)/(sqrt(a)*sin(f*x + e)))/sqrt(a))/((a*c^6 - a*c^5*d - a*c^4*d^2 + a
*c^3*d^3)*f*cos(f*x + e)^2 + (a*c^6 - 2*a*c^4*d^2 + a*c^2*d^4)*f*cos(f*x +
e) + (a*c^5*d - a*c^4*d^2 - a*c^3*d^3 + a*c^2*d^4)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\left(c+d\sec\left(e+fx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*sec(f*x+e))**2/(a+a*sec(f*x+e))**(1/2),x)

[Out] Integral(1/(sqrt(a*(sec(e + f*x) + 1))*(c + d*sec(e + f*x))**2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")

[Out] Timed out
```

3.171 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3}\,dx$

Optimal. Leaf size=653

$$\frac{d^2(2c-d)\tan(e+fx)}{c^2f(c-d)^2(c+d)\sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))}+\frac{d^2\tan(e+fx)}{2cf\left(c^2-d^2\right)\sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))^2}+\frac{2\sqrt{a}d}{c^3}$$

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(c^3*f*S
qrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (Sqrt[2]*Sqrt[a]*ArcTan
h[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/((c - d)^3*f*Sq
rt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (3*Sqrt[a]*d^(3/2)*ArcTa
nh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])/
(4*c*(c - d)*(c + d)^(5/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*
x]]) + (Sqrt[a]*(2*c - d)*d^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]]
)/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])/(c^2*(c - d)^2*(c + d)^(3/2)*f*Sqrt[
a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (2*Sqrt[a]*d^(3/2)*(3*c^2 -
 3*c*d + d^2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c +
d])]*Tan[e + f*x])/(c^3*(c - d)^3*Sqrt[c + d]*f*Sqrt[a - a*Sec[e + f*x]]*Sq
rt[a + a*Sec[e + f*x]]) + (d^2*Tan[e + f*x])/(2*c*(c^2 - d^2)*f*Sqrt[a + a*
Sec[e + f*x]]*(c + d*Sec[e + f*x])^2) + (3*d^2*Tan[e + f*x])/(4*c*(c - d)*(
c + d)^2*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])) + ((2*c - d)*d^2*
Tan[e + f*x])/(c^2*(c - d)^2*(c + d)*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[
e + f*x]))
```

Rubi [A] time = 0.646345, antiderivative size = 653, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3940, 180, 63, 206, 51, 208}

$$\frac{d^2(2c-d)\tan(e+fx)}{c^2f(c-d)^2(c+d)\sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))}+\frac{d^2\tan(e+fx)}{2cf\left(c^2-d^2\right)\sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))^2}+\frac{2\sqrt{a}d}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^3),x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(c^3*f*S
qrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (Sqrt[2]*Sqrt[a]*ArcTan
h[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/((c - d)^3*f*Sq
```

```
rt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (3*Sqrt[a]*d^(3/2)*ArcTa
nh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])/
(4*c*(c - d)*(c + d)^(5/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*
x]]) + (Sqrt[a]*(2*c - d)*d^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]]
)/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])/(c^2*(c - d)^2*(c + d)^(3/2)*f*Sqrt[
a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (2*Sqrt[a]*d^(3/2)*(3*c^2 -
 3*c*d + d^2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c +
d])]*Tan[e + f*x])/(c^3*(c - d)^3*Sqrt[c + d]*f*Sqrt[a - a*Sec[e + f*x]]*Sq
rt[a + a*Sec[e + f*x]]) + (d^2*Tan[e + f*x])/(2*c*(c^2 - d^2)*f*Sqrt[a + a*
Sec[e + f*x]]*(c + d*Sec[e + f*x])^2) + (3*d^2*Tan[e + f*x])/(4*c*(c - d)*(
c + d)^2*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])) + ((2*c - d)*d^2*
Tan[e + f*x])/(c^2*(c - d)^2*(c + d)*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[
e + f*x]))
```

Rule 3940

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(a^2*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
x)^n)/(x*Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rule 180

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a-ax}(a+ax)(c+dx)^3}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\left(a^2\tan(e+fx)\right)\operatorname{Subst}\left(\int\left(\frac{1}{ac^3x\sqrt{a-ax}} - \frac{1}{a(c-d)^3(1+x)\sqrt{a-ax}} + \frac{d^2}{ac(c-d)\sqrt{a-ax}(\cdots}\right.\right.}{f\sqrt{a-a\sec(e+fx)\cdots}}$$

$$= -\frac{(a\tan(e+fx))\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{(a\tan(e+fx))\cdots}{(c-d)^3f\sqrt{a\cdots}}$$

$$= \frac{d^2\tan(e+fx)}{2c\left(c^2-d^2\right)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2} + \frac{\cdots}{c^2(c-d)^2(c+d\cdots}$$

$$= \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{\sqrt{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e\cdots}}{\sqrt{2}\sqrt{a}\cdots}\right.}{(c-d)^3f\sqrt{a-a\sec(e+fx\cdots}}$$

$$= \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{\sqrt{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e\cdots}}{\sqrt{2}\sqrt{a}\cdots}\right.}{(c-d)^3f\sqrt{a-a\sec(e+fx\cdots}}$$

$$= \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{\sqrt{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e\cdots}}{\sqrt{2}\sqrt{a}\cdots}\right.}{(c-d)^3f\sqrt{a-a\sec(e+fx\cdots}}$$

Mathematica [C] time = 38.6795, size = 652560, normalized size = 999.33

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^3),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 20.612, size = 402966, normalized size = 617.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima"
)
```

```
[Out] Timed out
```

Fricas [A] time = 132.514, size = 10966, normalized size = 16.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas"
)
```

```
[Out] [-1/16*(4*sqrt(2)*(a*c^5*d^2 + 2*a*c^4*d^3 + a*c^3*d^4 + (a*c^7 + 2*a*c^6*d
 + a*c^5*d^2)*cos(f*x + e)^3 + (a*c^7 + 4*a*c^6*d + 5*a*c^5*d^2 + 2*a*c^4*d
^3)*cos(f*x + e)^2 + (2*a*c^6*d + 5*a*c^5*d^2 + 4*a*c^4*d^3 + a*c^3*d^4)*co
s(f*x + e))*sqrt(-1/a)*log(-(4*sqrt(2)*(3*cos(f*x + e)^2 - cos(f*x + e))*sq
rt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*sin(f*x + e) - 17*cos(f*x
+ e)^3 - 3*cos(f*x + e)^2 + 13*cos(f*x + e) - 1)/(cos(f*x + e)^3 + 3*cos(f*
x + e)^2 + 3*cos(f*x + e) + 1)) + (35*a*c^4*d^3 + 14*a*c^3*d^4 - 21*a*c^2*d
^5 - 4*a*c*d^6 + 8*a*d^7 + (35*a*c^6*d + 14*a*c^5*d^2 - 21*a*c^4*d^3 - 4*a*
c^3*d^4 + 8*a*c^2*d^5)*cos(f*x + e)^3 + (35*a*c^6*d + 84*a*c^5*d^2 + 7*a*c^
4*d^3 - 46*a*c^3*d^4 + 16*a*c*d^6)*cos(f*x + e)^2 + (70*a*c^5*d^2 + 63*a*c^
4*d^3 - 28*a*c^3*d^4 - 29*a*c^2*d^5 + 12*a*c*d^6 + 8*a*d^7)*cos(f*x + e))*s
qrt(-d/(a*c + a*d))*log(((c^2 + 8*c*d + 8*d^2)*cos(f*x + e)^3 + (c^2 + 2*c*
d)*cos(f*x + e)^2 + 4*((c^2 + 3*c*d + 2*d^2)*cos(f*x + e)^2 - (c*d + d^2)*c
os(f*x + e))*sqrt(-d/(a*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*s
in(f*x + e) + d^2 - (6*c*d + 7*d^2)*cos(f*x + e))/(c^2*cos(f*x + e)^3 + (c^
2 + 2*c*d)*cos(f*x + e)^2 + d^2 + (2*c*d + d^2)*cos(f*x + e))) + 8*(c^5*d^2
 - c^4*d^3 - 2*c^3*d^4 + 2*c^2*d^5 + c*d^6 - d^7 + (c^7 - c^6*d - 2*c^5*d^2
 + 2*c^4*d^3 + c^3*d^4 - c^2*d^5)*cos(f*x + e)^3 + (c^7 + c^6*d - 4*c^5*d^2
 - 2*c^4*d^3 + 5*c^3*d^4 + c^2*d^5 - 2*c*d^6)*cos(f*x + e)^2 + (2*c^6*d - c
^5*d^2 - 5*c^4*d^3 + 2*c^3*d^4 + 4*c^2*d^5 - c*d^6 - d^7)*cos(f*x + e))*sqr
t(-a)*log((8*a*cos(f*x + e)^3 + 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a
)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x + e) +
 a)/(cos(f*x + e) + 1)) - 4*((13*c^5*d^2 - 12*c^4*d^3 - 7*c^3*d^4 + 6*c^2*d
^5)*cos(f*x + e)^2 + (11*c^4*d^3 - 10*c^3*d^4 - 5*c^2*d^5 + 4*c*d^6)*cos(f*
x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/((a*c^10 - a*
c^9*d - 2*a*c^8*d^2 + 2*a*c^7*d^3 + a*c^6*d^4 - a*c^5*d^5)*f*cos(f*x + e)^3
 + (a*c^10 + a*c^9*d - 4*a*c^8*d^2 - 2*a*c^7*d^3 + 5*a*c^6*d^4 + a*c^5*d^5
- 2*a*c^4*d^6)*f*cos(f*x + e)^2 + (2*a*c^9*d - a*c^8*d^2 - 5*a*c^7*d^3 + 2*
a*c^6*d^4 + 4*a*c^5*d^5 - a*c^4*d^6 - a*c^3*d^7)*f*cos(f*x + e) + (a*c^8*d^
2 - a*c^7*d^3 - 2*a*c^6*d^4 + 2*a*c^5*d^5 + a*c^4*d^6 - a*c^3*d^7)*f), -1/8
*(2*sqrt(2)*(a*c^5*d^2 + 2*a*c^4*d^3 + a*c^3*d^4 + (a*c^7 + 2*a*c^6*d + a*c
^5*d^2)*cos(f*x + e)^3 + (a*c^7 + 4*a*c^6*d + 5*a*c^5*d^2 + 2*a*c^4*d^3)*co
s(f*x + e)^2 + (2*a*c^6*d + 5*a*c^5*d^2 + 4*a*c^4*d^3 + a*c^3*d^4)*cos(f*x
+ e))*sqrt(-1/a)*log(-(4*sqrt(2)*(3*cos(f*x + e)^2 - cos(f*x + e))*sqrt((a*
cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*sin(f*x + e) - 17*cos(f*x + e)^3
 - 3*cos(f*x + e)^2 + 13*cos(f*x + e) - 1)/(cos(f*x + e)^3 + 3*cos(f*x + e)
^2 + 3*cos(f*x + e) + 1)) + (35*a*c^4*d^3 + 14*a*c^3*d^4 - 21*a*c^2*d^5 - 4
*a*c*d^6 + 8*a*d^7 + (35*a*c^6*d + 14*a*c^5*d^2 - 21*a*c^4*d^3 - 4*a*c^3*d^
4 + 8*a*c^2*d^5)*cos(f*x + e)^3 + (35*a*c^6*d + 84*a*c^5*d^2 + 7*a*c^4*d^3
- 46*a*c^3*d^4 + 16*a*c*d^6)*cos(f*x + e)^2 + (70*a*c^5*d^2 + 63*a*c^4*d^3
- 28*a*c^3*d^4 - 29*a*c^2*d^5 + 12*a*c*d^6 + 8*a*d^7)*cos(f*x + e))*sqrt(d/
```

(a*c + a*d))*arctan(1/2*((c + 2*d)*cos(f*x + e) - d)*sqrt(d/(a*c + a*d))*sq
rt((a*cos(f*x + e) + a)/cos(f*x + e))/(d*sin(f*x + e))) + 4*(c^5*d^2 - c^4*
d^3 - 2*c^3*d^4 + 2*c^2*d^5 + c*d^6 - d^7 + (c^7 - c^6*d - 2*c^5*d^2 + 2*c^
4*d^3 + c^3*d^4 - c^2*d^5)*cos(f*x + e)^3 + (c^7 + c^6*d - 4*c^5*d^2 - 2*c^
4*d^3 + 5*c^3*d^4 + c^2*d^5 - 2*c*d^6)*cos(f*x + e)^2 + (2*c^6*d - c^5*d^2
- 5*c^4*d^3 + 2*c^3*d^4 + 4*c^2*d^5 - c*d^6 - d^7)*cos(f*x + e))*sqrt(-a)*l
og((8*a*cos(f*x + e)^3 + 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt(
(a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(co
s(f*x + e) + 1)) - 2*((13*c^5*d^2 - 12*c^4*d^3 - 7*c^3*d^4 + 6*c^2*d^5)*cos
(f*x + e)^2 + (11*c^4*d^3 - 10*c^3*d^4 - 5*c^2*d^5 + 4*c*d^6)*cos(f*x + e))
*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/((a*c^10 - a*c^9*d -
 2*a*c^8*d^2 + 2*a*c^7*d^3 + a*c^6*d^4 - a*c^5*d^5)*f*cos(f*x + e)^3 + (a*c
^10 + a*c^9*d - 4*a*c^8*d^2 - 2*a*c^7*d^3 + 5*a*c^6*d^4 + a*c^5*d^5 - 2*a*c
^4*d^6)*f*cos(f*x + e)^2 + (2*a*c^9*d - a*c^8*d^2 - 5*a*c^7*d^3 + 2*a*c^6*d
^4 + 4*a*c^5*d^5 - a*c^4*d^6 - a*c^3*d^7)*f*cos(f*x + e) + (a*c^8*d^2 - a*c
^7*d^3 - 2*a*c^6*d^4 + 2*a*c^5*d^5 + a*c^4*d^6 - a*c^3*d^7)*f), -1/16*(16*(
c^5*d^2 - c^4*d^3 - 2*c^3*d^4 + 2*c^2*d^5 + c*d^6 - d^7 + (c^7 - c^6*d - 2*
c^5*d^2 + 2*c^4*d^3 + c^3*d^4 - c^2*d^5)*cos(f*x + e)^3 + (c^7 + c^6*d - 4*
c^5*d^2 - 2*c^4*d^3 + 5*c^3*d^4 + c^2*d^5 - 2*c*d^6)*cos(f*x + e)^2 + (2*c^
6*d - c^5*d^2 - 5*c^4*d^3 + 2*c^3*d^4 + 4*c^2*d^5 - c*d^6 - d^7)*cos(f*x +
e))*sqrt(a)*arctan(1/2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*(2*cos(f*x +
 e) - 1)/(sqrt(a)*sin(f*x + e))) + (35*a*c^4*d^3 + 14*a*c^3*d^4 - 21*a*c^2*
d^5 - 4*a*c*d^6 + 8*a*d^7 + (35*a*c^6*d + 14*a*c^5*d^2 - 21*a*c^4*d^3 - 4*a
*c^3*d^4 + 8*a*c^2*d^5)*cos(f*x + e)^3 + (35*a*c^6*d + 84*a*c^5*d^2 + 7*a*c
^4*d^3 - 46*a*c^3*d^4 + 16*a*c*d^6)*cos(f*x + e)^2 + (70*a*c^5*d^2 + 63*a*c
^4*d^3 - 28*a*c^3*d^4 - 29*a*c^2*d^5 + 12*a*c*d^6 + 8*a*d^7)*cos(f*x + e))*
sqrt(-d/(a*c + a*d))*log(((c^2 + 8*c*d + 8*d^2)*cos(f*x + e)^3 + (c^2 + 2*c
*d)*cos(f*x + e)^2 + 4*((c^2 + 3*c*d + 2*d^2)*cos(f*x + e)^2 - (c*d + d^2)*
cos(f*x + e))*sqrt(-d/(a*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*
sin(f*x + e) + d^2 - (6*c*d + 7*d^2)*cos(f*x + e))/(c^2*cos(f*x + e)^3 + (c
^2 + 2*c*d)*cos(f*x + e)^2 + d^2 + (2*c*d + d^2)*cos(f*x + e))) - 4*((13*c^
5*d^2 - 12*c^4*d^3 - 7*c^3*d^4 + 6*c^2*d^5)*cos(f*x + e)^2 + (11*c^4*d^3 -
10*c^3*d^4 - 5*c^2*d^5 + 4*c*d^6)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/c
os(f*x + e))*sin(f*x + e) - 8*sqrt(2)*(a*c^5*d^2 + 2*a*c^4*d^3 + a*c^3*d^4
+ (a*c^7 + 2*a*c^6*d + a*c^5*d^2)*cos(f*x + e)^3 + (a*c^7 + 4*a*c^6*d + 5*a
*c^5*d^2 + 2*a*c^4*d^3)*cos(f*x + e)^2 + (2*a*c^6*d + 5*a*c^5*d^2 + 4*a*c^4
*d^3 + a*c^3*d^4)*cos(f*x + e))*arctan(1/4*sqrt(2)*sqrt((a*cos(f*x + e) + a
)/cos(f*x + e))*(3*cos(f*x + e) - 1)/(sqrt(a)*sin(f*x + e)))/sqrt(a))/((a*c
^10 - a*c^9*d - 2*a*c^8*d^2 + 2*a*c^7*d^3 + a*c^6*d^4 - a*c^5*d^5)*f*cos(f*
x + e)^3 + (a*c^10 + a*c^9*d - 4*a*c^8*d^2 - 2*a*c^7*d^3 + 5*a*c^6*d^4 + a*
c^5*d^5 - 2*a*c^4*d^6)*f*cos(f*x + e)^2 + (2*a*c^9*d - a*c^8*d^2 - 5*a*c^7*
d^3 + 2*a*c^6*d^4 + 4*a*c^5*d^5 - a*c^4*d^6 - a*c^3*d^7)*f*cos(f*x + e) + (
a*c^8*d^2 - a*c^7*d^3 - 2*a*c^6*d^4 + 2*a*c^5*d^5 + a*c^4*d^6 - a*c^3*d^7)*
f), -1/8*((35*a*c^4*d^3 + 14*a*c^3*d^4 - 21*a*c^2*d^5 - 4*a*c*d^6 + 8*a*d^7
 + (35*a*c^6*d + 14*a*c^5*d^2 - 21*a*c^4*d^3 - 4*a*c^3*d^4 + 8*a*c^2*d^5)*c

```
os(f*x + e)^3 + (35*a*c^6*d + 84*a*c^5*d^2 + 7*a*c^4*d^3 - 46*a*c^3*d^4 + 1
6*a*c*d^6)*cos(f*x + e)^2 + (70*a*c^5*d^2 + 63*a*c^4*d^3 - 28*a*c^3*d^4 - 2
9*a*c^2*d^5 + 12*a*c*d^6 + 8*a*d^7)*cos(f*x + e))*sqrt(d/(a*c + a*d))*arcta
n(1/2*((c + 2*d)*cos(f*x + e) - d)*sqrt(d/(a*c + a*d))*sqrt((a*cos(f*x + e)
 + a)/cos(f*x + e))/(d*sin(f*x + e))) + 8*(c^5*d^2 - c^4*d^3 - 2*c^3*d^4 +
2*c^2*d^5 + c*d^6 - d^7 + (c^7 - c^6*d - 2*c^5*d^2 + 2*c^4*d^3 + c^3*d^4 -
c^2*d^5)*cos(f*x + e)^3 + (c^7 + c^6*d - 4*c^5*d^2 - 2*c^4*d^3 + 5*c^3*d^4
+ c^2*d^5 - 2*c*d^6)*cos(f*x + e)^2 + (2*c^6*d - c^5*d^2 - 5*c^4*d^3 + 2*c^
3*d^4 + 4*c^2*d^5 - c*d^6 - d^7)*cos(f*x + e))*sqrt(a)*arctan(1/2*sqrt((a*c
os(f*x + e) + a)/cos(f*x + e))*(2*cos(f*x + e) - 1)/(sqrt(a)*sin(f*x + e)))
 - 2*((13*c^5*d^2 - 12*c^4*d^3 - 7*c^3*d^4 + 6*c^2*d^5)*cos(f*x + e)^2 + (1
1*c^4*d^3 - 10*c^3*d^4 - 5*c^2*d^5 + 4*c*d^6)*cos(f*x + e))*sqrt((a*cos(f*x
 + e) + a)/cos(f*x + e))*sin(f*x + e) - 4*sqrt(2)*(a*c^5*d^2 + 2*a*c^4*d^3
+ a*c^3*d^4 + (a*c^7 + 2*a*c^6*d + a*c^5*d^2)*cos(f*x + e)^3 + (a*c^7 + 4*a
*c^6*d + 5*a*c^5*d^2 + 2*a*c^4*d^3)*cos(f*x + e)^2 + (2*a*c^6*d + 5*a*c^5*d
^2 + 4*a*c^4*d^3 + a*c^3*d^4)*cos(f*x + e))*arctan(1/4*sqrt(2)*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*(3*cos(f*x + e) - 1)/(sqrt(a)*sin(f*x + e)))/sq
rt(a))/((a*c^10 - a*c^9*d - 2*a*c^8*d^2 + 2*a*c^7*d^3 + a*c^6*d^4 - a*c^5*d
^5)*f*cos(f*x + e)^3 + (a*c^10 + a*c^9*d - 4*a*c^8*d^2 - 2*a*c^7*d^3 + 5*a*
c^6*d^4 + a*c^5*d^5 - 2*a*c^4*d^6)*f*cos(f*x + e)^2 + (2*a*c^9*d - a*c^8*d^
2 - 5*a*c^7*d^3 + 2*a*c^6*d^4 + 4*a*c^5*d^5 - a*c^4*d^6 - a*c^3*d^7)*f*cos(
f*x + e) + (a*c^8*d^2 - a*c^7*d^3 - 2*a*c^6*d^4 + 2*a*c^5*d^5 + a*c^4*d^6 -
 a*c^3*d^7)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\left(c+d\sec\left(e+fx\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*sec(f*x+e))**3/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sec(e + f*x) + 1))*(c + d*sec(e + f*x))**3), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.172 $\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=324

$$\frac{2c^3\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{(c-d)^3\tan(e+fx)}{2af(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}}-\frac{(c-d)^3\tan(e+fx)\tanh^{-1}\left(\right.}{2\sqrt{2}\sqrt{a}f\sqrt{a-a\sec(e+fx)}\sqrt{a}}$$

```
[Out] (2*d^3*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e + f*x]]) - ((c - d)^3*Tan[e + f*
x])/(2*a*f*(1 + Sec[e + f*x])*Sqrt[a + a*Sec[e + f*x]]) + (2*c^3*ArcTanh[Sq
rt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(Sqrt[a]*f*Sqrt[a - a*Sec[e +
 f*x]]*Sqrt[a + a*Sec[e + f*x]]) - ((c - d)^3*ArcTanh[Sqrt[a - a*Sec[e + f*
x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(2*Sqrt[2]*Sqrt[a]*f*Sqrt[a - a*Sec[e
+ f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (Sqrt[2]*(c - d)^2*(c + 2*d)*ArcTanh[Sq
rt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(Sqrt[a]*f*Sqrt[a -
 a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])
```

Rubi [A] time = 0.236516, antiderivative size = 324, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {3940, 180, 63, 206, 51}

$$\frac{2c^3\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{(c-d)^3\tan(e+fx)}{2af(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}}-\frac{(c-d)^3\tan(e+fx)\tanh^{-1}\left(\right.}{2\sqrt{2}\sqrt{a}f\sqrt{a-a\sec(e+fx)}\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^(3/2),x]
```


```
[Out] (2*d^3*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e + f*x]]) - ((c - d)^3*Tan[e + f*
x])/(2*a*f*(1 + Sec[e + f*x])*Sqrt[a + a*Sec[e + f*x]]) + (2*c^3*ArcTanh[Sq
rt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(Sqrt[a]*f*Sqrt[a - a*Sec[e +
 f*x]]*Sqrt[a + a*Sec[e + f*x]]) - ((c - d)^3*ArcTanh[Sqrt[a - a*Sec[e + f*
x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(2*Sqrt[2]*Sqrt[a]*f*Sqrt[a - a*Sec[e
+ f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (Sqrt[2]*(c - d)^2*(c + 2*d)*ArcTanh[Sq
rt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(Sqrt[a]*f*Sqrt[a -
 a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])
```


Rule 3940

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(a^2*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
```

```
e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
x)^n)/(x*Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rule 180

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rubi steps

$$\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(c+dx)^3}{x\sqrt{a-ax}(a+ax)^2}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int\left(\frac{d^3}{a^2\sqrt{a-ax}}+\frac{c^3}{a^2x\sqrt{a-ax}}-\frac{(c-d)^3}{a^2(1+x)^2\sqrt{a-ax}}-\frac{(c-d)^2(c+2d)}{a^2(1+x)\sqrt{a-ax}}\right)dx, x, \sec\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2d^3\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}} - \frac{\left(c^3\tan(e+fx)\right)\text{Subst}\left(\int\frac{1}{x\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{\left((c-d)^3\right.}{}$$

$$= \frac{2d^3\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}} - \frac{(c-d)^3\tan(e+fx)}{2af(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + \frac{\left(2c^3\tan(e+fx)\right)\text{Su}}{af\sqrt{a-a\,\text{se}}}$$

$$= \frac{2d^3\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}} - \frac{(c-d)^3\tan(e+fx)}{2af(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + \frac{2c^3\tanh^{-1}\left(\frac{\sqrt{a-a\,\text{se}}}{\sqrt{}}\right.}{\sqrt{a}f\sqrt{a-a\sec(e+f}}$$

$$= \frac{2d^3\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}} - \frac{(c-d)^3\tan(e+fx)}{2af(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + \frac{2c^3\tanh^{-1}\left(\frac{\sqrt{a-a\,\text{se}}}{\sqrt{}}\right.}{\sqrt{a}f\sqrt{a-a\sec(e+f}}$$

Mathematica [C] time = 6.5066, size = 856, normalized size = 2.64

$$\frac{2\cos^3\left(\frac{1}{2}(e+fx)\right)(c+d\sec(e+fx))^3\sqrt{\frac{1}{1-2\sin^2\left(\frac{1}{2}(e+fx)\right)}}\sqrt{1-2\sin^2\left(\frac{1}{2}(e+fx)\right)}\left(-\frac{2\left(2\sqrt{2}\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin^2\left(\frac{1}{2}(e+fx)\right)-\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*Cos[(e + f*x)/2]^3*(c + d*Sec[e + f*x])^3*Sqrt[(1 - 2*Sin[(e + f*x)/2]^2
)^(-1)]*Sqrt[1 - 2*Sin[(e + f*x)/2]^2]*((-3*(c - d)^3*ArcTan[(1 - 2*Sin[(e
+ f*x)/2])/Sqrt[1 - 2*Sin[(e + f*x)/2]^2]])/2 + (3*(c - d)^3*ArcTan[(1 + 2*
```

```
Sin[(e + f*x)/2])/Sqrt[1 - 2*Sin[(e + f*x)/2]^2]])/2 - (4*c^2*(c - 3*d)*Sin
[(e + f*x)/2])/Sqrt[1 - 2*Sin[(e + f*x)/2]^2] + ((c - d)^3*(1 - 2*Sin[(e +
f*x)/2]))/(4*(1 + Sin[(e + f*x)/2])*Sqrt[1 - 2*Sin[(e + f*x)/2]^2]) - ((c -
 d)^3*(1 + 2*Sin[(e + f*x)/2]))/(4*(1 - Sin[(e + f*x)/2])*Sqrt[1 - 2*Sin[(e
 + f*x)/2]^2]) - ((c - d)^3*Sqrt[1 - 2*Sin[(e + f*x)/2]^2])/(1 - Sin[(e + f
*x)/2]) + ((c - d)^3*Sqrt[1 - 2*Sin[(e + f*x)/2]^2])/(1 + Sin[(e + f*x)/2])
 - (2*c^3*(-(Sqrt[2]*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]]) + 2*Sqrt[2]*ArcSin[S
qrt[2]*Sin[(e + f*x)/2]]*Sin[(e + f*x)/2]^2 + 2*Sin[(e + f*x)/2]*Sqrt[1 - 2
*Sin[(e + f*x)/2]^2]))/(1 - 2*Sin[(e + f*x)/2]^2) - ((c - d)^2*(11*c + d)*S
in[(e + f*x)/2]*((2*Cos[(e + f*x)/2]^2*Hypergeometric2F1[2, 5/2, 7/2, -(Sin
[(e + f*x)/2]^2/(1 - 2*Sin[(e + f*x)/2]^2))]*Sin[(e + f*x)/2]^2)/(1 - 2*Sin
[(e + f*x)/2]^2) + 5*Csc[(e + f*x)/2]^4*Sqrt[-(Sin[(e + f*x)/2]^2/(1 - 2*Si
n[(e + f*x)/2]^2))]*(1 - 2*Sin[(e + f*x)/2]^2)^2*(3 - 2*Sin[(e + f*x)/2]^2)
*(-ArcTanh[Sqrt[-(Sin[(e + f*x)/2]^2/(1 - 2*Sin[(e + f*x)/2]^2))]] + Sqrt[-
(Sin[(e + f*x)/2]^2/(1 - 2*Sin[(e + f*x)/2]^2))])))/(10*(1 - 2*Sin[(e + f*x
)/2]^2)^(3/2))))/(f*(d + c*Cos[e + f*x])^3*Sec[e + f*x]^(3/2)*(a*(1 + Sec[e
 + f*x]))^(3/2))
```

Maple [B] time = 0.268, size = 957, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(3/2),x)
```

```
[Out] 1/4/f/a^2*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(-1+cos(f*x+e))*(4*(-2*cos(
f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)*cos(f*x+e)*2^(1/2)*arctanh(1/2*2^(1
/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*c^3+5*(-2*c
os(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)*cos(f*x+e)*ln(-(-(-2*cos(f*x+e)/
(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))*c^3-3*(-2*cos(f*
x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)*cos(f*x+e)*ln(-(-(-2*cos(f*x+e)/(1+co
s(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))*c^2*d-9*(-2*cos(f*x+e
)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)*cos(f*x+e)*ln(-(-(-2*cos(f*x+e)/(1+cos(f
*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))*c*d^2+7*(-2*cos(f*x+e)/(
1+cos(f*x+e)))^(1/2)*sin(f*x+e)*cos(f*x+e)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+
e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))*d^3+4*2^(1/2)*(-2*cos(f*x+e
)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^
(1/2)*sin(f*x+e)/cos(f*x+e))*c^3*sin(f*x+e)+5*(-2*cos(f*x+e)/(1+cos(f*x+e))
)^(1/2)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)
/sin(f*x+e))*c^3*sin(f*x+e)-3*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(-(-(-
2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))*c^2
```

```
*d*sin(f*x+e)-9*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(-(-(-2*cos(f*x+e)/(
1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))*c*d^2*sin(f*x+e)+
7*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))
^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))*d^3*sin(f*x+e)-2*cos(f*x+e)^2*c
^3+6*cos(f*x+e)^2*c^2*d-6*cos(f*x+e)^2*c*d^2+10*cos(f*x+e)^2*d^3+2*cos(f*x+
e)*c^3-6*cos(f*x+e)*c^2*d+6*cos(f*x+e)*c*d^2-2*cos(f*x+e)*d^3-8*d^3)/sin(f*
x+e)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 66.1323, size = 1740, normalized size = 5.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(2)*(5*c^3 - 3*c^2*d - 9*c*d^2 + 7*d^3 + (5*c^3 - 3*c^2*d - 9*c*
d^2 + 7*d^3)*cos(f*x + e)^2 + 2*(5*c^3 - 3*c^2*d - 9*c*d^2 + 7*d^3)*cos(f*x
 + e))*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x
+ e))*cos(f*x + e)*sin(f*x + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f*x + e) + a
)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 8*(c^3*cos(f*x + e)^2 + 2*c^3*co
s(f*x + e) + c^3)*sqrt(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a
)/(cos(f*x + e) + 1)) - 4*(4*d^3 - (c^3 - 3*c^2*d + 3*c*d^2 - 5*d^3)*cos(f*
x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*
x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f), 1/4*(sqrt(2)*(5*c^3 - 3*c^2*d - 9
*c*d^2 + 7*d^3 + (5*c^3 - 3*c^2*d - 9*c*d^2 + 7*d^3)*cos(f*x + e)^2 + 2*(5*
c^3 - 3*c^2*d - 9*c*d^2 + 7*d^3)*cos(f*x + e))*sqrt(a)*arctan(sqrt(2)*sqrt(
(a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 8
*(c^3*cos(f*x + e)^2 + 2*c^3*cos(f*x + e) + c^3)*sqrt(a)*arctan(sqrt((a*cos
```

```
(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + 2*(4*d^
3 - (c^3 - 3*c^2*d + 3*c*d^2 - 5*d^3)*cos(f*x + e))*sqrt((a*cos(f*x + e) +
a)/cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e)
 + a^2*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c+d\sec\left(e+fx\right)\right)^{3}}{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))**3/(a+a*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((c + d*sec(e + f*x))**3/(a*(sec(e + f*x) + 1))**(3/2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.173 $\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=290

$$-\frac{\sqrt{2}\left(c^2-d^2\right)\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2c^2\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{(c-d)^2\tan}{2af(\sec(e+fx)+1)}$$

```
[Out] -((c - d)^2*Tan[e + f*x])/(2*a*f*(1 + Sec[e + f*x])*Sqrt[a + a*Sec[e + f*x]
]) + (2*c^2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(Sqrt[a
]*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - ((c - d)^2*ArcTanh
[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(2*Sqrt[2]*Sqrt[
a]*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (Sqrt[2]*(c^2 - d
^2)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(Sqrt
[a]*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])
```

Rubi [A] time = 0.216937, antiderivative size = 290, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {3940, 180, 63, 206, 51}

$$-\frac{\sqrt{2}\left(c^2-d^2\right)\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2c^2\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{(c-d)^2\tan}{2af(\sec(e+fx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] -((c - d)^2*Tan[e + f*x])/(2*a*f*(1 + Sec[e + f*x])*Sqrt[a + a*Sec[e + f*x]
]) + (2*c^2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(Sqrt[a
]*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - ((c - d)^2*ArcTanh
[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(2*Sqrt[2]*Sqrt[
a]*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (Sqrt[2]*(c^2 - d
^2)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(Sqrt
[a]*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])
```

Rule 3940

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(a^2*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
```

```
x)^n)/(x*Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rule 180

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rubi steps

$$\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(c+dx)^2}{x\sqrt{a-ax}(a+ax)^2}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \left(\frac{c^2}{a^2x\sqrt{a-ax}}-\frac{(c-d)^2}{a^2(1+x)^2\sqrt{a-ax}}+\frac{-c^2+d^2}{a^2(1+x)\sqrt{a-ax}}\right)dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\left(c^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{\left((c-d)^2\tan(e+fx)\right)\text{Subst}\left(\int\right.}{f\sqrt{a-a\sec(e+fx)}}$$

$$= -\frac{(c-d)^2\tan(e+fx)}{2af(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + \frac{\left(2c^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx,x,\sqrt{a-}\right.}{af\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e}}$$

$$= -\frac{(c-d)^2\tan(e+fx)}{2af(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + \frac{2c^2\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} -$$

$$= -\frac{(c-d)^2\tan(e+fx)}{2af(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + \frac{2c^2\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} -$$

Mathematica [C] time = 27.7059, size = 16163, normalized size = 55.73

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.19, size = 756, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(3/2),x)
```

```
[Out] -1/4/f/a^2*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(4*2^(1/2)*sin(f*x+e)*(-2*
cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(
f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*c^2*cos(f*x+e)+4*(-2*cos(f*x+e)/(1+co
s(f*x+e)))^(1/2)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))
^(1/2)*sin(f*x+e)/cos(f*x+e))*c^2*sin(f*x+e)+5*sin(f*x+e)*(-2*cos(f*x+e)/(1
+cos(f*x+e)))^(1/2)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+c
os(f*x+e)-1)/sin(f*x+e))*c^2*cos(f*x+e)-2*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(
f*x+e)))^(1/2)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*
x+e)-1)/sin(f*x+e))*c*d*cos(f*x+e)-3*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e
)))^(1/2)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-
1)/sin(f*x+e))*d^2*cos(f*x+e)+5*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(-(-
(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))*c
^2*sin(f*x+e)-2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(-(-(-2*cos(f*x+e)/(
1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))*c*d*sin(f*x+e)-3*
(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(
1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))*d^2*sin(f*x+e)-2*cos(f*x+e)^2*c^2
+4*cos(f*x+e)^2*c*d-2*cos(f*x+e)^2*d^2+2*c^2*cos(f*x+e)-4*cos(f*x+e)*c*d+2*
cos(f*x+e)*d^2)/(1+cos(f*x+e))/sin(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(fx+e\right)+c\right)^2}{\left(a\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e) + c)^2/(a*sec(f*x + e) + a)^(3/2), x)
```

Fricas [A] time = 29.9784, size = 1578, normalized size = 5.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(c^2 - 2*c*d + d^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*
x + e)*sin(f*x + e) - sqrt(2)*((5*c^2 - 2*c*d - 3*d^2)*cos(f*x + e)^2 + 5*c
^2 - 2*c*d - 3*d^2 + 2*(5*c^2 - 2*c*d - 3*d^2)*cos(f*x + e))*sqrt(-a)*log((
2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin
(f*x + e) + 3*a*cos(f*x + e)^2 + 2*a*cos(f*x + e) - a)/(cos(f*x + e)^2 + 2*
cos(f*x + e) + 1)) + 8*(c^2*cos(f*x + e)^2 + 2*c^2*cos(f*x + e) + c^2)*sqrt
(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x
 + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))
/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f), -1/4*(2*(c^2 - 2*c*
d + d^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)
- sqrt(2)*((5*c^2 - 2*c*d - 3*d^2)*cos(f*x + e)^2 + 5*c^2 - 2*c*d - 3*d^2 +
 2*(5*c^2 - 2*c*d - 3*d^2)*cos(f*x + e))*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + 8*(c^2*
cos(f*x + e)^2 + 2*c^2*cos(f*x + e) + c^2)*sqrt(a)*arctan(sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))))/(a^2*f*cos(f*x
 + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c + d\sec\left(e + fx\right)\right)^2}{\left(a\left(\sec\left(e + fx\right) + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))**2/(a+a*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((c + d*sec(e + f*x))**2/(a*(sec(e + f*x) + 1))**(3/2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.174 $\int \frac{c+d\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=127

$$-\frac{(5c-d)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f}+\frac{2c\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{3/2}f}-\frac{(c-d)\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}}$$

```
[Out] (2*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(3/2)*f) -
 ((5*c - d)*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]]
)])/(2*Sqrt[2]*a^(3/2)*f) - ((c - d)*Tan[e + f*x])/(2*f*(a + a*Sec[e + f*x]
)^(3/2))
```

Rubi [A] time = 0.183273, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3922, 3920, 3774, 203, 3795}

$$-\frac{(5c-d)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f}+\frac{2c\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{3/2}f}-\frac{(c-d)\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sec[e + f*x])/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(3/2)*f) -
 ((5*c - d)*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]]
)])/(2*Sqrt[2]*a^(3/2)*f) - ((c - d)*Tan[e + f*x])/(2*f*(a + a*Sec[e + f*x]
)^(3/2))
```

Rule 3922

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*
x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x
])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ
[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+d\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx &= -\frac{(c-d)\tan(e+fx)}{2f(a+a\sec(e+fx))^{3/2}}-\frac{\int \frac{-2ac+\frac{1}{2}a(c-d)\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx}{2a^2}\\
&= -\frac{(c-d)\tan(e+fx)}{2f(a+a\sec(e+fx))^{3/2}}+\frac{c\int \sqrt{a+a\sec(e+fx)}\,dx}{a^2}-\frac{(5c-d)\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx}{4a}\\
&= -\frac{(c-d)\tan(e+fx)}{2f(a+a\sec(e+fx))^{3/2}}-\frac{(2c)\,\mathrm{Subst}\left(\int \frac{1}{a+x^2}\,dx,x,-\frac{a\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{af}+\frac{(5c-d)\,\mathrm{Subst}\left(\ldots\right.}{\ldots}\\
&= \frac{2c\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}f}-\frac{(5c-d)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f}-\frac{(c-d)\tan(e+fx)}{2f(a+a\sec(e+fx))^{3/2}}
\end{aligned}
$$

Mathematica [C] time = 26.6113, size = 10115, normalized size = 79.65

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sec[e + f*x])/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.174, size = 554, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(3/2),x)
```

```
[Out] -1/4/f/a^2*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(4*2^(1/2)*sin(f*x+e)*(-2*
cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(
f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*c*cos(f*x+e)+4*(-2*cos(f*x+e)/(1+cos(
f*x+e)))^(1/2)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(
1/2)*sin(f*x+e)/cos(f*x+e))*c*sin(f*x+e)+5*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos
(f*x+e)))^(1/2)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f
*x+e)-1)/sin(f*x+e))*c*cos(f*x+e)-sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))
^(1/2)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/
sin(f*x+e))*d*cos(f*x+e)+5*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*
ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x
+e))*c-sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(-(-(-2*cos(f*x+e)
/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))*d-2*cos(f*x+e)^
2*c+2*cos(f*x+e)^2*d+2*c*cos(f*x+e)-2*d*cos(f*x+e))/(1+cos(f*x+e))/sin(f*x+
e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d \sec\left(fx+e\right)+c}{\left(a \sec\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e) + c)/(a*sec(f*x + e) + a)^(3/2), x)
```

Fricas [B] time = 7.86022, size = 1416, normalized size = 11.15

$$\left[-\frac{4\,(c-d)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)\sin\left(fx+e\right)-\sqrt{2}\left((5\,c-d)\cos\left(fx+e\right)^{2}+2\,(5\,c-d)\cos\left(fx+e\right)+5\,c-d\right)\sqrt{-a}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(c - d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f
*x + e) - sqrt(2)*((5*c - d)*cos(f*x + e)^2 + 2*(5*c - d)*cos(f*x + e) + 5*
c - d)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x +
 e))*cos(f*x + e)*sin(f*x + e) + 3*a*cos(f*x + e)^2 + 2*a*cos(f*x + e) - a)
/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 8*(c*cos(f*x + e)^2 + 2*c*cos(f*x
 + e) + c)*sqrt(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x +
e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(
f*x + e) + 1)))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f), -1/4
*(2*(c - d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x +
e) - sqrt(2)*((5*c - d)*cos(f*x + e)^2 + 2*(5*c - d)*cos(f*x + e) + 5*c - d
)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x +
e)/(sqrt(a)*sin(f*x + e))) + 8*(c*cos(f*x + e)^2 + 2*c*cos(f*x + e) + c)*sq
rt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*
sin(f*x + e))))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{c+d\sec\left(e+fx\right)}{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((c + d*sec(e + f*x))/(a*(sec(e + f*x) + 1))**(3/2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.175 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))}\,dx$

Optimal. Leaf size=394

$$-\frac{2d^{5/2}\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{\sqrt{a}cf(c-d)^2\sqrt{c+d}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{\tan(e+fx)}{2af(c-d)(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}}-\frac{}{2\sqrt{2}\sqrt{a}}$$

```
[Out] -Tan[e + f*x]/(2*a*(c - d)*f*(1 + Sec[e + f*x])*Sqrt[a + a*Sec[e + f*x]]) +
 (2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(Sqrt[a]*c*f*Sq
rt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (Sqrt[2]*(c - 2*d)*ArcTa
nh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(Sqrt[a]*(c -
d)^2*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (ArcTanh[Sqrt[a
 - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(2*Sqrt[2]*Sqrt[a]*(c -
 d)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (2*d^(5/2)*ArcTa
nh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])/
(Sqrt[a]*c*(c - d)^2*Sqrt[c + d]*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[
e + f*x]])
```

Rubi [A] time = 0.334993, antiderivative size = 394, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3940, 180, 63, 206, 51, 208}

$$-\frac{2d^{5/2}\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{\sqrt{a}cf(c-d)^2\sqrt{c+d}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{\tan(e+fx)}{2af(c-d)(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}}-\frac{}{2\sqrt{2}\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])),x]
```


```
[Out] -Tan[e + f*x]/(2*a*(c - d)*f*(1 + Sec[e + f*x])*Sqrt[a + a*Sec[e + f*x]]) +
 (2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(Sqrt[a]*c*f*Sq
rt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (Sqrt[2]*(c - 2*d)*ArcTa
nh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(Sqrt[a]*(c -
d)^2*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (ArcTanh[Sqrt[a
 - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(2*Sqrt[2]*Sqrt[a]*(c -
 d)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (2*d^(5/2)*ArcTa
nh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])/
(Sqrt[a]*c*(c - d)^2*Sqrt[c + d]*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[
```

```
e + f*x]])
```

Rule 3940

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(a^2*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
x)^n)/(x*Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rule 180

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}(a+ax)^2(c+dx)}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int\left(\frac{1}{a^2cx\sqrt{a-ax}} - \frac{1}{a^2(c-d)(1+x)^2\sqrt{a-ax}} + \frac{-c+2}{a^2(c-d)^2(1+}\right.\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\,\text{se}}}$$

$$= -\frac{\tan(e+fx)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{cf\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{((c-2d)\tan(e+}{(c-d)^2f\sqrt{}}$$

$$= -\frac{\tan(e+fx)}{2a(c-d)f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + \frac{(2\tan(e+fx))\,\text{Su}}{acf\sqrt{a-a\,\text{s}}}$$

$$= -\frac{\tan(e+fx)}{2a(c-d)f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + \frac{2\tanh^{-1}\left(\frac{\sqrt{a-a\,\text{s}}}{}\right.}{\sqrt{a}cf\sqrt{a-a\sec(e}}$$

$$= -\frac{\tan(e+fx)}{2a(c-d)f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + \frac{2\tanh^{-1}\left(\frac{\sqrt{a-a\,\text{s}}}{}\right.}{\sqrt{a}cf\sqrt{a-a\sec(e}}$$

Mathematica [C] time = 35.0797, size = 377837, normalized size = 958.98

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.294, size = 2076, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x)
```

```
[Out] 1/4/f/(d/(c-d))^(1/2)/(c-d)^2/c/((c+d)*(c-d))^(1/2)/a^2*(-1+cos(f*x+e))*(4*
2^(1/2)*((c+d)*(c-d))^(1/2)*sin(f*x+e)*cos(f*x+e)*(d/(c-d))^(1/2)*(-2*cos(f
*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e
)))^(1/2)*sin(f*x+e)/cos(f*x+e))*c^2-8*2^(1/2)*((c+d)*(c-d))^(1/2)*sin(f*x+
e)*cos(f*x+e)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(
1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*c*d
+4*2^(1/2)*((c+d)*(c-d))^(1/2)*sin(f*x+e)*cos(f*x+e)*(d/(c-d))^(1/2)*(-2*co
s(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*
x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*d^2+4*2^(1/2)*((c+d)*(c-d))^(1/2)*sin(f
*x+e)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*2^(1
/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*c^2-8*2^(1/
2)*((c+d)*(c-d))^(1/2)*sin(f*x+e)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x
+e)))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*
x+e)/cos(f*x+e))*c*d+4*2^(1/2)*((c+d)*(c-d))^(1/2)*sin(f*x+e)*(d/(c-d))^(1/
2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/
(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*d^2-2*2^(1/2)*sin(f*x+e)*cos(f
*x+e)*ln(2*((-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(d/(c-d))^(1/2)*2^(1/2)*c*
sin(f*x+e)-2^(1/2)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*d*s
in(f*x+e)+((c+d)*(c-d))^(1/2)*cos(f*x+e)-c*sin(f*x+e)+d*sin(f*x+e)-((c+d)*(
c-d))^(1/2))/(((c+d)*(c-d))^(1/2)*sin(f*x+e)-c*cos(f*x+e)+d*cos(f*x+e)+c-d)
)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*d^3+2*2^(1/2)*sin(f*x+e)*cos(f*x+e)*
(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(-2*((-2*cos(f*x+e)/(1+cos(f*x+e)))^
(1/2)*(d/(c-d))^(1/2)*2^(1/2)*c*sin(f*x+e)-2^(1/2)*(d/(c-d))^(1/2)*(-2*cos(
f*x+e)/(1+cos(f*x+e)))^(1/2)*d*sin(f*x+e)-((c+d)*(c-d))^(1/2)*cos(f*x+e)-c*
sin(f*x+e)+d*sin(f*x+e)+((c+d)*(c-d))^(1/2))/(((c+d)*(c-d))^(1/2)*sin(f*x+e
)+c*cos(f*x+e)-d*cos(f*x+e)-c+d))*d^3+5*((c+d)*(c-d))^(1/2)*sin(f*x+e)*cos(
f*x+e)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(-(-(-2*cos(f
*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))*c^2-9*((c+
d)*(c-d))^(1/2)*sin(f*x+e)*cos(f*x+e)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(1+cos
(f*x+e)))^(1/2)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f
*x+e)-1)/sin(f*x+e))*c*d-2*2^(1/2)*sin(f*x+e)*ln(2*((-2*cos(f*x+e)/(1+cos(f
*x+e)))^(1/2)*(d/(c-d))^(1/2)*2^(1/2)*c*sin(f*x+e)-2^(1/2)*(d/(c-d))^(1/2)*
(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*d*sin(f*x+e)+((c+d)*(c-d))^(1/2)*cos(f
*x+e)-c*sin(f*x+e)+d*sin(f*x+e)-((c+d)*(c-d))^(1/2))/(((c+d)*(c-d))^(1/2)*s
in(f*x+e)-c*cos(f*x+e)+d*cos(f*x+e)+c-d))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1
/2)*d^3+2*2^(1/2)*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(-2*((-
2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(d/(c-d))^(1/2)*2^(1/2)*c*sin(f*x+e)-2^(
1/2)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*d*sin(f*x+e)-((c+
d)*(c-d))^(1/2)*cos(f*x+e)-c*sin(f*x+e)+d*sin(f*x+e)+((c+d)*(c-d))^(1/2))/(
((c+d)*(c-d))^(1/2)*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d))*d^3+5*((c+d)
*(c-d))^(1/2)*sin(f*x+e)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/
2)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(
```

```
f*x+e))*c^2-9*((c+d)*(c-d))^(1/2)*sin(f*x+e)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)
/(1+cos(f*x+e)))^(1/2)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e
)+cos(f*x+e)-1)/sin(f*x+e))*c*d-2*((c+d)*(c-d))^(1/2)*(d/(c-d))^(1/2)*cos(f
*x+e)^2*c^2+2*((c+d)*(c-d))^(1/2)*cos(f*x+e)^2*(d/(c-d))^(1/2)*c*d+2*((c+d)
*(c-d))^(1/2)*(d/(c-d))^(1/2)*cos(f*x+e)*c^2-2*((c+d)*(c-d))^(1/2)*(d/(c-d)
)^(1/2)*cos(f*x+e)*c*d)*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)/sin(f*x+e)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sec\left(fx+e\right)+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e) + c)), x)
```

Fricas [A] time = 54.4344, size = 5720, normalized size = 14.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] [-1/16*(8*(c^2 - c*d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*
sin(f*x + e) - sqrt(2)*((5*c^2 - 9*c*d)*cos(f*x + e)^2 + 5*c^2 - 9*c*d + 2*
(5*c^2 - 9*c*d)*cos(f*x + e))*sqrt(-a)*log((17*a*cos(f*x + e)^3 + 4*sqrt(2)
*(3*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f
*x + e))*sin(f*x + e) + 3*a*cos(f*x + e)^2 - 13*a*cos(f*x + e) + a)/(cos(f*
x + e)^3 + 3*cos(f*x + e)^2 + 3*cos(f*x + e) + 1)) - 8*(a*d^2*cos(f*x + e)^
2 + 2*a*d^2*cos(f*x + e) + a*d^2)*sqrt(-d/(a*c + a*d))*log(((c^2 + 8*c*d +
8*d^2)*cos(f*x + e)^3 + (c^2 + 2*c*d)*cos(f*x + e)^2 + 4*((c^2 + 3*c*d + 2*
d^2)*cos(f*x + e)^2 - (c*d + d^2)*cos(f*x + e))*sqrt(-d/(a*c + a*d))*sqrt((
a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) + d^2 - (6*c*d + 7*d^2)*cos(
f*x + e))/(c^2*cos(f*x + e)^3 + (c^2 + 2*c*d)*cos(f*x + e)^2 + d^2 + (2*c*d
 + d^2)*cos(f*x + e))) + 8*((c^2 - 2*c*d + d^2)*cos(f*x + e)^2 + c^2 - 2*c*
d + d^2 + 2*(c^2 - 2*c*d + d^2)*cos(f*x + e))*sqrt(-a)*log((8*a*cos(f*x + e
```

```
)^3 + 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a
)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1)))/(
(a^2*c^3 - 2*a^2*c^2*d + a^2*c*d^2)*f*cos(f*x + e)^2 + 2*(a^2*c^3 - 2*a^2*c
^2*d + a^2*c*d^2)*f*cos(f*x + e) + (a^2*c^3 - 2*a^2*c^2*d + a^2*c*d^2)*f),
-1/16*(8*(c^2 - c*d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*s
in(f*x + e) - sqrt(2)*((5*c^2 - 9*c*d)*cos(f*x + e)^2 + 5*c^2 - 9*c*d + 2*(
5*c^2 - 9*c*d)*cos(f*x + e))*sqrt(-a)*log((17*a*cos(f*x + e)^3 + 4*sqrt(2)*
(3*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*
x + e))*sin(f*x + e) + 3*a*cos(f*x + e)^2 - 13*a*cos(f*x + e) + a)/(cos(f*x
 + e)^3 + 3*cos(f*x + e)^2 + 3*cos(f*x + e) + 1)) - 16*(a*d^2*cos(f*x + e)^
2 + 2*a*d^2*cos(f*x + e) + a*d^2)*sqrt(d/(a*c + a*d))*arctan(1/2*((c + 2*d)
*cos(f*x + e) - d)*sqrt(d/(a*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x +
e))/(d*sin(f*x + e))) + 8*((c^2 - 2*c*d + d^2)*cos(f*x + e)^2 + c^2 - 2*c*d
 + d^2 + 2*(c^2 - 2*c*d + d^2)*cos(f*x + e))*sqrt(-a)*log((8*a*cos(f*x + e)
^3 + 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)
/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1)))/((
a^2*c^3 - 2*a^2*c^2*d + a^2*c*d^2)*f*cos(f*x + e)^2 + 2*(a^2*c^3 - 2*a^2*c^
2*d + a^2*c*d^2)*f*cos(f*x + e) + (a^2*c^3 - 2*a^2*c^2*d + a^2*c*d^2)*f), -
1/8*(4*(c^2 - c*d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin
(f*x + e) - sqrt(2)*((5*c^2 - 9*c*d)*cos(f*x + e)^2 + 5*c^2 - 9*c*d + 2*(5*
c^2 - 9*c*d)*cos(f*x + e))*sqrt(a)*arctan(1/4*sqrt(2)*sqrt((a*cos(f*x + e)
+ a)/cos(f*x + e))*(3*cos(f*x + e) - 1)/(sqrt(a)*sin(f*x + e))) + 8*((c^2 -
 2*c*d + d^2)*cos(f*x + e)^2 + c^2 - 2*c*d + d^2 + 2*(c^2 - 2*c*d + d^2)*co
s(f*x + e))*sqrt(a)*arctan(1/2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*(2*c
os(f*x + e) - 1)/(sqrt(a)*sin(f*x + e))) - 4*(a*d^2*cos(f*x + e)^2 + 2*a*d^
2*cos(f*x + e) + a*d^2)*sqrt(-d/(a*c + a*d))*log(((c^2 + 8*c*d + 8*d^2)*cos
(f*x + e)^3 + (c^2 + 2*c*d)*cos(f*x + e)^2 + 4*((c^2 + 3*c*d + 2*d^2)*cos(f
*x + e)^2 - (c*d + d^2)*cos(f*x + e))*sqrt(-d/(a*c + a*d))*sqrt((a*cos(f*x
+ e) + a)/cos(f*x + e))*sin(f*x + e) + d^2 - (6*c*d + 7*d^2)*cos(f*x + e))/
(c^2*cos(f*x + e)^3 + (c^2 + 2*c*d)*cos(f*x + e)^2 + d^2 + (2*c*d + d^2)*co
s(f*x + e))))/((a^2*c^3 - 2*a^2*c^2*d + a^2*c*d^2)*f*cos(f*x + e)^2 + 2*(a^
2*c^3 - 2*a^2*c^2*d + a^2*c*d^2)*f*cos(f*x + e) + (a^2*c^3 - 2*a^2*c^2*d +
a^2*c*d^2)*f), -1/8*(4*(c^2 - c*d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*
cos(f*x + e)*sin(f*x + e) - sqrt(2)*((5*c^2 - 9*c*d)*cos(f*x + e)^2 + 5*c^2
 - 9*c*d + 2*(5*c^2 - 9*c*d)*cos(f*x + e))*sqrt(a)*arctan(1/4*sqrt(2)*sqrt(
(a*cos(f*x + e) + a)/cos(f*x + e))*(3*cos(f*x + e) - 1)/(sqrt(a)*sin(f*x +
e))) - 8*(a*d^2*cos(f*x + e)^2 + 2*a*d^2*cos(f*x + e) + a*d^2)*sqrt(d/(a*c
+ a*d))*arctan(1/2*((c + 2*d)*cos(f*x + e) - d)*sqrt(d/(a*c + a*d))*sqrt((a
*cos(f*x + e) + a)/cos(f*x + e))/(d*sin(f*x + e))) + 8*((c^2 - 2*c*d + d^2)
*cos(f*x + e)^2 + c^2 - 2*c*d + d^2 + 2*(c^2 - 2*c*d + d^2)*cos(f*x + e))*s
qrt(a)*arctan(1/2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*(2*cos(f*x + e) -
 1)/(sqrt(a)*sin(f*x + e))))/((a^2*c^3 - 2*a^2*c^2*d + a^2*c*d^2)*f*cos(f*x
 + e)^2 + 2*(a^2*c^3 - 2*a^2*c^2*d + a^2*c*d^2)*f*cos(f*x + e) + (a^2*c^3 -
 2*a^2*c^2*d + a^2*c*d^2)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\left(c+d\sec\left(e+fx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e)),x)
```

```
[Out] Integral(1/((a*(sec(e + f*x) + 1))**(3/2)*(c + d*sec(e + f*x))), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.176 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2}\,dx$

Optimal. Leaf size=560

$$-\frac{2d^{5/2}(3c-d)\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{\sqrt{a}c^2f(c-d)^3\sqrt{c+d}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{}{acf(c-d)}$$

```
[Out] -Tan[e + f*x]/(2*a*(c - d)^2*f*(1 + Sec[e + f*x])*Sqrt[a + a*Sec[e + f*x]])
 + (2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(Sqrt[a]*c^2*
f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (Sqrt[2]*(c - 3*d)*A
rcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(Sqrt[a]*(
c - d)^3*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (ArcTanh[Sq
rt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(2*Sqrt[2]*Sqrt[a]*
(c - d)^2*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (d^(5/2)*A
rcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*
x])/(Sqrt[a]*c*(c - d)^2*(c + d)^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a +
a*Sec[e + f*x]]) - (2*(3*c - d)*d^(5/2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e +
 f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])/(Sqrt[a]*c^2*(c - d)^3*Sqrt[c
+ d]*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (d^3*Tan[e + f*
x])/(a*c*(c - d)^2*(c + d)*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x]))
```

Rubi [A] time = 0.510682, antiderivative size = 560, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3940, 180, 63, 206, 51, 208}

$$-\frac{2d^{5/2}(3c-d)\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{\sqrt{a}c^2f(c-d)^3\sqrt{c+d}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{}{acf(c-d)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])^2),x]
```

```
[Out] -Tan[e + f*x]/(2*a*(c - d)^2*f*(1 + Sec[e + f*x])*Sqrt[a + a*Sec[e + f*x]])
 + (2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(Sqrt[a]*c^2*
f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (Sqrt[2]*(c - 3*d)*A
rcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(Sqrt[a]*(
c - d)^3*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (ArcTanh[Sq
rt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(2*Sqrt[2]*Sqrt[a]*
(c - d)^2*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (d^(5/2)*A
```

```
rcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*
x])/(Sqrt[a]*c*(c - d)^2*(c + d)^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a +
a*Sec[e + f*x]]) - (2*(3*c - d)*d^(5/2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e +
 f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])/(Sqrt[a]*c^2*(c - d)^3*Sqrt[c
+ d]*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (d^3*Tan[e + f*
x])/(a*c*(c - d)^2*(c + d)*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x]))
```

Rule 3940

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(a^2*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
x)^n)/(x*Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rule 180

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
```

```
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}(a+ax)^2(c+dx)^2}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \left(\frac{1}{a^2c^2x\sqrt{a-ax}} - \frac{1}{a^2(c-d)^2(1+x)^2\sqrt{a-ax}} + \frac{-c+}{a^2(c-d)^3(1\cdots}\right.\right.}{f\sqrt{a-a\sec(e+f\cdots}}$$

$$= -\frac{\tan(e+fx)\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{((c-3d)\tan(e+\cdots}{(c-d)^3f\sqrt{\cdots}}$$

$$= -\frac{\tan(e+fx)}{2a(c-d)^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} - \frac{\cdots}{ac(c-d)^2(c+d)f\cdots}$$

$$= -\frac{\tan(e+fx)}{2a(c-d)^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + \frac{2\tanh^{-1}\left(\frac{\sqrt{a-\cdots}}{\cdots}\right.}{\sqrt{a}c^2f\sqrt{a-a\sec(\cdots}}$$

$$= -\frac{\tan(e+fx)}{2a(c-d)^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + \frac{2\tanh^{-1}\left(\frac{\sqrt{a-\cdots}}{\cdots}\right.}{\sqrt{a}c^2f\sqrt{a-a\sec(\cdots}}$$

Mathematica [C] time = 37.741, size = 581056, normalized size = 1037.6

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])^2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 3.779, size = 164796, normalized size = 294.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^2,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^2,x, algorithm="maxima"
)
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^2,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\left(c+d\sec\left(e+fx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e))**2,x)
```

```
[Out] Integral(1/((a*(sec(e + f*x) + 1))**(3/2)*(c + d*sec(e + f*x))**2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.177 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^3}\,dx$

Optimal. Leaf size=802

$$-\frac{(3c-d)\tan(e+fx)d^3}{ac^2(c-d)^3(c+d)f\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))}-\frac{3\tan(e+fx)d^3}{4ac\left(c^2-d^2\right)^2f\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))}-\frac{}{2a}$$

```
[Out] -Tan[e + f*x]/(2*a*(c - d)^3*f*(1 + Sec[e + f*x])*Sqrt[a + a*Sec[e + f*x]])
 + (2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(Sqrt[a]*c^3*
f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (Sqrt[2]*(c - 4*d)*A
rcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(Sqrt[a]*(
c - d)^4*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (ArcTanh[Sq
rt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(2*Sqrt[2]*Sqrt[a]*
(c - d)^3*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (3*d^(5/2)
*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e +
f*x])/(4*Sqrt[a]*c*(c - d)^2*(c + d)^(5/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[
a + a*Sec[e + f*x]]) - ((3*c - d)*d^(5/2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e
 + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])/(Sqrt[a]*c^2*(c - d)^3*(c +
d)^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (2*d^(5/2)*
(6*c^2 - 4*c*d + d^2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*S
qrt[c + d])]*Tan[e + f*x])/(Sqrt[a]*c^3*(c - d)^4*Sqrt[c + d]*f*Sqrt[a - a*
Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (d^3*Tan[e + f*x])/(2*a*c*(c - d)
^2*(c + d)*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^2) - ((3*c - d)*
d^3*Tan[e + f*x])/(a*c^2*(c - d)^3*(c + d)*f*Sqrt[a + a*Sec[e + f*x]]*(c +
d*Sec[e + f*x])) - (3*d^3*Tan[e + f*x])/(4*a*c*(c^2 - d^2)^2*f*Sqrt[a + a*S
ec[e + f*x]]*(c + d*Sec[e + f*x]))
```

Rubi [A] time = 0.796566, antiderivative size = 802, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3940, 180, 63, 206, 51, 208}

$$-\frac{(3c-d)\tan(e+fx)d^3}{ac^2(c-d)^3(c+d)f\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))}-\frac{3\tan(e+fx)d^3}{4ac\left(c^2-d^2\right)^2f\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))}-\frac{}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])^3),x]
```

```
[Out] -Tan[e + f*x]/(2*a*(c - d)^3*f*(1 + Sec[e + f*x])*Sqrt[a + a*Sec[e + f*x]])
 + (2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(Sqrt[a]*c^3*
f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (Sqrt[2]*(c - 4*d)*A
rcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(Sqrt[a]*(
c - d)^4*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (ArcTanh[Sq
rt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(2*Sqrt[2]*Sqrt[a]*
(c - d)^3*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (3*d^(5/2)
*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e +
f*x])/(4*Sqrt[a]*c*(c - d)^2*(c + d)^(5/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[
a + a*Sec[e + f*x]]) - ((3*c - d)*d^(5/2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e
 + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])/(Sqrt[a]*c^2*(c - d)^3*(c +
d)^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (2*d^(5/2)*
(6*c^2 - 4*c*d + d^2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*S
qrt[c + d])]*Tan[e + f*x])/(Sqrt[a]*c^3*(c - d)^4*Sqrt[c + d]*f*Sqrt[a - a*
Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (d^3*Tan[e + f*x])/(2*a*c*(c - d)
^2*(c + d)*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^2) - ((3*c - d)*
d^3*Tan[e + f*x])/(a*c^2*(c - d)^3*(c + d)*f*Sqrt[a + a*Sec[e + f*x]]*(c +
d*Sec[e + f*x])) - (3*d^3*Tan[e + f*x])/(4*a*c*(c^2 - d^2)^2*f*Sqrt[a + a*S
ec[e + f*x]]*(c + d*Sec[e + f*x]))
```

Rule 3940

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(a^2*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
x)^n)/(x*Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rule 180

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^3}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a-ax}(a+ax)^2(c+dx)^3}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\left(a^2\tan(e+fx)\right)\operatorname{Subst}\left(\int \left(\frac{1}{a^2c^3x\sqrt{a-ax}} - \frac{1}{a^2(c-d)^3(1+x)^2\sqrt{a-ax}} + \frac{-c+\cdots}{a^2(c-d)^4(1\cdots}\right.\right.}{f\sqrt{a-\cdots}}$$

$$= -\frac{\tan(e+fx)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{((c-4d)\tan(e+\cdots}{(c-d)^4f\sqrt{\cdots}}$$

$$= -\frac{\tan(e+fx)}{2a(c-d)^3f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} - \frac{\cdots}{2ac(c-d)^2(c+d)f\cdots}$$

$$= -\frac{\tan(e+fx)}{2a(c-d)^3f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + \frac{2\tanh^{-1}\left(\frac{\sqrt{a-\cdots}}{\cdots}\right.}{\sqrt{a}c^3f\sqrt{a-a\sec(\cdots}}$$

$$= -\frac{\tan(e+fx)}{2a(c-d)^3f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + \frac{2\tanh^{-1}\left(\frac{\sqrt{a-\cdots}}{\cdots}\right.}{\sqrt{a}c^3f\sqrt{a-a\sec(\cdots}}$$

$$= -\frac{\tan(e+fx)}{2a(c-d)^3f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + \frac{2\tanh^{-1}\left(\frac{\sqrt{a-\cdots}}{\cdots}\right.}{\sqrt{a}c^3f\sqrt{a-a\sec(\cdots}}$$

Mathematica [C] time = 40.8732, size = 774154, normalized size = 965.28

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])^3),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 28.445, size = 480553, normalized size = 599.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^3,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^3,x, algorithm="maxima"
)
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^3,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.178 $\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=480

$$-\frac{\sqrt{2}\left(c^3-d^3\right)\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2c^3\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{3(c-d)^3\,\mathrm{t}}{16a^2f(\sec(e+fx)+}$$

```
[Out] -((c - d)^3*Tan[e + f*x])/(4*a^2*f*(1 + Sec[e + f*x])^2*Sqrt[a + a*Sec[e +
f*x]]) - (3*(c - d)^3*Tan[e + f*x])/(16*a^2*f*(1 + Sec[e + f*x])*Sqrt[a + a
*Sec[e + f*x]]) - ((c - d)^2*(c + 2*d)*Tan[e + f*x])/(2*a^2*f*(1 + Sec[e +
f*x])*Sqrt[a + a*Sec[e + f*x]]) + (2*c^3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/S
qrt[a]]*Tan[e + f*x])/(a^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e
+ f*x]]) - (3*(c - d)^3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]
*Tan[e + f*x])/(16*Sqrt[2]*a^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Se
c[e + f*x]]) - ((c - d)^2*(c + 2*d)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[
2]*Sqrt[a])]*Tan[e + f*x])/(2*Sqrt[2]*a^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sq
rt[a + a*Sec[e + f*x]]) - (Sqrt[2]*(c^3 - d^3)*ArcTanh[Sqrt[a - a*Sec[e + f
*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(a^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*S
qrt[a + a*Sec[e + f*x]])
```

Rubi [A] time = 0.315644, antiderivative size = 480, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {3940, 180, 63, 206, 51}

$$-\frac{\sqrt{2}\left(c^3-d^3\right)\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2c^3\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{3(c-d)^3\,\mathrm{t}}{16a^2f(\sec(e+fx)+}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] -((c - d)^3*Tan[e + f*x])/(4*a^2*f*(1 + Sec[e + f*x])^2*Sqrt[a + a*Sec[e +
f*x]]) - (3*(c - d)^3*Tan[e + f*x])/(16*a^2*f*(1 + Sec[e + f*x])*Sqrt[a + a
*Sec[e + f*x]]) - ((c - d)^2*(c + 2*d)*Tan[e + f*x])/(2*a^2*f*(1 + Sec[e +
f*x])*Sqrt[a + a*Sec[e + f*x]]) + (2*c^3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/S
qrt[a]]*Tan[e + f*x])/(a^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e
+ f*x]]) - (3*(c - d)^3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]
*Tan[e + f*x])/(16*Sqrt[2]*a^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Se
c[e + f*x]]) - ((c - d)^2*(c + 2*d)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[
```

```
2]*Sqrt[a])]*Tan[e + f*x])/(2*Sqrt[2]*a^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sq
rt[a + a*Sec[e + f*x]]) - (Sqrt[2]*(c^3 - d^3)*ArcTanh[Sqrt[a - a*Sec[e + f
*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(a^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*S
qrt[a + a*Sec[e + f*x]])
```

Rule 3940

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(a^2*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
x)^n)/(x*Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rule 180

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rubi steps

$$\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(c+dx)^3}{x\sqrt{a-ax}(a+ax)^3}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \left(\frac{c^3}{a^3x\sqrt{a-ax}} - \frac{(c-d)^3}{a^3(1+x)^3\sqrt{a-ax}} - \frac{(c-d)^2(c+2d)}{a^3(1+x)^2\sqrt{a-ax}} + \frac{-c^3+d^3}{a^3(1+x)\sqrt{a-ax}}\right)dx, x$$

$$f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}$$

$$= -\frac{\left(c^3\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{af\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{\left((c-d)^3\tan(e+fx)\right)\text{Subst}\left(\int\right.}{af\sqrt{a-a\sec(e+fx)}}$$

$$= -\frac{(c-d)^3\tan(e+fx)}{4a^2f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} - \frac{(c-d)^2(c+2d)\tan(e+fx)}{2a^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{(c-d)^3\tan(e+fx)}{4a^2f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} - \frac{3(c-d)^3\tan(e+fx)}{16a^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{(c-d)^3\tan(e+fx)}{4a^2f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} - \frac{3(c-d)^3\tan(e+fx)}{16a^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{(c-d)^3\tan(e+fx)}{4a^2f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} - \frac{3(c-d)^3\tan(e+fx)}{16a^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}}$$

Mathematica [C] time = 29.3114, size = 21204, normalized size = 44.18

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.278, size = 1444, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(5/2),x)
```

```
[Out] 1/32/f/a^3*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(-1+cos(f*x+e))*(86*(-2*co
s(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)*cos(f*x+e)*ln(-(-(-2*cos(f*x+e)/(
1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))*c^3-38*(-2*cos(f*
x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)*cos(f*x+e)*ln(-(-(-2*cos(f*x+e)/(1+co
s(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))*d^3+32*2^(1/2)*(-2*co
s(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*
x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*c^3*sin(f*x+e)-9*(-2*cos(f*x+e)/(1+cos(
f*x+e)))^(1/2)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*
x+e)-1)/sin(f*x+e))*c^2*d*sin(f*x+e)-15*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2
)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f
*x+e))*c*d^2*sin(f*x+e)+32*2^(1/2)*sin(f*x+e)*cos(f*x+e)^2*arctanh(1/2*2^(1
/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*(-2*cos(f*x
+e)/(1+cos(f*x+e)))^(1/2)*c^3+42*cos(f*x+e)^3*c^2*d-19*sin(f*x+e)*cos(f*x+e
)^2*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin
(f*x+e))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*d^3+43*sin(f*x+e)*cos(f*x+e)^
2*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f
*x+e))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*c^3-18*cos(f*x+e)*c^2*d+64*(-2*
cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)*cos(f*x+e)*2^(1/2)*arctanh(1/2*
2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*c^3-30*
cos(f*x+e)^3*c^3+43*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(-(-(-2*cos(f*x+
e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))*c^3*sin(f*x+e
)-19*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e
)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))*d^3*sin(f*x+e)-9*sin(f*x+e)*
cos(f*x+e)^2*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+
e)-1)/sin(f*x+e))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*c^2*d-15*sin(f*x+e)*
cos(f*x+e)^2*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+
e)-1)/sin(f*x+e))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*c*d^2+6*cos(f*x+e)^3
*c*d^2-24*cos(f*x+e)^2*c^2*d+24*cos(f*x+e)^2*c*d^2-30*cos(f*x+e)*c*d^2+8*co
s(f*x+e)^2*c^3+22*cos(f*x+e)*c^3-18*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*si
n(f*x+e)*cos(f*x+e)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+c
os(f*x+e)-1)/sin(f*x+e))*c^2*d-30*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(
f*x+e)*cos(f*x+e)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos
(f*x+e)-1)/sin(f*x+e))*c*d^2-18*cos(f*x+e)^3*d^3-8*cos(f*x+e)^2*d^3+26*cos(
f*x+e)*d^3)/(1+cos(f*x+e))/sin(f*x+e)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 145.17, size = 2188, normalized size = 4.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/64*(sqrt(2)*((43*c^3 - 9*c^2*d - 15*c*d^2 - 19*d^3)*cos(f*x + e)^3 + 43*
c^3 - 9*c^2*d - 15*c*d^2 - 19*d^3 + 3*(43*c^3 - 9*c^2*d - 15*c*d^2 - 19*d^3
)*cos(f*x + e)^2 + 3*(43*c^3 - 9*c^2*d - 15*c*d^2 - 19*d^3)*cos(f*x + e))*s
qrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos
(f*x + e)*sin(f*x + e) + 3*a*cos(f*x + e)^2 + 2*a*cos(f*x + e) - a)/(cos(f*
x + e)^2 + 2*cos(f*x + e) + 1)) - 64*(c^3*cos(f*x + e)^3 + 3*c^3*cos(f*x +
e)^2 + 3*c^3*cos(f*x + e) + c^3)*sqrt(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(
-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*c
os(f*x + e) - a)/(cos(f*x + e) + 1)) - 4*(3*(5*c^3 - 7*c^2*d - c*d^2 + 3*d^
3)*cos(f*x + e)^2 + (11*c^3 - 9*c^2*d - 15*c*d^2 + 13*d^3)*cos(f*x + e))*sq
rt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 +
 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f), 1/32*(sqrt(2)*((43
*c^3 - 9*c^2*d - 15*c*d^2 - 19*d^3)*cos(f*x + e)^3 + 43*c^3 - 9*c^2*d - 15*
c*d^2 - 19*d^3 + 3*(43*c^3 - 9*c^2*d - 15*c*d^2 - 19*d^3)*cos(f*x + e)^2 +
3*(43*c^3 - 9*c^2*d - 15*c*d^2 - 19*d^3)*cos(f*x + e))*sqrt(a)*arctan(sqrt(
2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x +
e))) - 64*(c^3*cos(f*x + e)^3 + 3*c^3*cos(f*x + e)^2 + 3*c^3*cos(f*x + e) +
 c^3)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(
sqrt(a)*sin(f*x + e))) - 2*(3*(5*c^3 - 7*c^2*d - c*d^2 + 3*d^3)*cos(f*x + e
)^2 + (11*c^3 - 9*c^2*d - 15*c*d^2 + 13*d^3)*cos(f*x + e))*sqrt((a*cos(f*x
+ e) + a)/cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f
*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c+d\sec\left(e+fx\right)\right)^{3}}{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))**3/(a+a*sec(f*x+e))**(5/2),x)
```

```
[Out] Integral((c + d*sec(e + f*x))**3/(a*(sec(e + f*x) + 1))**(5/2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.179 $\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=468

$$-\frac{\left(c^2-d^2\right)\tan(e+fx)}{2a^2f(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}}-\frac{\left(c^2-d^2\right)\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2c^2\tan(e+fx)\tanh}{a^{3/2}f\sqrt{a-a\sec(e+fx}}$$

```
[Out] -((c - d)^2*Tan[e + f*x])/(4*a^2*f*(1 + Sec[e + f*x])^2*Sqrt[a + a*Sec[e +
f*x]]) - (3*(c - d)^2*Tan[e + f*x])/(16*a^2*f*(1 + Sec[e + f*x])*Sqrt[a + a
*Sec[e + f*x]]) - ((c^2 - d^2)*Tan[e + f*x])/(2*a^2*f*(1 + Sec[e + f*x])*Sq
rt[a + a*Sec[e + f*x]]) + (2*c^2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*
Tan[e + f*x])/(a^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])
 - (Sqrt[2]*c^2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e +
 f*x])/(a^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (3*(
c - d)^2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/
(16*Sqrt[2]*a^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) -
((c^2 - d^2)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*
x])/(2*Sqrt[2]*a^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])
```

Rubi [A] time = 0.294868, antiderivative size = 468, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {3940, 180, 63, 206, 51}

$$-\frac{\left(c^2-d^2\right)\tan(e+fx)}{2a^2f(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}}-\frac{\left(c^2-d^2\right)\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2c^2\tan(e+fx)\tanh}{a^{3/2}f\sqrt{a-a\sec(e+fx}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^(5/2),x]
```


```
[Out] -((c - d)^2*Tan[e + f*x])/(4*a^2*f*(1 + Sec[e + f*x])^2*Sqrt[a + a*Sec[e +
f*x]]) - (3*(c - d)^2*Tan[e + f*x])/(16*a^2*f*(1 + Sec[e + f*x])*Sqrt[a + a
*Sec[e + f*x]]) - ((c^2 - d^2)*Tan[e + f*x])/(2*a^2*f*(1 + Sec[e + f*x])*Sq
rt[a + a*Sec[e + f*x]]) + (2*c^2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*
Tan[e + f*x])/(a^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])
 - (Sqrt[2]*c^2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e +
 f*x])/(a^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (3*(
c - d)^2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/
(16*Sqrt[2]*a^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) -
```

```
((c^2 - d^2)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*
x])/(2*Sqrt[2]*a^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])
```

Rule 3940

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(a^2*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
x)^n)/(x*Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rule 180

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rubi steps

$$\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(c+dx)^2}{x\sqrt{a-ax}(a+ax)^3}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int\left(\frac{c^2}{a^3x\sqrt{a-ax}} - \frac{(c-d)^2}{a^3(1+x)^3\sqrt{a-ax}} + \frac{-c^2+d^2}{a^3(1+x)^2\sqrt{a-ax}} - \frac{c^2}{a^3(1+x)\sqrt{a-ax}}\right)dx,x\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\left(c^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{af\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{\left(c^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{(1+x)\cdots}\right.}{af\sqrt{a-a\sec(e+fx)}\sqrt{a\cdots}}$$

$$= -\frac{(c-d)^2\tan(e+fx)}{4a^2f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} - \frac{\left(c^2-d^2\right)\tan(e+fx)}{2a^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{(c-d)^2\tan(e+fx)}{4a^2f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} - \frac{3(c-d)^2\tan(e+fx)}{16a^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{(c-d)^2\tan(e+fx)}{4a^2f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} - \frac{3(c-d)^2\tan(e+fx)}{16a^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{(c-d)^2\tan(e+fx)}{4a^2f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} - \frac{3(c-d)^2\tan(e+fx)}{16a^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}}$$

Mathematica [C] time = 27.9488, size = 16259, normalized size = 34.74

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.208, size = 1133, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(5/2),x)
```

```
[Out] -1/32/f/a^3*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(32*2^(1/2)*(-2*cos(f*x+e
)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^
(1/2)*sin(f*x+e)/cos(f*x+e))*cos(f*x+e)^2*sin(f*x+e)*c^2+64*2^(1/2)*sin(f*x
+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)
/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*c^2*cos(f*x+e)+43*(-2*cos(f*x
+e)/(1+cos(f*x+e)))^(1/2)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*
x+e)+cos(f*x+e)-1)/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)*c^2-6*(-2*cos(f*x+e)
/(1+cos(f*x+e)))^(1/2)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e
)+cos(f*x+e)-1)/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)*c*d-5*(-2*cos(f*x+e)/(1
+cos(f*x+e)))^(1/2)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+c
os(f*x+e)-1)/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)*d^2+32*(-2*cos(f*x+e)/(1+c
os(f*x+e)))^(1/2)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e))
)^(1/2)*sin(f*x+e)/cos(f*x+e))*c^2*sin(f*x+e)+86*sin(f*x+e)*(-2*cos(f*x+e)/
(1+cos(f*x+e)))^(1/2)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)
+cos(f*x+e)-1)/sin(f*x+e))*c^2*cos(f*x+e)-12*sin(f*x+e)*(-2*cos(f*x+e)/(1+c
os(f*x+e)))^(1/2)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos
(f*x+e)-1)/sin(f*x+e))*c*d*cos(f*x+e)-10*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f
*x+e)))^(1/2)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x
+e)-1)/sin(f*x+e))*d^2*cos(f*x+e)+43*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*l
n(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+
e))*c^2*sin(f*x+e)-6*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(-(-(-2*cos(f*x
+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))*c*d*sin(f*x+
e)-5*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e
)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))*d^2*sin(f*x+e)-30*cos(f*x+e)
^3*c^2+28*cos(f*x+e)^3*c*d+2*cos(f*x+e)^3*d^2+8*cos(f*x+e)^2*c^2-16*cos(f*x
+e)^2*c*d+8*cos(f*x+e)^2*d^2+22*c^2*cos(f*x+e)-12*cos(f*x+e)*c*d-10*cos(f*x
+e)*d^2)/(1+cos(f*x+e))^2/sin(f*x+e)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 65.139, size = 1967, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/64*(sqrt(2)*((43*c^2 - 6*c*d - 5*d^2)*cos(f*x + e)^3 + 3*(43*c^2 - 6*c*d
 - 5*d^2)*cos(f*x + e)^2 + 43*c^2 - 6*c*d - 5*d^2 + 3*(43*c^2 - 6*c*d - 5*d
^2)*cos(f*x + e))*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a
)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + 3*a*cos(f*x + e)^2 + 2*a*cos(f*
x + e) - a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) - 64*(c^2*cos(f*x + e)^3
 + 3*c^2*cos(f*x + e)^2 + 3*c^2*cos(f*x + e) + c^2)*sqrt(-a)*log((2*a*cos(f
*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)
*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 4*((15*c^2 - 14*c
*d - d^2)*cos(f*x + e)^2 + (11*c^2 - 6*c*d - 5*d^2)*cos(f*x + e))*sqrt((a*c
os(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*
f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f), 1/32*(sqrt(2)*((43*c^2 -
6*c*d - 5*d^2)*cos(f*x + e)^3 + 3*(43*c^2 - 6*c*d - 5*d^2)*cos(f*x + e)^2 +
 43*c^2 - 6*c*d - 5*d^2 + 3*(43*c^2 - 6*c*d - 5*d^2)*cos(f*x + e))*sqrt(a)*
arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a
)*sin(f*x + e))) - 64*(c^2*cos(f*x + e)^3 + 3*c^2*cos(f*x + e)^2 + 3*c^2*co
s(f*x + e) + c^2)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*co
s(f*x + e)/(sqrt(a)*sin(f*x + e))) - 2*((15*c^2 - 14*c*d - d^2)*cos(f*x + e
)^2 + (11*c^2 - 6*c*d - 5*d^2)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(
f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*
a^3*f*cos(f*x + e) + a^3*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))**2/(a+a*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.180 $\int \frac{c+d\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=164

$$-\frac{(43c-3d)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{16\sqrt{2}a^{5/2}f}+\frac{2c\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{5/2}f}-\frac{(11c-3d)\tan(e+fx)}{16af(a\sec(e+fx)+a)^{3/2}}-\frac{(c-d)\tan(e+fx)}{4f(a\sec(e+fx)+a)^{5/2}}$$

```
[Out] (2*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(5/2)*f) -
 ((43*c - 3*d)*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*
x]])])/(16*Sqrt[2]*a^(5/2)*f) - ((c - d)*Tan[e + f*x])/(4*f*(a + a*Sec[e +
f*x])^(5/2)) - ((11*c - 3*d)*Tan[e + f*x])/(16*a*f*(a + a*Sec[e + f*x])^(3/
2))
```

Rubi [A] time = 0.26493, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3922, 3920, 3774, 203, 3795}

$$-\frac{(43c-3d)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{16\sqrt{2}a^{5/2}f}+\frac{2c\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{5/2}f}-\frac{(11c-3d)\tan(e+fx)}{16af(a\sec(e+fx)+a)^{3/2}}-\frac{(c-d)\tan(e+fx)}{4f(a\sec(e+fx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sec[e + f*x])/(a + a*Sec[e + f*x])^(5/2),x]
```


```
[Out] (2*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(5/2)*f) -
 ((43*c - 3*d)*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*
x]])])/(16*Sqrt[2]*a^(5/2)*f) - ((c - d)*Tan[e + f*x])/(4*f*(a + a*Sec[e +
f*x])^(5/2)) - ((11*c - 3*d)*Tan[e + f*x])/(16*a*f*(a + a*Sec[e + f*x])^(3/
2))
```


Rule 3922

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*
x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x
])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ
[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{c+d\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{(c-d)\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}} - \frac{\int \frac{-4ac+\frac{3}{2}a(c-d)\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{(c-d)\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}} - \frac{(11c-3d)\tan(e+fx)}{16af(a+a\sec(e+fx))^{3/2}} + \frac{\int \frac{8a^2c-\frac{1}{4}a^2(11c-3d)\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx}{8a^4}$$

$$= -\frac{(c-d)\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}} - \frac{(11c-3d)\tan(e+fx)}{16af(a+a\sec(e+fx))^{3/2}} + \frac{c\int \sqrt{a+a\sec(e+fx)}\,dx}{a^3} - \frac{(43}{}$$

$$= -\frac{(c-d)\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}} - \frac{(11c-3d)\tan(e+fx)}{16af(a+a\sec(e+fx))^{3/2}} - \frac{(2c)\,\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{a\tan}{\sqrt{a+a\,\text{se}}}\right.}{a^2 f}$$

$$= \frac{2c\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}f} - \frac{(43c-3d)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f} - \frac{(c-d)\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}} -$$

Mathematica [C] time = 26.8806, size = 10177, normalized size = 62.05

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sec[e + f*x])/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.185, size = 824, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(5/2),x)
```

```
[Out] -1/32/f/a^3*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(32*2^(1/2)*sin(f*x+e)*co
s(f*x+e)^2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos
(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*c+64*2^(1/2)*sin(f*x+e
)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(
1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*c*cos(f*x+e)+43*sin(f*x+e)*cos(
f*x+e)^2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(-(-(-2*cos(f*x+e)/(1+cos(f
*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))*c-3*sin(f*x+e)*cos(f*x+e
)^2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)
))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))*d+32*(-2*cos(f*x+e)/(1+cos(f*
x+e)))^(1/2)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/
2)*sin(f*x+e)/cos(f*x+e))*c*sin(f*x+e)+86*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(
f*x+e)))^(1/2)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*
x+e)-1)/sin(f*x+e))*c*cos(f*x+e)-6*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e))
)^(1/2)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)
/sin(f*x+e))*d*cos(f*x+e)+43*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2
)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f
*x+e))*c-3*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(-(-(-2*cos(f*
x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))*d-30*cos(f*
x+e)^3*c+14*cos(f*x+e)^3*d+8*cos(f*x+e)^2*c-8*cos(f*x+e)^2*d+22*c*cos(f*x+e
)-6*d*cos(f*x+e))/(1+cos(f*x+e))^2/sin(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d \sec \left(fx+e\right)+c}{\left(a \sec \left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e) + c)/(a*sec(f*x + e) + a)^(5/2), x)
```

Fricas [B] time = 16.7046, size = 1754, normalized size = 10.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/64*(sqrt(2)*((43*c - 3*d)*cos(f*x + e)^3 + 3*(43*c - 3*d)*cos(f*x + e)^2
 + 3*(43*c - 3*d)*cos(f*x + e) + 43*c - 3*d)*sqrt(-a)*log((2*sqrt(2)*sqrt(-
a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + 3*a*
cos(f*x + e)^2 + 2*a*cos(f*x + e) - a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1
)) - 64*(c*cos(f*x + e)^3 + 3*c*cos(f*x + e)^2 + 3*c*cos(f*x + e) + c)*sqrt
(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x
 + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1))
- 4*((15*c - 7*d)*cos(f*x + e)^2 + (11*c - 3*d)*cos(f*x + e))*sqrt((a*cos(f
*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*co
s(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f), 1/32*(sqrt(2)*((43*c - 3*d)*c
os(f*x + e)^3 + 3*(43*c - 3*d)*cos(f*x + e)^2 + 3*(43*c - 3*d)*cos(f*x + e)
 + 43*c - 3*d)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e
))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 64*(c*cos(f*x + e)^3 + 3*c*cos(f*
x + e)^2 + 3*c*cos(f*x + e) + c)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/c
os(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 2*((15*c - 7*d)*cos(f*x
 + e)^2 + (11*c - 3*d)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)
)*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*co
s(f*x + e) + a^3*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.181 $\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))}\,dx$

Optimal. Leaf size=592

$$-\frac{\sqrt{2}\left(c^2-3cd+3d^2\right)\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{a^{3/2}f(c-d)^3\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2d^{7/2}\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{a^{3/2}cf(c-d)^3\sqrt{c+d}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-$$

```
[Out] -Tan[e + f*x]/(4*a^2*(c - d)*f*(1 + Sec[e + f*x])^2*Sqrt[a + a*Sec[e + f*x]
]) - ((c - 2*d)*Tan[e + f*x])/(2*a^2*(c - d)^2*f*(1 + Sec[e + f*x])*Sqrt[a
+ a*Sec[e + f*x]]) - (3*Tan[e + f*x])/(16*a^2*(c - d)*f*(1 + Sec[e + f*x])*
Sqrt[a + a*Sec[e + f*x]]) + (2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Ta
n[e + f*x])/(a^(3/2)*c*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])
 - ((c - 2*d)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f
*x])/(2*Sqrt[2]*a^(3/2)*(c - d)^2*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec
[e + f*x]]) - (3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e
+ f*x])/(16*Sqrt[2]*a^(3/2)*(c - d)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*S
ec[e + f*x]]) - (Sqrt[2]*(c^2 - 3*c*d + 3*d^2)*ArcTanh[Sqrt[a - a*Sec[e + f
*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(a^(3/2)*(c - d)^3*f*Sqrt[a - a*Sec[e
 + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (2*d^(7/2)*ArcTanh[(Sqrt[d]*Sqrt[a - a
*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])/(a^(3/2)*c*(c - d)^3*S
qrt[c + d]*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])
```

Rubi [A] time = 0.456727, antiderivative size = 592, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3940, 180, 63, 206, 51, 208}

$$-\frac{\sqrt{2}\left(c^2-3cd+3d^2\right)\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{a^{3/2}f(c-d)^3\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2d^{7/2}\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{a^{3/2}cf(c-d)^3\sqrt{c+d}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])),x]
```


```
[Out] -Tan[e + f*x]/(4*a^2*(c - d)*f*(1 + Sec[e + f*x])^2*Sqrt[a + a*Sec[e + f*x]
]) - ((c - 2*d)*Tan[e + f*x])/(2*a^2*(c - d)^2*f*(1 + Sec[e + f*x])*Sqrt[a
+ a*Sec[e + f*x]]) - (3*Tan[e + f*x])/(16*a^2*(c - d)*f*(1 + Sec[e + f*x])*
Sqrt[a + a*Sec[e + f*x]]) + (2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Ta
n[e + f*x])/(a^(3/2)*c*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])
 - ((c - 2*d)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f
```

```
*x])/(2*Sqrt[2]*a^(3/2)*(c - d)^2*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec
[e + f*x]]) - (3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e
+ f*x])/(16*Sqrt[2]*a^(3/2)*(c - d)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*S
ec[e + f*x]]) - (Sqrt[2]*(c^2 - 3*c*d + 3*d^2)*ArcTanh[Sqrt[a - a*Sec[e + f
*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(a^(3/2)*(c - d)^3*f*Sqrt[a - a*Sec[e
 + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (2*d^(7/2)*ArcTanh[(Sqrt[d]*Sqrt[a - a
*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])/(a^(3/2)*c*(c - d)^3*S
qrt[c + d]*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])
```

Rule 3940

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(a^2*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
x)^n)/(x*Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rule 180

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
```

```
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}(a+ax)^3(c+dx)}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int\left(\frac{1}{a^3cx\sqrt{a-ax}} - \frac{1}{a^3(c-d)(1+x)^3\sqrt{a-ax}} + \frac{-c+2d}{a^3(c-d)^2(1+x)^2}\right.\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{}}$$

$$= -\frac{\tan(e+fx)\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{acf\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{((c-2d)\tan(e+f}{a(c-d)^2f\sqrt{}}$$

$$= -\frac{\tan(e+fx)}{4a^2(c-d)f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} - \frac{(c-}{2a^2(c-d)^2f(1+\text{se}}$$

$$= -\frac{\tan(e+fx)}{4a^2(c-d)f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} - \frac{(c-}{2a^2(c-d)^2f(1+\text{se}}$$

$$= -\frac{\tan(e+fx)}{4a^2(c-d)f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} - \frac{(c-}{2a^2(c-d)^2f(1+\text{se}}$$

$$= -\frac{\tan(e+fx)}{4a^2(c-d)f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} - \frac{(c-}{2a^2(c-d)^2f(1+\text{se}}$$

Mathematica [C] time = 36.9869, size = 484869, normalized size = 819.04

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.29, size = 3860, normalized size = 6.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e)),x)
```

```
[Out] 1/32/f/(d/(c-d))^(1/2)/(c-d)^3/c/((c+d)*(c-d))^(1/2)/a^3*(-1+cos(f*x+e))^2*
(-16*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(2*((-2*cos(f*x+e)/(
1+cos(f*x+e)))^(1/2)*(d/(c-d))^(1/2)*2^(1/2)*c*sin(f*x+e)-2^(1/2)*(d/(c-d))
^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*d*sin(f*x+e)+((c+d)*(c-d))^(1/2
)*cos(f*x+e)-c*sin(f*x+e)+d*sin(f*x+e)-((c+d)*(c-d))^(1/2))/(((c+d)*(c-d))^
(1/2)*sin(f*x+e)-c*cos(f*x+e)+d*cos(f*x+e)+c-d))*2^(1/2)*d^4+16*(d/(c-d))^(
1/2)*cos(f*x+e)^2*((c+d)*(c-d))^(1/2)*c^2*d-8*(d/(c-d))^(1/2)*cos(f*x+e)^2*
((c+d)*(c-d))^(1/2)*c*d^2+60*(d/(c-d))^(1/2)*((c+d)*(c-d))^(1/2)*cos(f*x+e)
*c^2*d-38*(d/(c-d))^(1/2)*((c+d)*(c-d))^(1/2)*cos(f*x+e)*c*d^2-76*(d/(c-d))
^(1/2)*cos(f*x+e)^3*((c+d)*(c-d))^(1/2)*c^2*d+46*(d/(c-d))^(1/2)*cos(f*x+e)
^3*((c+d)*(c-d))^(1/2)*c*d^2+16*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(
1/2)*ln(-2*((-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(d/(c-d))^(1/2)*2^(1/2)*c*
sin(f*x+e)-2^(1/2)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*d*s
in(f*x+e)-((c+d)*(c-d))^(1/2)*cos(f*x+e)-c*sin(f*x+e)+d*sin(f*x+e)+((c+d)*(
c-d))^(1/2))/(((c+d)*(c-d))^(1/2)*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)
)*2^(1/2)*d^4+30*(d/(c-d))^(1/2)*cos(f*x+e)^3*((c+d)*(c-d))^(1/2)*c^3-8*(d/
(c-d))^(1/2)*cos(f*x+e)^2*((c+d)*(c-d))^(1/2)*c^3-22*(d/(c-d))^(1/2)*((c+d)
*(c-d))^(1/2)*cos(f*x+e)*c^3-192*(d/(c-d))^(1/2)*cos(f*x+e)*sin(f*x+e)*((c+
d)*(c-d))^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*2^(1/2)*(-
2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*2^(1/2)*c*d^2+64*
(d/(c-d))^(1/2)*cos(f*x+e)*sin(f*x+e)*((c+d)*(c-d))^(1/2)*(-2*cos(f*x+e)/(1
+cos(f*x+e)))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2
)*sin(f*x+e)/cos(f*x+e))*2^(1/2)*d^3+252*(d/(c-d))^(1/2)*cos(f*x+e)*sin(f*x
+e)*((c+d)*(c-d))^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(-(-(-2*cos(
f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))*c^2*d-230
*(d/(c-d))^(1/2)*cos(f*x+e)*sin(f*x+e)*((c+d)*(c-d))^(1/2)*(-2*cos(f*x+e)/(
1+cos(f*x+e)))^(1/2)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+
cos(f*x+e)-1)/sin(f*x+e))*c*d^2+96*(d/(c-d))^(1/2)*sin(f*x+e)*((c+d)*(c-d))
^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x
+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*2^(1/2)*c^2*d-96*(d/(c-d))
```

^(1/2)*sin(f*x+e)*((c+d)*(c-d))^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*
arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x
+e))*2^(1/2)*c*d^2+32*(d/(c-d))^(1/2)*cos(f*x+e)^2*sin(f*x+e)*((c+d)*(c-d))
^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x
+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*2^(1/2)*d^3+126*(d/(c-d))^
(1/2)*cos(f*x+e)^2*sin(f*x+e)*((c+d)*(c-d))^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x
+e)))^(1/2)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e
)-1)/sin(f*x+e))*c^2*d-115*(d/(c-d))^(1/2)*cos(f*x+e)^2*sin(f*x+e)*((c+d)*(
c-d))^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(-(-(-2*cos(f*x+e)/(1+co
s(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))*c*d^2-32*(d/(c-d))^(1
/2)*cos(f*x+e)^2*sin(f*x+e)*((c+d)*(c-d))^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e
)))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+
e)/cos(f*x+e))*2^(1/2)*c^3+16*cos(f*x+e)^2*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos
(f*x+e)))^(1/2)*ln(-2*((-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(d/(c-d))^(1/2)
*2^(1/2)*c*sin(f*x+e)-2^(1/2)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e))
)^(1/2)*d*sin(f*x+e)-((c+d)*(c-d))^(1/2)*cos(f*x+e)-c*sin(f*x+e)+d*sin(f*x+
e)+((c+d)*(c-d))^(1/2))/(((c+d)*(c-d))^(1/2)*sin(f*x+e)+c*cos(f*x+e)-d*cos(
f*x+e)-c+d))*2^(1/2)*d^4-16*cos(f*x+e)^2*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f
*x+e)))^(1/2)*ln(2*((-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(d/(c-d))^(1/2)*2^
(1/2)*c*sin(f*x+e)-2^(1/2)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(
1/2)*d*sin(f*x+e)+((c+d)*(c-d))^(1/2)*cos(f*x+e)-c*sin(f*x+e)+d*sin(f*x+e)-
((c+d)*(c-d))^(1/2))/(((c+d)*(c-d))^(1/2)*sin(f*x+e)-c*cos(f*x+e)+d*cos(f*x
+e)+c-d))*2^(1/2)*d^4+32*cos(f*x+e)*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)
))^(1/2)*ln(-2*((-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(d/(c-d))^(1/2)*2^(1/2
)*c*sin(f*x+e)-2^(1/2)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)
*d*sin(f*x+e)-((c+d)*(c-d))^(1/2)*cos(f*x+e)-c*sin(f*x+e)+d*sin(f*x+e)+((c+
d)*(c-d))^(1/2))/(((c+d)*(c-d))^(1/2)*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-
c+d))*2^(1/2)*d^4-32*cos(f*x+e)*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(
1/2)*ln(2*((-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(d/(c-d))^(1/2)*2^(1/2)*c*s
in(f*x+e)-2^(1/2)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*d*si
n(f*x+e)+((c+d)*(c-d))^(1/2)*cos(f*x+e)-c*sin(f*x+e)+d*sin(f*x+e)-((c+d)*(c
-d))^(1/2))/(((c+d)*(c-d))^(1/2)*sin(f*x+e)-c*cos(f*x+e)+d*cos(f*x+e)+c-d))
*2^(1/2)*d^4-43*(d/(c-d))^(1/2)*sin(f*x+e)*((c+d)*(c-d))^(1/2)*(-2*cos(f*x+
e)/(1+cos(f*x+e)))^(1/2)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x
+e)+cos(f*x+e)-1)/sin(f*x+e))*c^3-43*(d/(c-d))^(1/2)*cos(f*x+e)^2*sin(f*x+e
)*((c+d)*(c-d))^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(-(-(-2*cos(f*
x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))*c^3-86*(d/(
c-d))^(1/2)*cos(f*x+e)*sin(f*x+e)*((c+d)*(c-d))^(1/2)*(-2*cos(f*x+e)/(1+cos
(f*x+e)))^(1/2)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f
*x+e)-1)/sin(f*x+e))*c^3-32*(d/(c-d))^(1/2)*sin(f*x+e)*((c+d)*(c-d))^(1/2)*
(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+
cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*2^(1/2)*c^3+32*(d/(c-d))^(1/2)*si
n(f*x+e)*((c+d)*(c-d))^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1
/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*2^(1
/2)*d^3+126*(d/(c-d))^(1/2)*sin(f*x+e)*((c+d)*(c-d))^(1/2)*(-2*cos(f*x+e)/(

```
1+cos(f*x+e)))^(1/2)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+
cos(f*x+e)-1)/sin(f*x+e))*c^2*d-115*(d/(c-d))^(1/2)*sin(f*x+e)*((c+d)*(c-d)
)^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*
x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))*c*d^2-64*(d/(c-d))^(1/2)*
cos(f*x+e)*sin(f*x+e)*((c+d)*(c-d))^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1
/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos
(f*x+e))*2^(1/2)*c^3+96*(d/(c-d))^(1/2)*cos(f*x+e)^2*sin(f*x+e)*((c+d)*(c-d
))^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(f
*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*2^(1/2)*c^2*d-96*(d/(c-d
))^(1/2)*cos(f*x+e)^2*sin(f*x+e)*((c+d)*(c-d))^(1/2)*(-2*cos(f*x+e)/(1+cos(
f*x+e)))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin
(f*x+e)/cos(f*x+e))*2^(1/2)*c*d^2+192*(d/(c-d))^(1/2)*cos(f*x+e)*sin(f*x+e)
*((c+d)*(c-d))^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*2^(1/
2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e))*2^(1/2)*c^2*
d)*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)/sin(f*x+e)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{5}{2}}\left(c+d\sec\left(e+fx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e)),x)
```

```
[Out] Integral(1/((a*(sec(e + f*x) + 1))**(5/2)*(c + d*sec(e + f*x))), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.182 $\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2}\,dx$

Optimal. Leaf size=756

$$\frac{2d^{7/2}(4c-d)\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{a^{3/2}c^2f(c-d)^4\sqrt{c+d}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{\sqrt{2}\left(c^2-4cd+6d^2\right)\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{a^{3/2}f(c-d)^4\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}\cdot$$

```
[Out] -Tan[e + f*x]/(4*a^2*(c - d)^2*f*(1 + Sec[e + f*x])^2*Sqrt[a + a*Sec[e + f*
x]]) - ((c - 3*d)*Tan[e + f*x])/(2*a^2*(c - d)^3*f*(1 + Sec[e + f*x])*Sqrt[
a + a*Sec[e + f*x]]) - (3*Tan[e + f*x])/(16*a^2*(c - d)^2*f*(1 + Sec[e + f*
x])*Sqrt[a + a*Sec[e + f*x]]) + (2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]
]*Tan[e + f*x])/(a^(3/2)*c^2*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e +
f*x]]) - ((c - 3*d)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan
[e + f*x])/(2*Sqrt[2]*a^(3/2)*(c - d)^3*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a +
 a*Sec[e + f*x]]) - (3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*
Tan[e + f*x])/(16*Sqrt[2]*a^(3/2)*(c - d)^2*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt
[a + a*Sec[e + f*x]]) - (Sqrt[2]*(c^2 - 4*c*d + 6*d^2)*ArcTanh[Sqrt[a - a*S
ec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(a^(3/2)*(c - d)^4*f*Sqrt[a -
 a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (d^(7/2)*ArcTanh[(Sqrt[d]*Sqrt
[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])/(a^(3/2)*c*(c -
d)^3*(c + d)^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (
2*(4*c - d)*d^(7/2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqr
t[c + d])]*Tan[e + f*x])/(a^(3/2)*c^2*(c - d)^4*Sqrt[c + d]*f*Sqrt[a - a*Se
c[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (d^4*Tan[e + f*x])/(a^2*c*(c - d)^3
*(c + d)*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x]))
```

Rubi [A] time = 0.633782, antiderivative size = 756, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3940, 180, 63, 206, 51, 208}

$$\frac{2d^{7/2}(4c-d)\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{a^{3/2}c^2f(c-d)^4\sqrt{c+d}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{\sqrt{2}\left(c^2-4cd+6d^2\right)\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{a^{3/2}f(c-d)^4\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}\cdot$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])^2),x]
```

```
[Out] -Tan[e + f*x]/(4*a^2*(c - d)^2*f*(1 + Sec[e + f*x])^2*Sqrt[a + a*Sec[e + f*
x]]) - ((c - 3*d)*Tan[e + f*x])/(2*a^2*(c - d)^3*f*(1 + Sec[e + f*x])*Sqrt[
```

```
a + a*Sec[e + f*x]]) - (3*Tan[e + f*x])/(16*a^2*(c - d)^2*f*(1 + Sec[e + f*
x])*Sqrt[a + a*Sec[e + f*x]]) + (2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]
]*Tan[e + f*x])/(a^(3/2)*c^2*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e +
f*x]]) - ((c - 3*d)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan
[e + f*x])/(2*Sqrt[2]*a^(3/2)*(c - d)^3*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a +
 a*Sec[e + f*x]]) - (3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*
Tan[e + f*x])/(16*Sqrt[2]*a^(3/2)*(c - d)^2*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt
[a + a*Sec[e + f*x]]) - (Sqrt[2]*(c^2 - 4*c*d + 6*d^2)*ArcTanh[Sqrt[a - a*S
ec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(a^(3/2)*(c - d)^4*f*Sqrt[a -
 a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (d^(7/2)*ArcTanh[(Sqrt[d]*Sqrt
[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])/(a^(3/2)*c*(c -
d)^3*(c + d)^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (
2*(4*c - d)*d^(7/2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqr
t[c + d])]*Tan[e + f*x])/(a^(3/2)*c^2*(c - d)^4*Sqrt[c + d]*f*Sqrt[a - a*Se
c[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (d^4*Tan[e + f*x])/(a^2*c*(c - d)^3
*(c + d)*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x]))
```

Rule 3940

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(a^2*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
x)^n)/(x*Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rule 180

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a-ax}(a+ax)^3(c+dx)^2}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\left(a^2\tan(e+fx)\right)\operatorname{Subst}\left(\int\left(\frac{1}{a^3c^2x\sqrt{a-ax}} - \frac{1}{a^3(c-d)^2(1+x)^3\sqrt{a-ax}} + \frac{-}{a^3(c-d)^3(}\right.\right.}{f\sqrt{a-}}$$

$$= -\frac{\tan(e+fx)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{ac^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{((c-3d)\tan(e+}{a(c-d)^3f}$$

$$= -\frac{\tan(e+fx)}{4a^2(c-d)^2f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} - \frac{(}{2a^2(c-d)^3f(1}$$

$$= -\frac{\tan(e+fx)}{4a^2(c-d)^2f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} - \frac{(}{2a^2(c-d)^3f(1}$$

$$= -\frac{\tan(e+fx)}{4a^2(c-d)^2f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} - \frac{(}{2a^2(c-d)^3f(1}$$

$$= -\frac{\tan(e+fx)}{4a^2(c-d)^2f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} - \frac{(}{2a^2(c-d)^3f(1}$$

Mathematica [C] time = 39.9961, size = 686248, normalized size = 907.74

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])^2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 5.676, size = 197500, normalized size = 261.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^2,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^2,x, algorithm="maxima"
)
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^2,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.183 $\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3}\,dx$

Optimal. Leaf size=999

$$\frac{(4c-d)\tan(e+fx)d^4}{a^2c^2(c-d)^4(c+d)f\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))}+\frac{3\tan(e+fx)d^4}{4a^2c(c-d)^3(c+d)^2f\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))}+$$

```
[Out] -Tan[e + f*x]/(4*a^2*(c - d)^3*f*(1 + Sec[e + f*x])^2*Sqrt[a + a*Sec[e + f*
x]]) - ((c - 4*d)*Tan[e + f*x])/(2*a^2*(c - d)^4*f*(1 + Sec[e + f*x])*Sqrt[
a + a*Sec[e + f*x]]) - (3*Tan[e + f*x])/(16*a^2*(c - d)^3*f*(1 + Sec[e + f*
x])*Sqrt[a + a*Sec[e + f*x]]) + (2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]
]*Tan[e + f*x])/(a^(3/2)*c^3*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e +
f*x]]) - ((c - 4*d)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan
[e + f*x])/(2*Sqrt[2]*a^(3/2)*(c - d)^4*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a +
 a*Sec[e + f*x]]) - (3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*
Tan[e + f*x])/(16*Sqrt[2]*a^(3/2)*(c - d)^3*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt
[a + a*Sec[e + f*x]]) - (Sqrt[2]*(c^2 - 5*c*d + 10*d^2)*ArcTanh[Sqrt[a - a*
Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(a^(3/2)*(c - d)^5*f*Sqrt[a
- a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (3*d^(7/2)*ArcTanh[(Sqrt[d]*S
qrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])/(4*a^(3/2)*c*
(c - d)^3*(c + d)^(5/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]
) + ((4*c - d)*d^(7/2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*
Sqrt[c + d])]*Tan[e + f*x])/(a^(3/2)*c^2*(c - d)^4*(c + d)^(3/2)*f*Sqrt[a -
 a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (2*d^(7/2)*(10*c^2 - 5*c*d + d
^2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e
 + f*x])/(a^(3/2)*c^3*(c - d)^5*Sqrt[c + d]*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt
[a + a*Sec[e + f*x]]) + (d^4*Tan[e + f*x])/(2*a^2*c*(c - d)^3*(c + d)*f*Sqr
t[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^2) + (3*d^4*Tan[e + f*x])/(4*a^2
*c*(c - d)^3*(c + d)^2*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])) + (
(4*c - d)*d^4*Tan[e + f*x])/(a^2*c^2*(c - d)^4*(c + d)*f*Sqrt[a + a*Sec[e +
 f*x]]*(c + d*Sec[e + f*x]))
```

Rubi [A] time = 0.914162, antiderivative size = 999, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3940, 180, 63, 206, 51, 208}

$$\frac{(4c-d)\tan(e+fx)d^4}{a^2c^2(c-d)^4(c+d)f\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))}+\frac{3\tan(e+fx)d^4}{4a^2c(c-d)^3(c+d)^2f\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))}+$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])^3),x]
```

```
[Out] -Tan[e + f*x]/(4*a^2*(c - d)^3*f*(1 + Sec[e + f*x])^2*Sqrt[a + a*Sec[e + f*
x]]) - ((c - 4*d)*Tan[e + f*x])/(2*a^2*(c - d)^4*f*(1 + Sec[e + f*x])*Sqrt[
a + a*Sec[e + f*x]]) - (3*Tan[e + f*x])/(16*a^2*(c - d)^3*f*(1 + Sec[e + f*
x])*Sqrt[a + a*Sec[e + f*x]]) + (2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]
]*Tan[e + f*x])/(a^(3/2)*c^3*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e +
f*x]]) - ((c - 4*d)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan
[e + f*x])/(2*Sqrt[2]*a^(3/2)*(c - d)^4*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a +
 a*Sec[e + f*x]]) - (3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*
Tan[e + f*x])/(16*Sqrt[2]*a^(3/2)*(c - d)^3*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt
[a + a*Sec[e + f*x]]) - (Sqrt[2]*(c^2 - 5*c*d + 10*d^2)*ArcTanh[Sqrt[a - a*
Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(a^(3/2)*(c - d)^5*f*Sqrt[a
- a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (3*d^(7/2)*ArcTanh[(Sqrt[d]*S
qrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])/(4*a^(3/2)*c*
(c - d)^3*(c + d)^(5/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]
) + ((4*c - d)*d^(7/2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*
Sqrt[c + d])]*Tan[e + f*x])/(a^(3/2)*c^2*(c - d)^4*(c + d)^(3/2)*f*Sqrt[a -
 a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (2*d^(7/2)*(10*c^2 - 5*c*d + d
^2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e
 + f*x])/(a^(3/2)*c^3*(c - d)^5*Sqrt[c + d]*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt
[a + a*Sec[e + f*x]]) + (d^4*Tan[e + f*x])/(2*a^2*c*(c - d)^3*(c + d)*f*Sqr
t[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^2) + (3*d^4*Tan[e + f*x])/(4*a^2
*c*(c - d)^3*(c + d)^2*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])) + (
(4*c - d)*d^4*Tan[e + f*x])/(a^2*c^2*(c - d)^4*(c + d)*f*Sqrt[a + a*Sec[e +
 f*x]]*(c + d*Sec[e + f*x]))
```

Rule 3940

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(a^2*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
x)^n)/(x*Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rule 180

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}(a+ax)^3(c+dx)^3}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \left(\frac{1}{a^3c^3x\sqrt{a-ax}} - \frac{1}{a^3(c-d)^3(1+x)^3\sqrt{a-ax}} + \frac{}{a^3(c-d)^4}\right.\right.}{}$$

$$= -\frac{\tan(e+fx)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{ac^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{((c-4d)\tan(e+}{a(c-d)^4f}$$

$$= -\frac{\tan(e+fx)}{4a^2(c-d)^3f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} - \frac{(}{2a^2(c-d)^4f(1}$$

$$= -\frac{\tan(e+fx)}{4a^2(c-d)^3f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} - \frac{(}{2a^2(c-d)^4f(1}$$

$$= -\frac{\tan(e+fx)}{4a^2(c-d)^3f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} - \frac{(}{2a^2(c-d)^4f(1}$$

$$= -\frac{\tan(e+fx)}{4a^2(c-d)^3f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} - \frac{(}{2a^2(c-d)^4f(1}$$

Mathematica [C] time = 43.5952, size = 891356, normalized size = 892.25

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])^3),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 37.042, size = 556423, normalized size = 557.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^3,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^3,x, algorithm="maxima"
)
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^3,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.184 $\int \sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx$

Optimal. Leaf size=123

$$\frac{2\sqrt{a}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{f}+\frac{2\sqrt{a}\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{f}$$

```
[Out] (2*Sqrt[a]*Sqrt[c]*ArcTan[(Sqrt[a]*Sqrt[c]*Tan[e + f*x])/(Sqrt[a + a*Sec[e
+ f*x]]*Sqrt[c + d*Sec[e + f*x]])])/f + (2*Sqrt[a]*Sqrt[d]*ArcTanh[(Sqrt[a]
*Sqrt[d]*Tan[e + f*x])/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])]
)/f
```

Rubi [A] time = 0.338634, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {3932, 3934, 203, 3980, 206}

$$\frac{2\sqrt{a}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{f}+\frac{2\sqrt{a}\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]],x]
```

```
[Out] (2*Sqrt[a]*Sqrt[c]*ArcTan[(Sqrt[a]*Sqrt[c]*Tan[e + f*x])/(Sqrt[a + a*Sec[e
+ f*x]]*Sqrt[c + d*Sec[e + f*x]])])/f + (2*Sqrt[a]*Sqrt[d]*ArcTanh[(Sqrt[a]
*Sqrt[d]*Tan[e + f*x])/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])]
)/f
```

Rule 3932

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Dist[c, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[c +
d*Csc[e + f*x]], x], x] + Dist[d, Int[(Csc[e + f*x]*Sqrt[a + b*Csc[e + f*x]
])/Sqrt[c + d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
b*c - a*d, 0]
```

Rule 3934

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(1 + a*c*x^2), x],
```

```
x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3980

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Dist[(-2*b)/f, Subst
[Int[1/(1 - b*d*x^2), x], x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c
+ d*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0
] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx = c\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx + d\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx$$

$$= -\frac{(2ac)\,\text{Subst}\left(\int \frac{1}{1+acx^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{f} - \frac{(2ad)\,\text{Subst}}{}$$

$$= \frac{2\sqrt{a}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{f} + \frac{2\sqrt{a}\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{a+a\sec(e+f}}\right)}{f}$$

Mathematica [A] time = 18.1594, size = 240, normalized size = 1.95

$$-\frac{2\cot(e+fx)\sqrt{a(\sec(e+fx)+1)}\sqrt{c+d\sec(e+fx)}\left(\sqrt{c^2\sin^2(e+fx)}\sqrt{c-c\cos(e+fx)}\tan^{-1}\left(\frac{\sqrt{c(\cos(e+fx)+1)}\sqrt{c\cos(e+}}{\sqrt{c^2\sin^2(e+fx)}}\right.\right.}{f\sqrt{c(\cos(e+fx)+1)}\sqrt{c-c\cos(e+fx)}\sqrt{c\cos(e}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]],x]
```

```
[Out] (-2*Cot[e + f*x]*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c + d*Sec[e + f*x]]*(-2*Sq
rt[c]*Sqrt[d]*ArcTanh[(Sqrt[c]*Sqrt[d + c*Cos[e + f*x]])/(Sqrt[d]*Sqrt[c -
c*Cos[e + f*x]])]*Sqrt[c*(1 + Cos[e + f*x])]*Sin[(e + f*x)/2]^2 + ArcTan[(S
qrt[c*(1 + Cos[e + f*x])]*Sqrt[d + c*Cos[e + f*x]])/Sqrt[c^2*Sin[e + f*x]^2
]]*Sqrt[c - c*Cos[e + f*x]]*Sqrt[c^2*Sin[e + f*x]^2]))/(f*Sqrt[c*(1 + Cos[e
 + f*x])]*Sqrt[c - c*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])
```

Maple [B] time = 0.42, size = 1563, normalized size = 12.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/f*2^(1/2)/(c-d)^(1/2)/(c^2-2*c*d+d^2)/(-d)^(1/2)*((d+c*cos(f*x+e))/cos(f
*x+e))^(1/2)*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*cos(f*x+e)*(-1+cos(f*x+e
))*(2^(1/2)*(-d)^(1/2)*ln(-((c-d)^(1/2)*cos(f*x+e)-(-2*(d+c*cos(f*x+e))/(1+
cos(f*x+e)))^(1/2)*sin(f*x+e)-(c-d)^(1/2))/sin(f*x+e))*c^3-3*2^(1/2)*(-d)^(
1/2)*ln(-((c-d)^(1/2)*cos(f*x+e)-(-2*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)
*sin(f*x+e)-(c-d)^(1/2))/sin(f*x+e))*c^2*d+3*2^(1/2)*(-d)^(1/2)*ln(-((c-d)^
(1/2)*cos(f*x+e)-(-2*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-(c-d
)^(1/2))/sin(f*x+e))*c*d^2-2^(1/2)*(-d)^(1/2)*ln(-((c-d)^(1/2)*cos(f*x+e)-(
-2*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-(c-d)^(1/2))/sin(f*x+e
))*d^3-2^(1/2)*(-d)^(1/2)*ln(1/(c-d)^(1/2)*((-2*(d+c*cos(f*x+e))/(1+cos(f*x
+e)))^(1/2)*(c-d)^(1/2)*sin(f*x+e)-c*cos(f*x+e)+d*cos(f*x+e)+c-d)/sin(f*x+e
))*c^3+3*2^(1/2)*(-d)^(1/2)*ln(1/(c-d)^(1/2)*((-2*(d+c*cos(f*x+e))/(1+cos(f
*x+e)))^(1/2)*(c-d)^(1/2)*sin(f*x+e)-c*cos(f*x+e)+d*cos(f*x+e)+c-d)/sin(f*x
+e))*c^2*d-3*2^(1/2)*(-d)^(1/2)*ln(1/(c-d)^(1/2)*((-2*(d+c*cos(f*x+e))/(1+c
os(f*x+e)))^(1/2)*(c-d)^(1/2)*sin(f*x+e)-c*cos(f*x+e)+d*cos(f*x+e)+c-d)/sin
(f*x+e))*c*d^2+2^(1/2)*(-d)^(1/2)*ln(1/(c-d)^(1/2)*((-2*(d+c*cos(f*x+e))/(1
+cos(f*x+e)))^(1/2)*(c-d)^(1/2)*sin(f*x+e)-c*cos(f*x+e)+d*cos(f*x+e)+c-d)/s
in(f*x+e))*d^3+(c-d)^(1/2)*ln(2*(2^(1/2)*(-d)^(1/2)*(-2*(d+c*cos(f*x+e))/(1
+cos(f*x+e)))^(1/2)*sin(f*x+e)-c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos
(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c^2*d-2*(c-d)^(1/2)*ln(2*(2^(1/2)*(
-d)^(1/2)*(-2*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-c*sin(f*x+e
)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c*
d^2+(c-d)^(1/2)*ln(2*(2^(1/2)*(-d)^(1/2)*(-2*(d+c*cos(f*x+e))/(1+cos(f*x+e)
```

```
))^(1/2)*sin(f*x+e)-c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d
)/(1-cos(f*x+e)+sin(f*x+e)))*d^3-(c-d)^(1/2)*ln(-2*(2^(1/2)*(-d)^(1/2)*(-2*
(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-c*sin(f*x+e)-d*sin(f*x+e)
-c*cos(f*x+e)+d*cos(f*x+e)+c-d)/(-1+cos(f*x+e)+sin(f*x+e)))*c^2*d+2*(c-d)^(
1/2)*ln(-2*(2^(1/2)*(-d)^(1/2)*(-2*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*s
in(f*x+e)-c*sin(f*x+e)-d*sin(f*x+e)-c*cos(f*x+e)+d*cos(f*x+e)+c-d)/(-1+cos(
f*x+e)+sin(f*x+e)))*c*d^2-(c-d)^(1/2)*ln(-2*(2^(1/2)*(-d)^(1/2)*(-2*(d+c*co
s(f*x+e))/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-c*sin(f*x+e)-d*sin(f*x+e)-c*cos(
f*x+e)+d*cos(f*x+e)+c-d)/(-1+cos(f*x+e)+sin(f*x+e)))*d^3+2*(-(c-d)^4*c)^(1/
2)*arctan((c-d)^2*c*2^(1/2)/(-(c-d)^4*c)^(1/2)*(-1+cos(f*x+e))/(-2*(d+c*cos
(f*x+e))/(1+cos(f*x+e)))^(1/2)/sin(f*x+e))*(c-d)^(1/2)*(-d)^(1/2))/sin(f*x+
e)^2/(-2*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sec\left(fx+e\right)+a}\sqrt{d\sec\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(a*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c), x)
```

Fricas [A] time = 1.40323, size = 2034, normalized size = 16.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] [(sqrt(a*d)*log((2*sqrt(a*d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((
c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c - a*d)*c
os(f*x + e)^2 + 2*a*d + (a*c + a*d)*cos(f*x + e))/(cos(f*x + e)^2 + cos(f*x
 + e))) + sqrt(-a*c)*log((2*a*c*cos(f*x + e)^2 - 2*sqrt(-a*c)*sqrt((a*cos(f
*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x
+ e)*sin(f*x + e) - a*c + a*d + (a*c + a*d)*cos(f*x + e))/(cos(f*x + e) + 1
```

```
)))/f, -(2*sqrt(a*c)*arctan(sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e
))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)/(a*c*sin(f*x + e)))
 - sqrt(a*d)*log((2*sqrt(a*d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(
(c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c - a*d)*
cos(f*x + e)^2 + 2*a*d + (a*c + a*d)*cos(f*x + e))/(cos(f*x + e)^2 + cos(f*
x + e))))/f, -(2*sqrt(-a*d)*arctan(sqrt(-a*d)*sqrt((a*cos(f*x + e) + a)/cos
(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)/(a*d*sin(f*
x + e))) - sqrt(-a*c)*log((2*a*c*cos(f*x + e)^2 - 2*sqrt(-a*c)*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x
 + e)*sin(f*x + e) - a*c + a*d + (a*c + a*d)*cos(f*x + e))/(cos(f*x + e) +
1)))/f, -2*(sqrt(a*c)*arctan(sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x +
e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)/(a*c*sin(f*x + e))
) + sqrt(-a*d)*arctan(sqrt(-a*d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sq
rt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)/(a*d*sin(f*x + e))))/f]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\left(\sec\left(e+fx\right)+1\right)}\sqrt{c+d\sec\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))**(1/2)*(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*sqrt(c + d*sec(e + f*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

3.185 $\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx$

Optimal. Leaf size=61

$$\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{c}f}$$

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Sqrt[c]*Tan[e + f*x])/(Sqrt[a + a*Sec[e + f*x]]*
Sqrt[c + d*Sec[e + f*x]])])/(Sqrt[c]*f)
```

Rubi [A] time = 0.0966942, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {3934, 203}

$$\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{c}f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]/Sqrt[c + d*Sec[e + f*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Sqrt[c]*Tan[e + f*x])/(Sqrt[a + a*Sec[e + f*x]]*
Sqrt[c + d*Sec[e + f*x]])])/(Sqrt[c]*f)
```

Rule 3934

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(1 + a*c*x^2), x],
x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx = -\frac{(2a)\,\text{Subst}\left(\int \frac{1}{1+acx^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{f}$$
$$= \frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{c}f}$$

Mathematica [A] time = 0.214166, size = 102, normalized size = 1.67

$$\frac{\sqrt{2}\sec\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}\sqrt{c\cos(e+fx)+d}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c\cos(e+fx)+d}}\right)}{\sqrt{c}f\sqrt{c+d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]/Sqrt[c + d*Sec[e + f*x]],x]
```

```
[Out] (Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[c]*Sin[(e + f*x)/2])/Sqrt[d + c*Cos[e + f*x]]
]*Sqrt[d + c*Cos[e + f*x]]*Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])])/(Sq
rt[c]*f*Sqrt[c + d*Sec[e + f*x]])
```

Maple [B] time = 0.318, size = 189, normalized size = 3.1

$$-2\,\frac{\sqrt{2}\sqrt{-(c-d)^4c}\cos\left(fx+e\right)\left(-1+\cos\left(fx+e\right)\right)}{f\left(c^2-2\,cd+d^2\right)c\left(\sin\left(fx+e\right)\right)^2}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\sqrt{\frac{d+c\cos\left(fx+e\right)}{\cos\left(fx+e\right)}}\arctan\left(\frac{c\,(c-d)^2\sqrt{2}\,(-}{\sqrt{-(c-d)^4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x)
```

```
[Out] -2/f*2^(1/2)*(-(c-d)^4*c)^(1/2)/(c^2-2*c*d+d^2)/c*(1/cos(f*x+e)*a*(1+cos(f*
x+e)))^(1/2)*cos(f*x+e)*(-1+cos(f*x+e))*((d+c*cos(f*x+e))/cos(f*x+e))^(1/2)
*arctan((c-d)^2*c*2^(1/2)/(-(c-d)^4*c)^(1/2)*(-1+cos(f*x+e))/(-2*(d+c*cos(f
*x+e))/(1+cos(f*x+e)))^(1/2)/sin(f*x+e))/sin(f*x+e)^2/(-2*(d+c*cos(f*x+e))/
```

```
(1+cos(f*x+e)))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.798218, size = 502, normalized size = 8.23

$$\left[\frac{\sqrt{-\frac{a}{c}}\log\left(-\frac{2c\sqrt{-\frac{a}{c}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)+d}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)-2ac\cos(fx+e)^2+ac-ad-(ac+ad)\cos(fx+e)}{\cos(fx+e)+1}\right)}{f}, -\frac{2\sqrt{\frac{a}{c}}\arctan\left(\frac{\sqrt{\frac{a}{c}}\sqrt{\frac{a\cos\ldots}{c\ldots}}\ldots}{\ldots}\right)}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] [sqrt(-a/c)*log(-(2*c*sqrt(-a/c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sq
rt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 2*a*c*cos
(f*x + e)^2 + a*c - a*d - (a*c + a*d)*cos(f*x + e))/(cos(f*x + e) + 1))/f,
-2*sqrt(a/c)*arctan(sqrt(a/c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(
(c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)/(a*sin(f*x + e)))/f]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}}{\sqrt{c+d\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))/sqrt(c + d*sec(e + f*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

3.186 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=111

$$\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{c^{3/2}f}-\frac{2ad\tan(e+fx)}{cf(c+d)\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}$$

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Sqrt[c]*Tan[e + f*x])/(Sqrt[a + a*Sec[e + f*x]]*
Sqrt[c + d*Sec[e + f*x]])])/(c^(3/2)*f) - (2*a*d*Tan[e + f*x])/(c*(c + d)*f
*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])
```

Rubi [A] time = 0.357118, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {3939, 3934, 203, 3987, 37}

$$\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{c^{3/2}f}-\frac{2ad\tan(e+fx)}{cf(c+d)\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]/(c + d*Sec[e + f*x])^(3/2),x]
```


```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Sqrt[c]*Tan[e + f*x])/(Sqrt[a + a*Sec[e + f*x]]*
Sqrt[c + d*Sec[e + f*x]])])/(c^(3/2)*f) - (2*a*d*Tan[e + f*x])/(c*(c + d)*f
*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])
```


Rule 3939

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(3/2), x_Symbol] :> Dist[1/c, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[
c + d*Csc[e + f*x]], x], x] - Dist[d/c, Int[(Csc[e + f*x]*Sqrt[a + b*Csc[e
+ f*x]])/(c + d*Csc[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3934

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(1 + a*c*x^2), x],
x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /;
```

```
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx &= \frac{\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx}{c} - \frac{d\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx}{c} \\
&= -\frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{1+acx^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{cf} + \frac{\left(a^2d\tan(e+fx)\right)\mathrm{Subst}\left(\int \frac{}{\sqrt{a}}\right.}{cf\sqrt{a-a\sec(e+fx)}} \\
&= \frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{c^{3/2}f} - \frac{2ad\tan(e+fx)}{c(c+d)f\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}
\end{aligned}$$

Mathematica [A] time = 0.955859, size = 135, normalized size = 1.22

$$-\frac{\sec\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}\left(2\sqrt{c}d\sin\left(\frac{1}{2}(e+fx)\right)-\sqrt{2}(c+d)^{3/2}\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\right)}{c^{3/2}f(c+d)\sqrt{c+d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]/(c + d*Sec[e + f*x])^(3/2),x]
```

```
[Out] -((Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])]*(-(Sqrt[2]*(c + d)^(3/2)*Arc
Sin[(Sqrt[2]*Sqrt[c]*Sin[(e + f*x)/2])/Sqrt[c + d]]*Sqrt[(d + c*Cos[e + f*x
])/(c + d)]) + 2*Sqrt[c]*d*Sin[(e + f*x)/2]))/(c^(3/2)*(c + d)*f*Sqrt[c + d
*Sec[e + f*x]]))
```

Maple [B] time = 0.338, size = 377, normalized size = 3.4

$$-\frac{\cos\left(fx+e\right)}{f\left(c+d\right)c^{2}\left(c^{2}-2\,cd+d^{2}\right)\left(d+c\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}\left(\sqrt{2}\sqrt{-\left(c-d\right)^{4}c}\arctan\left(\frac{c\left(c-d\right)^{2}\sqrt{2}\left(-1+\cos\left(fx+e\right.\right.}{\sin\left(fx+e\right)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(3/2),x)
```

```
[Out] -1/f/(c+d)/c^2/(c^2-2*c*d+d^2)*(2^(1/2)*(-(c-d)^4*c)^(1/2)*arctan((c-d)^2*c
*2^(1/2)/(-(c-d)^4*c)^(1/2)*(-1+cos(f*x+e))/(-2*(d+c*cos(f*x+e))/(1+cos(f*x
+e)))^(1/2)/sin(f*x+e))*(-2*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*c*sin(f*
x+e)+2^(1/2)*(-(c-d)^4*c)^(1/2)*arctan((c-d)^2*c*2^(1/2)/(-(c-d)^4*c)^(1/2)
*(-1+cos(f*x+e))/(-2*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)/sin(f*x+e))*(-2
*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*d*sin(f*x+e)-2*c^3*d*cos(f*x+e)+4*c
^2*d^2*cos(f*x+e)-2*cos(f*x+e)*c*d^3+2*c^3*d-4*c^2*d^2+2*c*d^3)*cos(f*x+e)*
(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*((d+c*cos(f*x+e))/cos(f*x+e))^(1/2)/(
d+c*cos(f*x+e))/sin(f*x+e)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.919819, size = 1262, normalized size = 11.37

$$\left[-\frac{2\,d\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)+d}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)\sin\left(fx+e\right)-\left(\left(c^2+cd\right)\cos\left(fx+e\right)^2+cd+d^2+\left(c^2+2\,cd+d^2\right)\cos\left(f\right.\right.}{\left(c^3+c^2d\right)f\cos\left(fx+e\right)^2+\left(c^3+2\,c^2d+cd^2\right)f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] [-(2*d*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/co
s(f*x + e))*cos(f*x + e)*sin(f*x + e) - ((c^2 + c*d)*cos(f*x + e)^2 + c*d +
 d^2 + (c^2 + 2*c*d + d^2)*cos(f*x + e))*sqrt(-a/c)*log(-(2*c*sqrt(-a/c)*sq
rt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e
))*cos(f*x + e)*sin(f*x + e) - 2*a*c*cos(f*x + e)^2 + a*c - a*d - (a*c + a*
d)*cos(f*x + e))/(cos(f*x + e) + 1)))/((c^3 + c^2*d)*f*cos(f*x + e)^2 + (c^
3 + 2*c^2*d + c*d^2)*f*cos(f*x + e) + (c^2*d + c*d^2)*f), -2*(d*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*
x + e)*sin(f*x + e) + ((c^2 + c*d)*cos(f*x + e)^2 + c*d + d^2 + (c^2 + 2*c*
d + d^2)*cos(f*x + e))*sqrt(a/c)*arctan(sqrt(a/c)*sqrt((a*cos(f*x + e) + a)
/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)/(a*sin(
f*x + e))))/((c^3 + c^2*d)*f*cos(f*x + e)^2 + (c^3 + 2*c^2*d + c*d^2)*f*cos
(f*x + e) + (c^2*d + c*d^2)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}}{\left(c+d\sec\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))/(c + d*sec(e + f*x))**(3/2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

3.187 $\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal. Leaf size=141

$$\frac{2\sqrt{c}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f}-\frac{\sqrt{2}\sqrt{c-d}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f}$$

```
[Out] (2*Sqrt[c]*ArcTan[(Sqrt[a]*Sqrt[c]*Tan[e + f*x])/(Sqrt[a + a*Sec[e + f*x]]*
Sqrt[c + d*Sec[e + f*x]])])/(Sqrt[a]*f) - (Sqrt[2]*Sqrt[c - d]*ArcTan[(Sqrt
[a]*Sqrt[c - d]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*
Sec[e + f*x]])])/(Sqrt[a]*f)
```

Rubi [A] time = 0.368627, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {3935, 3934, 203, 3983}

$$\frac{2\sqrt{c}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f}-\frac{\sqrt{2}\sqrt{c-d}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Sec[e + f*x]]/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*Sqrt[c]*ArcTan[(Sqrt[a]*Sqrt[c]*Tan[e + f*x])/(Sqrt[a + a*Sec[e + f*x]]*
Sqrt[c + d*Sec[e + f*x]])])/(Sqrt[a]*f) - (Sqrt[2]*Sqrt[c - d]*ArcTan[(Sqrt
[a]*Sqrt[c - d]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*
Sec[e + f*x]])])/(Sqrt[a]*f)
```

Rule 3935

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Dist[a/c, Int[Sqrt[c + d*Csc[e + f*x]]/Sqrt[a
+ b*Csc[e + f*x]], x], x] + Dist[(b*c - a*d)/c, Int[Csc[e + f*x]/(Sqrt[a +
b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

Rule 3934

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(1 + a*c*x^2), x],
x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3983

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqr
t[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Dist[(-2*a)/(b*f), S
ubst[Int[1/(2 + (a*c - b*d)*x^2), x], x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f
*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx &= \frac{c\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx}{a} + (-c+d)\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx\\ &= -\frac{(2c)\,\mathrm{Subst}\left(\int \frac{1}{1+acx^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{f} + \frac{(2(c-d))\,\mathrm{Subst}\left(\int \frac{1}{2+(ac-ad)x^2}\,d\right.}{}\\ &= \frac{2\sqrt{c}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f} - \frac{\sqrt{2}\sqrt{c-d}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f}\end{aligned}$$

Mathematica [A] time = 14.5044, size = 184, normalized size = 1.3

$$\frac{2\cos\left(\frac{1}{2}(e+fx)\right)\sqrt{c+d\sec(e+fx)}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{c+d}\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)\sqrt{\frac{c\cos(e+fx)+d}{c+d}}}{\sqrt{c\cos(e+fx)+d}} + \sqrt{d-c}\tanh^{-1}\left(\frac{\sqrt{d-c}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c\cos(e+fx)+d}}\right)\right)}{f\sqrt{a(\sec(e+fx)+1)}\sqrt{c\cos(e+fx)+d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*Sec[e + f*x]]/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*Cos[(e + f*x)/2]*(Sqrt[-c + d]*ArcTanh[(Sqrt[-c + d]*Sin[(e + f*x)/2])/S
qrt[d + c*Cos[e + f*x]]] + (Sqrt[2]*Sqrt[c]*Sqrt[c + d]*ArcSin[(Sqrt[2]*Sqr
t[c]*Sin[(e + f*x)/2])/Sqrt[c + d]]*Sqrt[(d + c*Cos[e + f*x])/(c + d)])/Sqr
t[d + c*Cos[e + f*x]])*Sqrt[c + d*Sec[e + f*x]])/(f*Sqrt[d + c*Cos[e + f*x]
]*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [B] time = 0.33, size = 494, normalized size = 3.5

$$-2\,\frac{\cos\left(fx+e\right)\left(-1+\cos\left(fx+e\right)\right)}{af\sqrt{c-d}\left(c^{2}-2\,cd+d^{2}\right)\left(\sin\left(fx+e\right)\right)^{2}}\sqrt{\frac{d+c\cos\left(fx+e\right)}{\cos\left(fx+e\right)}}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\left(\sqrt{2}\sqrt{-\left(c-d\right)^{4}c}\,\mathrm{arctan}\left(\frac{c\,(c-}{\sqrt{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] -2/f/a/(c-d)^(1/2)/(c^2-2*c*d+d^2)*((d+c*cos(f*x+e))/cos(f*x+e))^(1/2)*(1/c
os(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*cos(f*x+e)*(2^(1/2)*(-(c-d)^4*c)^(1/2)*ar
ctan((c-d)^2*c*2^(1/2)/(-(c-d)^4*c)^(1/2)*(-1+cos(f*x+e))/(-2*(d+c*cos(f*x+
e))/(1+cos(f*x+e)))^(1/2)/sin(f*x+e))*(c-d)^(1/2)+ln(-((c-d)^(1/2)*cos(f*x+
e)-(-2*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-(c-d)^(1/2))/sin(f
*x+e))*c^3-3*ln(-((c-d)^(1/2)*cos(f*x+e)-(-2*(d+c*cos(f*x+e))/(1+cos(f*x+e)
))^(1/2)*sin(f*x+e)-(c-d)^(1/2))/sin(f*x+e))*c^2*d+3*ln(-((c-d)^(1/2)*cos(f
*x+e)-(-2*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-(c-d)^(1/2))/si
n(f*x+e))*c*d^2-ln(-((c-d)^(1/2)*cos(f*x+e)-(-2*(d+c*cos(f*x+e))/(1+cos(f*x
+e)))^(1/2)*sin(f*x+e)-(c-d)^(1/2))/sin(f*x+e))*d^3)*(-1+cos(f*x+e))/sin(f*
x+e)^2/(-2*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.89197, size = 2190, normalized size = 15.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] [1/2*(sqrt(2)*sqrt(-(c - d)/a)*log((2*sqrt(2)*sqrt(-(c - d)/a)*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x
 + e)*sin(f*x + e) + (3*c - d)*cos(f*x + e)^2 + 2*(c + d)*cos(f*x + e) - c
+ 3*d)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 2*sqrt(-c/a)*log(-(2*sqrt(-
c/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(
f*x + e))*cos(f*x + e)*sin(f*x + e) - 2*c*cos(f*x + e)^2 - (c + d)*cos(f*x
+ e) + c - d)/(cos(f*x + e) + 1)))/f, 1/2*(sqrt(2)*sqrt(-(c - d)/a)*log((2*
sqrt(2)*sqrt(-(c - d)/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*co
s(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (3*c - d)*cos(f*x
 + e)^2 + 2*(c + d)*cos(f*x + e) - c + 3*d)/(cos(f*x + e)^2 + 2*cos(f*x + e
) + 1)) - 4*sqrt(c/a)*arctan(sqrt(c/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x +
e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)/(c*sin(f*x + e))))
/f, -(sqrt(2)*sqrt((c - d)/a)*arctan(-sqrt(2)*sqrt((c - d)/a)*sqrt((a*cos(f
*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x
+ e)/((c - d)*sin(f*x + e))) - sqrt(-c/a)*log(-(2*sqrt(-c/a)*sqrt((a*cos(f*
x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x +
 e)*sin(f*x + e) - 2*c*cos(f*x + e)^2 - (c + d)*cos(f*x + e) + c - d)/(cos(
f*x + e) + 1)))/f, -(sqrt(2)*sqrt((c - d)/a)*arctan(-sqrt(2)*sqrt((c - d)/a
)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x
 + e))*cos(f*x + e)/((c - d)*sin(f*x + e))) + 2*sqrt(c/a)*arctan(sqrt(c/a)*
sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x +
 e))*cos(f*x + e)/(c*sin(f*x + e))))/f]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c + d \sec \left(e + f x\right)}}{\sqrt{a \left(\sec \left(e + f x\right) + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*sec(e + f*x))/sqrt(a*(sec(e + f*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\sec\left(fx+e\right)+c}}{\sqrt{a\sec\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(d*sec(f*x + e) + c)/sqrt(a*sec(f*x + e) + a), x)
```

3.188 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$

Optimal. Leaf size=141

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\sqrt{c}f}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f\sqrt{c-d}}$$

```
[Out] (2*ArcTan[(Sqrt[a]*Sqrt[c]*Tan[e + f*x])/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c +
 d*Sec[e + f*x]])])/(Sqrt[a]*Sqrt[c]*f) - (Sqrt[2]*ArcTan[(Sqrt[a]*Sqrt[c -
 d]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]
])])/(Sqrt[a]*Sqrt[c - d]*f)
```

Rubi [A] time = 0.338635, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {3938, 3934, 203, 3983}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\sqrt{c}f}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f\sqrt{c-d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]),x]
```


```
[Out] (2*ArcTan[(Sqrt[a]*Sqrt[c]*Tan[e + f*x])/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c +
 d*Sec[e + f*x]])])/(Sqrt[a]*Sqrt[c]*f) - (Sqrt[2]*ArcTan[(Sqrt[a]*Sqrt[c -
 d]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]
])])/(Sqrt[a]*Sqrt[c - d]*f)
```


Rule 3938

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqrt[csc[(e_.) + (f_.)*(x
_)]*(d_.) + (c_)]), x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqr
t[c + d*Csc[e + f*x]], x], x] - Dist[b/a, Int[Csc[e + f*x]/(Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0]
```

Rule 3934

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(1 + a*c*x^2), x],
x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3983

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqr
t[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Dist[(-2*a)/(b*f), S
ubst[Int[1/(2 + (a*c - b*d)*x^2), x], x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f
*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \frac{\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx}{a} - \int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$$

$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{1+acx^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{f} + \frac{2\,\text{Subst}\left(\int \frac{1}{2+(ac-\ldots}\right.}{\ldots}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\sqrt{c}f} - \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+\ldots}}\right.}{\sqrt{a}\sqrt{c-d}f}$$

Mathematica [A] time = 0.342739, size = 171, normalized size = 1.21

$$\frac{2\cos\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\sqrt{c\cos(e+fx)+d}\left(\sqrt{2}\sqrt{c-d}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c\cos(e+fx)+d}}\right)-\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c-d}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c\cos(e+fx)+d}}\right)\right)}{\sqrt{c}f\sqrt{c-d}\sqrt{a(\sec(e+fx)+1)}\sqrt{c+d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]),x]
```

```
[Out] (2*(Sqrt[2]*Sqrt[c - d]*ArcTan[(Sqrt[2]*Sqrt[c]*Sin[(e + f*x)/2])/Sqrt[d +
c*Cos[e + f*x]]] - Sqrt[c]*ArcTan[(Sqrt[c - d]*Sin[(e + f*x)/2])/Sqrt[d + c
*Cos[e + f*x]]])*Cos[(e + f*x)/2]*Sqrt[d + c*Cos[e + f*x]]*Sec[e + f*x])/(S
qrt[c]*Sqrt[c - d]*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c + d*Sec[e + f*x]])
```

Maple [B] time = 0.332, size = 424, normalized size = 3.

$$-2\frac{\cos\left(fx+e\right)\left(-1+\cos\left(fx+e\right)\right)}{af\sqrt{c-d}\left(c^{2}-2\,cd+d^{2}\right)c\left(\sin\left(fx+e\right)\right)^{2}}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\sqrt{\frac{d+c\cos\left(fx+e\right)}{\cos\left(fx+e\right)}}\left(\sqrt{2}\sqrt{-\left(c-d\right)^{4}c}\arctan\left(\frac{c(\cdots}{\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x)
```

```
[Out] -2/f/a/(c-d)^(1/2)/(c^2-2*c*d+d^2)/c*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*
((d+c*cos(f*x+e))/cos(f*x+e))^(1/2)*cos(f*x+e)*(-1+cos(f*x+e))*(2^(1/2)*(-(
c-d)^4*c)^(1/2)*arctan((c-d)^2*c*2^(1/2)/(-(c-d)^4*c)^(1/2)*(-1+cos(f*x+e))
/(-2*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)/sin(f*x+e))*(c-d)^(1/2)+ln(-((c
-d)^(1/2)*cos(f*x+e)-(-2*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-
(c-d)^(1/2))/sin(f*x+e))*c^3-2*ln(-((c-d)^(1/2)*cos(f*x+e)-(-2*(d+c*cos(f*x
+e))/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-(c-d)^(1/2))/sin(f*x+e))*c^2*d+ln(-((
c-d)^(1/2)*cos(f*x+e)-(-2*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)
-(c-d)^(1/2))/sin(f*x+e))*c*d^2)/sin(f*x+e)^2/(-2*(d+c*cos(f*x+e))/(1+cos(f
*x+e)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{a\sec\left(fx+e\right)+a}\sqrt{d\sec\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/(sqrt(a*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)), x)
```

Fricas [A] time = 3.86766, size = 2290, normalized size = 16.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] [1/2*(sqrt(2)*a*c*sqrt(-1/(a*c - a*d))*log((2*sqrt(2)*(c - d)*sqrt((a*cos(f
*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*sqrt(-1/
(a*c - a*d))*cos(f*x + e)*sin(f*x + e) + (3*c - d)*cos(f*x + e)^2 + 2*(c +
d)*cos(f*x + e) - c + 3*d)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) - 2*sqrt(
-a*c)*log((2*a*c*cos(f*x + e)^2 + 2*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/co
s(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)*sin(f*x +
e) - a*c + a*d + (a*c + a*d)*cos(f*x + e))/(cos(f*x + e) + 1)))/(a*c*f), 1/
2*(sqrt(2)*a*c*sqrt(-1/(a*c - a*d))*log((2*sqrt(2)*(c - d)*sqrt((a*cos(f*x
+ e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*sqrt(-1/(a*
c - a*d))*cos(f*x + e)*sin(f*x + e) + (3*c - d)*cos(f*x + e)^2 + 2*(c + d)*
cos(f*x + e) - c + 3*d)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) - 4*sqrt(a*c
)*arctan(sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x
+ e) + d)/cos(f*x + e))*cos(f*x + e)/(a*c*sin(f*x + e))))/(a*c*f), (sqrt(2)
*a*c*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x
 + e) + d)/cos(f*x + e))*cos(f*x + e)/(sqrt(a*c - a*d)*sin(f*x + e)))/sqrt(
a*c - a*d) - sqrt(-a*c)*log((2*a*c*cos(f*x + e)^2 + 2*sqrt(-a*c)*sqrt((a*co
s(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f
*x + e)*sin(f*x + e) - a*c + a*d + (a*c + a*d)*cos(f*x + e))/(cos(f*x + e)
+ 1)))/(a*c*f), (sqrt(2)*a*c*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f
*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)/(sqrt(a*c - a
*d)*sin(f*x + e)))/sqrt(a*c - a*d) - 2*sqrt(a*c)*arctan(sqrt(a*c)*sqrt((a*c
os(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(
f*x + e)/(a*c*sin(f*x + e))))/(a*c*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\sqrt{c+d\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sec(e + f*x) + 1))*sqrt(c + d*sec(e + f*x))), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] Timed out
```

3.189 $\int \frac{a+b\sec(e+fx)}{c+d\sec(e+fx)}\,dx$

Optimal. Leaf size=67

$$\frac{2(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{cf\sqrt{c-d}\sqrt{c+d}}+\frac{ax}{c}$$

```
[Out] (a*x)/c + (2*(b*c - a*d)*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]
])/(c*Sqrt[c - d]*Sqrt[c + d]*f)
```

Rubi [A] time = 0.125556, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3919, 3831, 2659, 208}

$$\frac{2(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{cf\sqrt{c-d}\sqrt{c+d}}+\frac{ax}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x])/(c + d*Sec[e + f*x]),x]
```

```
[Out] (a*x)/c + (2*(b*c - a*d)*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]
])/(c*Sqrt[c - d]*Sqrt[c + d]*f)
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\sec(e+fx)}{c+d\sec(e+fx)}\,dx &= \frac{ax}{c} - \frac{(-bc+ad)\int \frac{\sec(e+fx)}{c+d\sec(e+fx)}\,dx}{c} \\
&= \frac{ax}{c} - \frac{(-bc+ad)\int \frac{1}{1+\frac{c\cos(e+fx)}{d}}\,dx}{cd} \\
&= \frac{ax}{c} + \frac{(2(bc-ad))\,\text{Subst}\left(\int \frac{1}{1+\frac{c}{d}+\left(1-\frac{c}{d}\right)x^2}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{cdf} \\
&= \frac{ax}{c} + \frac{2(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c\sqrt{c-d}\sqrt{c+d}f}
\end{aligned}$$

Mathematica [A] time = 0.151059, size = 68, normalized size = 1.01

$$\frac{\frac{2(ad-bc)\tanh^{-1}\left(\frac{(d-c)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}}+a(e+fx)}{cf}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x])/(c + d*Sec[e + f*x]),x]
```

```
[Out] (a*(e + f*x) + (2*(-(b*c) + a*d)*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c
^2 - d^2]])/Sqrt[c^2 - d^2])/(c*f)
```

Maple [A] time = 0.066, size = 113, normalized size = 1.7

$$2\frac{a\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)}{fc}-2\,\frac{ad}{fc\sqrt{(c+d)\,(c-d)}}\mathrm{Artanh}\left(\frac{\tan\left(1/2\,fx+e/2\right)(c-d)}{\sqrt{(c+d)\,(c-d)}}\right)+2\,\frac{b}{f\sqrt{(c+d)\,(c-d)}}\mathrm{Artanh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x)
```

```
[Out] 2/f*a/c*arctan(tan(1/2*f*x+1/2*e))-2/f/c/((c+d)*(c-d))^(1/2)*arctanh(tan(1/
2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))^(1/2))*a*d+2/f/((c+d)*(c-d))^(1/2)*arctanh
(tan(1/2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))^(1/2))*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.59345, size = 540, normalized size = 8.06

$$\left[\frac{2\left(ac^2-ad^2\right)fx-(bc-ad)\sqrt{c^2-d^2}\log\left(\frac{2\,cd\cos\left(fx+e\right)-\left(c^2-2\,d^2\right)\cos\left(fx+e\right)^2-2\,\sqrt{c^2-d^2}\left(d\cos\left(fx+e\right)+c\right)\sin\left(fx+e\right)+2\,c^2-d^2}{c^2\cos\left(fx+e\right)^2+2\,cd\cos\left(fx+e\right)+d^2}\right)}{2\left(c^3-cd^2\right)f},\frac{\left(ac^2-ad^2\right)}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a*c^2 - a*d^2)*f*x - (b*c - a*d)*sqrt(c^2 - d^2)*log((2*c*d*cos(f*
x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) +
```

```
 c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) +
d^2)))/((c^3 - c*d^2)*f), ((a*c^2 - a*d^2)*f*x + (b*c - a*d)*sqrt(-c^2 + d^
2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))
))/((c^3 - c*d^2)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b\sec\left(e + fx\right)}{c + d\sec\left(e + fx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x)
```

```
[Out] Integral((a + b*sec(e + f*x))/(c + d*sec(e + f*x)), x)
```

Giac [A] time = 1.47724, size = 142, normalized size = 2.12

$$\frac{\frac{(fx+e)a}{c} + \frac{2\left(\pi\left\lfloor \frac{fx+e}{2\pi} + \frac{1}{2}\right\rfloor \mathrm{sgn}(-2\,c+2\,d) + \arctan\left(-\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right) - d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-c^2+d^2}}\right)\right)(bc-ad)}{\sqrt{-c^2+d^2}\,c}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] ((f*x + e)*a/c + 2*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c + 2*d) + arct
an(-(c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*(b
*c - a*d)/(sqrt(-c^2 + d^2)*c))/f
```

3.190 $\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^2}\,dx$

Optimal. Leaf size=123

$$\frac{2\left(-2ac^2d+ad^3+bc^3\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^2f(c-d)^{3/2}(c+d)^{3/2}}-\frac{d(bc-ad)\tan(e+fx)}{cf\left(c^2-d^2\right)(c+d\sec(e+fx))}+\frac{ax}{c^2}$$

```
[Out] (a*x)/c^2 + (2*(b*c^3 - 2*a*c^2*d + a*d^3)*ArcTanh[(Sqrt[c - d]*Tan[(e + f*
x)/2])/Sqrt[c + d]])/(c^2*(c - d)^(3/2)*(c + d)^(3/2)*f) - (d*(b*c - a*d)*T
an[e + f*x])/(c*(c^2 - d^2)*f*(c + d*Sec[e + f*x]))
```

Rubi [A] time = 0.2468, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3923, 3919, 3831, 2659, 208}

$$\frac{2\left(-2ac^2d+ad^3+bc^3\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^2f(c-d)^{3/2}(c+d)^{3/2}}-\frac{d(bc-ad)\tan(e+fx)}{cf\left(c^2-d^2\right)(c+d\sec(e+fx))}+\frac{ax}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x])/(c + d*Sec[e + f*x])^2,x]
```


```
[Out] (a*x)/c^2 + (2*(b*c^3 - 2*a*c^2*d + a*d^3)*ArcTanh[(Sqrt[c - d]*Tan[(e + f*
x)/2])/Sqrt[c + d]])/(c^2*(c - d)^(3/2)*(c + d)^(3/2)*f) - (d*(b*c - a*d)*T
an[e + f*x])/(c*(c^2 - d^2)*f*(c + d*Sec[e + f*x]))
```


Rule 3923

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(b*(b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f
*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)
), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && Ne
Q[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^2}\,dx = -\frac{d(bc-ad)\tan(e+fx)}{c\left(c^2-d^2\right)f(c+d\sec(e+fx))} - \frac{\int \frac{-a\left(c^2-d^2\right)-c(bc-ad)\sec(e+fx)}{c+d\sec(e+fx)}\,dx}{c\left(c^2-d^2\right)}$$

$$= \frac{ax}{c^2} - \frac{d(bc-ad)\tan(e+fx)}{c\left(c^2-d^2\right)f(c+d\sec(e+fx))} + \frac{\left(c^2(bc-ad)-ad\left(c^2-d^2\right)\right)\int \frac{\sec(e+fx)}{c+d\sec(e+fx)}\,dx}{c^2\left(c^2-d^2\right)}$$

$$= \frac{ax}{c^2} - \frac{d(bc-ad)\tan(e+fx)}{c\left(c^2-d^2\right)f(c+d\sec(e+fx))} + \frac{\left(c^2(bc-ad)-ad\left(c^2-d^2\right)\right)\int \frac{1}{1+\frac{c\cos(e+fx)}{d}}\,dx}{c^2d\left(c^2-d^2\right)}$$

$$= \frac{ax}{c^2} - \frac{d(bc-ad)\tan(e+fx)}{c\left(c^2-d^2\right)f(c+d\sec(e+fx))} + \frac{\left(2\left(c^2(bc-ad)-ad\left(c^2-d^2\right)\right)\right)\text{Subst}\left(\int \frac{1}{1+\frac{c}{d}+\left(1-\frac{c}{d}\right.}\right.}{c^2d\left(c^2-d^2\right)f}$$

$$= \frac{ax}{c^2} + \frac{2\left(bc^3-2ac^2d+ad^3\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^2(c-d)^{3/2}(c+d)^{3/2}f} - \frac{d(bc-ad)\tan(e+fx)}{c\left(c^2-d^2\right)f(c+d\sec(e+fx))}$$

Mathematica [A] time = 0.643414, size = 155, normalized size = 1.26

$$\frac{\frac{-cd(bc-ad)\sin(e+fx)+ad\left(c^2-d^2\right)(e+fx)+ac\left(c^2-d^2\right)(e+fx)\cos(e+fx)}{c\cos(e+fx)+d}-\frac{2\left(ad\left(d^2-2c^2\right)+bc^3\right)\tanh^{-1}\left(\frac{(d-c)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}}}{c^2f(c-d)(c+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x])/(c + d*Sec[e + f*x])^2,x]
```

```
[Out] ((-2*(b*c^3 + a*d*(-2*c^2 + d^2))*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[
c^2 - d^2]])/Sqrt[c^2 - d^2] + (a*d*(c^2 - d^2)*(e + f*x) + a*c*(c^2 - d^2)
*(e + f*x)*Cos[e + f*x] - c*d*(b*c - a*d)*Sin[e + f*x])/(d + c*Cos[e + f*x]
))/(c^2*(c - d)*(c + d)*f)
```

Maple [B] time = 0.086, size = 328, normalized size = 2.7

$$2\,\frac{a\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)}{fc^2}-2\,\frac{d^2\tan\left(1/2\,fx+e/2\right)a}{fc\left(c^2-d^2\right)\left(\left(\tan\left(1/2\,fx+e/2\right)\right)^2c-\left(\tan\left(1/2\,fx+e/2\right)\right)^2d-c-d\right)}+2\,\frac{}{f\left(c^2-d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x)
```

```
[Out] 2/f*a/c^2*arctan(tan(1/2*f*x+1/2*e))-2/f/c*d^2/(c^2-d^2)*tan(1/2*f*x+1/2*e)
/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)*a+2/f*d/(c^2-d^2)*tan(
1/2*f*x+1/2*e)/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)*b-4/f/(c
+d)/(c-d)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*f*x+1/2*e)*(c-d)/((c+d)*(c-d)
)^(1/2))*a*d+2/f/c^2/(c+d)/(c-d)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*f*x+1/
2*e)*(c-d)/((c+d)*(c-d))^(1/2))*a*d^3+2/f*c/(c+d)/(c-d)/((c+d)*(c-d))^(1/2)
*arctanh(tan(1/2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))^(1/2))*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.602482, size = 1226, normalized size = 9.97

$$\left[\frac{2\left(ac^5-2\,ac^3d^2+acd^4\right)fx\cos\left(fx+e\right)+2\left(ac^4d-2\,ac^2d^3+ad^5\right)fx-\left(bc^3d-2\,ac^2d^2+ad^4+\left(bc^4-2\,ac^3d+acd^3\right)c\right.}{2\left(\left(c^7-2\,c^5d^2+c^3d^4\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a*c^5 - 2*a*c^3*d^2 + a*c*d^4)*f*x*cos(f*x + e) + 2*(a*c^4*d - 2*a
*c^2*d^3 + a*d^5)*f*x - (b*c^3*d - 2*a*c^2*d^2 + a*d^4 + (b*c^4 - 2*a*c^3*d
 + a*c*d^3)*cos(f*x + e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 -
2*d^2)*cos(f*x + e)^2 - 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e)
 + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) - 2*(b*c^4
*d - a*c^3*d^2 - b*c^2*d^3 + a*c*d^4)*sin(f*x + e))/((c^7 - 2*c^5*d^2 + c^3
*d^4)*f*cos(f*x + e) + (c^6*d - 2*c^4*d^3 + c^2*d^5)*f), ((a*c^5 - 2*a*c^3*
d^2 + a*c*d^4)*f*x*cos(f*x + e) + (a*c^4*d - 2*a*c^2*d^3 + a*d^5)*f*x + (b*
c^3*d - 2*a*c^2*d^2 + a*d^4 + (b*c^4 - 2*a*c^3*d + a*c*d^3)*cos(f*x + e))*s
qrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*
sin(f*x + e))) - (b*c^4*d - a*c^3*d^2 - b*c^2*d^3 + a*c*d^4)*sin(f*x + e))/
((c^7 - 2*c^5*d^2 + c^3*d^4)*f*cos(f*x + e) + (c^6*d - 2*c^4*d^3 + c^2*d^5)
*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+b\sec\left(e+fx\right)}{\left(c+d\sec\left(e+fx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e))**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x))/(c + d*sec(e + f*x))**2, x)
```

<u>Giac</u> [A] time = 1.46028, size = 282, normalized size = 2.29

$$\frac{\frac{2\left(bc^{3}-2\,ac^{2}d+ad^{3}\right)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,c+2\,d)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-c^{2}+d^{2}}}\right)\right)}{\left(c^{4}-c^{2}d^{2}\right)\sqrt{-c^{2}+d^{2}}}+\frac{\left(fx+e\right)a}{c^{2}}+\frac{2\left(bcd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-ad^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(c^{3}-cd^{2}\right)\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c-\right.}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] (2*(b*c^3 - 2*a*c^2*d + a*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c +
 2*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2
 + d^2)))/((c^4 - c^2*d^2)*sqrt(-c^2 + d^2)) + (f*x + e)*a/c^2 + 2*(b*c*d*t
an(1/2*f*x + 1/2*e) - a*d^2*tan(1/2*f*x + 1/2*e))/((c^3 - c*d^2)*(c*tan(1/2
*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)))/f
```

3.191 $\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^3}\,dx$

Optimal. Leaf size=204

$$\frac{\left(bc^3\left(2c^2+d^2\right)-ad\left(-5c^2d^2+6c^4+2d^4\right)\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^3f(c-d)^{5/2}(c+d)^{5/2}}-\frac{d\left(-5ac^2d+2ad^3+3bc^3\right)\tan(e+fx)}{2c^2f\left(c^2-d^2\right)^2(c+d\sec(e+fx))}-\frac{a}{2cf\,(c}$$

```
[Out] (a*x)/c^3 + ((b*c^3*(2*c^2 + d^2) - a*d*(6*c^4 - 5*c^2*d^2 + 2*d^4))*ArcTan
h[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/(c^3*(c - d)^(5/2)*(c + d)^(
5/2)*f) - (d*(b*c - a*d)*Tan[e + f*x])/(2*c*(c^2 - d^2)*f*(c + d*Sec[e + f*
x])^2) - (d*(3*b*c^3 - 5*a*c^2*d + 2*a*d^3)*Tan[e + f*x])/(2*c^2*(c^2 - d^2
)^2*f*(c + d*Sec[e + f*x]))
```

Rubi [A] time = 0.509632, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3923, 4060, 3919, 3831, 2659, 208}

$$\frac{\left(bc^3\left(2c^2+d^2\right)-ad\left(-5c^2d^2+6c^4+2d^4\right)\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^3f(c-d)^{5/2}(c+d)^{5/2}}-\frac{d\left(-5ac^2d+2ad^3+3bc^3\right)\tan(e+fx)}{2c^2f\left(c^2-d^2\right)^2(c+d\sec(e+fx))}-\frac{a}{2cf\,(c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x])/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (a*x)/c^3 + ((b*c^3*(2*c^2 + d^2) - a*d*(6*c^4 - 5*c^2*d^2 + 2*d^4))*ArcTan
h[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/(c^3*(c - d)^(5/2)*(c + d)^(
5/2)*f) - (d*(b*c - a*d)*Tan[e + f*x])/(2*c*(c^2 - d^2)*f*(c + d*Sec[e + f*
x])^2) - (d*(3*b*c^3 - 5*a*c^2*d + 2*a*d^3)*Tan[e + f*x])/(2*c^2*(c^2 - d^2
)^2*f*(c + d*Sec[e + f*x]))
```

Rule 3923

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(b*(b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f
*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)
), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
```

```
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && Ne
Q[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^3}\,dx = -\frac{d(bc-ad)\tan(e+fx)}{2c\left(c^2-d^2\right)f(c+d\sec(e+fx))^2} - \frac{\int \frac{-2a\left(c^2-d^2\right)-2c(bc-ad)\sec(e+fx)+d(bc-ad)\sec^2(e+fx)}{(c+d\sec(e+fx))^2}\,dx}{2c\left(c^2-d^2\right)}$$

$$= -\frac{d(bc-ad)\tan(e+fx)}{2c\left(c^2-d^2\right)f(c+d\sec(e+fx))^2} - \frac{d\left(3bc^3-5ac^2d+2ad^3\right)\tan(e+fx)}{2c^2\left(c^2-d^2\right)^2 f(c+d\sec(e+fx))} + \frac{\int \frac{2a\left(c^2-d^2\right)^2-c}{}}{}$$

$$= \frac{ax}{c^3} - \frac{d(bc-ad)\tan(e+fx)}{2c\left(c^2-d^2\right)f(c+d\sec(e+fx))^2} - \frac{d\left(3bc^3-5ac^2d+2ad^3\right)\tan(e+fx)}{2c^2\left(c^2-d^2\right)^2 f(c+d\sec(e+fx))} + \frac{\left(bc^3\left(2c^2\right.\right.}{}$$

$$= \frac{ax}{c^3} - \frac{d(bc-ad)\tan(e+fx)}{2c\left(c^2-d^2\right)f(c+d\sec(e+fx))^2} - \frac{d\left(3bc^3-5ac^2d+2ad^3\right)\tan(e+fx)}{2c^2\left(c^2-d^2\right)^2 f(c+d\sec(e+fx))} + \frac{\left(bc^3\left(2c^2\right.\right.}{}$$

$$= \frac{ax}{c^3} - \frac{d(bc-ad)\tan(e+fx)}{2c\left(c^2-d^2\right)f(c+d\sec(e+fx))^2} - \frac{d\left(3bc^3-5ac^2d+2ad^3\right)\tan(e+fx)}{2c^2\left(c^2-d^2\right)^2 f(c+d\sec(e+fx))} + \frac{\left(bc^3\left(2c^2\right.\right.}{}$$

$$= \frac{ax}{c^3} + \frac{\left(2bc^5-6ac^4d+bc^3d^2+5ac^2d^3-2ad^5\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^3(c-d)^{5/2}(c+d)^{5/2}f} - \frac{d(bc-ad)}{2c\left(c^2-d^2\right)f(c}$$

Mathematica [A] time = 1.31107, size = 267, normalized size = 1.31

$$\frac{\sec^2(e+fx)(a+b\sec(e+fx))(c\cos(e+fx)+d)\left(-\frac{cd\left(-6ac^2d+3ad^3+4bc^3-bcd^2\right)\sin(e+fx)(c\cos(e+fx)+d)}{(c-d)^2(c+d)^2} - \frac{2\left(ad\left(5c^2d^2-6c^4-2d^4\right)+bc^3\right(}{}\right.}{2c^3f(a\cos(e+fx)+b)(c+d\sec(}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x])/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] ((d + c*Cos[e + f*x])*Sec[e + f*x]^2*(a + b*Sec[e + f*x])*(2*a*(e + f*x)*(d
 + c*Cos[e + f*x])^2 - (2*(b*c^3*(2*c^2 + d^2) + a*d*(-6*c^4 + 5*c^2*d^2 -
2*d^4))*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(d + c*Cos[e +
 f*x])^2)/(c^2 - d^2)^(5/2) + (c*d^2*(b*c - a*d)*Sin[e + f*x])/((c - d)*(c
+ d)) - (c*d*(4*b*c^3 - 6*a*c^2*d - b*c*d^2 + 3*a*d^3)*(d + c*Cos[e + f*x])
*Sin[e + f*x])/((c - d)^2*(c + d)^2)))/(2*c^3*f*(b + a*Cos[e + f*x])*(c + d
```

```
*Sec[e + f*x])^3)
```

Maple [B] time = 0.091, size = 1063, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))/(c+d*sec(f*x+e))^3,x)

[Out] 2/f*a/c^3*arctan(tan(1/2*f*x+1/2*e))-6/f/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*
x+1/2*e)^2*d-c-d)^2*d^2/(c-d)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3*a-1/f/c/
(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^2*d^3/(c-d)/(c^2+2*c*d+
d^2)*tan(1/2*f*x+1/2*e)^3*a+2/f/c^2/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2
*e)^2*d-c-d)^2*d^4/(c-d)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3*a+4/f*c/(tan(
1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^2*d/(c-d)/(c^2+2*c*d+d^2)*ta
n(1/2*f*x+1/2*e)^3*b+1/f/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d
)^2*d^2/(c-d)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3*b+6/f/(tan(1/2*f*x+1/2*e
)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^2*d^2/(c+d)/(c-d)^2*tan(1/2*f*x+1/2*e)*a-
1/f/c/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^2*d^3/(c+d)/(c-d)
^2*tan(1/2*f*x+1/2*e)*a-2/f/c^2/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^
2*d-c-d)^2*d^4/(c+d)/(c-d)^2*tan(1/2*f*x+1/2*e)*a-4/f*c/(tan(1/2*f*x+1/2*e)
^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^2*d/(c+d)/(c-d)^2*tan(1/2*f*x+1/2*e)*b+1/f
/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^2*d^2/(c+d)/(c-d)^2*ta
n(1/2*f*x+1/2*e)*b-6/f*c/(c^4-2*c^2*d^2+d^4)/((c+d)*(c-d))^(1/2)*arctanh(ta
n(1/2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))^(1/2))*a*d+5/f/c/(c^4-2*c^2*d^2+d^4)/(
(c+d)*(c-d))^(1/2)*arctanh(tan(1/2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))^(1/2))*a*
d^3-2/f/c^3/(c^4-2*c^2*d^2+d^4)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*f*x+1/2
*e)*(c-d)/((c+d)*(c-d))^(1/2))*a*d^5+2/f*c^2/(c^4-2*c^2*d^2+d^4)/((c+d)*(c-
d))^(1/2)*arctanh(tan(1/2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))^(1/2))*b+1/f/(c^4-
2*c^2*d^2+d^4)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*f*x+1/2*e)*(c-d)/((c+d)*
(c-d))^(1/2))*b*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.74749, size = 2479, normalized size = 12.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a*c^8 - 3*a*c^6*d^2 + 3*a*c^4*d^4 - a*c^2*d^6)*f*x*cos(f*x + e)^2
+ 8*(a*c^7*d - 3*a*c^5*d^3 + 3*a*c^3*d^5 - a*c*d^7)*f*x*cos(f*x + e) + 4*(a
*c^6*d^2 - 3*a*c^4*d^4 + 3*a*c^2*d^6 - a*d^8)*f*x - (2*b*c^5*d^2 - 6*a*c^4*
d^3 + b*c^3*d^4 + 5*a*c^2*d^5 - 2*a*d^7 + (2*b*c^7 - 6*a*c^6*d + b*c^5*d^2
+ 5*a*c^4*d^3 - 2*a*c^2*d^5)*cos(f*x + e)^2 + 2*(2*b*c^6*d - 6*a*c^5*d^2 +
b*c^4*d^3 + 5*a*c^3*d^4 - 2*a*c*d^6)*cos(f*x + e))*sqrt(c^2 - d^2)*log((2*c
*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*sqrt(c^2 - d^2)*(d*cos(f
*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*
x + e) + d^2)) - 2*(3*b*c^6*d^2 - 5*a*c^5*d^3 - 3*b*c^4*d^4 + 7*a*c^3*d^5 -
 2*a*c*d^7 + (4*b*c^7*d - 6*a*c^6*d^2 - 5*b*c^5*d^3 + 9*a*c^4*d^4 + b*c^3*d
^5 - 3*a*c^2*d^6)*cos(f*x + e))*sin(f*x + e))/((c^11 - 3*c^9*d^2 + 3*c^7*d^
4 - c^5*d^6)*f*cos(f*x + e)^2 + 2*(c^10*d - 3*c^8*d^3 + 3*c^6*d^5 - c^4*d^7
)*f*cos(f*x + e) + (c^9*d^2 - 3*c^7*d^4 + 3*c^5*d^6 - c^3*d^8)*f), 1/2*(2*(
a*c^8 - 3*a*c^6*d^2 + 3*a*c^4*d^4 - a*c^2*d^6)*f*x*cos(f*x + e)^2 + 4*(a*c^
7*d - 3*a*c^5*d^3 + 3*a*c^3*d^5 - a*c*d^7)*f*x*cos(f*x + e) + 2*(a*c^6*d^2
- 3*a*c^4*d^4 + 3*a*c^2*d^6 - a*d^8)*f*x + (2*b*c^5*d^2 - 6*a*c^4*d^3 + b*c
^3*d^4 + 5*a*c^2*d^5 - 2*a*d^7 + (2*b*c^7 - 6*a*c^6*d + b*c^5*d^2 + 5*a*c^4
*d^3 - 2*a*c^2*d^5)*cos(f*x + e)^2 + 2*(2*b*c^6*d - 6*a*c^5*d^2 + b*c^4*d^3
 + 5*a*c^3*d^4 - 2*a*c*d^6)*cos(f*x + e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^
2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) - (3*b*c^6*d^2 -
5*a*c^5*d^3 - 3*b*c^4*d^4 + 7*a*c^3*d^5 - 2*a*c*d^7 + (4*b*c^7*d - 6*a*c^6*
d^2 - 5*b*c^5*d^3 + 9*a*c^4*d^4 + b*c^3*d^5 - 3*a*c^2*d^6)*cos(f*x + e))*si
n(f*x + e))/((c^11 - 3*c^9*d^2 + 3*c^7*d^4 - c^5*d^6)*f*cos(f*x + e)^2 + 2*
(c^10*d - 3*c^8*d^3 + 3*c^6*d^5 - c^4*d^7)*f*cos(f*x + e) + (c^9*d^2 - 3*c^
7*d^4 + 3*c^5*d^6 - c^3*d^8)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b\sec\left(e + fx\right)}{\left(c + d\sec\left(e + fx\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e))**3,x)
```

```
[Out] Integral((a + b*sec(e + f*x))/(c + d*sec(e + f*x))**3, x)
```

Giac [B] time = 1.55178, size = 644, normalized size = 3.16

$$\frac{\left(2\,bc^5-6\,ac^4d+bc^3d^2+5\,ac^2d^3-2\,ad^5\right)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,c+2\,d)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-c^2+d^2}}\right)\right)}{\left(c^7-2\,c^5d^2+c^3d^4\right)\sqrt{-c^2+d^2}}+\frac{(fx+e)a}{c^3}+\frac{4\,bc^4d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-6\,ac^3d^2\tan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] ((2*b*c^5 - 6*a*c^4*d + b*c^3*d^2 + 5*a*c^2*d^3 - 2*a*d^5)*(pi*floor(1/2*(f
*x + e)/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*tan
(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))/((c^7 - 2*c^5*d^2 + c^3*d^4)*sqrt(-c^
2 + d^2)) + (f*x + e)*a/c^3 + (4*b*c^4*d*tan(1/2*f*x + 1/2*e)^3 - 6*a*c^3*d
^2*tan(1/2*f*x + 1/2*e)^3 - 3*b*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 + 5*a*c^2*d^
3*tan(1/2*f*x + 1/2*e)^3 - b*c^2*d^3*tan(1/2*f*x + 1/2*e)^3 + 3*a*c*d^4*tan
(1/2*f*x + 1/2*e)^3 - 2*a*d^5*tan(1/2*f*x + 1/2*e)^3 - 4*b*c^4*d*tan(1/2*f*
x + 1/2*e) + 6*a*c^3*d^2*tan(1/2*f*x + 1/2*e) - 3*b*c^3*d^2*tan(1/2*f*x + 1
/2*e) + 5*a*c^2*d^3*tan(1/2*f*x + 1/2*e) + b*c^2*d^3*tan(1/2*f*x + 1/2*e) -
 3*a*c*d^4*tan(1/2*f*x + 1/2*e) - 2*a*d^5*tan(1/2*f*x + 1/2*e))/((c^6 - 2*c
^4*d^2 + c^2*d^4)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c
- d)^2))/f
```

3.192 $\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx$

Optimal. Leaf size=133

$$\frac{a^2x}{c^2}+\frac{(bc-ad)^2\sin(e+fx)}{cf\left(c^2-d^2\right)(c\cos(e+fx)+d)}+\frac{2(bc-ad)\left(2ac^2-ad^2-bcd\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^2f(c-d)^{3/2}(c+d)^{3/2}}$$

```
[Out] (a^2*x)/c^2 + (2*(b*c - a*d)*(2*a*c^2 - b*c*d - a*d^2)*ArcTanh[(Sqrt[c - d]
*Tan[(e + f*x)/2])/Sqrt[c + d]])/(c^2*(c - d)^(3/2)*(c + d)^(3/2)*f) + ((b*
c - a*d)^2*Sin[e + f*x])/(c*(c^2 - d^2)*f*(d + c*Cos[e + f*x]))
```

Rubi [A] time = 0.284952, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3941, 2790, 2735, 2659, 208}

$$\frac{a^2x}{c^2}+\frac{(bc-ad)^2\sin(e+fx)}{cf\left(c^2-d^2\right)(c\cos(e+fx)+d)}+\frac{2(bc-ad)\left(2ac^2-ad^2-bcd\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^2f(c-d)^{3/2}(c+d)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x])^2/(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (a^2*x)/c^2 + (2*(b*c - a*d)*(2*a*c^2 - b*c*d - a*d^2)*ArcTanh[(Sqrt[c - d]
*Tan[(e + f*x)/2])/Sqrt[c + d]])/(c^2*(c - d)^(3/2)*(c + d)^(3/2)*f) + ((b*
c - a*d)^2*Sin[e + f*x])/(c*(c^2 - d^2)*f*(d + c*Cos[e + f*x]))
```

Rule 3941

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Int[((b + a*Sin[e + f*x])^m*(d + c*Sin[e + f
*x])^n)/Sin[e + f*x]^(m + n), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && N
eQ[b*c - a*d, 0] && IntegerQ[m] && IntegerQ[n] && LeQ[-2, m + n, 0]
```

Rule 2790

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e +
```

```
f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(2
*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1) +
c^2*(m + 2)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx = \int \frac{(b+a\cos(e+fx))^2}{(d+c\cos(e+fx))^2}\,dx$$

$$= \frac{(bc-ad)^2\sin(e+fx)}{c\left(c^2-d^2\right)f(d+c\cos(e+fx))} - \frac{\int \frac{-c\left(2abc-\left(a^2+b^2\right)d\right)-a^2\left(c^2-d^2\right)\cos(e+fx)}{d+c\cos(e+fx)}\,dx}{c\left(c^2-d^2\right)}$$

$$= \frac{a^2x}{c^2} + \frac{(bc-ad)^2\sin(e+fx)}{c\left(c^2-d^2\right)f(d+c\cos(e+fx))} + \frac{\left(c^2\left(2abc-\left(a^2+b^2\right)d\right)-a^2d\left(c^2-d^2\right)\right)\int \frac{1}{d+c\cos(e+}}{c^2\left(c^2-d^2\right)}$$

$$= \frac{a^2x}{c^2} + \frac{(bc-ad)^2\sin(e+fx)}{c\left(c^2-d^2\right)f(d+c\cos(e+fx))} + \frac{\left(2\left(c^2\left(2abc-\left(a^2+b^2\right)d\right)-a^2d\left(c^2-d^2\right)\right)\right)\text{Subst}}{c^2\left(c^2-d^2\right)f}$$

$$= \frac{a^2x}{c^2} + \frac{2(bc-ad)\left(2ac^2-bcd-ad^2\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^2(c-d)^{3/2}(c+d)^{3/2}f} + \frac{(bc-ad)^2\sin(e+fx)}{c\left(c^2-d^2\right)f(d+c\cos(e+}$$

Mathematica [A] time = 0.642527, size = 136, normalized size = 1.02

$$\frac{\frac{2\left(a^2\left(2c^2d-d^3\right)-2abc^3+b^2c^2d\right)\tanh^{-1}\left(\frac{(d-c)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\left(c^2-d^2\right)^{3/2}}+a^2(e+fx)+\frac{c(bc-ad)^2\sin(e+fx)}{(c-d)(c+d)(c\cos(e+fx)+d)}}{c^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x])^2/(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (a^2*(e + f*x) + (2*(-2*a*b*c^3 + b^2*c^2*d + a^2*(2*c^2*d - d^3))*ArcTanh[
((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(c^2 - d^2)^(3/2) + (c*(b*c -
 a*d)^2*Sin[e + f*x])/((c - d)*(c + d)*(d + c*Cos[e + f*x])))/(c^2*f)
```

Maple [B] time = 0.085, size = 462, normalized size = 3.5

$$2\,\frac{a^2\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)}{fc^2}-2\,\frac{\tan\left(1/2\,fx+e/2\right)a^2d^2}{fc\left(c^2-d^2\right)\left(\left(\tan\left(1/2\,fx+e/2\right)\right)^2c-\left(\tan\left(1/2\,fx+e/2\right)\right)^2d-c-d\right)}+4\,\frac{\cdots}{f\left(c^2-\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x)
```

```
[Out] 2/f*a^2/c^2*arctan(tan(1/2*f*x+1/2*e))-2/f/c/(c^2-d^2)*tan(1/2*f*x+1/2*e)/(
tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)*a^2*d^2+4/f/(c^2-d^2)*ta
n(1/2*f*x+1/2*e)/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)*a*b*d-
2/f*c/(c^2-d^2)*tan(1/2*f*x+1/2*e)/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*
e)^2*d-c-d)*b^2-4/f/(c+d)/(c-d)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*f*x+1/2
*e)*(c-d)/((c+d)*(c-d))^(1/2))*a^2*d+2/f/c^2/(c+d)/(c-d)/((c+d)*(c-d))^(1/2
)*arctanh(tan(1/2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))^(1/2))*a^2*d^3+4/f*c/(c+d)
/(c-d)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))^(
1/2))*a*b-2/f/(c+d)/(c-d)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*f*x+1/2*e)*(c
-d)/((c+d)*(c-d))^(1/2))*b^2*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.621149, size = 1426, normalized size = 10.72

$$\left[\frac{2\left(a^2c^5 - 2\,a^2c^3d^2 + a^2cd^4\right)fx\cos\left(fx+e\right) + 2\left(a^2c^4d - 2\,a^2c^2d^3 + a^2d^5\right)fx + \left(2\,abc^3d + a^2d^4 - \left(2\,a^2 + b^2\right)c^2d^2 + \left(2\,abc\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a^2*c^5 - 2*a^2*c^3*d^2 + a^2*c*d^4)*f*x*cos(f*x + e) + 2*(a^2*c^4
*d - 2*a^2*c^2*d^3 + a^2*d^5)*f*x + (2*a*b*c^3*d + a^2*d^4 - (2*a^2 + b^2)*
c^2*d^2 + (2*a*b*c^4 + a^2*c*d^3 - (2*a^2 + b^2)*c^3*d)*cos(f*x + e))*sqrt(
c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 + 2*sqrt(
c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x +
e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(b^2*c^5 - 2*a*b*c^4*d + 2*a*b*c^2*d^
3 - a^2*c*d^4 + (a^2 - b^2)*c^3*d^2)*sin(f*x + e))/((c^7 - 2*c^5*d^2 + c^3*
d^4)*f*cos(f*x + e) + (c^6*d - 2*c^4*d^3 + c^2*d^5)*f), ((a^2*c^5 - 2*a^2*c
^3*d^2 + a^2*c*d^4)*f*x*cos(f*x + e) + (a^2*c^4*d - 2*a^2*c^2*d^3 + a^2*d^5
)*f*x + (2*a*b*c^3*d + a^2*d^4 - (2*a^2 + b^2)*c^2*d^2 + (2*a*b*c^4 + a^2*c
*d^3 - (2*a^2 + b^2)*c^3*d)*cos(f*x + e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^
2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) + (b^2*c^5 - 2*a*
b*c^4*d + 2*a*b*c^2*d^3 - a^2*c*d^4 + (a^2 - b^2)*c^3*d^2)*sin(f*x + e))/((
c^7 - 2*c^5*d^2 + c^3*d^4)*f*cos(f*x + e) + (c^6*d - 2*c^4*d^3 + c^2*d^5)*f
)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + b\sec\left(e + fx\right)\right)^2}{\left(c + d\sec\left(e + fx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**2/(c+d*sec(f*x+e))**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x))**2/(c + d*sec(e + f*x))**2, x)
```

Giac [A] time = 1.45983, size = 332, normalized size = 2.5

$$\frac{\frac{(fx+e)a^2}{c^2} + \frac{2\left(2\,abc^3-2\,a^2c^2d-b^2c^2d+a^2d^3\right)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,c+2\,d)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-c^2+d^2}}\right)\right)}{\left(c^4-c^2d^2\right)\sqrt{-c^2+d^2}} - \frac{2\left(b^2c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-2\,abcd\tan\left(\frac{1}{2}\right.\right.}{\left(c^3-cd^2\right)\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-\right.}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] ((f*x + e)*a^2/c^2 + 2*(2*a*b*c^3 - 2*a^2*c^2*d - b^2*c^2*d + a^2*d^3)*(pi*
floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*f*x + 1/
2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))/((c^4 - c^2*d^2)*sqrt(-c^
2 + d^2)) - 2*(b^2*c^2*tan(1/2*f*x + 1/2*e) - 2*a*b*c*d*tan(1/2*f*x + 1/2*e
) + a^2*d^2*tan(1/2*f*x + 1/2*e))/((c^3 - c*d^2)*(c*tan(1/2*f*x + 1/2*e)^2
- d*tan(1/2*f*x + 1/2*e)^2 - c - d)))/f
```

3.193 $\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^3}\,dx$

Optimal. Leaf size=237

$$-\frac{\left(a^2\left(-5c^2d^3+6c^4d+2d^5\right)-2abc^3\left(2c^2+d^2\right)+3b^2c^4d\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^3f(c-d)^{5/2}(c+d)^{5/2}}+\frac{a^2x}{c^3}-\frac{(bc-ad)\left(3ad\left(2c^2-d^2\right)-\right.}{2c^2f\left(c^2-d^2\right)^2(c}$$

```
[Out] (a^2*x)/c^3 - ((3*b^2*c^4*d - 2*a*b*c^3*(2*c^2 + d^2) + a^2*(6*c^4*d - 5*c^
2*d^3 + 2*d^5))*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/(c^3*(
c - d)^(5/2)*(c + d)^(5/2)*f) - (d*(b*c - a*d)^2*Sin[e + f*x])/(2*c^2*(c^2
- d^2)*f*(d + c*Cos[e + f*x])^2) - ((b*c - a*d)*(3*a*d*(2*c^2 - d^2) - b*c*
(2*c^2 + d^2))*Sin[e + f*x])/(2*c^2*(c^2 - d^2)^2*f*(d + c*Cos[e + f*x]))
```

Rubi [A] time = 0.795608, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3941, 2988, 3021, 2735, 2659, 208}

$$-\frac{\left(a^2\left(-5c^2d^3+6c^4d+2d^5\right)-2abc^3\left(2c^2+d^2\right)+3b^2c^4d\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^3f(c-d)^{5/2}(c+d)^{5/2}}+\frac{a^2x}{c^3}-\frac{(bc-ad)\left(3ad\left(2c^2-d^2\right)-\right.}{2c^2f\left(c^2-d^2\right)^2(c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x])^2/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (a^2*x)/c^3 - ((3*b^2*c^4*d - 2*a*b*c^3*(2*c^2 + d^2) + a^2*(6*c^4*d - 5*c^
2*d^3 + 2*d^5))*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/(c^3*(
c - d)^(5/2)*(c + d)^(5/2)*f) - (d*(b*c - a*d)^2*Sin[e + f*x])/(2*c^2*(c^2
- d^2)*f*(d + c*Cos[e + f*x])^2) - ((b*c - a*d)*(3*a*d*(2*c^2 - d^2) - b*c*
(2*c^2 + d^2))*Sin[e + f*x])/(2*c^2*(c^2 - d^2)^2*f*(d + c*Cos[e + f*x]))
```

Rule 3941

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Int[((b + a*Sin[e + f*x])^m*(d + c*Sin[e + f
*x])^n)/Sin[e + f*x]^(m + n), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && N
eQ[b*c - a*d, 0] && IntegerQ[m] && IntegerQ[n] && LeQ[-2, m + n, 0]
```

Rule 2988

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
((B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(f*d^
2*(n + 1)*(c^2 - d^2)), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^3}\,dx = \int \frac{\cos(e+fx)(b+a\cos(e+fx))^2}{(d+c\cos(e+fx))^3}\,dx$$

$$= -\frac{d(bc-ad)^2\sin(e+fx)}{2c^2\left(c^2-d^2\right)f(d+c\cos(e+fx))^2} - \frac{\int \frac{-2c(bc-ad)^2+\left(b^2c^2d-2abc\left(2c^2-d^2\right)+a^2\left(2c^2d-d^3\right)\right)\cos(e+fx)-2a^2c}{(d+c\cos(e+fx))^2}}{2c^2\left(c^2-d^2\right)}$$

$$= -\frac{d(bc-ad)^2\sin(e+fx)}{2c^2\left(c^2-d^2\right)f(d+c\cos(e+fx))^2} - \frac{(bc-ad)\left(3ad\left(2c^2-d^2\right)-bc\left(2c^2+d^2\right)\right)\sin(e+fx)}{2c^2\left(c^2-d^2\right)^2f(d+c\cos(e+fx))}$$

$$= \frac{a^2x}{c^3} - \frac{d(bc-ad)^2\sin(e+fx)}{2c^2\left(c^2-d^2\right)f(d+c\cos(e+fx))^2} - \frac{(bc-ad)\left(3ad\left(2c^2-d^2\right)-bc\left(2c^2+d^2\right)\right)\sin(e-}{2c^2\left(c^2-d^2\right)^2f(d+c\cos(e+fx))}$$

$$= \frac{a^2x}{c^3} - \frac{d(bc-ad)^2\sin(e+fx)}{2c^2\left(c^2-d^2\right)f(d+c\cos(e+fx))^2} - \frac{(bc-ad)\left(3ad\left(2c^2-d^2\right)-bc\left(2c^2+d^2\right)\right)\sin(e-}{2c^2\left(c^2-d^2\right)^2f(d+c\cos(e+fx))}$$

$$= \frac{a^2x}{c^3} + \frac{\left(4abc^5-6a^2c^4d-3b^2c^4d+2abc^3d^2+5a^2c^2d^3-2a^2d^5\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^3(c-d)^{5/2}(c+d)^{5/2}f} -$$

Mathematica [B] time = 2.01916, size = 493, normalized size = 2.08

$$\frac{\sec(e+fx)(a+b\sec(e+fx))^2(c\cos(e+fx)+d)\left(\frac{6a^2c^4d^2\sin(2(e+fx))+10a^2c^3d^3\sin(e+fx)-3a^2c^2d^4\sin(2(e+fx))+2a^2c^2\left(c^2-d^2\right)^2(e+fx)\cos\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x])^2/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] ((d + c*Cos[e + f*x])*Sec[e + f*x]*(a + b*Sec[e + f*x])^2*((4*(3*b^2*c^4*d
- 2*a*b*c^3*(2*c^2 + d^2) + a^2*(6*c^4*d - 5*c^2*d^3 + 2*d^5))*ArcTanh[((-c
 + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(d + c*Cos[e + f*x])^2)/(c^2 - d^2
)^(5/2) + (2*a^2*c^6*e - 6*a^2*c^2*d^4*e + 4*a^2*d^6*e + 2*a^2*c^6*f*x - 6*
a^2*c^2*d^4*f*x + 4*a^2*d^6*f*x + 8*a^2*c*d*(c^2 - d^2)^2*(e + f*x)*Cos[e +
 f*x] + 2*a^2*c^2*(c^2 - d^2)^2*(e + f*x)*Cos[2*(e + f*x)] + 2*b^2*c^5*d*Si
n[e + f*x] - 12*a*b*c^4*d^2*Sin[e + f*x] + 10*a^2*c^3*d^3*Sin[e + f*x] + 4*
b^2*c^3*d^3*Sin[e + f*x] - 4*a^2*c*d^5*Sin[e + f*x] + 2*b^2*c^6*Sin[2*(e +
```

```
f*x)] - 8*a*b*c^5*d*Sin[2*(e + f*x)] + 6*a^2*c^4*d^2*Sin[2*(e + f*x)] + b^2
*c^4*d^2*Sin[2*(e + f*x)] + 2*a*b*c^3*d^3*Sin[2*(e + f*x)] - 3*a^2*c^2*d^4*
Sin[2*(e + f*x)])/(c^2 - d^2)^2))/(4*c^3*f*(b + a*Cos[e + f*x])^2*(c + d*Se
c[e + f*x])^3)
```

Maple [B] time = 0.098, size = 1593, normalized size = 6.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x)
```

```
[Out] 2/f*a^2/c^3*arctan(tan(1/2*f*x+1/2*e))-6/f/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*
f*x+1/2*e)^2*d-c-d)^2/(c-d)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3*a^2*d^2-1/
f/c/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^2/(c-d)/(c^2+2*c*d+
d^2)*tan(1/2*f*x+1/2*e)^3*a^2*d^3+2/f/c^2/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f
*x+1/2*e)^2*d-c-d)^2/(c-d)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3*a^2*d^4+8/f
*c/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^2/(c-d)/(c^2+2*c*d+d
^2)*tan(1/2*f*x+1/2*e)^3*a*b*d+2/f/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*
e)^2*d-c-d)^2/(c-d)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3*a*b*d^2-2/f*c^2/(t
an(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^2/(c-d)/(c^2+2*c*d+d^2)*t
an(1/2*f*x+1/2*e)^3*b^2-1/f*c/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*
d-c-d)^2/(c-d)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3*b^2*d-2/f/(tan(1/2*f*x+
1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^2/(c-d)/(c^2+2*c*d+d^2)*tan(1/2*f*x+
1/2*e)^3*b^2*d^2+6/f/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^2/
(c+d)/(c-d)^2*tan(1/2*f*x+1/2*e)*a^2*d^2-1/f/c/(tan(1/2*f*x+1/2*e)^2*c-tan(
1/2*f*x+1/2*e)^2*d-c-d)^2/(c+d)/(c-d)^2*tan(1/2*f*x+1/2*e)*a^2*d^3-2/f/c^2/
(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^2/(c+d)/(c-d)^2*tan(1/2
*f*x+1/2*e)*a^2*d^4-8/f*c/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-
d)^2/(c+d)/(c-d)^2*tan(1/2*f*x+1/2*e)*a*b*d+2/f/(tan(1/2*f*x+1/2*e)^2*c-tan
(1/2*f*x+1/2*e)^2*d-c-d)^2/(c+d)/(c-d)^2*tan(1/2*f*x+1/2*e)*a*b*d^2+2/f*c^2
/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^2/(c+d)/(c-d)^2*tan(1/
2*f*x+1/2*e)*b^2-1/f*c/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^
2/(c+d)/(c-d)^2*tan(1/2*f*x+1/2*e)*b^2*d+2/f/(tan(1/2*f*x+1/2*e)^2*c-tan(1/
2*f*x+1/2*e)^2*d-c-d)^2/(c+d)/(c-d)^2*tan(1/2*f*x+1/2*e)*b^2*d^2-6/f*c/(c^4
-2*c^2*d^2+d^4)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*f*x+1/2*e)*(c-d)/((c+d)
*(c-d))^(1/2))*a^2*d+5/f/c/(c^4-2*c^2*d^2+d^4)/((c+d)*(c-d))^(1/2)*arctanh(
tan(1/2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))^(1/2))*a^2*d^3-2/f/c^3/(c^4-2*c^2*d^
2+d^4)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))^(
1/2))*a^2*d^5+4/f*c^2/(c^4-2*c^2*d^2+d^4)/((c+d)*(c-d))^(1/2)*arctanh(tan(1
/2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))^(1/2))*a*b+2/f/(c^4-2*c^2*d^2+d^4)/((c+d)
```

```
*(c-d))^(1/2)*arctanh(tan(1/2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))^(1/2))*a*b*d^2
-3/f*c/(c^4-2*c^2*d^2+d^4)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*f*x+1/2*e)*(
c-d)/((c+d)*(c-d))^(1/2))*b^2*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.821454, size = 2946, normalized size = 12.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^2*c^8 - 3*a^2*c^6*d^2 + 3*a^2*c^4*d^4 - a^2*c^2*d^6)*f*x*cos(f*x
 + e)^2 + 8*(a^2*c^7*d - 3*a^2*c^5*d^3 + 3*a^2*c^3*d^5 - a^2*c*d^7)*f*x*cos
(f*x + e) + 4*(a^2*c^6*d^2 - 3*a^2*c^4*d^4 + 3*a^2*c^2*d^6 - a^2*d^8)*f*x -
 (4*a*b*c^5*d^2 + 2*a*b*c^3*d^4 + 5*a^2*c^2*d^5 - 2*a^2*d^7 - 3*(2*a^2 + b^
2)*c^4*d^3 + (4*a*b*c^7 + 2*a*b*c^5*d^2 + 5*a^2*c^4*d^3 - 2*a^2*c^2*d^5 - 3
*(2*a^2 + b^2)*c^6*d)*cos(f*x + e)^2 + 2*(4*a*b*c^6*d + 2*a*b*c^4*d^3 + 5*a
^2*c^3*d^4 - 2*a^2*c*d^6 - 3*(2*a^2 + b^2)*c^5*d^2)*cos(f*x + e))*sqrt(c^2
- d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*sqrt(c^2
- d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2
 + 2*c*d*cos(f*x + e) + d^2)) + 2*(b^2*c^7*d - 6*a*b*c^6*d^2 + 6*a*b*c^4*d^
4 + 2*a^2*c*d^7 + (5*a^2 + b^2)*c^5*d^3 - (7*a^2 + 2*b^2)*c^3*d^5 + (2*b^2*
c^8 - 8*a*b*c^7*d + 10*a*b*c^5*d^3 - 2*a*b*c^3*d^5 + 3*a^2*c^2*d^6 + (6*a^2
 - b^2)*c^6*d^2 - (9*a^2 + b^2)*c^4*d^4)*cos(f*x + e))*sin(f*x + e))/((c^11
 - 3*c^9*d^2 + 3*c^7*d^4 - c^5*d^6)*f*cos(f*x + e)^2 + 2*(c^10*d - 3*c^8*d^
3 + 3*c^6*d^5 - c^4*d^7)*f*cos(f*x + e) + (c^9*d^2 - 3*c^7*d^4 + 3*c^5*d^6
- c^3*d^8)*f), 1/2*(2*(a^2*c^8 - 3*a^2*c^6*d^2 + 3*a^2*c^4*d^4 - a^2*c^2*d^
6)*f*x*cos(f*x + e)^2 + 4*(a^2*c^7*d - 3*a^2*c^5*d^3 + 3*a^2*c^3*d^5 - a^2*
c*d^7)*f*x*cos(f*x + e) + 2*(a^2*c^6*d^2 - 3*a^2*c^4*d^4 + 3*a^2*c^2*d^6 -
```

```
a^2*d^8)*f*x + (4*a*b*c^5*d^2 + 2*a*b*c^3*d^4 + 5*a^2*c^2*d^5 - 2*a^2*d^7 -
 3*(2*a^2 + b^2)*c^4*d^3 + (4*a*b*c^7 + 2*a*b*c^5*d^2 + 5*a^2*c^4*d^3 - 2*a
^2*c^2*d^5 - 3*(2*a^2 + b^2)*c^6*d)*cos(f*x + e)^2 + 2*(4*a*b*c^6*d + 2*a*b
*c^4*d^3 + 5*a^2*c^3*d^4 - 2*a^2*c*d^6 - 3*(2*a^2 + b^2)*c^5*d^2)*cos(f*x +
 e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 -
 d^2)*sin(f*x + e))) + (b^2*c^7*d - 6*a*b*c^6*d^2 + 6*a*b*c^4*d^4 + 2*a^2*c
*d^7 + (5*a^2 + b^2)*c^5*d^3 - (7*a^2 + 2*b^2)*c^3*d^5 + (2*b^2*c^8 - 8*a*b
*c^7*d + 10*a*b*c^5*d^3 - 2*a*b*c^3*d^5 + 3*a^2*c^2*d^6 + (6*a^2 - b^2)*c^6
*d^2 - (9*a^2 + b^2)*c^4*d^4)*cos(f*x + e))*sin(f*x + e))/((c^11 - 3*c^9*d^
2 + 3*c^7*d^4 - c^5*d^6)*f*cos(f*x + e)^2 + 2*(c^10*d - 3*c^8*d^3 + 3*c^6*d
^5 - c^4*d^7)*f*cos(f*x + e) + (c^9*d^2 - 3*c^7*d^4 + 3*c^5*d^6 - c^3*d^8)*
f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + b\sec\left(e + fx\right)\right)^2}{\left(c + d\sec\left(e + fx\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**2/(c+d*sec(f*x+e))**3,x)
```

```
[Out] Integral((a + b*sec(e + f*x))**2/(c + d*sec(e + f*x))**3, x)
```

Giac [B] time = 1.57897, size = 926, normalized size = 3.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] ((4*a*b*c^5 - 6*a^2*c^4*d - 3*b^2*c^4*d + 2*a*b*c^3*d^2 + 5*a^2*c^2*d^3 - 2
*a^2*d^5)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*ta
n(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))/((c^7 - 2*c
^5*d^2 + c^3*d^4)*sqrt(-c^2 + d^2)) + (f*x + e)*a^2/c^3 - (2*b^2*c^5*tan(1/
2*f*x + 1/2*e)^3 - 8*a*b*c^4*d*tan(1/2*f*x + 1/2*e)^3 - b^2*c^4*d*tan(1/2*f
*x + 1/2*e)^3 + 6*a^2*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 + 6*a*b*c^3*d^2*tan(1/
2*f*x + 1/2*e)^3 + b^2*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 - 5*a^2*c^2*d^3*tan(1
```

/2*f*x + 1/2*e)^3 + 2*a*b*c^2*d^3*tan(1/2*f*x + 1/2*e)^3 - 2*b^2*c^2*d^3*ta
n(1/2*f*x + 1/2*e)^3 - 3*a^2*c*d^4*tan(1/2*f*x + 1/2*e)^3 + 2*a^2*d^5*tan(1
/2*f*x + 1/2*e)^3 - 2*b^2*c^5*tan(1/2*f*x + 1/2*e) + 8*a*b*c^4*d*tan(1/2*f*
x + 1/2*e) - b^2*c^4*d*tan(1/2*f*x + 1/2*e) - 6*a^2*c^3*d^2*tan(1/2*f*x + 1
/2*e) + 6*a*b*c^3*d^2*tan(1/2*f*x + 1/2*e) - b^2*c^3*d^2*tan(1/2*f*x + 1/2*
e) - 5*a^2*c^2*d^3*tan(1/2*f*x + 1/2*e) - 2*a*b*c^2*d^3*tan(1/2*f*x + 1/2*e
) - 2*b^2*c^2*d^3*tan(1/2*f*x + 1/2*e) + 3*a^2*c*d^4*tan(1/2*f*x + 1/2*e) +
 2*a^2*d^5*tan(1/2*f*x + 1/2*e))/((c^6 - 2*c^4*d^2 + c^2*d^4)*(c*tan(1/2*f*
x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)^2))/f

3.194 $\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx$

Optimal. Leaf size=377

$$-\frac{\left(-a^2d^2\left(-28c^2d^2+34c^4+9d^4\right)+2abcd\left(-5c^2d^2+18c^4+2d^4\right)+b^2\left(-\left(10c^4d^2-c^2d^4+6c^6\right)\right)\right)\sin(e+fx)}{6c^3f\left(c^2-d^2\right)^3(c\cos(e+fx)+d)}-\frac{\left(a^2\left(7c^2\right.\right.}{}$$

```
[Out] (a^2*x)/c^4 - ((b^2*c^4*d*(4*c^2 + d^2) - a*b*(4*c^7 + 6*c^5*d^2) + a^2*(8*
c^6*d - 8*c^4*d^3 + 7*c^2*d^5 - 2*d^7))*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/
2])/Sqrt[c + d]])/(c^4*(c - d)^(7/2)*(c + d)^(7/2)*f) + (d^2*(b + a*Cos[e +
 f*x])^2*Sin[e + f*x])/(3*c*(c^2 - d^2)*f*(d + c*Cos[e + f*x])^3) - (d*(b*c
 - a*d)*(6*b*c^3 - 8*a*c^2*d - b*c*d^2 + 3*a*d^3)*Sin[e + f*x])/(6*c^3*(c^2
 - d^2)^2*f*(d + c*Cos[e + f*x])^2) - ((2*a*b*c*d*(18*c^4 - 5*c^2*d^2 + 2*d
^4) - a^2*d^2*(34*c^4 - 28*c^2*d^2 + 9*d^4) - b^2*(6*c^6 + 10*c^4*d^2 - c^2
*d^4))*Sin[e + f*x])/(6*c^3*(c^2 - d^2)^3*f*(d + c*Cos[e + f*x]))
```

Rubi [A] time = 1.99896, antiderivative size = 377, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3941, 3048, 3031, 3021, 2735, 2659, 208}

$$-\frac{\left(-a^2d^2\left(-28c^2d^2+34c^4+9d^4\right)+2abcd\left(-5c^2d^2+18c^4+2d^4\right)+b^2\left(-\left(10c^4d^2-c^2d^4+6c^6\right)\right)\right)\sin(e+fx)}{6c^3f\left(c^2-d^2\right)^3(c\cos(e+fx)+d)}-\frac{\left(a^2\left(7c^2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x])^2/(c + d*Sec[e + f*x])^4,x]
```

```
[Out] (a^2*x)/c^4 - ((b^2*c^4*d*(4*c^2 + d^2) - a*b*(4*c^7 + 6*c^5*d^2) + a^2*(8*
c^6*d - 8*c^4*d^3 + 7*c^2*d^5 - 2*d^7))*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/
2])/Sqrt[c + d]])/(c^4*(c - d)^(7/2)*(c + d)^(7/2)*f) + (d^2*(b + a*Cos[e +
 f*x])^2*Sin[e + f*x])/(3*c*(c^2 - d^2)*f*(d + c*Cos[e + f*x])^3) - (d*(b*c
 - a*d)*(6*b*c^3 - 8*a*c^2*d - b*c*d^2 + 3*a*d^3)*Sin[e + f*x])/(6*c^3*(c^2
 - d^2)^2*f*(d + c*Cos[e + f*x])^2) - ((2*a*b*c*d*(18*c^4 - 5*c^2*d^2 + 2*d
^4) - a^2*d^2*(34*c^4 - 28*c^2*d^2 + 9*d^4) - b^2*(6*c^6 + 10*c^4*d^2 - c^2
*d^4))*Sin[e + f*x])/(6*c^3*(c^2 - d^2)^3*f*(d + c*Cos[e + f*x]))
```

Rule 3941

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Int[((b + a*Sin[e + f*x])^m*(d + c*Sin[e + f
*x])^n)/Sin[e + f*x]^(m + n), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && N
eQ[b*c - a*d, 0] && IntegerQ[m] && IntegerQ[n] && LeQ[-2, m + n, 0]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
```

```
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx = \int \frac{\cos^2(e+fx)(b+a\cos(e+fx))^2}{(d+c\cos(e+fx))^4}\,dx$$

$$= \frac{d^2(b+a\cos(e+fx))^2\sin(e+fx)}{3c\left(c^2-d^2\right)f(d+c\cos(e+fx))^3} + \frac{\int \frac{(b+a\cos(e+fx))\left(-d(3bc-2ad)+\left(3bc^2-3acd-bd^2\right)\cos(e+fx)+3a\left(c^2-\right.\right.}{(d+c\cos(e+fx))^3}}{3c\left(c^2-d^2\right)}$$

$$= \frac{d^2(b+a\cos(e+fx))^2\sin(e+fx)}{3c\left(c^2-d^2\right)f(d+c\cos(e+fx))^3} - \frac{d(bc-ad)\left(6bc^3-8ac^2d-bcd^2+3ad^3\right)\sin(e+fx)}{6c^3\left(c^2-d^2\right)^2f(d+c\cos(e+fx))^2} -$$

$$= \frac{d^2(b+a\cos(e+fx))^2\sin(e+fx)}{3c\left(c^2-d^2\right)f(d+c\cos(e+fx))^3} - \frac{d(bc-ad)\left(6bc^3-8ac^2d-bcd^2+3ad^3\right)\sin(e+fx)}{6c^3\left(c^2-d^2\right)^2f(d+c\cos(e+fx))^2} -$$

$$= \frac{a^2x}{c^4} + \frac{d^2(b+a\cos(e+fx))^2\sin(e+fx)}{3c\left(c^2-d^2\right)f(d+c\cos(e+fx))^3} - \frac{d(bc-ad)\left(6bc^3-8ac^2d-bcd^2+3ad^3\right)\sin(e}{6c^3\left(c^2-d^2\right)^2f(d+c\cos(e+fx))^2}$$

$$= \frac{a^2x}{c^4} + \frac{d^2(b+a\cos(e+fx))^2\sin(e+fx)}{3c\left(c^2-d^2\right)f(d+c\cos(e+fx))^3} - \frac{d(bc-ad)\left(6bc^3-8ac^2d-bcd^2+3ad^3\right)\sin(e}{6c^3\left(c^2-d^2\right)^2f(d+c\cos(e+fx))^2}$$

$$= \frac{a^2x}{c^4} + \frac{\left(4abc^7-8a^2c^6d-4b^2c^6d+6abc^5d^2+8a^2c^4d^3-b^2c^4d^3-7a^2c^2d^5+2a^2d^7\right)\tanh^{-1}}{c^4(c-d)^{7/2}(c+d)^{7/2}f}$$

Mathematica [A] time = 3.32157, size = 438, normalized size = 1.16

$$\frac{\sec^2(e+fx)(a+b\sec(e+fx))^2(c\cos(e+fx)+d)\left(\frac{c\left(a^2d^2\left(-32c^2d^2+36c^4+11d^4\right)-2abcd\left(-5c^2d^2+18c^4+2d^4\right)+b^2\left(10c^4d^2-c^2d^4+6c^6\right)\right)\sin(e+fx}{\left(c^2-d^2\right)^3}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x])^2/(c + d*Sec[e + f*x])^4,x]
```

```
[Out] ((d + c*Cos[e + f*x])*Sec[e + f*x]^2*(a + b*Sec[e + f*x])^2*(6*a^2*(e + f*x
)*(d + c*Cos[e + f*x])^3 + (6*(b^2*c^4*d*(4*c^2 + d^2) - 2*a*b*c^5*(2*c^2 +
 3*d^2) + a^2*(8*c^6*d - 8*c^4*d^3 + 7*c^2*d^5 - 2*d^7))*ArcTanh[((-c + d)*
Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(d + c*Cos[e + f*x])^3)/(c^2 - d^2)^(7/2
) + (2*c*d^2*(b*c - a*d)^2*Sin[e + f*x])/(c^2 - d^2) - (c*d*(a^2*d^2*(12*c^
2 - 7*d^2) + b^2*(6*c^4 - c^2*d^2) + a*b*(-18*c^3*d + 8*c*d^3))*(d + c*Cos[
e + f*x])*Sin[e + f*x])/(c^2 - d^2)^2 + (c*(-2*a*b*c*d*(18*c^4 - 5*c^2*d^2
+ 2*d^4) + a^2*d^2*(36*c^4 - 32*c^2*d^2 + 11*d^4) + b^2*(6*c^6 + 10*c^4*d^2
 - c^2*d^4))*(d + c*Cos[e + f*x])^2*Sin[e + f*x])/(c^2 - d^2)^3))/(6*c^4*f*
(b + a*Cos[e + f*x])^2*(c + d*Sec[e + f*x])^4)
```

Maple [B] time = 0.113, size = 3293, normalized size = 8.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^4,x)
```

```
[Out] 6/f/c/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^3/(c-d)/(c^3+3*c^
2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^5*a^2*d^4+1/f/c^2/(tan(1/2*f*x+1/2*e)^2
*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^3/(c-d)/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*
x+1/2*e)^5*a^2*d^5-2/f/c^3/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c
-d)^3/(c-d)/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^5*a^2*d^6-2/f*c^2/
(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^3/(c-d)/(c^3+3*c^2*d+3*
c*d^2+d^3)*tan(1/2*f*x+1/2*e)^5*b^2*d-6/f*c/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2
*f*x+1/2*e)^2*d-c-d)^3/(c-d)/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^5
*b^2*d^2+24/f*c/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^3/(c^2-
2*c*d+d^2)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3*a^2*d^2-44/3/f/c/(tan(1/2*f
```

```
*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^3/(c^2-2*c*d+d^2)/(c^2+2*c*d+d^2)
*tan(1/2*f*x+1/2*e)^3*a^2*d^4+4/f/c^3/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1
/2*e)^2*d-c-d)^3/(c^2-2*c*d+d^2)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3*a^2*d
^6+28/3/f*c/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^3/(c^2-2*c*
d+d^2)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3*b^2*d^2-12/f*c/(tan(1/2*f*x+1/2
*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^3/(c+d)/(c^3-3*c^2*d+3*c*d^2-d^3)*tan(1
/2*f*x+1/2*e)*a^2*d^2+6/f/c/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-
c-d)^3/(c+d)/(c^3-3*c^2*d+3*c*d^2-d^3)*tan(1/2*f*x+1/2*e)*a^2*d^4+6/f*c/(c^
6-3*c^4*d^2+3*c^2*d^4-d^6)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*f*x+1/2*e)*(
c-d)/((c+d)*(c-d))^(1/2))*a*b*d^2-1/f/c^2/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f
*x+1/2*e)^2*d-c-d)^3/(c+d)/(c^3-3*c^2*d+3*c*d^2-d^3)*tan(1/2*f*x+1/2*e)*a^2
*d^5-2/f/c^3/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^3/(c+d)/(c
^3-3*c^2*d+3*c*d^2-d^3)*tan(1/2*f*x+1/2*e)*a^2*d^6+2/f*c^2/(tan(1/2*f*x+1/2
*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^3/(c+d)/(c^3-3*c^2*d+3*c*d^2-d^3)*tan(1
/2*f*x+1/2*e)*b^2*d-6/f*c/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-
d)^3/(c+d)/(c^3-3*c^2*d+3*c*d^2-d^3)*tan(1/2*f*x+1/2*e)*b^2*d^2+4/f/(tan(1/
2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^3/(c-d)/(c^3+3*c^2*d+3*c*d^2+d
^3)*tan(1/2*f*x+1/2*e)^5*a*b*d^3-8/3/f/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+
1/2*e)^2*d-c-d)^3/(c^2-2*c*d+d^2)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3*a*b*
d^3+2/f*a^2/c^4*arctan(tan(1/2*f*x+1/2*e))+4/f/(tan(1/2*f*x+1/2*e)^2*c-tan(
1/2*f*x+1/2*e)^2*d-c-d)^3/(c+d)/(c^3-3*c^2*d+3*c*d^2-d^3)*tan(1/2*f*x+1/2*e
)*a*b*d^3-12/f*c/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^3/(c-d
)/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^5*a^2*d^2-1/f/(c^6-3*c^4*d^2
+3*c^2*d^4-d^6)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*f*x+1/2*e)*(c-d)/((c+d)
*(c-d))^(1/2))*b^2*d^3+8/f/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)/((c+d)*(c-d))^(1/2
)*arctanh(tan(1/2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))^(1/2))*a^2*d^3+6/f*c/(tan(
1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^3/(c-d)/(c^3+3*c^2*d+3*c*d^2
+d^3)*tan(1/2*f*x+1/2*e)^5*a*b*d^2-24/f*c^2/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2
*f*x+1/2*e)^2*d-c-d)^3/(c^2-2*c*d+d^2)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3
*a*b*d+12/f*c^2/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^3/(c+d)
/(c^3-3*c^2*d+3*c*d^2-d^3)*tan(1/2*f*x+1/2*e)*a*b*d-6/f*c/(tan(1/2*f*x+1/2*
e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^3/(c+d)/(c^3-3*c^2*d+3*c*d^2-d^3)*tan(1/
2*f*x+1/2*e)*a*b*d^2-4/f*c^2/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)/((c+d)*(c-d))^(1
/2)*arctanh(tan(1/2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))^(1/2))*b^2*d-8/f*c^2/(c^
6-3*c^4*d^2+3*c^2*d^4-d^6)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*f*x+1/2*e)*(
c-d)/((c+d)*(c-d))^(1/2))*a^2*d+2/f/c^4/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)/((c+d
)*(c-d))^(1/2)*arctanh(tan(1/2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))^(1/2))*a^2*d^
7-2/f*c^3/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^3/(c-d)/(c^3+
3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^5*b^2+4/f*c^3/(tan(1/2*f*x+1/2*e)^2
*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^3/(c^2-2*c*d+d^2)/(c^2+2*c*d+d^2)*tan(1/2*f*
x+1/2*e)^3*b^2-2/f*c^3/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^
3/(c+d)/(c^3-3*c^2*d+3*c*d^2-d^3)*tan(1/2*f*x+1/2*e)*b^2-7/f/c^2/(c^6-3*c^4
*d^2+3*c^2*d^4-d^6)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*f*x+1/2*e)*(c-d)/((
c+d)*(c-d))^(1/2))*a^2*d^5+4/f*c^3/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)/((c+d)*(c-
d))^(1/2)*arctanh(tan(1/2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))^(1/2))*a*b-4/f/(ta
```

```
n(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^3/(c-d)/(c^3+3*c^2*d+3*c*d
^2+d^3)*tan(1/2*f*x+1/2*e)^5*a^2*d^3-1/f/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*
x+1/2*e)^2*d-c-d)^3/(c-d)/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^5*b^
2*d^3+4/f/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^3/(c+d)/(c^3-
3*c^2*d+3*c*d^2-d^3)*tan(1/2*f*x+1/2*e)*a^2*d^3+1/f/(tan(1/2*f*x+1/2*e)^2*c
-tan(1/2*f*x+1/2*e)^2*d-c-d)^3/(c+d)/(c^3-3*c^2*d+3*c*d^2-d^3)*tan(1/2*f*x+
1/2*e)*b^2*d^3+12/f*c^2/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)
^3/(c-d)/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^5*a*b*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.10881, size = 5010, normalized size = 13.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] [1/12*(12*(a^2*c^11 - 4*a^2*c^9*d^2 + 6*a^2*c^7*d^4 - 4*a^2*c^5*d^6 + a^2*c
^3*d^8)*f*x*cos(f*x + e)^3 + 36*(a^2*c^10*d - 4*a^2*c^8*d^3 + 6*a^2*c^6*d^5
 - 4*a^2*c^4*d^7 + a^2*c^2*d^9)*f*x*cos(f*x + e)^2 + 36*(a^2*c^9*d^2 - 4*a^
2*c^7*d^4 + 6*a^2*c^5*d^6 - 4*a^2*c^3*d^8 + a^2*c*d^10)*f*x*cos(f*x + e) +
12*(a^2*c^8*d^3 - 4*a^2*c^6*d^5 + 6*a^2*c^4*d^7 - 4*a^2*c^2*d^9 + a^2*d^11)
*f*x - 3*(4*a*b*c^7*d^3 + 6*a*b*c^5*d^5 - 7*a^2*c^2*d^8 + 2*a^2*d^10 - 4*(2
*a^2 + b^2)*c^6*d^4 + (8*a^2 - b^2)*c^4*d^6 + (4*a*b*c^10 + 6*a*b*c^8*d^2 -
 7*a^2*c^5*d^5 + 2*a^2*c^3*d^7 - 4*(2*a^2 + b^2)*c^9*d + (8*a^2 - b^2)*c^7*
d^3)*cos(f*x + e)^3 + 3*(4*a*b*c^9*d + 6*a*b*c^7*d^3 - 7*a^2*c^4*d^6 + 2*a^
2*c^2*d^8 - 4*(2*a^2 + b^2)*c^8*d^2 + (8*a^2 - b^2)*c^6*d^4)*cos(f*x + e)^2
 + 3*(4*a*b*c^8*d^2 + 6*a*b*c^6*d^4 - 7*a^2*c^3*d^7 + 2*a^2*c*d^9 - 4*(2*a^
2 + b^2)*c^7*d^3 + (8*a^2 - b^2)*c^5*d^5)*cos(f*x + e))*sqrt(c^2 - d^2)*log
((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*sqrt(c^2 - d^2)*(d*
```

```
cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*c
os(f*x + e) + d^2)) + 2*(2*b^2*c^9*d^2 - 22*a*b*c^8*d^3 + 14*a*b*c^6*d^5 +
8*a*b*c^4*d^7 + 23*a^2*c^3*d^8 - 6*a^2*c*d^10 + (26*a^2 + 11*b^2)*c^7*d^4 -
 (43*a^2 + 13*b^2)*c^5*d^6 + (6*b^2*c^11 - 36*a*b*c^10*d + 46*a*b*c^8*d^3 -
 14*a*b*c^6*d^5 + 4*a*b*c^4*d^7 - 11*a^2*c^3*d^8 + 4*(9*a^2 + b^2)*c^9*d^2
- (68*a^2 + 11*b^2)*c^7*d^4 + (43*a^2 + b^2)*c^5*d^6)*cos(f*x + e)^2 + 3*(2
*b^2*c^10*d - 18*a*b*c^9*d^2 + 16*a*b*c^7*d^4 + 2*a*b*c^5*d^6 - 5*a^2*c^2*d
^9 + (20*a^2 + 7*b^2)*c^8*d^3 - 5*(7*a^2 + 2*b^2)*c^6*d^5 + (20*a^2 + b^2)*
c^4*d^7)*cos(f*x + e))*sin(f*x + e))/((c^15 - 4*c^13*d^2 + 6*c^11*d^4 - 4*c
^9*d^6 + c^7*d^8)*f*cos(f*x + e)^3 + 3*(c^14*d - 4*c^12*d^3 + 6*c^10*d^5 -
4*c^8*d^7 + c^6*d^9)*f*cos(f*x + e)^2 + 3*(c^13*d^2 - 4*c^11*d^4 + 6*c^9*d^
6 - 4*c^7*d^8 + c^5*d^10)*f*cos(f*x + e) + (c^12*d^3 - 4*c^10*d^5 + 6*c^8*d
^7 - 4*c^6*d^9 + c^4*d^11)*f), 1/6*(6*(a^2*c^11 - 4*a^2*c^9*d^2 + 6*a^2*c^7
*d^4 - 4*a^2*c^5*d^6 + a^2*c^3*d^8)*f*x*cos(f*x + e)^3 + 18*(a^2*c^10*d - 4
*a^2*c^8*d^3 + 6*a^2*c^6*d^5 - 4*a^2*c^4*d^7 + a^2*c^2*d^9)*f*x*cos(f*x + e
)^2 + 18*(a^2*c^9*d^2 - 4*a^2*c^7*d^4 + 6*a^2*c^5*d^6 - 4*a^2*c^3*d^8 + a^2
*c*d^10)*f*x*cos(f*x + e) + 6*(a^2*c^8*d^3 - 4*a^2*c^6*d^5 + 6*a^2*c^4*d^7
- 4*a^2*c^2*d^9 + a^2*d^11)*f*x + 3*(4*a*b*c^7*d^3 + 6*a*b*c^5*d^5 - 7*a^2*
c^2*d^8 + 2*a^2*d^10 - 4*(2*a^2 + b^2)*c^6*d^4 + (8*a^2 - b^2)*c^4*d^6 + (4
*a*b*c^10 + 6*a*b*c^8*d^2 - 7*a^2*c^5*d^5 + 2*a^2*c^3*d^7 - 4*(2*a^2 + b^2)
*c^9*d + (8*a^2 - b^2)*c^7*d^3)*cos(f*x + e)^3 + 3*(4*a*b*c^9*d + 6*a*b*c^7
*d^3 - 7*a^2*c^4*d^6 + 2*a^2*c^2*d^8 - 4*(2*a^2 + b^2)*c^8*d^2 + (8*a^2 - b
^2)*c^6*d^4)*cos(f*x + e)^2 + 3*(4*a*b*c^8*d^2 + 6*a*b*c^6*d^4 - 7*a^2*c^3*
d^7 + 2*a^2*c*d^9 - 4*(2*a^2 + b^2)*c^7*d^3 + (8*a^2 - b^2)*c^5*d^5)*cos(f*
x + e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^
2 - d^2)*sin(f*x + e))) + (2*b^2*c^9*d^2 - 22*a*b*c^8*d^3 + 14*a*b*c^6*d^5
+ 8*a*b*c^4*d^7 + 23*a^2*c^3*d^8 - 6*a^2*c*d^10 + (26*a^2 + 11*b^2)*c^7*d^4
 - (43*a^2 + 13*b^2)*c^5*d^6 + (6*b^2*c^11 - 36*a*b*c^10*d + 46*a*b*c^8*d^3
 - 14*a*b*c^6*d^5 + 4*a*b*c^4*d^7 - 11*a^2*c^3*d^8 + 4*(9*a^2 + b^2)*c^9*d^
2 - (68*a^2 + 11*b^2)*c^7*d^4 + (43*a^2 + b^2)*c^5*d^6)*cos(f*x + e)^2 + 3*
(2*b^2*c^10*d - 18*a*b*c^9*d^2 + 16*a*b*c^7*d^4 + 2*a*b*c^5*d^6 - 5*a^2*c^2
*d^9 + (20*a^2 + 7*b^2)*c^8*d^3 - 5*(7*a^2 + 2*b^2)*c^6*d^5 + (20*a^2 + b^2
)*c^4*d^7)*cos(f*x + e))*sin(f*x + e))/((c^15 - 4*c^13*d^2 + 6*c^11*d^4 - 4
*c^9*d^6 + c^7*d^8)*f*cos(f*x + e)^3 + 3*(c^14*d - 4*c^12*d^3 + 6*c^10*d^5
- 4*c^8*d^7 + c^6*d^9)*f*cos(f*x + e)^2 + 3*(c^13*d^2 - 4*c^11*d^4 + 6*c^9*
d^6 - 4*c^7*d^8 + c^5*d^10)*f*cos(f*x + e) + (c^12*d^3 - 4*c^10*d^5 + 6*c^8
*d^7 - 4*c^6*d^9 + c^4*d^11)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+b\sec\left(e+fx\right)\right)^2}{\left(c+d\sec\left(e+fx\right)\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**2/(c+d*sec(f*x+e))**4,x)
```

```
[Out] Integral((a + b*sec(e + f*x))**2/(c + d*sec(e + f*x))**4, x)
```

Giac [B] time = 1.63226, size = 1692, normalized size = 4.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(4*a*b*c^7 - 8*a^2*c^6*d - 4*b^2*c^6*d + 6*a*b*c^5*d^2 + 8*a^2*c^4*d
^3 - b^2*c^4*d^3 - 7*a^2*c^2*d^5 + 2*a^2*d^7)*(pi*floor(1/2*(f*x + e)/pi +
1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/
2*e))/sqrt(-c^2 + d^2)))/((c^10 - 3*c^8*d^2 + 3*c^6*d^4 - c^4*d^6)*sqrt(-c^
2 + d^2)) + 3*(f*x + e)*a^2/c^4 - (6*b^2*c^8*tan(1/2*f*x + 1/2*e)^5 - 36*a*
b*c^7*d*tan(1/2*f*x + 1/2*e)^5 - 6*b^2*c^7*d*tan(1/2*f*x + 1/2*e)^5 + 36*a^
2*c^6*d^2*tan(1/2*f*x + 1/2*e)^5 + 54*a*b*c^6*d^2*tan(1/2*f*x + 1/2*e)^5 +
12*b^2*c^6*d^2*tan(1/2*f*x + 1/2*e)^5 - 60*a^2*c^5*d^3*tan(1/2*f*x + 1/2*e)
^5 - 12*a*b*c^5*d^3*tan(1/2*f*x + 1/2*e)^5 - 27*b^2*c^5*d^3*tan(1/2*f*x + 1
/2*e)^5 - 6*a^2*c^4*d^4*tan(1/2*f*x + 1/2*e)^5 + 6*a*b*c^4*d^4*tan(1/2*f*x
+ 1/2*e)^5 + 12*b^2*c^4*d^4*tan(1/2*f*x + 1/2*e)^5 + 45*a^2*c^3*d^5*tan(1/2
*f*x + 1/2*e)^5 - 12*a*b*c^3*d^5*tan(1/2*f*x + 1/2*e)^5 + 3*b^2*c^3*d^5*tan
(1/2*f*x + 1/2*e)^5 - 6*a^2*c^2*d^6*tan(1/2*f*x + 1/2*e)^5 - 15*a^2*c*d^7*t
an(1/2*f*x + 1/2*e)^5 + 6*a^2*d^8*tan(1/2*f*x + 1/2*e)^5 - 12*b^2*c^8*tan(1
/2*f*x + 1/2*e)^3 + 72*a*b*c^7*d*tan(1/2*f*x + 1/2*e)^3 - 72*a^2*c^6*d^2*ta
n(1/2*f*x + 1/2*e)^3 - 16*b^2*c^6*d^2*tan(1/2*f*x + 1/2*e)^3 - 64*a*b*c^5*d
^3*tan(1/2*f*x + 1/2*e)^3 + 116*a^2*c^4*d^4*tan(1/2*f*x + 1/2*e)^3 + 28*b^2
*c^4*d^4*tan(1/2*f*x + 1/2*e)^3 - 8*a*b*c^3*d^5*tan(1/2*f*x + 1/2*e)^3 - 56
*a^2*c^2*d^6*tan(1/2*f*x + 1/2*e)^3 + 12*a^2*d^8*tan(1/2*f*x + 1/2*e)^3 + 6
*b^2*c^8*tan(1/2*f*x + 1/2*e) - 36*a*b*c^7*d*tan(1/2*f*x + 1/2*e) + 6*b^2*c
^7*d*tan(1/2*f*x + 1/2*e) + 36*a^2*c^6*d^2*tan(1/2*f*x + 1/2*e) - 54*a*b*c^
6*d^2*tan(1/2*f*x + 1/2*e) + 12*b^2*c^6*d^2*tan(1/2*f*x + 1/2*e) + 60*a^2*c
^5*d^3*tan(1/2*f*x + 1/2*e) - 12*a*b*c^5*d^3*tan(1/2*f*x + 1/2*e) + 27*b^2*
c^5*d^3*tan(1/2*f*x + 1/2*e) - 6*a^2*c^4*d^4*tan(1/2*f*x + 1/2*e) - 6*a*b*c
^4*d^4*tan(1/2*f*x + 1/2*e) + 12*b^2*c^4*d^4*tan(1/2*f*x + 1/2*e) - 45*a^2*
c^3*d^5*tan(1/2*f*x + 1/2*e) - 12*a*b*c^3*d^5*tan(1/2*f*x + 1/2*e) - 3*b^2*
c^3*d^5*tan(1/2*f*x + 1/2*e) - 6*a^2*c^2*d^6*tan(1/2*f*x + 1/2*e) + 15*a^2*
c*d^7*tan(1/2*f*x + 1/2*e) + 6*a^2*d^8*tan(1/2*f*x + 1/2*e))/((c^9 - 3*c^7*
```

```
d^2 + 3*c^5*d^4 - c^3*d^6)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*
e)^2 - c - d)^3))/f
```

3.195 $\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx$

Optimal. Leaf size=254

$$-\frac{(bc-ad)\left(a^2\left(-\left(-5c^2d^2+6c^4+2d^4\right)\right)+2abcd\left(4c^2-d^2\right)-b^2c^2\left(c^2+2d^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^3f(c-d)^{5/2}(c+d)^{5/2}}+\frac{a^3x}{c^3}+\frac{(bc-a\ldots}{2c\ldots}$$

```
[Out] (a^3*x)/c^3 - ((b*c - a*d)*(2*a*b*c*d*(4*c^2 - d^2) - b^2*c^2*(c^2 + 2*d^2)
 - a^2*(6*c^4 - 5*c^2*d^2 + 2*d^4))*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/
Sqrt[c + d]])/(c^3*(c - d)^(5/2)*(c + d)^(5/2)*f) + ((b*c - a*d)^2*(b + a*C
os[e + f*x])*Sin[e + f*x])/(2*c*(c^2 - d^2)*f*(d + c*Cos[e + f*x])^2) + ((b
*c - a*d)^2*(5*a*c^2 - 3*b*c*d - 2*a*d^2)*Sin[e + f*x])/(2*c^2*(c^2 - d^2)^
2*f*(d + c*Cos[e + f*x]))
```

Rubi [A] time = 1.13244, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3941, 2792, 3021, 2735, 2659, 208}

$$-\frac{(bc-ad)\left(a^2\left(-\left(-5c^2d^2+6c^4+2d^4\right)\right)+2abcd\left(4c^2-d^2\right)-b^2c^2\left(c^2+2d^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^3f(c-d)^{5/2}(c+d)^{5/2}}+\frac{a^3x}{c^3}+\frac{(bc-a\ldots}{2c\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x])^3/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (a^3*x)/c^3 - ((b*c - a*d)*(2*a*b*c*d*(4*c^2 - d^2) - b^2*c^2*(c^2 + 2*d^2)
 - a^2*(6*c^4 - 5*c^2*d^2 + 2*d^4))*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/
Sqrt[c + d]])/(c^3*(c - d)^(5/2)*(c + d)^(5/2)*f) + ((b*c - a*d)^2*(b + a*C
os[e + f*x])*Sin[e + f*x])/(2*c*(c^2 - d^2)*f*(d + c*Cos[e + f*x])^2) + ((b
*c - a*d)^2*(5*a*c^2 - 3*b*c*d - 2*a*d^2)*Sin[e + f*x])/(2*c^2*(c^2 - d^2)^
2*f*(d + c*Cos[e + f*x]))
```

Rule 3941

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Int[((b + a*Sin[e + f*x])^m*(d + c*Sin[e + f
*x])^n)/Sin[e + f*x]^(m + n), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && N
```

eQ[b*c - a*d, 0] && IntegerQ[m] && IntegerQ[n] && LeQ[-2, m + n, 0]

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx = \int \frac{(b+a\cos(e+fx))^3}{(d+c\cos(e+fx))^3}\,dx$$

$$= \frac{(bc-ad)^2(b+a\cos(e+fx))\sin(e+fx)}{2c\left(c^2-d^2\right)f(d+c\cos(e+fx))^2} + \frac{\int \frac{5ab^2c^2-4a^2bcd-2b^3cd+a^3d^2+\left(b^3c^2-2a^3cd-4ab^2cd+a^2b\left(6c^2-\right.\right.}{(d+c\cos(e+fx))^2}}{2c\left(c^2-d^2\right)}$$

$$= \frac{(bc-ad)^2(b+a\cos(e+fx))\sin(e+fx)}{2c\left(c^2-d^2\right)f(d+c\cos(e+fx))^2} + \frac{(bc-ad)^2\left(5ac^2-3bcd-2ad^2\right)\sin(e+fx)}{2c^2\left(c^2-d^2\right)^2f(d+c\cos(e+fx))} + \frac{\int}{-}$$

$$= \frac{a^3x}{c^3} + \frac{(bc-ad)^2(b+a\cos(e+fx))\sin(e+fx)}{2c\left(c^2-d^2\right)f(d+c\cos(e+fx))^2} + \frac{(bc-ad)^2\left(5ac^2-3bcd-2ad^2\right)\sin(e+f}{2c^2\left(c^2-d^2\right)^2f(d+c\cos(e+fx))}$$

$$= \frac{a^3x}{c^3} + \frac{(bc-ad)^2(b+a\cos(e+fx))\sin(e+fx)}{2c\left(c^2-d^2\right)f(d+c\cos(e+fx))^2} + \frac{(bc-ad)^2\left(5ac^2-3bcd-2ad^2\right)\sin(e+f}{2c^2\left(c^2-d^2\right)^2f(d+c\cos(e+fx))}$$

$$= \frac{a^3x}{c^3} + \frac{(bc-ad)\left(6a^2c^4+b^2c^4-8abc^3d-5a^2c^2d^2+2b^2c^2d^2+2abcd^3+2a^2d^4\right)\tanh^{-1}\left(\frac{\sqrt{c-}}{}\right.}{c^3(c-d)^{5/2}(c+d)^{5/2}f}$$

Mathematica [B] time = 2.25181, size = 517, normalized size = 2.04

$$\frac{-18a^2bc^4d^2\sin(e+fx)+3a^2bc^3d^3\sin(2(e+fx))-12a^2bc^5d\sin(2(e+fx))+6a^3c^4d^2\sin(2(e+fx))+10a^3c^3d^3\sin(e+fx)-3a^3c^2d^4\sin(2(e+fx))+8a^3cd\left(c^2-d^2\right)^2(e+fx)\,c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x])^3/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] ((-4*(-9*a*b^2*c^4*d + 3*a^2*b*c^3*(2*c^2 + d^2) + b^3*c^3*(c^2 + 2*d^2) +
a^3*(-6*c^4*d + 5*c^2*d^3 - 2*d^5))*ArcTanh[((-c + d)*Tan[(e + f*x)/2]]/Sqr
t[c^2 - d^2]])/(c^2 - d^2)^(5/2) + (2*a^3*c^6*e - 6*a^3*c^2*d^4*e + 4*a^3*d
^6*e + 2*a^3*c^6*f*x - 6*a^3*c^2*d^4*f*x + 4*a^3*d^6*f*x + 8*a^3*c*d*(c^2 -
 d^2)^2*(e + f*x)*Cos[e + f*x] + 2*a^3*(c^3 - c*d^2)^2*(e + f*x)*Cos[2*(e +
 f*x)] + 2*b^3*c^6*Sin[e + f*x] + 6*a*b^2*c^5*d*Sin[e + f*x] - 18*a^2*b*c^4
*d^2*Sin[e + f*x] - 8*b^3*c^4*d^2*Sin[e + f*x] + 10*a^3*c^3*d^3*Sin[e + f*x
```

```
] + 12*a*b^2*c^3*d^3*Sin[e + f*x] - 4*a^3*c*d^5*Sin[e + f*x] + 6*a*b^2*c^6*
Sin[2*(e + f*x)] - 12*a^2*b*c^5*d*Sin[2*(e + f*x)] - 3*b^3*c^5*d*Sin[2*(e +
 f*x)] + 6*a^3*c^4*d^2*Sin[2*(e + f*x)] + 3*a*b^2*c^4*d^2*Sin[2*(e + f*x)]
+ 3*a^2*b*c^3*d^3*Sin[2*(e + f*x)] - 3*a^3*c^2*d^4*Sin[2*(e + f*x)])/((c^2
- d^2)^2*(d + c*Cos[e + f*x])^2))/(4*c^3*f)
```

Maple [B] time = 0.111, size = 2031, normalized size = 8.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x)
```

```
[Out] -9/f*c/(c^4-2*c^2*d^2+d^4)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*f*x+1/2*e)*(
c-d)/((c+d)*(c-d))^(1/2))*a*b^2*d-1/f/c/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x
+1/2*e)^2*d-c-d)^2/(c-d)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3*a^3*d^3+2/f/c
^2/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^2/(c-d)/(c^2+2*c*d+d
^2)*tan(1/2*f*x+1/2*e)^3*a^3*d^4-6/f*c^2/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*
x+1/2*e)^2*d-c-d)^2/(c-d)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3*a*b^2+4/f*c/
(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^2/(c-d)/(c^2+2*c*d+d^2)
*tan(1/2*f*x+1/2*e)^3*b^3*d-1/f/c/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e
)^2*d-c-d)^2/(c+d)/(c-d)^2*tan(1/2*f*x+1/2*e)*a^3*d^3-2/f/c^2/(tan(1/2*f*x+
1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^2/(c+d)/(c-d)^2*tan(1/2*f*x+1/2*e)*a
^3*d^4+6/f*c^2/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^2/(c+d)/
(c-d)^2*tan(1/2*f*x+1/2*e)*a*b^2-4/f*c/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+
1/2*e)^2*d-c-d)^2/(c+d)/(c-d)^2*tan(1/2*f*x+1/2*e)*b^3*d+3/f/(tan(1/2*f*x+1
/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^2/(c-d)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1
/2*e)^3*a^2*b*d^2-6/f/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^2
/(c-d)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3*a*b^2*d^2+2/f*a^3/c^3*arctan(ta
n(1/2*f*x+1/2*e))+3/f/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^2
/(c+d)/(c-d)^2*tan(1/2*f*x+1/2*e)*a^2*b*d^2+6/f/(tan(1/2*f*x+1/2*e)^2*c-tan
(1/2*f*x+1/2*e)^2*d-c-d)^2/(c+d)/(c-d)^2*tan(1/2*f*x+1/2*e)*a*b^2*d^2+2/f/(
c^4-2*c^2*d^2+d^4)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*f*x+1/2*e)*(c-d)/((c
+d)*(c-d))^(1/2))*b^3*d^2+1/f*c^2/(c^4-2*c^2*d^2+d^4)/((c+d)*(c-d))^(1/2)*a
rctanh(tan(1/2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))^(1/2))*b^3+5/f/c/(c^4-2*c^2*d
^2+d^4)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))^
(1/2))*a^3*d^3-6/f*c/(c^4-2*c^2*d^2+d^4)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/
2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))^(1/2))*a^3*d-2/f/c^3/(c^4-2*c^2*d^2+d^4)/(
(c+d)*(c-d))^(1/2)*arctanh(tan(1/2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))^(1/2))*a^
3*d^5+1/f*c^2/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^2/(c-d)/(
c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3*b^3+1/f*c^2/(tan(1/2*f*x+1/2*e)^2*c-tan
```

```
(1/2*f*x+1/2*e)^2*d-c-d)^2/(c+d)/(c-d)^2*tan(1/2*f*x+1/2*e)*b^3+6/f*c^2/(c^
4-2*c^2*d^2+d^4)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*f*x+1/2*e)*(c-d)/((c+d
)*(c-d))^(1/2))*a^2*b+3/f/(c^4-2*c^2*d^2+d^4)/((c+d)*(c-d))^(1/2)*arctanh(t
an(1/2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))^(1/2))*a^2*b*d^2-6/f/(tan(1/2*f*x+1/2
*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^2/(c-d)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2
*e)^3*a^3*d^2+6/f/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^2/(c+
d)/(c-d)^2*tan(1/2*f*x+1/2*e)*a^3*d^2-3/f*c/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2
*f*x+1/2*e)^2*d-c-d)^2/(c-d)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3*a*b^2*d+1
2/f*c/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^2/(c-d)/(c^2+2*c*
d+d^2)*tan(1/2*f*x+1/2*e)^3*a^2*b*d-12/f*c/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*
f*x+1/2*e)^2*d-c-d)^2/(c+d)/(c-d)^2*tan(1/2*f*x+1/2*e)*a^2*b*d-3/f*c/(tan(1
/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^2/(c+d)/(c-d)^2*tan(1/2*f*x+1
/2*e)*a*b^2*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.928389, size = 3343, normalized size = 13.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^3*c^8 - 3*a^3*c^6*d^2 + 3*a^3*c^4*d^4 - a^3*c^2*d^6)*f*x*cos(f*x
 + e)^2 + 8*(a^3*c^7*d - 3*a^3*c^5*d^3 + 3*a^3*c^3*d^5 - a^3*c*d^7)*f*x*cos
(f*x + e) + 4*(a^3*c^6*d^2 - 3*a^3*c^4*d^4 + 3*a^3*c^2*d^6 - a^3*d^8)*f*x -
 (5*a^3*c^2*d^5 - 2*a^3*d^7 + (6*a^2*b + b^3)*c^5*d^2 - 3*(2*a^3 + 3*a*b^2)
*c^4*d^3 + (3*a^2*b + 2*b^3)*c^3*d^4 + (5*a^3*c^4*d^3 - 2*a^3*c^2*d^5 + (6*
a^2*b + b^3)*c^7 - 3*(2*a^3 + 3*a*b^2)*c^6*d + (3*a^2*b + 2*b^3)*c^5*d^2)*c
os(f*x + e)^2 + 2*(5*a^3*c^3*d^4 - 2*a^3*c*d^6 + (6*a^2*b + b^3)*c^6*d - 3*
(2*a^3 + 3*a*b^2)*c^5*d^2 + (3*a^2*b + 2*b^3)*c^4*d^3)*cos(f*x + e))*sqrt(c
```

```
^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*sqrt(c
^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e
)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(b^3*c^8 + 3*a*b^2*c^7*d + 2*a^3*c*d^7
 - (9*a^2*b + 5*b^3)*c^6*d^2 + (5*a^3 + 3*a*b^2)*c^5*d^3 + (9*a^2*b + 4*b^3
)*c^4*d^4 - (7*a^3 + 6*a*b^2)*c^3*d^5 + 3*(2*a*b^2*c^8 - a^2*b*c^3*d^5 + a^
3*c^2*d^6 - (4*a^2*b + b^3)*c^7*d + (2*a^3 - a*b^2)*c^6*d^2 + (5*a^2*b + b^
3)*c^5*d^3 - (3*a^3 + a*b^2)*c^4*d^4)*cos(f*x + e))*sin(f*x + e))/((c^11 -
3*c^9*d^2 + 3*c^7*d^4 - c^5*d^6)*f*cos(f*x + e)^2 + 2*(c^10*d - 3*c^8*d^3 +
 3*c^6*d^5 - c^4*d^7)*f*cos(f*x + e) + (c^9*d^2 - 3*c^7*d^4 + 3*c^5*d^6 - c
^3*d^8)*f), 1/2*(2*(a^3*c^8 - 3*a^3*c^6*d^2 + 3*a^3*c^4*d^4 - a^3*c^2*d^6)*
f*x*cos(f*x + e)^2 + 4*(a^3*c^7*d - 3*a^3*c^5*d^3 + 3*a^3*c^3*d^5 - a^3*c*d
^7)*f*x*cos(f*x + e) + 2*(a^3*c^6*d^2 - 3*a^3*c^4*d^4 + 3*a^3*c^2*d^6 - a^3
*d^8)*f*x + (5*a^3*c^2*d^5 - 2*a^3*d^7 + (6*a^2*b + b^3)*c^5*d^2 - 3*(2*a^3
 + 3*a*b^2)*c^4*d^3 + (3*a^2*b + 2*b^3)*c^3*d^4 + (5*a^3*c^4*d^3 - 2*a^3*c^
2*d^5 + (6*a^2*b + b^3)*c^7 - 3*(2*a^3 + 3*a*b^2)*c^6*d + (3*a^2*b + 2*b^3)
*c^5*d^2)*cos(f*x + e)^2 + 2*(5*a^3*c^3*d^4 - 2*a^3*c*d^6 + (6*a^2*b + b^3)
*c^6*d - 3*(2*a^3 + 3*a*b^2)*c^5*d^2 + (3*a^2*b + 2*b^3)*c^4*d^3)*cos(f*x +
 e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 -
 d^2)*sin(f*x + e))) + (b^3*c^8 + 3*a*b^2*c^7*d + 2*a^3*c*d^7 - (9*a^2*b +
5*b^3)*c^6*d^2 + (5*a^3 + 3*a*b^2)*c^5*d^3 + (9*a^2*b + 4*b^3)*c^4*d^4 - (7
*a^3 + 6*a*b^2)*c^3*d^5 + 3*(2*a*b^2*c^8 - a^2*b*c^3*d^5 + a^3*c^2*d^6 - (4
*a^2*b + b^3)*c^7*d + (2*a^3 - a*b^2)*c^6*d^2 + (5*a^2*b + b^3)*c^5*d^3 - (
3*a^3 + a*b^2)*c^4*d^4)*cos(f*x + e))*sin(f*x + e))/((c^11 - 3*c^9*d^2 + 3*
c^7*d^4 - c^5*d^6)*f*cos(f*x + e)^2 + 2*(c^10*d - 3*c^8*d^3 + 3*c^6*d^5 - c
^4*d^7)*f*cos(f*x + e) + (c^9*d^2 - 3*c^7*d^4 + 3*c^5*d^6 - c^3*d^8)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + b\sec\left(e + fx\right)\right)^3}{\left(c + d\sec\left(e + fx\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**3/(c+d*sec(f*x+e))**3,x)
```

```
[Out] Integral((a + b*sec(e + f*x))**3/(c + d*sec(e + f*x))**3, x)
```

Giac [B] time = 1.56538, size = 1150, normalized size = 4.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] ((f*x + e)*a^3/c^3 + (6*a^2*b*c^5 + b^3*c^5 - 6*a^3*c^4*d - 9*a*b^2*c^4*d +
 3*a^2*b*c^3*d^2 + 2*b^3*c^3*d^2 + 5*a^3*c^2*d^3 - 2*a^3*d^5)*(pi*floor(1/2
*(f*x + e)/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*
tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))/((c^7 - 2*c^5*d^2 + c^3*d^4)*sqrt(
-c^2 + d^2)) - (6*a*b^2*c^5*tan(1/2*f*x + 1/2*e)^3 - b^3*c^5*tan(1/2*f*x +
1/2*e)^3 - 12*a^2*b*c^4*d*tan(1/2*f*x + 1/2*e)^3 - 3*a*b^2*c^4*d*tan(1/2*f*
x + 1/2*e)^3 - 3*b^3*c^4*d*tan(1/2*f*x + 1/2*e)^3 + 6*a^3*c^3*d^2*tan(1/2*f
*x + 1/2*e)^3 + 9*a^2*b*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 + 3*a*b^2*c^3*d^2*ta
n(1/2*f*x + 1/2*e)^3 + 4*b^3*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 - 5*a^3*c^2*d^3
*tan(1/2*f*x + 1/2*e)^3 + 3*a^2*b*c^2*d^3*tan(1/2*f*x + 1/2*e)^3 - 6*a*b^2*
c^2*d^3*tan(1/2*f*x + 1/2*e)^3 - 3*a^3*c*d^4*tan(1/2*f*x + 1/2*e)^3 + 2*a^3
*d^5*tan(1/2*f*x + 1/2*e)^3 - 6*a*b^2*c^5*tan(1/2*f*x + 1/2*e) - b^3*c^5*ta
n(1/2*f*x + 1/2*e) + 12*a^2*b*c^4*d*tan(1/2*f*x + 1/2*e) - 3*a*b^2*c^4*d*ta
n(1/2*f*x + 1/2*e) + 3*b^3*c^4*d*tan(1/2*f*x + 1/2*e) - 6*a^3*c^3*d^2*tan(1
/2*f*x + 1/2*e) + 9*a^2*b*c^3*d^2*tan(1/2*f*x + 1/2*e) - 3*a*b^2*c^3*d^2*ta
n(1/2*f*x + 1/2*e) + 4*b^3*c^3*d^2*tan(1/2*f*x + 1/2*e) - 5*a^3*c^2*d^3*tan
(1/2*f*x + 1/2*e) - 3*a^2*b*c^2*d^3*tan(1/2*f*x + 1/2*e) - 6*a*b^2*c^2*d^3*
tan(1/2*f*x + 1/2*e) + 3*a^3*c*d^4*tan(1/2*f*x + 1/2*e) + 2*a^3*d^5*tan(1/2
*f*x + 1/2*e))/((c^6 - 2*c^4*d^2 + c^2*d^4)*(c*tan(1/2*f*x + 1/2*e)^2 - d*t
an(1/2*f*x + 1/2*e)^2 - c - d)^2))/f
```

3.196 $\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx$

Optimal. Leaf size=412

$$-\frac{(bc-ad)\left(a^2\left(-28c^2d^3+34c^4d+9d^5\right)-abc\left(17c^2d^2+18c^4-5d^4\right)+b^2c^2d\left(13c^2+2d^2\right)\right)\sin(e+fx)}{6c^3f\left(c^2-d^2\right)^3(c\cos(e+fx)+d)}-\frac{\left(-a^2b\left(9c^5d^2\right.\right.}{}$$

```
[Out] (a^3*x)/c^4 - ((3*a*b^2*c^4*d*(4*c^2 + d^2) - b^3*c^5*(c^2 + 4*d^2) - a^2*b
*(6*c^7 + 9*c^5*d^2) + a^3*(8*c^6*d - 8*c^4*d^3 + 7*c^2*d^5 - 2*d^7))*ArcTa
nh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/(c^4*Sqrt[c - d]*Sqrt[c + d
]*(c^2 - d^2)^3*f) - (d*(b*c - a*d)*(b + a*Cos[e + f*x])^2*Sin[e + f*x])/(3
*c*(c^2 - d^2)*f*(d + c*Cos[e + f*x])^3) + ((b*c - a*d)^2*(3*b*c^3 - 8*a*c^
2*d + 2*b*c*d^2 + 3*a*d^3)*Sin[e + f*x])/(6*c^3*(c^2 - d^2)^2*f*(d + c*Cos[
e + f*x])^2) - ((b*c - a*d)*(b^2*c^2*d*(13*c^2 + 2*d^2) - a*b*c*(18*c^4 + 1
7*c^2*d^2 - 5*d^4) + a^2*(34*c^4*d - 28*c^2*d^3 + 9*d^5))*Sin[e + f*x])/(6*
c^3*(c^2 - d^2)^3*f*(d + c*Cos[e + f*x]))
```

Rubi [A] time = 1.0623, antiderivative size = 412, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$, Rules used = {3941, 2989, 3031, 3021, 2735, 2659, 208}

$$-\frac{(bc-ad)\left(a^2\left(-28c^2d^3+34c^4d+9d^5\right)-abc\left(17c^2d^2+18c^4-5d^4\right)+b^2c^2d\left(13c^2+2d^2\right)\right)\sin(e+fx)}{6c^3f\left(c^2-d^2\right)^3(c\cos(e+fx)+d)}-\frac{\left(-a^2b\left(9c^5d^2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x])^3/(c + d*Sec[e + f*x])^4,x]
```

```
[Out] (a^3*x)/c^4 - ((3*a*b^2*c^4*d*(4*c^2 + d^2) - b^3*c^5*(c^2 + 4*d^2) - a^2*b
*(6*c^7 + 9*c^5*d^2) + a^3*(8*c^6*d - 8*c^4*d^3 + 7*c^2*d^5 - 2*d^7))*ArcTa
nh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/(c^4*Sqrt[c - d]*Sqrt[c + d
]*(c^2 - d^2)^3*f) - (d*(b*c - a*d)*(b + a*Cos[e + f*x])^2*Sin[e + f*x])/(3
*c*(c^2 - d^2)*f*(d + c*Cos[e + f*x])^3) + ((b*c - a*d)^2*(3*b*c^3 - 8*a*c^
2*d + 2*b*c*d^2 + 3*a*d^3)*Sin[e + f*x])/(6*c^3*(c^2 - d^2)^2*f*(d + c*Cos[
e + f*x])^2) - ((b*c - a*d)*(b^2*c^2*d*(13*c^2 + 2*d^2) - a*b*c*(18*c^4 + 1
7*c^2*d^2 - 5*d^4) + a^2*(34*c^4*d - 28*c^2*d^3 + 9*d^5))*Sin[e + f*x])/(6*
c^3*(c^2 - d^2)^3*f*(d + c*Cos[e + f*x]))
```

Rule 3941

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Int[((b + a*Sin[e + f*x])^m*(d + c*Sin[e + f
*x])^n)/Sin[e + f*x]^(m + n), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && N
eQ[b*c - a*d, 0] && IntegerQ[m] && IntegerQ[n] && LeQ[-2, m + n, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx = \int \frac{\cos(e+fx)(b+a\cos(e+fx))^3}{(d+c\cos(e+fx))^4}\,dx$$

$$= -\frac{d(bc-ad)(b+a\cos(e+fx))^2\sin(e+fx)}{3c\left(c^2-d^2\right)f(d+c\cos(e+fx))^3} + \frac{\int \frac{(b+a\cos(e+fx))\left((3bc-2ad)(bc-ad)-\left(3a^2cd+2b^2cd-ab\right.\right.}{(d+c\cos(e+f}}{3c\left(c^2-\right.}$$

$$= -\frac{d(bc-ad)(b+a\cos(e+fx))^2\sin(e+fx)}{3c\left(c^2-d^2\right)f(d+c\cos(e+fx))^3} + \frac{(bc-ad)^2\left(3bc^3-8ac^2d+2bcd^2+3ad^3\right)\sin}{6c^3\left(c^2-d^2\right)^2f(d+c\cos(e+fx))^2}$$

$$= -\frac{d(bc-ad)(b+a\cos(e+fx))^2\sin(e+fx)}{3c\left(c^2-d^2\right)f(d+c\cos(e+fx))^3} + \frac{(bc-ad)^2\left(3bc^3-8ac^2d+2bcd^2+3ad^3\right)\sin}{6c^3\left(c^2-d^2\right)^2f(d+c\cos(e+fx))^2}$$

$$= \frac{a^3x}{c^4} - \frac{d(bc-ad)(b+a\cos(e+fx))^2\sin(e+fx)}{3c\left(c^2-d^2\right)f(d+c\cos(e+fx))^3} + \frac{(bc-ad)^2\left(3bc^3-8ac^2d+2bcd^2+3ad\right.}{6c^3\left(c^2-d^2\right)^2f(d+c\cos(e+\right.}$$

$$= \frac{a^3x}{c^4} - \frac{d(bc-ad)(b+a\cos(e+fx))^2\sin(e+fx)}{3c\left(c^2-d^2\right)f(d+c\cos(e+fx))^3} + \frac{(bc-ad)^2\left(3bc^3-8ac^2d+2bcd^2+3ad\right.}{6c^3\left(c^2-d^2\right)^2f(d+c\cos(e+\right.}$$

$$= \frac{a^3x}{c^4} - \frac{\left(3ab^2c^4d\left(4c^2+d^2\right)-b^3c^5\left(c^2+4d^2\right)-a^2b\left(6c^7+9c^5d^2\right)+a^3\left(8c^6d-8c^4d^3+7c^2d\right.\right.}{c^4\sqrt{c-d}\sqrt{c+d}\left(c^2-d^2\right)^3f}$$

Mathematica [A] time = 3.80536, size = 459, normalized size = 1.11

$$\sec(e+fx)(a+b\sec(e+fx))^3(c\cos(e+fx)+d)\left(\frac{c\left(-3a^2bcd\left(-5c^2d^2+18c^4+2d^4\right)+a^3\left(-32c^2d^4+36c^4d^2+11d^6\right)+3ab^2c^2\left(10c^2d^2+6c^4-d^4\right)-b^3c^3d\right.}{\left(c^2-d^2\right)^3}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x])^3/(c + d*Sec[e + f*x])^4,x]
```

```
[Out] ((d + c*Cos[e + f*x])*Sec[e + f*x]*(a + b*Sec[e + f*x])^3*(6*a^3*(e + f*x)*
(d + c*Cos[e + f*x])^3 - (6*(-3*a*b^2*c^4*d*(4*c^2 + d^2) + b^3*c^5*(c^2 +
4*d^2) + a^2*b*(6*c^7 + 9*c^5*d^2) + a^3*(-8*c^6*d + 8*c^4*d^3 - 7*c^2*d^5
+ 2*d^7))*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(d + c*Cos[e
 + f*x])^3)/(c^2 - d^2)^(7/2) - (2*c*d*(b*c - a*d)^3*Sin[e + f*x])/(c^2 - d
^2) + (c*(b*c - a*d)^2*(3*b*c^3 - 12*a*c^2*d + 2*b*c*d^2 + 7*a*d^3)*(d + c*
Cos[e + f*x])*Sin[e + f*x])/(c^2 - d^2)^2 + (c*(-(b^3*c^3*d*(13*c^2 + 2*d^2
)) + 3*a*b^2*c^2*(6*c^4 + 10*c^2*d^2 - d^4) - 3*a^2*b*c*d*(18*c^4 - 5*c^2*d
^2 + 2*d^4) + a^3*(36*c^4*d^2 - 32*c^2*d^4 + 11*d^6))*(d + c*Cos[e + f*x])^
2*Sin[e + f*x])/(c^2 - d^2)^3))/(6*c^4*f*(b + a*Cos[e + f*x])^3*(c + d*Sec[
e + f*x])^4)
```

Maple [B] time = 0.167, size = 4330, normalized size = 10.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^4,x)
```

```
[Out] 12/f*c^3/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^3/(c^2-2*c*d+d
^2)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3*a*b^2-28/3/f*c^2/(tan(1/2*f*x+1/2*
e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^3/(c^2-2*c*d+d^2)/(c^2+2*c*d+d^2)*tan(1/
2*f*x+1/2*e)^3*b^3*d-12/f*c/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-
c-d)^3/(c+d)/(c^3-3*c^2*d+3*c*d^2-d^3)*tan(1/2*f*x+1/2*e)*a^3*d^2+6/f/c/(ta
n(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^3/(c+d)/(c^3-3*c^2*d+3*c*d
^2-d^3)*tan(1/2*f*x+1/2*e)*a^3*d^4-1/f/c^2/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*
f*x+1/2*e)^2*d-c-d)^3/(c+d)/(c^3-3*c^2*d+3*c*d^2-d^3)*tan(1/2*f*x+1/2*e)*a^
3*d^5-2/f/c^3/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^3/(c+d)/(
```

c^3-3*c^2*d+3*c*d^2-d^3)*tan(1/2*f*x+1/2*e)*a^3*d^6-6/f*c^3/(tan(1/2*f*x+1/
2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^3/(c+d)/(c^3-3*c^2*d+3*c*d^2-d^3)*tan(
1/2*f*x+1/2*e)*a*b^2+6/f*c^2/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d
-c-d)^3/(c+d)/(c^3-3*c^2*d+3*c*d^2-d^3)*tan(1/2*f*x+1/2*e)*b^3*d-2/f*c/(tan
(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^3/(c+d)/(c^3-3*c^2*d+3*c*d^
2-d^3)*tan(1/2*f*x+1/2*e)*b^3*d^2+9/f*c/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)/((c+d
)*(c-d))^(1/2)*arctanh(tan(1/2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))^(1/2))*a^2*b*
d^2-12/f*c^2/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)/((c+d)*(c-d))^(1/2)*arctanh(tan(
1/2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))^(1/2))*a*b^2*d+2/f*a^3/c^4*arctan(tan(1/
2*f*x+1/2*e))-9/f*c/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^3/(
c+d)/(c^3-3*c^2*d+3*c*d^2-d^3)*tan(1/2*f*x+1/2*e)*a^2*b*d^2+6/f*c^2/(tan(1/
2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^3/(c+d)/(c^3-3*c^2*d+3*c*d^2-d
^3)*tan(1/2*f*x+1/2*e)*a*b^2*d-18/f*c/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1
/2*e)^2*d-c-d)^3/(c+d)/(c^3-3*c^2*d+3*c*d^2-d^3)*tan(1/2*f*x+1/2*e)*a*b^2*d
^2+1/f*c^3/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/
2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))^(1/2))*b^3+8/f/(c^6-3*c^4*d^2+3*c^2*d^4-d^
6)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))^(1/2)
)*a^3*d^3-12/f*c/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^3/(c-d
)/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^5*a^3*d^2+6/f/c/(tan(1/2*f*x
+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^3/(c-d)/(c^3+3*c^2*d+3*c*d^2+d^3)*t
an(1/2*f*x+1/2*e)^5*a^3*d^4+1/f/c^2/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2
*e)^2*d-c-d)^3/(c-d)/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^5*a^3*d^5
-2/f/c^3/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^3/(c-d)/(c^3+3
*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^5*a^3*d^6-6/f*c^3/(tan(1/2*f*x+1/2*e
)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^3/(c-d)/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2
*f*x+1/2*e)^5*a*b^2+6/f*c^2/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-
c-d)^3/(c-d)/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^5*b^3*d+2/f*c/(ta
n(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^3/(c-d)/(c^3+3*c^2*d+3*c*d
^2+d^3)*tan(1/2*f*x+1/2*e)^5*b^3*d^2+6/f/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*
x+1/2*e)^2*d-c-d)^3/(c-d)/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^5*a^
2*b*d^3-3/f/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^3/(c-d)/(c^
3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^5*a*b^2*d^3-4/f/(tan(1/2*f*x+1/2*
e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^3/(c^2-2*c*d+d^2)/(c^2+2*c*d+d^2)*tan(1/
2*f*x+1/2*e)^3*a^2*b*d^3+6/f/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d
-c-d)^3/(c+d)/(c^3-3*c^2*d+3*c*d^2-d^3)*tan(1/2*f*x+1/2*e)*a^2*b*d^3+3/f/(t
an(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^3/(c+d)/(c^3-3*c^2*d+3*c*
d^2-d^3)*tan(1/2*f*x+1/2*e)*a*b^2*d^3+24/f*c/(tan(1/2*f*x+1/2*e)^2*c-tan(1/
2*f*x+1/2*e)^2*d-c-d)^3/(c^2-2*c*d+d^2)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^
3*a^3*d^2-44/3/f/c/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^3/(c
^2-2*c*d+d^2)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3*a^3*d^4+4/f/c^3/(tan(1/2
*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^3/(c^2-2*c*d+d^2)/(c^2+2*c*d+d^
2)*tan(1/2*f*x+1/2*e)^3*a^3*d^6+2/f/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2
*e)^2*d-c-d)^3/(c+d)/(c^3-3*c^2*d+3*c*d^2-d^3)*tan(1/2*f*x+1/2*e)*b^3*d^3+4
/f/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^3/(c+d)/(c^3-3*c^2*d
+3*c*d^2-d^3)*tan(1/2*f*x+1/2*e)*a^3*d^3-8/f*c^2/(c^6-3*c^4*d^2+3*c^2*d^4-d

```
^6)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))^(1/2
))*a^3*d+2/f/c^4/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)/((c+d)*(c-d))^(1/2)*arctanh(
tan(1/2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))^(1/2))*a^3*d^7+4/f*c/(c^6-3*c^4*d^2+
3*c^2*d^4-d^6)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*f*x+1/2*e)*(c-d)/((c+d)*
(c-d))^(1/2))*b^3*d^2+1/f*c^3/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*
d-c-d)^3/(c-d)/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^5*b^3-1/f*c^3/(
tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^3/(c+d)/(c^3-3*c^2*d+3*c
*d^2-d^3)*tan(1/2*f*x+1/2*e)*b^3-7/f/c^2/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)/((c+
d)*(c-d))^(1/2)*arctanh(tan(1/2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))^(1/2))*a^3*d
^5+6/f*c^3/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/
2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))^(1/2))*a^2*b-3/f/(c^6-3*c^4*d^2+3*c^2*d^4-
d^6)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))^(1/
2))*a*b^2*d^3-4/f/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^3/(c-
d)/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^5*a^3*d^3+2/f/(tan(1/2*f*x+
1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^3/(c-d)/(c^3+3*c^2*d+3*c*d^2+d^3)*ta
n(1/2*f*x+1/2*e)^5*b^3*d^3-4/f/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2
*d-c-d)^3/(c^2-2*c*d+d^2)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3*b^3*d^3-6/f*
c^2/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^3/(c-d)/(c^3+3*c^2*
d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^5*a*b^2*d+9/f*c/(tan(1/2*f*x+1/2*e)^2*c-t
an(1/2*f*x+1/2*e)^2*d-c-d)^3/(c-d)/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/
2*e)^5*a^2*b*d^2+18/f*c^2/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-
d)^3/(c-d)/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^5*a^2*b*d-18/f*c/(t
an(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^3/(c-d)/(c^3+3*c^2*d+3*c*
d^2+d^3)*tan(1/2*f*x+1/2*e)^5*a*b^2*d^2-36/f*c^2/(tan(1/2*f*x+1/2*e)^2*c-ta
n(1/2*f*x+1/2*e)^2*d-c-d)^3/(c^2-2*c*d+d^2)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2
*e)^3*a^2*b*d+28/f*c/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^3/
(c^2-2*c*d+d^2)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3*a*b^2*d^2+18/f*c^2/(ta
n(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^3/(c+d)/(c^3-3*c^2*d+3*c*d
^2-d^3)*tan(1/2*f*x+1/2*e)*a^2*b*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.27304, size = 5782, normalized size = 14.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] [1/12*(12*(a^3*c^11 - 4*a^3*c^9*d^2 + 6*a^3*c^7*d^4 - 4*a^3*c^5*d^6 + a^3*c
^3*d^8)*f*x*cos(f*x + e)^3 + 36*(a^3*c^10*d - 4*a^3*c^8*d^3 + 6*a^3*c^6*d^5
 - 4*a^3*c^4*d^7 + a^3*c^2*d^9)*f*x*cos(f*x + e)^2 + 36*(a^3*c^9*d^2 - 4*a^
3*c^7*d^4 + 6*a^3*c^5*d^6 - 4*a^3*c^3*d^8 + a^3*c*d^10)*f*x*cos(f*x + e) +
12*(a^3*c^8*d^3 - 4*a^3*c^6*d^5 + 6*a^3*c^4*d^7 - 4*a^3*c^2*d^9 + a^3*d^11)
*f*x + 3*(7*a^3*c^2*d^8 - 2*a^3*d^10 - (6*a^2*b + b^3)*c^7*d^3 + 4*(2*a^3 +
 3*a*b^2)*c^6*d^4 - (9*a^2*b + 4*b^3)*c^5*d^5 - (8*a^3 - 3*a*b^2)*c^4*d^6 +
 (7*a^3*c^5*d^5 - 2*a^3*c^3*d^7 - (6*a^2*b + b^3)*c^10 + 4*(2*a^3 + 3*a*b^2
)*c^9*d - (9*a^2*b + 4*b^3)*c^8*d^2 - (8*a^3 - 3*a*b^2)*c^7*d^3)*cos(f*x +
e)^3 + 3*(7*a^3*c^4*d^6 - 2*a^3*c^2*d^8 - (6*a^2*b + b^3)*c^9*d + 4*(2*a^3
+ 3*a*b^2)*c^8*d^2 - (9*a^2*b + 4*b^3)*c^7*d^3 - (8*a^3 - 3*a*b^2)*c^6*d^4)
*cos(f*x + e)^2 + 3*(7*a^3*c^3*d^7 - 2*a^3*c*d^9 - (6*a^2*b + b^3)*c^8*d^2
+ 4*(2*a^3 + 3*a*b^2)*c^7*d^3 - (9*a^2*b + 4*b^3)*c^6*d^4 - (8*a^3 - 3*a*b^
2)*c^5*d^5)*cos(f*x + e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 -
2*d^2)*cos(f*x + e)^2 - 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e)
 + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(b^3*c
^10*d + 6*a*b^2*c^9*d^2 + 23*a^3*c^3*d^8 - 6*a^3*c*d^10 - 11*(3*a^2*b + b^3
)*c^8*d^3 + (26*a^3 + 33*a*b^2)*c^7*d^4 + (21*a^2*b + 4*b^3)*c^6*d^5 - (43*
a^3 + 39*a*b^2)*c^5*d^6 + 6*(2*a^2*b + b^3)*c^4*d^7 + (18*a*b^2*c^11 + 6*a^
2*b*c^4*d^7 - 11*a^3*c^3*d^8 - (54*a^2*b + 13*b^3)*c^10*d + 12*(3*a^3 + a*b
^2)*c^9*d^2 + (69*a^2*b + 11*b^3)*c^8*d^3 - (68*a^3 + 33*a*b^2)*c^7*d^4 - (
21*a^2*b - 2*b^3)*c^6*d^5 + (43*a^3 + 3*a*b^2)*c^5*d^6)*cos(f*x + e)^2 + 3*
(b^3*c^11 + 6*a*b^2*c^10*d - 5*a^3*c^2*d^9 - (27*a^2*b + 10*b^3)*c^9*d^2 +
(20*a^3 + 21*a*b^2)*c^8*d^3 + (24*a^2*b + 7*b^3)*c^7*d^4 - 5*(7*a^3 + 6*a*b
^2)*c^6*d^5 + (3*a^2*b + 2*b^3)*c^5*d^6 + (20*a^3 + 3*a*b^2)*c^4*d^7)*cos(f
*x + e))*sin(f*x + e))/((c^15 - 4*c^13*d^2 + 6*c^11*d^4 - 4*c^9*d^6 + c^7*d
^8)*f*cos(f*x + e)^3 + 3*(c^14*d - 4*c^12*d^3 + 6*c^10*d^5 - 4*c^8*d^7 + c^
6*d^9)*f*cos(f*x + e)^2 + 3*(c^13*d^2 - 4*c^11*d^4 + 6*c^9*d^6 - 4*c^7*d^8
+ c^5*d^10)*f*cos(f*x + e) + (c^12*d^3 - 4*c^10*d^5 + 6*c^8*d^7 - 4*c^6*d^9
 + c^4*d^11)*f), 1/6*(6*(a^3*c^11 - 4*a^3*c^9*d^2 + 6*a^3*c^7*d^4 - 4*a^3*c
^5*d^6 + a^3*c^3*d^8)*f*x*cos(f*x + e)^3 + 18*(a^3*c^10*d - 4*a^3*c^8*d^3 +
 6*a^3*c^6*d^5 - 4*a^3*c^4*d^7 + a^3*c^2*d^9)*f*x*cos(f*x + e)^2 + 18*(a^3*
c^9*d^2 - 4*a^3*c^7*d^4 + 6*a^3*c^5*d^6 - 4*a^3*c^3*d^8 + a^3*c*d^10)*f*x*c
os(f*x + e) + 6*(a^3*c^8*d^3 - 4*a^3*c^6*d^5 + 6*a^3*c^4*d^7 - 4*a^3*c^2*d^
9 + a^3*d^11)*f*x - 3*(7*a^3*c^2*d^8 - 2*a^3*d^10 - (6*a^2*b + b^3)*c^7*d^3
 + 4*(2*a^3 + 3*a*b^2)*c^6*d^4 - (9*a^2*b + 4*b^3)*c^5*d^5 - (8*a^3 - 3*a*b
```

```
^2)*c^4*d^6 + (7*a^3*c^5*d^5 - 2*a^3*c^3*d^7 - (6*a^2*b + b^3)*c^10 + 4*(2*
a^3 + 3*a*b^2)*c^9*d - (9*a^2*b + 4*b^3)*c^8*d^2 - (8*a^3 - 3*a*b^2)*c^7*d^
3)*cos(f*x + e)^3 + 3*(7*a^3*c^4*d^6 - 2*a^3*c^2*d^8 - (6*a^2*b + b^3)*c^9*
d + 4*(2*a^3 + 3*a*b^2)*c^8*d^2 - (9*a^2*b + 4*b^3)*c^7*d^3 - (8*a^3 - 3*a*
b^2)*c^6*d^4)*cos(f*x + e)^2 + 3*(7*a^3*c^3*d^7 - 2*a^3*c*d^9 - (6*a^2*b +
b^3)*c^8*d^2 + 4*(2*a^3 + 3*a*b^2)*c^7*d^3 - (9*a^2*b + 4*b^3)*c^6*d^4 - (8
*a^3 - 3*a*b^2)*c^5*d^5)*cos(f*x + e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 +
 d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) + (b^3*c^10*d + 6*a*
b^2*c^9*d^2 + 23*a^3*c^3*d^8 - 6*a^3*c*d^10 - 11*(3*a^2*b + b^3)*c^8*d^3 +
(26*a^3 + 33*a*b^2)*c^7*d^4 + (21*a^2*b + 4*b^3)*c^6*d^5 - (43*a^3 + 39*a*b
^2)*c^5*d^6 + 6*(2*a^2*b + b^3)*c^4*d^7 + (18*a*b^2*c^11 + 6*a^2*b*c^4*d^7
- 11*a^3*c^3*d^8 - (54*a^2*b + 13*b^3)*c^10*d + 12*(3*a^3 + a*b^2)*c^9*d^2
+ (69*a^2*b + 11*b^3)*c^8*d^3 - (68*a^3 + 33*a*b^2)*c^7*d^4 - (21*a^2*b - 2
*b^3)*c^6*d^5 + (43*a^3 + 3*a*b^2)*c^5*d^6)*cos(f*x + e)^2 + 3*(b^3*c^11 +
6*a*b^2*c^10*d - 5*a^3*c^2*d^9 - (27*a^2*b + 10*b^3)*c^9*d^2 + (20*a^3 + 21
*a*b^2)*c^8*d^3 + (24*a^2*b + 7*b^3)*c^7*d^4 - 5*(7*a^3 + 6*a*b^2)*c^6*d^5
+ (3*a^2*b + 2*b^3)*c^5*d^6 + (20*a^3 + 3*a*b^2)*c^4*d^7)*cos(f*x + e))*sin
(f*x + e))/((c^15 - 4*c^13*d^2 + 6*c^11*d^4 - 4*c^9*d^6 + c^7*d^8)*f*cos(f*
x + e)^3 + 3*(c^14*d - 4*c^12*d^3 + 6*c^10*d^5 - 4*c^8*d^7 + c^6*d^9)*f*cos
(f*x + e)^2 + 3*(c^13*d^2 - 4*c^11*d^4 + 6*c^9*d^6 - 4*c^7*d^8 + c^5*d^10)*
f*cos(f*x + e) + (c^12*d^3 - 4*c^10*d^5 + 6*c^8*d^7 - 4*c^6*d^9 + c^4*d^11)
*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+b\sec\left(e+fx\right)\right)^3}{\left(c+d\sec\left(e+fx\right)\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**3/(c+d*sec(f*x+e))**4,x)
```

```
[Out] Integral((a + b*sec(e + f*x))**3/(c + d*sec(e + f*x))**4, x)
```

Giac [B] time = 1.7899, size = 2213, normalized size = 5.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(6*a^2*b*c^7 + b^3*c^7 - 8*a^3*c^6*d - 12*a*b^2*c^6*d + 9*a^2*b*c^5*
d^2 + 4*b^3*c^5*d^2 + 8*a^3*c^4*d^3 - 3*a*b^2*c^4*d^3 - 7*a^3*c^2*d^5 + 2*a
^3*d^7)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(
1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))/((c^10 - 3*c^
8*d^2 + 3*c^6*d^4 - c^4*d^6)*sqrt(-c^2 + d^2)) + 3*(f*x + e)*a^3/c^4 - (18*
a*b^2*c^8*tan(1/2*f*x + 1/2*e)^5 - 3*b^3*c^8*tan(1/2*f*x + 1/2*e)^5 - 54*a^
2*b*c^7*d*tan(1/2*f*x + 1/2*e)^5 - 18*a*b^2*c^7*d*tan(1/2*f*x + 1/2*e)^5 -
12*b^3*c^7*d*tan(1/2*f*x + 1/2*e)^5 + 36*a^3*c^6*d^2*tan(1/2*f*x + 1/2*e)^5
 + 81*a^2*b*c^6*d^2*tan(1/2*f*x + 1/2*e)^5 + 36*a*b^2*c^6*d^2*tan(1/2*f*x +
 1/2*e)^5 + 27*b^3*c^6*d^2*tan(1/2*f*x + 1/2*e)^5 - 60*a^3*c^5*d^3*tan(1/2*
f*x + 1/2*e)^5 - 18*a^2*b*c^5*d^3*tan(1/2*f*x + 1/2*e)^5 - 81*a*b^2*c^5*d^3
*tan(1/2*f*x + 1/2*e)^5 - 12*b^3*c^5*d^3*tan(1/2*f*x + 1/2*e)^5 - 6*a^3*c^4
*d^4*tan(1/2*f*x + 1/2*e)^5 + 9*a^2*b*c^4*d^4*tan(1/2*f*x + 1/2*e)^5 + 36*a
*b^2*c^4*d^4*tan(1/2*f*x + 1/2*e)^5 + 6*b^3*c^4*d^4*tan(1/2*f*x + 1/2*e)^5
+ 45*a^3*c^3*d^5*tan(1/2*f*x + 1/2*e)^5 - 18*a^2*b*c^3*d^5*tan(1/2*f*x + 1/
2*e)^5 + 9*a*b^2*c^3*d^5*tan(1/2*f*x + 1/2*e)^5 - 6*b^3*c^3*d^5*tan(1/2*f*x
 + 1/2*e)^5 - 6*a^3*c^2*d^6*tan(1/2*f*x + 1/2*e)^5 - 15*a^3*c*d^7*tan(1/2*f
*x + 1/2*e)^5 + 6*a^3*d^8*tan(1/2*f*x + 1/2*e)^5 - 36*a*b^2*c^8*tan(1/2*f*x
 + 1/2*e)^3 + 108*a^2*b*c^7*d*tan(1/2*f*x + 1/2*e)^3 + 28*b^3*c^7*d*tan(1/2
*f*x + 1/2*e)^3 - 72*a^3*c^6*d^2*tan(1/2*f*x + 1/2*e)^3 - 48*a*b^2*c^6*d^2*
tan(1/2*f*x + 1/2*e)^3 - 96*a^2*b*c^5*d^3*tan(1/2*f*x + 1/2*e)^3 - 16*b^3*c
^5*d^3*tan(1/2*f*x + 1/2*e)^3 + 116*a^3*c^4*d^4*tan(1/2*f*x + 1/2*e)^3 + 84
*a*b^2*c^4*d^4*tan(1/2*f*x + 1/2*e)^3 - 12*a^2*b*c^3*d^5*tan(1/2*f*x + 1/2*
e)^3 - 12*b^3*c^3*d^5*tan(1/2*f*x + 1/2*e)^3 - 56*a^3*c^2*d^6*tan(1/2*f*x +
 1/2*e)^3 + 12*a^3*d^8*tan(1/2*f*x + 1/2*e)^3 + 18*a*b^2*c^8*tan(1/2*f*x +
1/2*e) + 3*b^3*c^8*tan(1/2*f*x + 1/2*e) - 54*a^2*b*c^7*d*tan(1/2*f*x + 1/2*
e) + 18*a*b^2*c^7*d*tan(1/2*f*x + 1/2*e) - 12*b^3*c^7*d*tan(1/2*f*x + 1/2*e
) + 36*a^3*c^6*d^2*tan(1/2*f*x + 1/2*e) - 81*a^2*b*c^6*d^2*tan(1/2*f*x + 1/
2*e) + 36*a*b^2*c^6*d^2*tan(1/2*f*x + 1/2*e) - 27*b^3*c^6*d^2*tan(1/2*f*x +
 1/2*e) + 60*a^3*c^5*d^3*tan(1/2*f*x + 1/2*e) - 18*a^2*b*c^5*d^3*tan(1/2*f*
x + 1/2*e) + 81*a*b^2*c^5*d^3*tan(1/2*f*x + 1/2*e) - 12*b^3*c^5*d^3*tan(1/2
*f*x + 1/2*e) - 6*a^3*c^4*d^4*tan(1/2*f*x + 1/2*e) - 9*a^2*b*c^4*d^4*tan(1/
2*f*x + 1/2*e) + 36*a*b^2*c^4*d^4*tan(1/2*f*x + 1/2*e) - 6*b^3*c^4*d^4*tan(
1/2*f*x + 1/2*e) - 45*a^3*c^3*d^5*tan(1/2*f*x + 1/2*e) - 18*a^2*b*c^3*d^5*t
an(1/2*f*x + 1/2*e) - 9*a*b^2*c^3*d^5*tan(1/2*f*x + 1/2*e) - 6*b^3*c^3*d^5*
tan(1/2*f*x + 1/2*e) - 6*a^3*c^2*d^6*tan(1/2*f*x + 1/2*e) + 15*a^3*c*d^7*ta
n(1/2*f*x + 1/2*e) + 6*a^3*d^8*tan(1/2*f*x + 1/2*e))/((c^9 - 3*c^7*d^2 + 3*
c^5*d^4 - c^3*d^6)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c
 - d)^3))/f
```

3.197 $\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx$

Optimal. Leaf size=622

$$-\frac{\left(a^2bcd\left(10c^4d^2+49c^2d^4+272c^6-16d^6\right)+a^3\left(-\left(139c^2d^6-210c^4d^4+212c^6d^2-36d^8\right)\right)-3ab^2c^2\left(84c^4d^2-5c^2d^4+24\right.\right.}{24c^4f\left(c^2-d^2\right)^4(c\cos(e+fx)+d)}$$

```
[Out] (a^3*x)/c^5 - ((15*a*b^2*c^6*d*(4*c^2 + 3*d^2) - 3*a^2*b*c^5*(8*c^4 + 24*c^
2*d^2 + 3*d^4) - b^3*c^5*(4*c^4 + 27*c^2*d^2 + 4*d^4) + a^3*(40*c^8*d - 40*
c^6*d^3 + 63*c^4*d^5 - 36*c^2*d^7 + 8*d^9))*ArcTanh[(Sqrt[c - d]*Tan[(e + f
*x)/2])/Sqrt[c + d]])/(4*c^5*Sqrt[c - d]*Sqrt[c + d]*(c^2 - d^2)^4*f) + (d^
2*(b + a*Cos[e + f*x])^3*Sin[e + f*x])/(4*c*(c^2 - d^2)*f*(d + c*Cos[e + f*
x])^4) - (d*(8*b*c^3 - 11*a*c^2*d - b*c*d^2 + 4*a*d^3)*(b + a*Cos[e + f*x])
^2*Sin[e + f*x])/(12*c^2*(c^2 - d^2)^2*f*(d + c*Cos[e + f*x])^3) - ((b*c -
a*d)*(2*a*b*c*d*(32*c^4 + c^2*d^2 + 2*d^4) - a^2*d^2*(58*c^4 - 35*c^2*d^2 +
 12*d^4) - b^2*(12*c^6 + 25*c^4*d^2 - 2*c^2*d^4))*Sin[e + f*x])/(24*c^4*(c^
2 - d^2)^3*f*(d + c*Cos[e + f*x])^2) - ((b^3*c^3*d*(68*c^4 + 39*c^2*d^2 - 2
*d^4) + a^2*b*c*d*(272*c^6 + 10*c^4*d^2 + 49*c^2*d^4 - 16*d^6) - 3*a*b^2*c^
2*(24*c^6 + 84*c^4*d^2 - 5*c^2*d^4 + 2*d^6) - a^3*(212*c^6*d^2 - 210*c^4*d^
4 + 139*c^2*d^6 - 36*d^8))*Sin[e + f*x])/(24*c^4*(c^2 - d^2)^4*f*(d + c*Cos
[e + f*x]))
```

Rubi [A] time = 1.7689, antiderivative size = 622, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.32$, Rules used = {3941, 3048, 3047, 3031, 3021, 2735, 2659, 208}

$$-\frac{\left(a^2bcd\left(10c^4d^2+49c^2d^4+272c^6-16d^6\right)+a^3\left(-\left(139c^2d^6-210c^4d^4+212c^6d^2-36d^8\right)\right)-3ab^2c^2\left(84c^4d^2-5c^2d^4+24\right.\right.}{24c^4f\left(c^2-d^2\right)^4(c\cos(e+fx)+d)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x])^3/(c + d*Sec[e + f*x])^5,x]
```

```
[Out] (a^3*x)/c^5 - ((15*a*b^2*c^6*d*(4*c^2 + 3*d^2) - 3*a^2*b*c^5*(8*c^4 + 24*c^
2*d^2 + 3*d^4) - b^3*c^5*(4*c^4 + 27*c^2*d^2 + 4*d^4) + a^3*(40*c^8*d - 40*
c^6*d^3 + 63*c^4*d^5 - 36*c^2*d^7 + 8*d^9))*ArcTanh[(Sqrt[c - d]*Tan[(e + f
*x)/2])/Sqrt[c + d]])/(4*c^5*Sqrt[c - d]*Sqrt[c + d]*(c^2 - d^2)^4*f) + (d^
```

```
2*(b + a*Cos[e + f*x])^3*Sin[e + f*x])/(4*c*(c^2 - d^2)*f*(d + c*Cos[e + f*
x])^4) - (d*(8*b*c^3 - 11*a*c^2*d - b*c*d^2 + 4*a*d^3)*(b + a*Cos[e + f*x])
^2*Sin[e + f*x])/(12*c^2*(c^2 - d^2)^2*f*(d + c*Cos[e + f*x])^3) - ((b*c -
a*d)*(2*a*b*c*d*(32*c^4 + c^2*d^2 + 2*d^4) - a^2*d^2*(58*c^4 - 35*c^2*d^2 +
 12*d^4) - b^2*(12*c^6 + 25*c^4*d^2 - 2*c^2*d^4))*Sin[e + f*x])/(24*c^4*(c^
2 - d^2)^3*f*(d + c*Cos[e + f*x])^2) - ((b^3*c^3*d*(68*c^4 + 39*c^2*d^2 - 2
*d^4) + a^2*b*c*d*(272*c^6 + 10*c^4*d^2 + 49*c^2*d^4 - 16*d^6) - 3*a*b^2*c^
2*(24*c^6 + 84*c^4*d^2 - 5*c^2*d^4 + 2*d^6) - a^3*(212*c^6*d^2 - 210*c^4*d^
4 + 139*c^2*d^6 - 36*d^8))*Sin[e + f*x])/(24*c^4*(c^2 - d^2)^4*f*(d + c*Cos
[e + f*x]))
```

Rule 3941

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Int[((b + a*Sin[e + f*x])^m*(d + c*Sin[e + f
*x])^n)/Sin[e + f*x]^(m + n), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && N
eQ[b*c - a*d, 0] && IntegerQ[m] && IntegerQ[n] && LeQ[-2, m + n, 0]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx = \int \frac{\cos^2(e+fx)(b+a\cos(e+fx))^3}{(d+c\cos(e+fx))^5}\,dx$$

$$= \frac{d^2(b+a\cos(e+fx))^3\sin(e+fx)}{4c\left(c^2-d^2\right)f(d+c\cos(e+fx))^4} + \frac{\int \frac{(b+a\cos(e+fx))^2\left(-d(4bc-3ad)+\left(4bc^2-4acd-bd^2\right)\cos(e+fx)+4a\left(c^2\right.\right.}{(d+c\cos(e+fx))^4}}{4c\left(c^2-d^2\right)}$$

$$= \frac{d^2(b+a\cos(e+fx))^3\sin(e+fx)}{4c\left(c^2-d^2\right)f(d+c\cos(e+fx))^4} - \frac{d\left(8bc^3-11ac^2d-bcd^2+4ad^3\right)(b+a\cos(e+fx))^2 \mathrm{s}}{12c^2\left(c^2-d^2\right)^2 f(d+c\cos(e+fx))^3}$$

$$= \frac{d^2(b+a\cos(e+fx))^3\sin(e+fx)}{4c\left(c^2-d^2\right)f(d+c\cos(e+fx))^4} - \frac{d\left(8bc^3-11ac^2d-bcd^2+4ad^3\right)(b+a\cos(e+fx))^2 \mathrm{s}}{12c^2\left(c^2-d^2\right)^2 f(d+c\cos(e+fx))^3}$$

$$= \frac{d^2(b+a\cos(e+fx))^3\sin(e+fx)}{4c\left(c^2-d^2\right)f(d+c\cos(e+fx))^4} - \frac{d\left(8bc^3-11ac^2d-bcd^2+4ad^3\right)(b+a\cos(e+fx))^2 \mathrm{s}}{12c^2\left(c^2-d^2\right)^2 f(d+c\cos(e+fx))^3}$$

$$= \frac{a^3x}{c^5} + \frac{d^2(b+a\cos(e+fx))^3\sin(e+fx)}{4c\left(c^2-d^2\right)f(d+c\cos(e+fx))^4} - \frac{d\left(8bc^3-11ac^2d-bcd^2+4ad^3\right)(b+a\cos(e+}{12c^2\left(c^2-d^2\right)^2 f(d+c\cos(e+f}$$

$$= \frac{a^3x}{c^5} + \frac{d^2(b+a\cos(e+fx))^3\sin(e+fx)}{4c\left(c^2-d^2\right)f(d+c\cos(e+fx))^4} - \frac{d\left(8bc^3-11ac^2d-bcd^2+4ad^3\right)(b+a\cos(e+}{12c^2\left(c^2-d^2\right)^2 f(d+c\cos(e+f}$$

$$= \frac{a^3x}{c^5} - \frac{\left(15ab^2c^6d\left(4c^2+3d^2\right)-3a^2bc^5\left(8c^4+24c^2d^2+3d^4\right)-b^3c^5\left(4c^4+27c^2d^2+4d^4\right)+\right.}{4c^5\sqrt{c-d}\sqrt{c+d}\left(c^2\right.}$$

Mathematica [B] time = 6.84163, size = 1285, normalized size = 2.07

$$\frac{a^3(e+fx)\sec^2(e+fx)(a+b\sec(e+fx))^3(d+c\cos(e+fx))^5}{c^5f(b+a\cos(e+fx))^3(c+d\sec(e+fx))^5} + \frac{\left(-4b^3c^9-24a^2bc^9+40a^3dc^8+60ab^2dc^8-27b^3d^2c^7\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x])^3/(c + d*Sec[e + f*x])^5,x]
```

```
[Out] (a^3*(e + f*x)*(d + c*Cos[e + f*x])^5*Sec[e + f*x]^2*(a + b*Sec[e + f*x])^3
)/(c^5*f*(b + a*Cos[e + f*x])^3*(c + d*Sec[e + f*x])^5) + ((-24*a^2*b*c^9 -
```

```
 4*b^3*c^9 + 40*a^3*c^8*d + 60*a*b^2*c^8*d - 72*a^2*b*c^7*d^2 - 27*b^3*c^7*
d^2 - 40*a^3*c^6*d^3 + 45*a*b^2*c^6*d^3 - 9*a^2*b*c^5*d^4 - 4*b^3*c^5*d^4 +
 63*a^3*c^4*d^5 - 36*a^3*c^2*d^7 + 8*a^3*d^9)*ArcTanh[((-c + d)*Tan[(e + f*
x)/2])/Sqrt[c^2 - d^2]]*(d + c*Cos[e + f*x])^5*Sec[e + f*x]^2*(a + b*Sec[e
+ f*x])^3)/(4*c^5*Sqrt[c^2 - d^2]*(-c^2 + d^2)^4*f*(b + a*Cos[e + f*x])^3*(
c + d*Sec[e + f*x])^5) + ((d + c*Cos[e + f*x])*Sec[e + f*x]^2*(a + b*Sec[e
+ f*x])^3*(b^3*c^3*d^2*Sin[e + f*x] - 3*a*b^2*c^2*d^3*Sin[e + f*x] + 3*a^2*
b*c*d^4*Sin[e + f*x] - a^3*d^5*Sin[e + f*x]))/(4*c^4*(c^2 - d^2)*f*(b + a*C
os[e + f*x])^3*(c + d*Sec[e + f*x])^5) + ((d + c*Cos[e + f*x])^2*Sec[e + f*
x]^2*(a + b*Sec[e + f*x])^3*(-8*b^3*c^5*d*Sin[e + f*x] + 36*a*b^2*c^4*d^2*S
in[e + f*x] - 48*a^2*b*c^3*d^3*Sin[e + f*x] + b^3*c^3*d^3*Sin[e + f*x] + 20
*a^3*c^2*d^4*Sin[e + f*x] - 15*a*b^2*c^2*d^4*Sin[e + f*x] + 27*a^2*b*c*d^5*
Sin[e + f*x] - 13*a^3*d^6*Sin[e + f*x]))/(12*c^4*(c^2 - d^2)^2*f*(b + a*Cos
[e + f*x])^3*(c + d*Sec[e + f*x])^5) + ((d + c*Cos[e + f*x])^3*Sec[e + f*x]
^2*(a + b*Sec[e + f*x])^3*(12*b^3*c^7*Sin[e + f*x] - 108*a*b^2*c^6*d*Sin[e
+ f*x] + 216*a^2*b*c^5*d^2*Sin[e + f*x] + 25*b^3*c^5*d^2*Sin[e + f*x] - 120
*a^3*c^4*d^3*Sin[e + f*x] + 9*a*b^2*c^4*d^3*Sin[e + f*x] - 165*a^2*b*c^3*d^
4*Sin[e + f*x] - 2*b^3*c^3*d^4*Sin[e + f*x] + 131*a^3*c^2*d^5*Sin[e + f*x]
- 6*a*b^2*c^2*d^5*Sin[e + f*x] + 54*a^2*b*c*d^6*Sin[e + f*x] - 46*a^3*d^7*S
in[e + f*x]))/(24*c^4*(c^2 - d^2)^3*f*(b + a*Cos[e + f*x])^3*(c + d*Sec[e +
 f*x])^5) + ((d + c*Cos[e + f*x])^4*Sec[e + f*x]^2*(a + b*Sec[e + f*x])^3*(
72*a*b^2*c^8*Sin[e + f*x] - 288*a^2*b*c^7*d*Sin[e + f*x] - 68*b^3*c^7*d*Sin
[e + f*x] + 240*a^3*c^6*d^2*Sin[e + f*x] + 252*a*b^2*c^6*d^2*Sin[e + f*x] +
 24*a^2*b*c^5*d^3*Sin[e + f*x] - 39*b^3*c^5*d^3*Sin[e + f*x] - 280*a^3*c^4*
d^4*Sin[e + f*x] - 15*a*b^2*c^4*d^4*Sin[e + f*x] - 69*a^2*b*c^3*d^5*Sin[e +
 f*x] + 2*b^3*c^3*d^5*Sin[e + f*x] + 195*a^3*c^2*d^6*Sin[e + f*x] + 6*a*b^2
*c^2*d^6*Sin[e + f*x] + 18*a^2*b*c*d^7*Sin[e + f*x] - 50*a^3*d^8*Sin[e + f*
x]))/(24*c^4*(c^2 - d^2)^4*f*(b + a*Cos[e + f*x])^3*(c + d*Sec[e + f*x])^5)
```

Maple [B] time = 0.159, size = 8573, normalized size = 13.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^5,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.99851, size = 9385, normalized size = 15.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^5,x, algorithm="fricas")
```

```
[Out] [1/48*(48*(a^3*c^14 - 5*a^3*c^12*d^2 + 10*a^3*c^10*d^4 - 10*a^3*c^8*d^6 + 5
*a^3*c^6*d^8 - a^3*c^4*d^10)*f*x*cos(f*x + e)^4 + 192*(a^3*c^13*d - 5*a^3*c
^11*d^3 + 10*a^3*c^9*d^5 - 10*a^3*c^7*d^7 + 5*a^3*c^5*d^9 - a^3*c^3*d^11)*f
*x*cos(f*x + e)^3 + 288*(a^3*c^12*d^2 - 5*a^3*c^10*d^4 + 10*a^3*c^8*d^6 - 1
0*a^3*c^6*d^8 + 5*a^3*c^4*d^10 - a^3*c^2*d^12)*f*x*cos(f*x + e)^2 + 192*(a^
3*c^11*d^3 - 5*a^3*c^9*d^5 + 10*a^3*c^7*d^7 - 10*a^3*c^5*d^9 + 5*a^3*c^3*d^
11 - a^3*c*d^13)*f*x*cos(f*x + e) + 48*(a^3*c^10*d^4 - 5*a^3*c^8*d^6 + 10*a
^3*c^6*d^8 - 10*a^3*c^4*d^10 + 5*a^3*c^2*d^12 - a^3*d^14)*f*x + 3*(63*a^3*c
^4*d^9 - 36*a^3*c^2*d^11 + 8*a^3*d^13 - 4*(6*a^2*b + b^3)*c^9*d^4 + 20*(2*a
^3 + 3*a*b^2)*c^8*d^5 - 9*(8*a^2*b + 3*b^3)*c^7*d^6 - 5*(8*a^3 - 9*a*b^2)*c
^6*d^7 - (9*a^2*b + 4*b^3)*c^5*d^8 + (63*a^3*c^8*d^5 - 36*a^3*c^6*d^7 + 8*a
^3*c^4*d^9 - 4*(6*a^2*b + b^3)*c^13 + 20*(2*a^3 + 3*a*b^2)*c^12*d - 9*(8*a^
2*b + 3*b^3)*c^11*d^2 - 5*(8*a^3 - 9*a*b^2)*c^10*d^3 - (9*a^2*b + 4*b^3)*c^
9*d^4)*cos(f*x + e)^4 + 4*(63*a^3*c^7*d^6 - 36*a^3*c^5*d^8 + 8*a^3*c^3*d^10
 - 4*(6*a^2*b + b^3)*c^12*d + 20*(2*a^3 + 3*a*b^2)*c^11*d^2 - 9*(8*a^2*b +
3*b^3)*c^10*d^3 - 5*(8*a^3 - 9*a*b^2)*c^9*d^4 - (9*a^2*b + 4*b^3)*c^8*d^5)*
cos(f*x + e)^3 + 6*(63*a^3*c^6*d^7 - 36*a^3*c^4*d^9 + 8*a^3*c^2*d^11 - 4*(6
*a^2*b + b^3)*c^11*d^2 + 20*(2*a^3 + 3*a*b^2)*c^10*d^3 - 9*(8*a^2*b + 3*b^3
)*c^9*d^4 - 5*(8*a^3 - 9*a*b^2)*c^8*d^5 - (9*a^2*b + 4*b^3)*c^7*d^6)*cos(f*
x + e)^2 + 4*(63*a^3*c^5*d^8 - 36*a^3*c^3*d^10 + 8*a^3*c*d^12 - 4*(6*a^2*b
+ b^3)*c^10*d^3 + 20*(2*a^3 + 3*a*b^2)*c^9*d^4 - 9*(8*a^2*b + 3*b^3)*c^8*d^
5 - 5*(8*a^3 - 9*a*b^2)*c^7*d^6 - (9*a^2*b + 4*b^3)*c^6*d^7)*cos(f*x + e))*
sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*
sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f
```

*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(2*b^3*c^12*d^2 + 18*a*b^2*c^11*
d^3 - 116*a^3*c^3*d^11 + 24*a^3*c*d^13 - (150*a^2*b + 41*b^3)*c^10*d^4 + 77
*(2*a^3 + 3*a*b^2)*c^9*d^5 - (15*a^2*b + 29*b^3)*c^8*d^6 - (271*a^3 + 201*a
*b^2)*c^7*d^7 + (165*a^2*b + 68*b^3)*c^6*d^8 + (209*a^3 - 48*a*b^2)*c^5*d^9
 + (72*a*b^2*c^14 - 18*a^2*b*c^5*d^9 + 50*a^3*c^4*d^10 - 4*(72*a^2*b + 17*b
^3)*c^13*d + 60*(4*a^3 + 3*a*b^2)*c^12*d^2 + (312*a^2*b + 29*b^3)*c^11*d^3
- (520*a^3 + 267*a*b^2)*c^10*d^4 - (93*a^2*b - 41*b^3)*c^9*d^5 + (475*a^3 +
 21*a*b^2)*c^8*d^6 + (87*a^2*b - 2*b^3)*c^7*d^7 - (245*a^3 + 6*a*b^2)*c^6*d
^8)*cos(f*x + e)^3 + (12*b^3*c^14 + 108*a*b^2*c^13*d + 104*a^3*c^3*d^11 - (
648*a^2*b + 203*b^3)*c^12*d^2 + 15*(40*a^3 + 51*a*b^2)*c^11*d^3 + (339*a^2*
b + 47*b^3)*c^10*d^4 - (1189*a^3 + 933*a*b^2)*c^9*d^5 + (321*a^2*b + 152*b^
3)*c^8*d^6 + (997*a^3 + 84*a*b^2)*c^7*d^7 - 4*(3*a^2*b + 2*b^3)*c^6*d^8 - 8
*(64*a^3 + 3*a*b^2)*c^5*d^9)*cos(f*x + e)^2 + (8*b^3*c^13*d + 72*a*b^2*c^12
*d^2 - 407*a^3*c^4*d^10 + 84*a^3*c^2*d^12 - 8*(66*a^2*b + 19*b^3)*c^11*d^3
+ 8*(65*a^3 + 93*a*b^2)*c^10*d^4 + (84*a^2*b - 47*b^3)*c^9*d^5 - (964*a^3 +
 759*a*b^2)*c^8*d^6 + (471*a^2*b + 203*b^3)*c^7*d^7 + (767*a^3 - 57*a*b^2)*
c^6*d^8 - 3*(9*a^2*b + 4*b^3)*c^5*d^9)*cos(f*x + e))*sin(f*x + e))/((c^19 -
 5*c^17*d^2 + 10*c^15*d^4 - 10*c^13*d^6 + 5*c^11*d^8 - c^9*d^10)*f*cos(f*x
+ e)^4 + 4*(c^18*d - 5*c^16*d^3 + 10*c^14*d^5 - 10*c^12*d^7 + 5*c^10*d^9 -
c^8*d^11)*f*cos(f*x + e)^3 + 6*(c^17*d^2 - 5*c^15*d^4 + 10*c^13*d^6 - 10*c^
11*d^8 + 5*c^9*d^10 - c^7*d^12)*f*cos(f*x + e)^2 + 4*(c^16*d^3 - 5*c^14*d^5
 + 10*c^12*d^7 - 10*c^10*d^9 + 5*c^8*d^11 - c^6*d^13)*f*cos(f*x + e) + (c^1
5*d^4 - 5*c^13*d^6 + 10*c^11*d^8 - 10*c^9*d^10 + 5*c^7*d^12 - c^5*d^14)*f),
 1/24*(24*(a^3*c^14 - 5*a^3*c^12*d^2 + 10*a^3*c^10*d^4 - 10*a^3*c^8*d^6 + 5
*a^3*c^6*d^8 - a^3*c^4*d^10)*f*x*cos(f*x + e)^4 + 96*(a^3*c^13*d - 5*a^3*c^
11*d^3 + 10*a^3*c^9*d^5 - 10*a^3*c^7*d^7 + 5*a^3*c^5*d^9 - a^3*c^3*d^11)*f*
x*cos(f*x + e)^3 + 144*(a^3*c^12*d^2 - 5*a^3*c^10*d^4 + 10*a^3*c^8*d^6 - 10
*a^3*c^6*d^8 + 5*a^3*c^4*d^10 - a^3*c^2*d^12)*f*x*cos(f*x + e)^2 + 96*(a^3*
c^11*d^3 - 5*a^3*c^9*d^5 + 10*a^3*c^7*d^7 - 10*a^3*c^5*d^9 + 5*a^3*c^3*d^11
 - a^3*c*d^13)*f*x*cos(f*x + e) + 24*(a^3*c^10*d^4 - 5*a^3*c^8*d^6 + 10*a^3
*c^6*d^8 - 10*a^3*c^4*d^10 + 5*a^3*c^2*d^12 - a^3*d^14)*f*x - 3*(63*a^3*c^4
*d^9 - 36*a^3*c^2*d^11 + 8*a^3*d^13 - 4*(6*a^2*b + b^3)*c^9*d^4 + 20*(2*a^3
 + 3*a*b^2)*c^8*d^5 - 9*(8*a^2*b + 3*b^3)*c^7*d^6 - 5*(8*a^3 - 9*a*b^2)*c^6
*d^7 - (9*a^2*b + 4*b^3)*c^5*d^8 + (63*a^3*c^8*d^5 - 36*a^3*c^6*d^7 + 8*a^3
*c^4*d^9 - 4*(6*a^2*b + b^3)*c^13 + 20*(2*a^3 + 3*a*b^2)*c^12*d - 9*(8*a^2*
b + 3*b^3)*c^11*d^2 - 5*(8*a^3 - 9*a*b^2)*c^10*d^3 - (9*a^2*b + 4*b^3)*c^9*
d^4)*cos(f*x + e)^4 + 4*(63*a^3*c^7*d^6 - 36*a^3*c^5*d^8 + 8*a^3*c^3*d^10 -
 4*(6*a^2*b + b^3)*c^12*d + 20*(2*a^3 + 3*a*b^2)*c^11*d^2 - 9*(8*a^2*b + 3*
b^3)*c^10*d^3 - 5*(8*a^3 - 9*a*b^2)*c^9*d^4 - (9*a^2*b + 4*b^3)*c^8*d^5)*co
s(f*x + e)^3 + 6*(63*a^3*c^6*d^7 - 36*a^3*c^4*d^9 + 8*a^3*c^2*d^11 - 4*(6*a
^2*b + b^3)*c^11*d^2 + 20*(2*a^3 + 3*a*b^2)*c^10*d^3 - 9*(8*a^2*b + 3*b^3)*
c^9*d^4 - 5*(8*a^3 - 9*a*b^2)*c^8*d^5 - (9*a^2*b + 4*b^3)*c^7*d^6)*cos(f*x
+ e)^2 + 4*(63*a^3*c^5*d^8 - 36*a^3*c^3*d^10 + 8*a^3*c*d^12 - 4*(6*a^2*b +
b^3)*c^10*d^3 + 20*(2*a^3 + 3*a*b^2)*c^9*d^4 - 9*(8*a^2*b + 3*b^3)*c^8*d^5
- 5*(8*a^3 - 9*a*b^2)*c^7*d^6 - (9*a^2*b + 4*b^3)*c^6*d^7)*cos(f*x + e))*sq

```
rt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*s
in(f*x + e))) + (2*b^3*c^12*d^2 + 18*a*b^2*c^11*d^3 - 116*a^3*c^3*d^11 + 24
*a^3*c*d^13 - (150*a^2*b + 41*b^3)*c^10*d^4 + 77*(2*a^3 + 3*a*b^2)*c^9*d^5
- (15*a^2*b + 29*b^3)*c^8*d^6 - (271*a^3 + 201*a*b^2)*c^7*d^7 + (165*a^2*b
+ 68*b^3)*c^6*d^8 + (209*a^3 - 48*a*b^2)*c^5*d^9 + (72*a*b^2*c^14 - 18*a^2*
b*c^5*d^9 + 50*a^3*c^4*d^10 - 4*(72*a^2*b + 17*b^3)*c^13*d + 60*(4*a^3 + 3*
a*b^2)*c^12*d^2 + (312*a^2*b + 29*b^3)*c^11*d^3 - (520*a^3 + 267*a*b^2)*c^1
0*d^4 - (93*a^2*b - 41*b^3)*c^9*d^5 + (475*a^3 + 21*a*b^2)*c^8*d^6 + (87*a^
2*b - 2*b^3)*c^7*d^7 - (245*a^3 + 6*a*b^2)*c^6*d^8)*cos(f*x + e)^3 + (12*b^
3*c^14 + 108*a*b^2*c^13*d + 104*a^3*c^3*d^11 - (648*a^2*b + 203*b^3)*c^12*d
^2 + 15*(40*a^3 + 51*a*b^2)*c^11*d^3 + (339*a^2*b + 47*b^3)*c^10*d^4 - (118
9*a^3 + 933*a*b^2)*c^9*d^5 + (321*a^2*b + 152*b^3)*c^8*d^6 + (997*a^3 + 84*
a*b^2)*c^7*d^7 - 4*(3*a^2*b + 2*b^3)*c^6*d^8 - 8*(64*a^3 + 3*a*b^2)*c^5*d^9
)*cos(f*x + e)^2 + (8*b^3*c^13*d + 72*a*b^2*c^12*d^2 - 407*a^3*c^4*d^10 + 8
4*a^3*c^2*d^12 - 8*(66*a^2*b + 19*b^3)*c^11*d^3 + 8*(65*a^3 + 93*a*b^2)*c^1
0*d^4 + (84*a^2*b - 47*b^3)*c^9*d^5 - (964*a^3 + 759*a*b^2)*c^8*d^6 + (471*
a^2*b + 203*b^3)*c^7*d^7 + (767*a^3 - 57*a*b^2)*c^6*d^8 - 3*(9*a^2*b + 4*b^
3)*c^5*d^9)*cos(f*x + e))*sin(f*x + e))/((c^19 - 5*c^17*d^2 + 10*c^15*d^4 -
 10*c^13*d^6 + 5*c^11*d^8 - c^9*d^10)*f*cos(f*x + e)^4 + 4*(c^18*d - 5*c^16
*d^3 + 10*c^14*d^5 - 10*c^12*d^7 + 5*c^10*d^9 - c^8*d^11)*f*cos(f*x + e)^3
+ 6*(c^17*d^2 - 5*c^15*d^4 + 10*c^13*d^6 - 10*c^11*d^8 + 5*c^9*d^10 - c^7*d
^12)*f*cos(f*x + e)^2 + 4*(c^16*d^3 - 5*c^14*d^5 + 10*c^12*d^7 - 10*c^10*d^
9 + 5*c^8*d^11 - c^6*d^13)*f*cos(f*x + e) + (c^15*d^4 - 5*c^13*d^6 + 10*c^1
1*d^8 - 10*c^9*d^10 + 5*c^7*d^12 - c^5*d^14)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + b\sec\left(e + fx\right)\right)^3}{\left(c + d\sec\left(e + fx\right)\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**3/(c+d*sec(f*x+e))**5,x)
```

```
[Out] Integral((a + b*sec(e + f*x))**3/(c + d*sec(e + f*x))**5, x)
```

Giac [B] time = 2.03642, size = 4470, normalized size = 7.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^5,x, algorithm="giac")
```

```
[Out] 1/12*(3*(24*a^2*b*c^9 + 4*b^3*c^9 - 40*a^3*c^8*d - 60*a*b^2*c^8*d + 72*a^2*
b*c^7*d^2 + 27*b^3*c^7*d^2 + 40*a^3*c^6*d^3 - 45*a*b^2*c^6*d^3 + 9*a^2*b*c^
5*d^4 + 4*b^3*c^5*d^4 - 63*a^3*c^4*d^5 + 36*a^3*c^2*d^7 - 8*a^3*d^9)*(pi*fl
oor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*f*x + 1/2*
e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))/((c^13 - 4*c^11*d^2 + 6*c^9
*d^4 - 4*c^7*d^6 + c^5*d^8)*sqrt(-c^2 + d^2)) + 12*(f*x + e)*a^3/c^5 - (72*
a*b^2*c^11*tan(1/2*f*x + 1/2*e)^7 - 12*b^3*c^11*tan(1/2*f*x + 1/2*e)^7 - 28
8*a^2*b*c^10*d*tan(1/2*f*x + 1/2*e)^7 - 108*a*b^2*c^10*d*tan(1/2*f*x + 1/2*
e)^7 - 60*b^3*c^10*d*tan(1/2*f*x + 1/2*e)^7 + 240*a^3*c^9*d^2*tan(1/2*f*x +
 1/2*e)^7 + 648*a^2*b*c^9*d^2*tan(1/2*f*x + 1/2*e)^7 + 324*a*b^2*c^9*d^2*ta
n(1/2*f*x + 1/2*e)^7 + 189*b^3*c^9*d^2*tan(1/2*f*x + 1/2*e)^7 - 600*a^3*c^8
*d^3*tan(1/2*f*x + 1/2*e)^7 - 504*a^2*b*c^8*d^3*tan(1/2*f*x + 1/2*e)^7 - 89
1*a*b^2*c^8*d^3*tan(1/2*f*x + 1/2*e)^7 - 183*b^3*c^8*d^3*tan(1/2*f*x + 1/2*
e)^7 + 240*a^3*c^7*d^4*tan(1/2*f*x + 1/2*e)^7 + 459*a^2*b*c^7*d^4*tan(1/2*f
*x + 1/2*e)^7 + 801*a*b^2*c^7*d^4*tan(1/2*f*x + 1/2*e)^7 + 183*b^3*c^7*d^4*
tan(1/2*f*x + 1/2*e)^7 + 435*a^3*c^6*d^5*tan(1/2*f*x + 1/2*e)^7 - 513*a^2*b
*c^6*d^5*tan(1/2*f*x + 1/2*e)^7 - 189*a*b^2*c^6*d^5*tan(1/2*f*x + 1/2*e)^7
- 189*b^3*c^6*d^5*tan(1/2*f*x + 1/2*e)^7 - 249*a^3*c^5*d^6*tan(1/2*f*x + 1/
2*e)^7 + 153*a^2*b*c^5*d^6*tan(1/2*f*x + 1/2*e)^7 + 63*a*b^2*c^5*d^6*tan(1/
2*f*x + 1/2*e)^7 + 60*b^3*c^5*d^6*tan(1/2*f*x + 1/2*e)^7 - 291*a^3*c^4*d^7*
tan(1/2*f*x + 1/2*e)^7 + 45*a^2*b*c^4*d^7*tan(1/2*f*x + 1/2*e)^7 - 72*a*b^2
*c^4*d^7*tan(1/2*f*x + 1/2*e)^7 + 12*b^3*c^4*d^7*tan(1/2*f*x + 1/2*e)^7 + 2
73*a^3*c^3*d^8*tan(1/2*f*x + 1/2*e)^7 + 12*a^3*c^2*d^9*tan(1/2*f*x + 1/2*e)
^7 - 84*a^3*c*d^10*tan(1/2*f*x + 1/2*e)^7 + 24*a^3*d^11*tan(1/2*f*x + 1/2*e
)^7 - 216*a*b^2*c^11*tan(1/2*f*x + 1/2*e)^5 + 12*b^3*c^11*tan(1/2*f*x + 1/2
*e)^5 + 864*a^2*b*c^10*d*tan(1/2*f*x + 1/2*e)^5 + 108*a*b^2*c^10*d*tan(1/2*
f*x + 1/2*e)^5 + 212*b^3*c^10*d*tan(1/2*f*x + 1/2*e)^5 - 720*a^3*c^9*d^2*ta
n(1/2*f*x + 1/2*e)^5 - 648*a^2*b*c^9*d^2*tan(1/2*f*x + 1/2*e)^5 - 684*a*b^2
*c^9*d^2*tan(1/2*f*x + 1/2*e)^5 - 197*b^3*c^9*d^2*tan(1/2*f*x + 1/2*e)^5 +
600*a^3*c^8*d^3*tan(1/2*f*x + 1/2*e)^5 - 600*a^2*b*c^8*d^3*tan(1/2*f*x + 1/
2*e)^5 + 819*a*b^2*c^8*d^3*tan(1/2*f*x + 1/2*e)^5 - 27*b^3*c^8*d^3*tan(1/2*
f*x + 1/2*e)^5 + 1360*a^3*c^7*d^4*tan(1/2*f*x + 1/2*e)^5 + 141*a^2*b*c^7*d^
4*tan(1/2*f*x + 1/2*e)^5 + 861*a*b^2*c^7*d^4*tan(1/2*f*x + 1/2*e)^5 - 27*b^
3*c^7*d^4*tan(1/2*f*x + 1/2*e)^5 - 1051*a^3*c^6*d^5*tan(1/2*f*x + 1/2*e)^5
- 237*a^2*b*c^6*d^5*tan(1/2*f*x + 1/2*e)^5 - 807*a*b^2*c^6*d^5*tan(1/2*f*x
+ 1/2*e)^5 - 197*b^3*c^6*d^5*tan(1/2*f*x + 1/2*e)^5 - 1029*a^3*c^5*d^6*tan(
1/2*f*x + 1/2*e)^5 + 507*a^2*b*c^5*d^6*tan(1/2*f*x + 1/2*e)^5 + 39*a*b^2*c^
5*d^6*tan(1/2*f*x + 1/2*e)^5 + 212*b^3*c^5*d^6*tan(1/2*f*x + 1/2*e)^5 + 759
*a^3*c^4*d^7*tan(1/2*f*x + 1/2*e)^5 - 27*a^2*b*c^4*d^7*tan(1/2*f*x + 1/2*e)
^5 - 120*a*b^2*c^4*d^7*tan(1/2*f*x + 1/2*e)^5 + 12*b^3*c^4*d^7*tan(1/2*f*x
```

+ 1/2*e)^5 + 473*a^3*c^3*d^8*tan(1/2*f*x + 1/2*e)^5 - 380*a^3*c^2*d^9*tan(1
/2*f*x + 1/2*e)^5 - 84*a^3*c*d^10*tan(1/2*f*x + 1/2*e)^5 + 72*a^3*d^11*tan(
1/2*f*x + 1/2*e)^5 + 216*a*b^2*c^11*tan(1/2*f*x + 1/2*e)^3 + 12*b^3*c^11*ta
n(1/2*f*x + 1/2*e)^3 - 864*a^2*b*c^10*d*tan(1/2*f*x + 1/2*e)^3 + 108*a*b^2*
c^10*d*tan(1/2*f*x + 1/2*e)^3 - 212*b^3*c^10*d*tan(1/2*f*x + 1/2*e)^3 + 720
*a^3*c^9*d^2*tan(1/2*f*x + 1/2*e)^3 - 648*a^2*b*c^9*d^2*tan(1/2*f*x + 1/2*e
)^3 + 684*a*b^2*c^9*d^2*tan(1/2*f*x + 1/2*e)^3 - 197*b^3*c^9*d^2*tan(1/2*f*
x + 1/2*e)^3 + 600*a^3*c^8*d^3*tan(1/2*f*x + 1/2*e)^3 + 600*a^2*b*c^8*d^3*t
an(1/2*f*x + 1/2*e)^3 + 819*a*b^2*c^8*d^3*tan(1/2*f*x + 1/2*e)^3 + 27*b^3*c
^8*d^3*tan(1/2*f*x + 1/2*e)^3 - 1360*a^3*c^7*d^4*tan(1/2*f*x + 1/2*e)^3 + 1
41*a^2*b*c^7*d^4*tan(1/2*f*x + 1/2*e)^3 - 861*a*b^2*c^7*d^4*tan(1/2*f*x + 1
/2*e)^3 - 27*b^3*c^7*d^4*tan(1/2*f*x + 1/2*e)^3 - 1051*a^3*c^6*d^5*tan(1/2*
f*x + 1/2*e)^3 + 237*a^2*b*c^6*d^5*tan(1/2*f*x + 1/2*e)^3 - 807*a*b^2*c^6*d
^5*tan(1/2*f*x + 1/2*e)^3 + 197*b^3*c^6*d^5*tan(1/2*f*x + 1/2*e)^3 + 1029*a
^3*c^5*d^6*tan(1/2*f*x + 1/2*e)^3 + 507*a^2*b*c^5*d^6*tan(1/2*f*x + 1/2*e)^
3 - 39*a*b^2*c^5*d^6*tan(1/2*f*x + 1/2*e)^3 + 212*b^3*c^5*d^6*tan(1/2*f*x +
 1/2*e)^3 + 759*a^3*c^4*d^7*tan(1/2*f*x + 1/2*e)^3 + 27*a^2*b*c^4*d^7*tan(1
/2*f*x + 1/2*e)^3 - 120*a*b^2*c^4*d^7*tan(1/2*f*x + 1/2*e)^3 - 12*b^3*c^4*d
^7*tan(1/2*f*x + 1/2*e)^3 - 473*a^3*c^3*d^8*tan(1/2*f*x + 1/2*e)^3 - 380*a^
3*c^2*d^9*tan(1/2*f*x + 1/2*e)^3 + 84*a^3*c*d^10*tan(1/2*f*x + 1/2*e)^3 + 7
2*a^3*d^11*tan(1/2*f*x + 1/2*e)^3 - 72*a*b^2*c^11*tan(1/2*f*x + 1/2*e) - 12
*b^3*c^11*tan(1/2*f*x + 1/2*e) + 288*a^2*b*c^10*d*tan(1/2*f*x + 1/2*e) - 10
8*a*b^2*c^10*d*tan(1/2*f*x + 1/2*e) + 60*b^3*c^10*d*tan(1/2*f*x + 1/2*e) -
240*a^3*c^9*d^2*tan(1/2*f*x + 1/2*e) + 648*a^2*b*c^9*d^2*tan(1/2*f*x + 1/2*
e) - 324*a*b^2*c^9*d^2*tan(1/2*f*x + 1/2*e) + 189*b^3*c^9*d^2*tan(1/2*f*x +
 1/2*e) - 600*a^3*c^8*d^3*tan(1/2*f*x + 1/2*e) + 504*a^2*b*c^8*d^3*tan(1/2*
f*x + 1/2*e) - 891*a*b^2*c^8*d^3*tan(1/2*f*x + 1/2*e) + 183*b^3*c^8*d^3*tan
(1/2*f*x + 1/2*e) - 240*a^3*c^7*d^4*tan(1/2*f*x + 1/2*e) + 459*a^2*b*c^7*d^
4*tan(1/2*f*x + 1/2*e) - 801*a*b^2*c^7*d^4*tan(1/2*f*x + 1/2*e) + 183*b^3*c
^7*d^4*tan(1/2*f*x + 1/2*e) + 435*a^3*c^6*d^5*tan(1/2*f*x + 1/2*e) + 513*a^
2*b*c^6*d^5*tan(1/2*f*x + 1/2*e) - 189*a*b^2*c^6*d^5*tan(1/2*f*x + 1/2*e) +
 189*b^3*c^6*d^5*tan(1/2*f*x + 1/2*e) + 249*a^3*c^5*d^6*tan(1/2*f*x + 1/2*e
) + 153*a^2*b*c^5*d^6*tan(1/2*f*x + 1/2*e) - 63*a*b^2*c^5*d^6*tan(1/2*f*x +
 1/2*e) + 60*b^3*c^5*d^6*tan(1/2*f*x + 1/2*e) - 291*a^3*c^4*d^7*tan(1/2*f*x
 + 1/2*e) - 45*a^2*b*c^4*d^7*tan(1/2*f*x + 1/2*e) - 72*a*b^2*c^4*d^7*tan(1/
2*f*x + 1/2*e) - 12*b^3*c^4*d^7*tan(1/2*f*x + 1/2*e) - 273*a^3*c^3*d^8*tan(
1/2*f*x + 1/2*e) + 12*a^3*c^2*d^9*tan(1/2*f*x + 1/2*e) + 84*a^3*c*d^10*tan(
1/2*f*x + 1/2*e) + 24*a^3*d^11*tan(1/2*f*x + 1/2*e))/((c^12 - 4*c^10*d^2 +
6*c^8*d^4 - 4*c^6*d^6 + c^4*d^8)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x
+ 1/2*e)^2 - c - d)^4))/f

3.198 $\int \sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))\,dx$

Optimal. Leaf size=320

$$\frac{2\sqrt{a+b}(ad+b(c-d))\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bf}-\frac{2c\sqrt{a+b}\cot(}{}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*d*Cot[e + f*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[e +
f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(b*f) + (2*Sqrt[a + b]*(b*(c - d) +
 a*d)*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]],
(a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e
+ f*x]))/(a - b))])/(b*f) - (2*Sqrt[a + b]*c*Cot[e + f*x]*EllipticPi[(a + b
)/a, ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b
*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/f
```

Rubi [A] time = 0.282933, antiderivative size = 320, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3916, 3784, 4005, 3832, 4004}

$$\frac{2\sqrt{a+b}(ad+b(c-d))\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bf}-\frac{2c\sqrt{a+b}\cot(e+fx)\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x]),x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*d*Cot[e + f*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[e +
f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(b*f) + (2*Sqrt[a + b]*(b*(c - d) +
 a*d)*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]],
(a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e
+ f*x]))/(a - b))])/(b*f) - (2*Sqrt[a + b]*c*Cot[e + f*x]*EllipticPi[(a + b
)/a, ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b
*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/f
```

Rule 3916

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[a*c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
```

```
 Int[(Csc[e + f*x]*(b*c + a*d + b*d*Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))\,dx = (ac)\int \frac{1}{\sqrt{a+b\sec(e+fx)}}\,dx + \int \frac{\sec(e+fx)(bc+ad+bd\sec(e+fx))}{\sqrt{a+b\sec(e+fx)}}\,d$$

$$= -\frac{2\sqrt{a+b}c\cot(e+fx)\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{}}{f}$$

$$= -\frac{2(a-b)\sqrt{a+b}d\cot(e+fx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}}{bf}$$

Mathematica [C] time = 17.8105, size = 913, normalized size = 2.85

$$\frac{2d\cos(e+fx)\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))\sin(e+fx)}{f(d+c\cos(e+fx))} + \frac{2\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))\left(a\sqrt{\frac{b-a}{a+b}}d\tan^5\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x]),x]
```

```
[Out] (2*d*Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])*Sin[e + f*x
])/(f*(d + c*Cos[e + f*x])) + (2*Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*
x])*(a*Sqrt[(-a + b)/(a + b)]*d*Tan[(e + f*x)/2] + b*Sqrt[(-a + b)/(a + b)]
*d*Tan[(e + f*x)/2] - 2*a*Sqrt[(-a + b)/(a + b)]*d*Tan[(e + f*x)/2]^3 + a*S
qrt[(-a + b)/(a + b)]*d*Tan[(e + f*x)/2]^5 - b*Sqrt[(-a + b)/(a + b)]*d*Tan
[(e + f*x)/2]^5 + (2*I)*a*c*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(
-a + b)/(a + b)]*Tan[(e + f*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(e + f*x)
/2]^2]*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)]
+ (2*I)*a*c*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]
*Tan[(e + f*x)/2]], (a + b)/(a - b)]*Tan[(e + f*x)/2]^2*Sqrt[1 - Tan[(e + f
*x)/2]^2]*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b
)] - I*(a - b)*d*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2
]], (a + b)/(a - b)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*(1 + Tan[(e + f*x)/2]^2)*
Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] - I*(a
- b)*(c - d)*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2]],
(a + b)/(a - b)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*(1 + Tan[(e + f*x)/2]^2)*Sqrt
[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)]))/(Sqrt[(-a
 + b)/(a + b)]*f*Sqrt[b + a*Cos[e + f*x]]*(d + c*Cos[e + f*x])*Sec[e + f*x]
^(3/2)*Sqrt[(1 - Tan[(e + f*x)/2]^2)^(-1)]*(-1 + Tan[(e + f*x)/2]^2)*(1 + T
```

```
an[(e + f*x)/2]^2)^(3/2)*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*
x)/2]^2)/(1 + Tan[(e + f*x)/2]^2)])
```

Maple [B] time = 0.38, size = 1372, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sec(f*x+e))*(a+b*sec(f*x+e))^(1/2),x)
```

```
[Out] 2/f*(1/cos(f*x+e)*(a*cos(f*x+e)+b))^(1/2)*(1+cos(f*x+e))^2*(-1+cos(f*x+e))^
2*(cos(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(
1+cos(f*x+e)))^(1/2)*sin(f*x+e)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)
/(a+b))^(1/2))*a*c-cos(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a
*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)*EllipticF((-1+cos(f*x+e))/s
in(f*x+e),((a-b)/(a+b))^(1/2))*a*d-cos(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(
1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)*EllipticF((
-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*b*c-cos(f*x+e)*(cos(f*x+e)/(
1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*sin(f*
x+e)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*b*d+cos(f*x+
e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e
)))^(1/2)*sin(f*x+e)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/
2))*a*d+cos(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)
+b)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),(
(a-b)/(a+b))^(1/2))*b*d-2*cos(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(
a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)*EllipticPi((-1+cos(f
*x+e))/sin(f*x+e),-1,((a-b)/(a+b))^(1/2))*a*c+(cos(f*x+e)/(1+cos(f*x+e)))^(
1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+
e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a*c*sin(f*x+e)-(cos(f*x+e)/(1+cos(f*x+e
)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos
(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a*d*sin(f*x+e)-(cos(f*x+e)/(1+cos(
f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-
1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*b*c*sin(f*x+e)-(cos(f*x+e)/(1
+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*Ellipti
cF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*b*d*sin(f*x+e)+(cos(f*x+
e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*si
n(f*x+e)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a*d+(cos
(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/
2)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*b*d*sin(f*x+e)
-2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e
)))^(1/2)*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),-1,((a-b)/(a+b))^(1/2))*a*c
```

```
*sin(f*x+e)-cos(f*x+e)^2*a*d+cos(f*x+e)*a*d-cos(f*x+e)*b*d+d*b)/sin(f*x+e)^
5/(a*cos(f*x+e)+b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec(fx+e)+a}(d\sec(fx+e)+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))*(a+b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\sec(fx+e)+a}(d\sec(fx+e)+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))*(a+b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\sec(e+fx)}\,(c+d\sec(e+fx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))*(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x))*(c + d*sec(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec \left(fx+e\right)+a}\left(d \sec \left(fx+e\right)+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))*(a+b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c), x)
```

3.199 $\int \frac{\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$

Optimal. Leaf size=220

$$\frac{2(bc-ad)\tan(e+fx)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\Pi\left(\frac{2d}{c+d};\sin^{-1}\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right)|\frac{2b}{a+b}\right)}{cf(c+d)\sqrt{-\tan^2(e+fx)}\sqrt{a+b\sec(e+fx)}}-\frac{2\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+}{a-b}}}{cf}$$

```
[Out] (-2*Sqrt[a + b]*Cot[e + f*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[e
+ f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]
*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(c*f) + (2*(b*c - a*d)*EllipticPi
[(2*d)/(c + d), ArcSin[Sqrt[1 - Sec[e + f*x]]/Sqrt[2]], (2*b)/(a + b)]*Sqrt
[(a + b*Sec[e + f*x])/(a + b)]*Tan[e + f*x])/(c*(c + d)*f*Sqrt[a + b*Sec[e
+ f*x]]*Sqrt[-Tan[e + f*x]^2])
```

Rubi [A] time = 0.243852, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3926, 3784, 3973}

$$\frac{2(bc-ad)\tan(e+fx)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\Pi\left(\frac{2d}{c+d};\sin^{-1}\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right)|\frac{2b}{a+b}\right)}{cf(c+d)\sqrt{-\tan^2(e+fx)}\sqrt{a+b\sec(e+fx)}}-\frac{2\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+}{a-b}}}{cf}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]]/(c + d*Sec[e + f*x]),x]
```

```
[Out] (-2*Sqrt[a + b]*Cot[e + f*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[e
+ f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]
*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(c*f) + (2*(b*c - a*d)*EllipticPi
[(2*d)/(c + d), ArcSin[Sqrt[1 - Sec[e + f*x]]/Sqrt[2]], (2*b)/(a + b)]*Sqrt
[(a + b*Sec[e + f*x])/(a + b)]*Tan[e + f*x])/(c*(c + d)*f*Sqrt[a + b*Sec[e
+ f*x]]*Sqrt[-Tan[e + f*x]^2])
```

Rule 3926

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[a/c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
 Dist[(b*c - a*d)/c, Int[Csc[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[
e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
```

```
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3973

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Simp[(-2*Cot[e + f*x]*Sq
rt[(a + b*Csc[e + f*x])/(a + b)]*EllipticPi[(2*d)/(c + d), ArcSin[Sqrt[1 -
Csc[e + f*x]]/Sqrt[2]], (2*b)/(a + b)])/(f*(c + d)*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[-Cot[e + f*x]^2]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \frac{a\int \frac{1}{\sqrt{a+b\sec(e+fx)}}\,dx}{c} + \frac{(bc-ad)\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx}{c}$$

$$= -\frac{2\sqrt{a+b}\cot(e+fx)\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{cf} + \cdots$$

Mathematica [A] time = 4.20486, size = 229, normalized size = 1.04

$$\frac{4\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{\cos(e+fx)}{\cos(e+fx)+1}}\sqrt{\frac{a\cos(e+fx)+b}{(a+b)(\cos(e+fx)+1)}}\sqrt{a+b\sec(e+fx)}\left(c(-(a-b))(c+d)\text{EllipticF}\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(e+f\cdots\right.\right.\right.\right.}{cf(c-d)(c+d)(a\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]]/(c + d*Sec[e + f*x]),x]
```

```
[Out] (4*Cos[(e + f*x)/2]^2*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[(b + a*Cos
[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*(-((a - b)*c*(c + d)*EllipticF[Arc
Sin[Tan[(e + f*x)/2]], (a - b)/(a + b)]) - 2*a*(c^2 - d^2)*EllipticPi[-1, -
```

```
ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)] + 2*d*(b*c - a*d)*EllipticPi[(c
- d)/(c + d), -ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)])*Sqrt[a + b*Sec[e
 + f*x]])/(c*(c - d)*(c + d)*f*(b + a*Cos[e + f*x]))
```

Maple [B] time = 0.335, size = 443, normalized size = 2.

$$-2\frac{\left(1+\cos\left(fx+e\right)\right)^{2}\left(-1+\cos\left(fx+e\right)\right)}{cf\left(c-d\right)\left(c+d\right)\left(a\cos\left(fx+e\right)+b\right)\left(\sin\left(fx+e\right)\right)^{2}}\sqrt{\frac{a\cos\left(fx+e\right)+b}{\cos\left(fx+e\right)}}\sqrt{\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\sqrt{\frac{a\cos\left(fx+e\right)+b}{\left(a+b\right)\left(1+\cos\left(fx+\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x)
```

```
[Out] -2/f/c/(c-d)/(c+d)*(1/cos(f*x+e)*(a*cos(f*x+e)+b))^(1/2)*(cos(f*x+e)/(1+cos
(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(1+cos(f*x+
e))^2*(EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a*c^2+Elli
pticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a*c*d-EllipticF((-1+c
os(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*b*c^2-EllipticF((-1+cos(f*x+e))/
sin(f*x+e),((a-b)/(a+b))^(1/2))*b*c*d-2*EllipticPi((-1+cos(f*x+e))/sin(f*x+
e),-1,((a-b)/(a+b))^(1/2))*a*c^2+2*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),-1
,((a-b)/(a+b))^(1/2))*a*d^2-2*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),(c-d)/(
c+d),((a-b)/(a+b))^(1/2))*a*d^2+2*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),(c-
d)/(c+d),((a-b)/(a+b))^(1/2))*b*c*d)*(-1+cos(f*x+e))/(a*cos(f*x+e)+b)/sin(f
*x+e)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{b\sec\left(fx+e\right)+a}}{d\sec\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)/(d*sec(f*x + e) + c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + b\sec\left(e + fx\right)}}{c + d\sec\left(e + fx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e)),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x))/(c + d*sec(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sec\left(fx + e\right) + a}}{d\sec\left(fx + e\right) + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)/(d*sec(f*x + e) + c), x)
```

3.200 $\int (a+b\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx$

Optimal. Leaf size=380

$$\frac{2\sqrt{a+b}\left(3a^2d+ab(6c-4d)-b^2(3c-d)\right)\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{3bf}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(3*b*c + 4*a*d)*Cot[e + f*x]*EllipticE[ArcSin[Sqrt[
a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(3*b*f) + (2*Sqrt[a +
 b]*(a*b*(6*c - 4*d) - b^2*(3*c - d) + 3*a^2*d)*Cot[e + f*x]*EllipticF[ArcS
in[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec
[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(3*b*f) - (2*
a*Sqrt[a + b]*c*Cot[e + f*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[e
+ f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]
*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/f + (2*b*d*Sqrt[a + b*Sec[e + f*x
]]*Tan[e + f*x])/(3*f)
```

Rubi [A] time = 0.433004, antiderivative size = 380, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3918, 4058, 3921, 3784, 3832, 4004}

$$\frac{2\sqrt{a+b}\left(3a^2d+ab(6c-4d)-b^2(3c-d)\right)\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3bf}-\frac{2(a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x]),x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*(3*b*c + 4*a*d)*Cot[e + f*x]*EllipticE[ArcSin[Sqrt[
a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(3*b*f) + (2*Sqrt[a +
 b]*(a*b*(6*c - 4*d) - b^2*(3*c - d) + 3*a^2*d)*Cot[e + f*x]*EllipticF[ArcS
in[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec
[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(3*b*f) - (2*
a*Sqrt[a + b]*c*Cot[e + f*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[e
+ f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]
*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/f + (2*b*d*Sqrt[a + b*Sec[e + f*x
]]*Tan[e + f*x])/(3*f)
```

Rule 3918

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 2)*Simp[a^2*c*m +
(b^2*d*(m - 1) + 2*a*b*c*m + a^2*d*m)*Csc[e + f*x] + b*(b*c*m + a*d*(2*m -
1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && GtQ[m, 1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
```

```
f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int (a+b\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx = \frac{2bd\sqrt{a+b\sec(e+fx)}\tan(e+fx)}{3f}+\frac{2}{3}\int \frac{\frac{3a^2c}{2}+\frac{1}{2}\left(6abc+3a^2d+b^2d\right)}{\sqrt{a}}$$

$$=\frac{2bd\sqrt{a+b\sec(e+fx)}\tan(e+fx)}{3f}+\frac{2}{3}\int \frac{\frac{3a^2c}{2}+\left(-\frac{1}{2}b(3bc+4ad)+\frac{1}{2}\right(}{\sqrt{a+b\,\mathrm{s}}}$$

$$=-\frac{2(a-b)\sqrt{a+b}(3bc+4ad)\cot(e+fx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{}}{3bf}$$

$$=-\frac{2(a-b)\sqrt{a+b}(3bc+4ad)\cot(e+fx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{}}{3bf}$$

Mathematica [B] time = 24.3461, size = 6093, normalized size = 16.03

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x]),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.424, size = 2337, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e)),x)
```

```
[Out] -2/3/f*(-1+cos(f*x+e))^2*(4*cos(f*x+e)^3*a^2*d-4*cos(f*x+e)^2*a^2*d-b^2*d+c
os(f*x+e)^2*b^2*d+3*cos(f*x+e)^2*b^2*c-3*cos(f*x+e)*b^2*c+4*cos(f*x+e)^2*(c
```

os(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(
1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*a
*b*d-3*cos(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+
b)/(1+cos(f*x+e)))^(1/2)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))
^(1/2))*sin(f*x+e)*a*b*c-4*cos(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/
(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*EllipticE((-1+cos(f*x+e))/sin(
f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*a*b*d+6*cos(f*x+e)*(cos(f*x+e)/(1+co
s(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*EllipticF(
(-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*a*b*c+4*cos(f*x+
e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e
)))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x
+e)*a*b*d-3*cos(f*x+e)^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(
f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)
/(a+b))^(1/2))*sin(f*x+e)*a*b*c-4*cos(f*x+e)^2*(cos(f*x+e)/(1+cos(f*x+e)))^
(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*EllipticE((-1+cos(f*x
+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*a*b*d+6*cos(f*x+e)^2*(cos(f
*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)
*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*a*b*c
-4*cos(f*x+e)^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)
/(1+cos(f*x+e)))^(1/2)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(
1/2))*sin(f*x+e)*a^2*d-3*cos(f*x+e)^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/
(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*EllipticE((-1+cos(f*x+e))/sin(
f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*b^2*c+3*cos(f*x+e)*(cos(f*x+e)/(1+co
s(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*EllipticF(
(-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*a^2*d+3*cos(f*x+
e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e
)))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x
+e)*b^2*c+cos(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+
e)+b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+
b))^(1/2))*sin(f*x+e)*b^2*d+6*cos(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*
(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))/
sin(f*x+e),-1,((a-b)/(a+b))^(1/2))*sin(f*x+e)*a^2*c-3*cos(f*x+e)*(cos(f*x+e
)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*Ell
ipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*a^2*c-3*c
os(f*x+e)^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+
cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2)
)*sin(f*x+e)*a^2*c+3*cos(f*x+e)^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b
)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+
e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*a^2*d+3*cos(f*x+e)^2*(cos(f*x+e)/(1+cos(
f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-
1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*b^2*c+3*cos(f*x+e)
^3*a*b*c+cos(f*x+e)^3*a*b*d-3*cos(f*x+e)^2*a*b*c+4*cos(f*x+e)^2*a*b*d-5*cos
(f*x+e)*a*b*d+cos(f*x+e)^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*co
s(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-
b)/(a+b))^(1/2))*sin(f*x+e)*b^2*d+6*cos(f*x+e)^2*(cos(f*x+e)/(1+cos(f*x+e))

```
)^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(
f*x+e))/sin(f*x+e),-1,((a-b)/(a+b))^(1/2))*sin(f*x+e)*a^2*c-4*cos(f*x+e)*(c
os(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(
1/2)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*a
^2*d-3*cos(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+
b)/(1+cos(f*x+e)))^(1/2)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))
^(1/2))*sin(f*x+e)*b^2*c)*(1/cos(f*x+e)*(a*cos(f*x+e)+b))^(1/2)*(1+cos(f*x+
e))^2/(a*cos(f*x+e)+b)/cos(f*x+e)/sin(f*x+e)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sec\left(fx+e\right)+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e) + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bd\sec\left(fx+e\right)^{2}+ac+(bc+ad)\sec\left(fx+e\right)\right)\sqrt{b\sec\left(fx+e\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((b*d*sec(f*x + e)^2 + a*c + (b*c + a*d)*sec(f*x + e))*sqrt(b*sec(f
*x + e) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\sec\left(e+fx\right)\right)^{\frac{3}{2}}\left(c+d\sec\left(e+fx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(3/2)*(c+d*sec(f*x+e)),x)
```

```
[Out] Integral((a + b*sec(e + f*x))**(3/2)*(c + d*sec(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sec\left(fx+e\right)+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e) + c), x)
```

3.201 $\int \frac{(a+b\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx$

Optimal. Leaf size=326

$$\frac{2b\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{df}-\frac{2(bc-ad)^2\tan(e+fx)\sqrt{\frac{a+\cdots}{}}}{cdf(c+d)\sqrt{-\tan\cdots}}$$

```
[Out] (2*b*Sqrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqr
t[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*
(1 + Sec[e + f*x]))/(a - b))])/(d*f) - (2*a*Sqrt[a + b]*Cot[e + f*x]*Ellipt
icPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a -
b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a -
 b))])/(c*f) - (2*(b*c - a*d)^2*EllipticPi[(2*d)/(c + d), ArcSin[Sqrt[1 - S
ec[e + f*x]]/Sqrt[2]], (2*b)/(a + b)]*Sqrt[(a + b*Sec[e + f*x])/(a + b)]*Ta
n[e + f*x])/(c*d*(c + d)*f*Sqrt[a + b*Sec[e + f*x]]*Sqrt[-Tan[e + f*x]^2])
```

Rubi [A] time = 0.361492, antiderivative size = 326, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {3928, 3921, 3784, 3832, 3973}

$$-\frac{2(bc-ad)^2\tan(e+fx)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\Pi\left(\frac{2d}{c+d};\sin^{-1}\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right)|\frac{2b}{a+b}\right)}{cdf(c+d)\sqrt{-\tan^2(e+fx)}\sqrt{a+b\sec(e+fx)}}-\frac{2a\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+\cdots}{a-\cdots}}}{cf}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x]),x]
```



```
[Out] (2*b*Sqrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqr
t[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*
(1 + Sec[e + f*x]))/(a - b))])/(d*f) - (2*a*Sqrt[a + b]*Cot[e + f*x]*Ellipt
icPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a -
b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a -
 b))])/(c*f) - (2*(b*c - a*d)^2*EllipticPi[(2*d)/(c + d), ArcSin[Sqrt[1 - S
ec[e + f*x]]/Sqrt[2]], (2*b)/(a + b)]*Sqrt[(a + b*Sec[e + f*x])/(a + b)]*Ta
n[e + f*x])/(c*d*(c + d)*f*Sqrt[a + b*Sec[e + f*x]]*Sqrt[-Tan[e + f*x]^2])
```


Rule 3928

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_)), x_Symbol] :> Dist[1/(c*d), Int[(a^2*d + b^2*c*Csc[e + f*x])/S
qrt[a + b*Csc[e + f*x]], x], x] - Dist[(b*c - a*d)^2/(c*d), Int[Csc[e + f*x
]/(Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])), x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3973

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Simp[(-2*Cot[e + f*x]*Sq
rt[(a + b*Csc[e + f*x])/(a + b)]*EllipticPi[(2*d)/(c + d), ArcSin[Sqrt[1 -
Csc[e + f*x]]/Sqrt[2]], (2*b)/(a + b)])/(f*(c + d)*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[-Cot[e + f*x]^2]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx = \frac{\int \frac{a^2d+b^2c\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx}{cd} - \frac{(bc-ad)^2\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx}{cd}$$

$$= -\frac{2(bc-ad)^2\Pi\left(\frac{2d}{c+d};\sin^{-1}\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\tan(e+fx)}{cd(c+d)f\sqrt{a+b\sec(e+fx)}\sqrt{-\tan^2(e+fx)}} + \frac{a^2\int \frac{1}{\sqrt{a+b\sec(e+}}}{c}$$

$$= \frac{2b\sqrt{a+b}\cot(e+fx)F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{df} - \frac{2a\sqrt{a}}{}$$

Mathematica [A] time = 5.62718, size = 233, normalized size = 0.71

$$-\frac{4\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{\cos(e+fx)}{\cos(e+fx)+1}}\sqrt{\frac{a\cos(e+fx)+b}{(a+b)(\cos(e+fx)+1)}}\sqrt{a+b\sec(e+fx)}\left(c(a-b)^2(c+d)\text{EllipticF}\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right.\right.\right.}{cf(c-d)(c+d)(a\,c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x]),x]
```

```
[Out] (-4*Cos[(e + f*x)/2]^2*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[(b + a*Co
s[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*((a - b)^2*c*(c + d)*EllipticF[Ar
cSin[Tan[(e + f*x)/2]], (a - b)/(a + b)] + 2*a^2*(c^2 - d^2)*EllipticPi[-1,
 -ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)] + 2*(b*c - a*d)^2*EllipticPi[(
c - d)/(c + d), -ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)])*Sqrt[a + b*Sec
[e + f*x]])/(c*(c - d)*(c + d)*f*(b + a*Cos[e + f*x]))
```

Maple [A] time = 0.305, size = 581, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x)
```

```
[Out] -2/f/c/(c+d)/(c-d)*(1/cos(f*x+e)*(a*cos(f*x+e)+b))^(1/2)*(cos(f*x+e)/(1+cos
(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(1+cos(f*x+
```

```
e))^2*(EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a^2*c^2+El
lipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a^2*c*d-2*EllipticF
((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a*b*c^2-2*EllipticF((-1+co
s(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a*b*c*d+EllipticF((-1+cos(f*x+e))
/sin(f*x+e),((a-b)/(a+b))^(1/2))*b^2*c^2+EllipticF((-1+cos(f*x+e))/sin(f*x+
e),((a-b)/(a+b))^(1/2))*b^2*c*d-2*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),-1,
((a-b)/(a+b))^(1/2))*a^2*c^2+2*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),-1,((a
-b)/(a+b))^(1/2))*a^2*d^2-2*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),(c-d)/(c+
d),((a-b)/(a+b))^(1/2))*a^2*d^2+4*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),(c-
d)/(c+d),((a-b)/(a+b))^(1/2))*a*b*c*d-2*EllipticPi((-1+cos(f*x+e))/sin(f*x+
e),(c-d)/(c+d),((a-b)/(a+b))^(1/2))*b^2*c^2)*(-1+cos(f*x+e))/(a*cos(f*x+e)+
b)/sin(f*x+e)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}}{d\sec\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(3/2)/(d*sec(f*x + e) + c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + b \sec \left(e + fx\right)\right)^{\frac{3}{2}}}{c + d \sec \left(e + fx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e)),x)
```

```
[Out] Integral((a + b*sec(e + f*x))**(3/2)/(c + d*sec(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \sec \left(fx + e\right) + a\right)^{\frac{3}{2}}}{d \sec \left(fx + e\right) + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(3/2)/(d*sec(f*x + e) + c), x)
```

3.202 $\int (a+b\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx$

Optimal. Leaf size=442

$$\frac{2\sqrt{a+b}\left(a^2b(45c-23d)+15a^3d-ab^2(35c-17d)+b^3(5c-9d)\right)\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\text{EllipticF}\Big(}{15bf}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(35*a*b*c + 23*a^2*d + 9*b^2*d)*Cot[e + f*x]*Ellipt
icE[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*
(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(15*b
*f) + (2*Sqrt[a + b]*(a^2*b*(45*c - 23*d) - a*b^2*(35*c - 17*d) + b^3*(5*c
- 9*d) + 15*a^3*d)*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/S
qrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((
b*(1 + Sec[e + f*x]))/(a - b))])/(15*b*f) - (2*a^2*Sqrt[a + b]*c*Cot[e + f*
x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a +
 b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*
x]))/(a - b))])/f + (2*b*(5*b*c + 8*a*d)*Sqrt[a + b*Sec[e + f*x]]*Tan[e + f
*x])/(15*f) + (2*b*d*(a + b*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(5*f)
```

Rubi [A] time = 0.629278, antiderivative size = 442, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3918, 4056, 4058, 3921, 3784, 3832, 4004}

$$\frac{2\sqrt{a+b}\left(a^2b(45c-23d)+15a^3d-ab^2(35c-17d)+b^3(5c-9d)\right)\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}F\left(\sin^{-1}\left(\right.\right.}{15bf}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x]),x]
```



```
[Out] (-2*(a - b)*Sqrt[a + b]*(35*a*b*c + 23*a^2*d + 9*b^2*d)*Cot[e + f*x]*Ellipt
icE[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*
(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(15*b
*f) + (2*Sqrt[a + b]*(a^2*b*(45*c - 23*d) - a*b^2*(35*c - 17*d) + b^3*(5*c
- 9*d) + 15*a^3*d)*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/S
qrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((
b*(1 + Sec[e + f*x]))/(a - b))])/(15*b*f) - (2*a^2*Sqrt[a + b]*c*Cot[e + f*
x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a +
 b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*
x]))/(a - b))])/f + (2*b*(5*b*c + 8*a*d)*Sqrt[a + b*Sec[e + f*x]]*Tan[e + f
```

```
*x])/(15*f) + (2*b*d*(a + b*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(5*f)
```

Rule 3918

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 2)*Simp[a^2*c*m +
(b^2*d*(m - 1) + 2*a*b*c*m + a^2*d*m)*Csc[e + f*x] + b*(b*c*m + a*d*(2*m -
1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && GtQ[m, 1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4056

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a +
 b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)*C
sc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int (a+b\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx = \frac{2bd(a+b\sec(e+fx))^{3/2}\tan(e+fx)}{5f}+\frac{2}{5}\int\sqrt{a+b\sec(e+fx)}\left(\frac{5a^2c}{2}\right.$$

$$=\frac{2b(5bc+8ad)\sqrt{a+b\sec(e+fx)}\tan(e+fx)}{15f}+\frac{2bd(a+b\sec(e+fx))}{5f}$$

$$=\frac{2b(5bc+8ad)\sqrt{a+b\sec(e+fx)}\tan(e+fx)}{15f}+\frac{2bd(a+b\sec(e+fx))}{5f}$$

$$=-\frac{2(a-b)\sqrt{a+b}\left(35abc+23a^2d+9b^2d\right)\cot(e+fx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(}}{\sqrt{a+b}}\right.\right.}{15bf}$$

$$=-\frac{2(a-b)\sqrt{a+b}\left(35abc+23a^2d+9b^2d\right)\cot(e+fx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(}}{\sqrt{a+b}}\right.\right.}{15bf}$$

Mathematica [B] time = 25.2858, size = 7168, normalized size = 16.22

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x]),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.687, size = 3285, normalized size = 7.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e)),x)
```

```
[Out] 2/15/f*(1+cos(f*x+e))^2*(1/cos(f*x+e)*(a*cos(f*x+e)+b))^(1/2)*(-1+cos(f*x+e
))^2*(-23*cos(f*x+e)^4*a^3*d+23*sin(f*x+e)*cos(f*x+e)^2*(cos(f*x+e)/(1+cos(
f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*EllipticE((-
1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a^2*b*d+35*sin(f*x+e)*cos(f*x
+e)^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*
x+e)))^(1/2)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a*b^
2*c+9*sin(f*x+e)*cos(f*x+e)^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a
*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),(
(a-b)/(a+b))^(1/2))*a*b^2*d-17*sin(f*x+e)*cos(f*x+e)^3*(cos(f*x+e)/(1+cos(f
*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1
+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a*b^2*d+35*sin(f*x+e)*cos(f*x+
e)^3*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x
+e)))^(1/2)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a^2*b
*c-35*cos(f*x+e)^4*a^2*b*c-11*cos(f*x+e)^4*a^2*b*d-5*cos(f*x+e)^4*a*b^2*c-9
*cos(f*x+e)^4*a*b^2*d+35*cos(f*x+e)^3*a^2*b*c-23*cos(f*x+e)^3*a^2*b*d-35*co
s(f*x+e)^3*a*b^2*c-5*cos(f*x+e)^3*a*b^2*d+34*cos(f*x+e)^2*a^2*b*d-9*sin(f*x
+e)*cos(f*x+e)^3*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b
)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^
(1/2))*b^3*d+23*sin(f*x+e)*cos(f*x+e)^3*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(
1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*EllipticE((-1+cos(f*x+e))/si
n(f*x+e),((a-b)/(a+b))^(1/2))*a^3*d+9*sin(f*x+e)*cos(f*x+e)^3*(cos(f*x+e)/(
1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*Ellipt
icE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*b^3*d-30*sin(f*x+e)*cos
(f*x+e)^3*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+co
s(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),-1,((a-b)/(a+b))^(1/
2))*a^3*c+15*sin(f*x+e)*cos(f*x+e)^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(
a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f
*x+e),((a-b)/(a+b))^(1/2))*a^3*c-15*sin(f*x+e)*cos(f*x+e)^2*(cos(f*x+e)/(1+
cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*Elliptic
F((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a^3*d-5*sin(f*x+e)*cos(f*
x+e)^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f
*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*b^3
*c-9*sin(f*x+e)*cos(f*x+e)^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*
cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((
a-b)/(a+b))^(1/2))*b^3*d+23*sin(f*x+e)*cos(f*x+e)^2*(cos(f*x+e)/(1+cos(f*x+
```

e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*EllipticE((-1+co
s(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a^3*d+9*sin(f*x+e)*cos(f*x+e)^2*(
cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^
(1/2)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*b^3*d-30*si
n(f*x+e)*cos(f*x+e)^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x
+e)+b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),-1,((a-b
)/(a+b))^(1/2))*a^3*c+15*sin(f*x+e)*cos(f*x+e)^3*(cos(f*x+e)/(1+cos(f*x+e))
)^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f
*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a^3*c-15*sin(f*x+e)*cos(f*x+e)^3*(co
s(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1
/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a^3*d-5*sin(f
*x+e)*cos(f*x+e)^3*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)
+b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b)
)^(1/2))*b^3*c-23*sin(f*x+e)*cos(f*x+e)^3*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)
(1/(a+b)(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))/
sin(f*x+e),((a-b)/(a+b))^(1/2))*a^2*b*d-35*sin(f*x+e)*cos(f*x+e)^3*(cos(f*x
+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*E
llipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a*b^2*c+23*cos(f*x
+e)^3*a^3*d-9*cos(f*x+e)^3*b^3*d+6*cos(f*x+e)^2*b^3*d-5*cos(f*x+e)^3*b^3*c+
5*cos(f*x+e)*b^3*c+40*cos(f*x+e)^2*a*b^2*c+14*cos(f*x+e)*a*b^2*d-45*sin(f*x
+e)*cos(f*x+e)^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b
)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^
(1/2))*a^2*b*c-23*sin(f*x+e)*cos(f*x+e)^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)
(1/(a+b)(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))/
sin(f*x+e),((a-b)/(a+b))^(1/2))*a^2*b*d-35*sin(f*x+e)*cos(f*x+e)^2*(cos(f*x
+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*E
llipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a*b^2*c-17*sin(f*x
+e)*cos(f*x+e)^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b
)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^
(1/2))*a*b^2*d+35*sin(f*x+e)*cos(f*x+e)^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)
(1/(a+b)(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*EllipticE((-1+cos(f*x+e))/
sin(f*x+e),((a-b)/(a+b))^(1/2))*a^2*b*c-45*sin(f*x+e)*cos(f*x+e)^3*(cos(f*x
+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*E
llipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a^2*b*c+3*b^3*d+23
*sin(f*x+e)*cos(f*x+e)^3*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(
f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)
/(a+b))^(1/2))*a^2*b*d+35*sin(f*x+e)*cos(f*x+e)^3*(cos(f*x+e)/(1+cos(f*x+e)
))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*EllipticE((-1+cos(
f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a*b^2*c+9*sin(f*x+e)*cos(f*x+e)^3*(
cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^
(1/2)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a*b^2*d)/(a
*cos(f*x+e)+b)/cos(f*x+e)^2/sin(f*x+e)^5

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(d\sec\left(fx+e\right)+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(5/2)*(d*sec(f*x + e) + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b^2d\sec\left(fx+e\right)^3+a^2c+\left(b^2c+2\,abd\right)\sec\left(fx+e\right)^2+\left(2\,abc+a^2d\right)\sec\left(fx+e\right)\right)\sqrt{b\sec\left(fx+e\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((b^2*d*sec(f*x + e)^3 + a^2*c + (b^2*c + 2*a*b*d)*sec(f*x + e)^2 +
 (2*a*b*c + a^2*d)*sec(f*x + e))*sqrt(b*sec(f*x + e) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(5/2)*(c+d*sec(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(d\sec\left(fx+e\right)+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(5/2)*(d*sec(f*x + e) + c), x)
```

3.203 $\int \frac{c+d\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx$

Optimal. Leaf size=208

$$\frac{2d\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bf}-\frac{2c\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-s}{}}}{}$$

```
[Out] (2*Sqrt[a + b]*d*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqr
t[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*
(1 + Sec[e + f*x]))/(a - b))])/(b*f) - (2*Sqrt[a + b]*c*Cot[e + f*x]*Ellipt
icPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a -
b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a -
 b))])/(a*f)
```

Rubi [A] time = 0.116644, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3921, 3784, 3832}

$$\frac{2d\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)\middle|\frac{a+b}{a-b}\right)}{bf}-\frac{2c\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] (2*Sqrt[a + b]*d*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqr
t[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*
(1 + Sec[e + f*x]))/(a - b))])/(b*f) - (2*Sqrt[a + b]*c*Cot[e + f*x]*Ellipt
icPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a -
b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a -
 b))])/(a*f)
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{c+d\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx = c\int \frac{1}{\sqrt{a+b\sec(e+fx)}}\,dx + d\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx$$

$$= \frac{2\sqrt{a+b}d\cot(e+fx)F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{bf} - \frac{2\sqrt{a+}}{}$$

Mathematica [A] time = 2.48339, size = 147, normalized size = 0.71

$$-\frac{4\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{\cos(e+fx)}{\cos(e+fx)+1}}\sec(e+fx)\sqrt{\frac{a\cos(e+fx)+b}{(a+b)(\cos(e+fx)+1)}}\left((c-d)\text{EllipticF}\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right),\frac{a-b}{a+b}\right)+2c\Pi\right.}{f\sqrt{a+b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] (-4*Cos[(e + f*x)/2]^2*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[(b + a*Co
s[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*((c - d)*EllipticF[ArcSin[Tan[(e
+ f*x)/2]], (a - b)/(a + b)] + 2*c*EllipticPi[-1, -ArcSin[Tan[(e + f*x)/2]]
, (a - b)/(a + b)])*Sec[e + f*x])/(f*Sqrt[a + b*Sec[e + f*x]])
```

Maple [A] time = 0.325, size = 215, normalized size = 1.

$$-2\frac{\left(1+\cos\left(fx+e\right)\right)^{2}\left(-1+\cos\left(fx+e\right)\right)}{f\left(a\cos\left(fx+e\right)+b\right)\left(\sin\left(fx+e\right)\right)^{2}}\sqrt{\frac{a\cos\left(fx+e\right)+b}{\cos\left(fx+e\right)}}\sqrt{\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\sqrt{\frac{a\cos\left(fx+e\right)+b}{\left(a+b\right)\left(1+\cos\left(fx+e\right)\right)}}\left(\text{Elliptic}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x)
```

```
[Out] -2/f*(1/cos(f*x+e)*(a*cos(f*x+e)+b))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2
)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(1+cos(f*x+e))^2*(Ellipti
cF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*c-EllipticF((-1+cos(f*x+
e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*d-2*EllipticPi((-1+cos(f*x+e))/sin(f*x+
e),-1,((a-b)/(a+b))^(1/2))*c)*(-1+cos(f*x+e))/(a*cos(f*x+e)+b)/sin(f*x+e)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d\sec\left(fx+e\right)+c}{\sqrt{b\sec\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e) + c)/sqrt(b*sec(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{d\sec\left(fx+e\right)+c}{\sqrt{b\sec\left(fx+e\right)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*sec(f*x + e) + c)/sqrt(b*sec(f*x + e) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{c + d\sec\left(e + fx\right)}{\sqrt{a + b\sec\left(e + fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((c + d*sec(e + f*x))/sqrt(a + b*sec(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d\sec\left(fx + e\right) + c}{\sqrt{b\sec\left(fx + e\right) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e) + c)/sqrt(b*sec(f*x + e) + a), x)
```

3.204 $\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx$

Optimal. Leaf size=216

$$-\frac{2d\tan(e+fx)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\Pi\left(\frac{2d}{c+d};\sin^{-1}\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right)|\frac{2b}{a+b}\right)}{cf(c+d)\sqrt{-\tan^2(e+fx)}\sqrt{a+b\sec(e+fx)}}-\frac{2\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\Pi\Big(}{acf}$$

```
[Out] (-2*Sqrt[a + b]*Cot[e + f*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[e
+ f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]
*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(a*c*f) - (2*d*EllipticPi[(2*d)/(
c + d), ArcSin[Sqrt[1 - Sec[e + f*x]]/Sqrt[2]], (2*b)/(a + b)]*Sqrt[(a + b*
Sec[e + f*x])/(a + b)]*Tan[e + f*x])/(c*(c + d)*f*Sqrt[a + b*Sec[e + f*x]]*
Sqrt[-Tan[e + f*x]^2])
```

Rubi [A] time = 0.238235, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3930, 3784, 3973}

$$-\frac{2d\tan(e+fx)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\Pi\left(\frac{2d}{c+d};\sin^{-1}\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right)|\frac{2b}{a+b}\right)}{cf(c+d)\sqrt{-\tan^2(e+fx)}\sqrt{a+b\sec(e+fx)}}-\frac{2\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\Pi\Big(}{acf}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

```
[Out] (-2*Sqrt[a + b]*Cot[e + f*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[e
+ f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]
*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(a*c*f) - (2*d*EllipticPi[(2*d)/(
c + d), ArcSin[Sqrt[1 - Sec[e + f*x]]/Sqrt[2]], (2*b)/(a + b)]*Sqrt[(a + b*
Sec[e + f*x])/(a + b)]*Tan[e + f*x])/(c*(c + d)*f*Sqrt[a + b*Sec[e + f*x]]*
Sqrt[-Tan[e + f*x]^2])
```

Rule 3930

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))), x_Symbol] :> Dist[1/c, Int[1/Sqrt[a + b*Csc[e + f*x]], x],
x] - Dist[d/c, Int[Csc[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*
x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^
```

```
2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3973

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Simp[(-2*Cot[e + f*x]*Sq
rt[(a + b*Csc[e + f*x])/(a + b)]*EllipticPi[(2*d)/(c + d), ArcSin[Sqrt[1 -
Csc[e + f*x]]/Sqrt[2]], (2*b)/(a + b)])/(f*(c + d)*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[-Cot[e + f*x]^2]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \frac{\int \frac{1}{\sqrt{a+b\sec(e+fx)}}\,dx}{c} - \frac{d\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx}{c}$$

$$= -\frac{2\sqrt{a+b}\cot(e+fx)\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{}}{acf}$$

Mathematica [A] time = 9.95483, size = 254, normalized size = 1.18

$$-\frac{2\sec^{\frac{3}{2}}(e+fx)\sqrt{\sec(e+fx)+1}\sqrt{\cos(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)}\sqrt{\frac{a\cos(e+fx)+b}{(a+b)(\cos(e+fx)+1)}}(c\cos(e+fx)+d)\left(c(c+d)\text{Elliptic}\right.}{cf(c-d)(c+d)\sqrt{\sec^2\left(\frac{1}{2}(e+\right.}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

```
[Out] (-2*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*(d + c*Cos[e +
f*x])*(c*(c + d)*EllipticF[ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)] + 2*(
```

```
c^2 - d^2)*EllipticPi[-1, -ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)] + 2*d
^2*EllipticPi[(c - d)/(c + d), -ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)])
*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^2]*Sec[e + f*x]^(3/2)*Sqrt[1 + Sec[e +
f*x]])/(c*(c - d)*(c + d)*f*Sqrt[Sec[(e + f*x)/2]^2]*Sqrt[a + b*Sec[e + f*x
]]*(c + d*Sec[e + f*x]))
```

Maple [A] time = 0.31, size = 318, normalized size = 1.5

$$-2\,\frac{\left(1+\cos\left(fx+e\right)\right)^{2}\left(-1+\cos\left(fx+e\right)\right)}{cf\left(c+d\right)\left(c-d\right)\left(a\cos\left(fx+e\right)+b\right)\left(\sin\left(fx+e\right)\right)^{2}}\sqrt{\frac{a\cos\left(fx+e\right)+b}{\cos\left(fx+e\right)}}\sqrt{\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\sqrt{\frac{a\cos\left(fx+e\right)+b}{\left(a+b\right)\left(1+\cos\left(fx+\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x)
```

```
[Out] -2/f/c/(c+d)/(c-d)*(1/cos(f*x+e)*(a*cos(f*x+e)+b))^(1/2)*(cos(f*x+e)/(1+cos
(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(1+cos(f*x+
e))^2*(c^2*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))+d*Elli
pticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*c-2*c^2*EllipticPi((-
1+cos(f*x+e))/sin(f*x+e),-1,((a-b)/(a+b))^(1/2))+2*EllipticPi((-1+cos(f*x+e
))/sin(f*x+e),-1,((a-b)/(a+b))^(1/2))*d^2-2*EllipticPi((-1+cos(f*x+e))/sin(
f*x+e),(c-d)/(c+d),((a-b)/(a+b))^(1/2))*d^2)*(-1+cos(f*x+e))/(a*cos(f*x+e)+
b)/sin(f*x+e)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sec\left(fx+e\right)+a}\left(d\sec\left(fx+e\right)+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + b\sec\left(e + fx\right)}\left(c + d\sec\left(e + fx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*sec(e + f*x))*(c + d*sec(e + f*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sec\left(fx + e\right) + a}\left(d\sec\left(fx + e\right) + c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c)), x)
```

3.205 $\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=376

$$-\frac{2(bc-ad)\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{abf\sqrt{a+b}}+\frac{2b(bc-ad)\tan(e+f}{af\left(a^2-b^2\right)\sqrt{a+b\sec(e}$$

```
[Out] (2*(b*c - a*d)*Cot[e + f*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[e + f*x]))/(a - b))])/(a*b*Sqrt[a + b]*f) - (2*(b*c - a*d)*Cot[e + f
*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)
]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b
))])/(a*b*Sqrt[a + b]*f) - (2*Sqrt[a + b]*c*Cot[e + f*x]*EllipticPi[(a + b)
/a, ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*
(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(a^2*
f) + (2*b*(b*c - a*d)*Tan[e + f*x])/(a*(a^2 - b^2)*f*Sqrt[a + b*Sec[e + f*x
]])
```

Rubi [A] time = 0.427559, antiderivative size = 376, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3923, 4058, 3921, 3784, 3832, 4004}

$$\frac{2b(bc-ad)\tan(e+fx)}{af\left(a^2-b^2\right)\sqrt{a+b\sec(e+fx)}}-\frac{2c\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sec[e + f*x])/(a + b*Sec[e + f*x])^(3/2),x]
```


```
[Out] (2*(b*c - a*d)*Cot[e + f*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[e + f*x]))/(a - b))])/(a*b*Sqrt[a + b]*f) - (2*(b*c - a*d)*Cot[e + f
*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)
]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b
))])/(a*b*Sqrt[a + b]*f) - (2*Sqrt[a + b]*c*Cot[e + f*x]*EllipticPi[(a + b)
/a, ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*
(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(a^2*
f) + (2*b*(b*c - a*d)*Tan[e + f*x])/(a*(a^2 - b^2)*f*Sqrt[a + b*Sec[e + f*x
]])
```

Rule 3923

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(b*(b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f
*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)
), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && Ne
Q[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
```

```
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx = \frac{2b(bc-ad)\tan(e+fx)}{a\left(a^2-b^2\right)f\sqrt{a+b\sec(e+fx)}} - \frac{2\int\frac{-\frac{1}{2}\left(a^2-b^2\right)c+\frac{1}{2}a(bc-ad)\sec(e+fx)+\frac{1}{2}b(bc-ad)\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx}{a\left(a^2-b^2\right)}$$

$$= \frac{2b(bc-ad)\tan(e+fx)}{a\left(a^2-b^2\right)f\sqrt{a+b\sec(e+fx)}} - \frac{2\int\frac{-\frac{1}{2}\left(a^2-b^2\right)c+\left(\frac{1}{2}a(bc-ad)-\frac{1}{2}b(bc-ad)\right)\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx}{a\left(a^2-b^2\right)} - \frac{(b(bc-a}{}$$

$$= \frac{2(bc-ad)\cot(e+fx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{ab\sqrt{a+b}f} + \frac{2}{a\left(a^2\right.}$$

$$= \frac{2(bc-ad)\cot(e+fx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{ab\sqrt{a+b}f} - \frac{2(bc}{}$$

Mathematica [C] time = 14.5611, size = 1491, normalized size = 3.97

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sec[e + f*x])/(a + b*Sec[e + f*x])^(3/2),x]
```

```
[Out] ((b + a*Cos[e + f*x])^2*Sec[e + f*x]*(c + d*Sec[e + f*x])*((2*(-(b*c) + a*d
)*Sin[e + f*x])/(a*(a^2 - b^2)) - (2*(-(b^2*c*Sin[e + f*x]) + a*b*d*Sin[e +
 f*x]))/(a*(a^2 - b^2)*(b + a*Cos[e + f*x]))))/(f*(d + c*Cos[e + f*x])*(a +
 b*Sec[e + f*x])^(3/2)) + (2*(b + a*Cos[e + f*x])^(3/2)*Sqrt[Sec[e + f*x]]*
(c + d*Sec[e + f*x])*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2
]^2)/(1 + Tan[(e + f*x)/2]^2)]*(a*b*Sqrt[(-a + b)/(a + b)]*c*Tan[(e + f*x)/
2] + b^2*Sqrt[(-a + b)/(a + b)]*c*Tan[(e + f*x)/2] - a^2*Sqrt[(-a + b)/(a +
 b)]*d*Tan[(e + f*x)/2] - a*b*Sqrt[(-a + b)/(a + b)]*d*Tan[(e + f*x)/2] - 2
*a*b*Sqrt[(-a + b)/(a + b)]*c*Tan[(e + f*x)/2]^3 + 2*a^2*Sqrt[(-a + b)/(a +
 b)]*d*Tan[(e + f*x)/2]^3 + a*b*Sqrt[(-a + b)/(a + b)]*c*Tan[(e + f*x)/2]^5
 - b^2*Sqrt[(-a + b)/(a + b)]*c*Tan[(e + f*x)/2]^5 - a^2*Sqrt[(-a + b)/(a +
 b)]*d*Tan[(e + f*x)/2]^5 + a*b*Sqrt[(-a + b)/(a + b)]*d*Tan[(e + f*x)/2]^5
```

```
 - (2*I)*a^2*c*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a +
b)]*Tan[(e + f*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[(
a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] + (2*I)*b^2*c
*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(e + f
*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[(a + b - a*Tan[
(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] - (2*I)*a^2*c*EllipticPi[-(
(a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2]], (a +
b)/(a - b)]*Tan[(e + f*x)/2]^2*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[(a + b - a
*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] + (2*I)*b^2*c*Elliptic
Pi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2]],
(a + b)/(a - b)]*Tan[(e + f*x)/2]^2*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[(a +
b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] + I*(a - b)*(-(b*
c) + a*d)*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2]], (a
+ b)/(a - b)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*(1 + Tan[(e + f*x)/2]^2)*Sqrt[(a
 + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] + I*(a - b)*(2
*b*c + a*(c - d))*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/
2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*(1 + Tan[(e + f*x)/2]^2)
*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)]))/(a*S
qrt[(-a + b)/(a + b)]*(a^2 - b^2)*f*(d + c*Cos[e + f*x])*(a + b*Sec[e + f*x
])^(3/2)*(-1 + Tan[(e + f*x)/2]^2)*Sqrt[(1 + Tan[(e + f*x)/2]^2)/(1 - Tan[(
e + f*x)/2]^2)]*(a*(-1 + Tan[(e + f*x)/2]^2) - b*(1 + Tan[(e + f*x)/2]^2)))
```

Maple [B] time = 0.327, size = 2009, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(3/2),x)
```

```
[Out] 1/f/a/(a+b)/(a-b)*4^(1/2)*(1/cos(f*x+e)*(a*cos(f*x+e)+b))^(1/2)*(EllipticE(
(-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*(cos(f*x+e)/(1+c
os(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*a^2*d+cos
(f*x+e)*a^2*d-cos(f*x+e)^2*a^2*d-EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b
)/(a+b))^(1/2))*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*co
s(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*b^2*c+EllipticF((-1+cos(f*x+e))/sin(f*x+e
),((a-b)/(a+b))^(1/2))*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b
)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*a*b*c-EllipticF((-1+cos(f*x+e))/si
n(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*
(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*a*b*d+2*sin(f*x+e)*cos(f*x+
e)*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),-1,((a-b)/(a+b))^(1/2))*(cos(f*x+e
)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*b^2
```

```
*c-cos(f*x+e)^2*b^2*c+cos(f*x+e)*b^2*c-cos(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)
))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*EllipticE((-1+cos(
f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*a*b*c+cos(f*x+e)*(cos(f*
x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*
EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*a*b*d+
cos(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+c
os(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))
*sin(f*x+e)*a*b*c-cos(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*
cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((
a-b)/(a+b))^(1/2))*sin(f*x+e)*a*b*d-EllipticF((-1+cos(f*x+e))/sin(f*x+e),((
a-b)/(a+b))^(1/2))*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a
*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*a^2*d-2*EllipticPi((-1+cos(f*x+e))/sin
(f*x+e),-1,((a-b)/(a+b))^(1/2))*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2
)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*a^2*c+2*EllipticPi((-1+co
s(f*x+e))/sin(f*x+e),-1,((a-b)/(a+b))^(1/2))*sin(f*x+e)*(cos(f*x+e)/(1+cos(
f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*b^2*c-cos(f*
x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x
+e)))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f
*x+e)*a^2*d-2*cos(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(
f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),-1,((
a-b)/(a+b))^(1/2))*sin(f*x+e)*a^2*c+cos(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^
(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x
+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*a^2*c+EllipticF((-1+cos(f*x
+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))
^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*a^2*c-cos(f*x+e)*a*b
*c+cos(f*x+e)^2*a*b*c+cos(f*x+e)^2*a*b*d-cos(f*x+e)*a*b*d+cos(f*x+e)*(cos(f
*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)
*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*a^2*d
-cos(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+
cos(f*x+e)))^(1/2)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2)
)*sin(f*x+e)*b^2*c-EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2)
)*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1
+cos(f*x+e)))^(1/2)*a*b*c+EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b)
)^(1/2))*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e
)+b)/(1+cos(f*x+e)))^(1/2)*a*b*d)/(a*cos(f*x+e)+b)/sin(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d\sec\left(fx+e\right)+c}{\left(b\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e) + c)/(b*sec(f*x + e) + a)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{c + d\sec\left(e + fx\right)}{\left(a + b\sec\left(e + fx\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((c + d*sec(e + f*x))/(a + b*sec(e + f*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d\sec\left(fx + e\right) + c}{\left(b\sec\left(fx + e\right) + a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e) + c)/(b*sec(f*x + e) + a)^(3/2), x)
```

3.206 $\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=495

$$-\frac{2\left(6a^2bc+a^2bd-3a^3d-ab^2c-3b^3c\right)\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{3a^2bf(a-b)(a+b)^{3/2}}$$

```
[Out] (2*(7*a^2*b*c - 3*b^3*c - 4*a^3*d)*Cot[e + f*x]*EllipticE[ArcSin[Sqrt[a + b
*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(
a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(3*a^2*(a - b)*b*(a + b)^(
3/2)*f) - (2*(6*a^2*b*c - a*b^2*c - 3*b^3*c - 3*a^3*d + a^2*b*d)*Cot[e + f*
x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]
*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b)
)])/(3*a^2*(a - b)*b*(a + b)^(3/2)*f) - (2*Sqrt[a + b]*c*Cot[e + f*x]*Ellip
ticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a -
 b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a
- b))])/(a^3*f) + (2*b*(b*c - a*d)*Tan[e + f*x])/(3*a*(a^2 - b^2)*f*(a + b*
Sec[e + f*x])^(3/2)) + (2*b*(7*a^2*b*c - 3*b^3*c - 4*a^3*d)*Tan[e + f*x])/(
3*a^2*(a^2 - b^2)^2*f*Sqrt[a + b*Sec[e + f*x]])
```

Rubi [A] time = 0.777503, antiderivative size = 495, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3923, 4060, 4058, 3921, 3784, 3832, 4004}

$$\frac{2b\left(7a^2bc-4a^3d-3b^3c\right)\tan(e+fx)}{3a^2f\left(a^2-b^2\right)^2\sqrt{a+b\sec(e+fx)}}+\frac{2b(bc-ad)\tan(e+fx)}{3af\left(a^2-b^2\right)(a+b\sec(e+fx))^{3/2}}-\frac{2\left(6a^2bc+a^2bd-3a^3d-ab^2c-3b^3c\right)\cot(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sec[e + f*x])/(a + b*Sec[e + f*x])^(5/2),x]
```


```
[Out] (2*(7*a^2*b*c - 3*b^3*c - 4*a^3*d)*Cot[e + f*x]*EllipticE[ArcSin[Sqrt[a + b
*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(
a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(3*a^2*(a - b)*b*(a + b)^(
3/2)*f) - (2*(6*a^2*b*c - a*b^2*c - 3*b^3*c - 3*a^3*d + a^2*b*d)*Cot[e + f*
x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]
*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b)
)])/(3*a^2*(a - b)*b*(a + b)^(3/2)*f) - (2*Sqrt[a + b]*c*Cot[e + f*x]*Ellip
ticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a -
```

```
 b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a
- b))]])/(a^3*f) + (2*b*(b*c - a*d)*Tan[e + f*x])/(3*a*(a^2 - b^2)*f*(a + b*
Sec[e + f*x])^(3/2)) + (2*b*(7*a^2*b*c - 3*b^3*c - 4*a^3*d)*Tan[e + f*x])/(
3*a^2*(a^2 - b^2)^2*f*Sqrt[a + b*Sec[e + f*x]])
```

Rule 3923

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(b*(b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f
*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)
), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && Ne
Q[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
```

```
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{5/2}}\,dx = \frac{2b(bc-ad)\tan(e+fx)}{3a\left(a^2-b^2\right)f(a+b\sec(e+fx))^{3/2}} - \frac{2\int \frac{-\frac{3}{2}\left(a^2-b^2\right)c+\frac{3}{2}a(bc-ad)\sec(e+fx)-\frac{1}{2}b(bc-ad)\sec^2(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx}{3a\left(a^2-b^2\right)}$$

$$= \frac{2b(bc-ad)\tan(e+fx)}{3a\left(a^2-b^2\right)f(a+b\sec(e+fx))^{3/2}} + \frac{2b\left(7a^2bc-3b^3c-4a^3d\right)\tan(e+fx)}{3a^2\left(a^2-b^2\right)^2 f\sqrt{a+b\sec(e+fx)}} + \frac{4\int \frac{\frac{3}{4}\left(a^2-b^2\right)^2 \cdots}{\cdots}}{\cdots}$$

$$= \frac{2b(bc-ad)\tan(e+fx)}{3a\left(a^2-b^2\right)f(a+b\sec(e+fx))^{3/2}} + \frac{2b\left(7a^2bc-3b^3c-4a^3d\right)\tan(e+fx)}{3a^2\left(a^2-b^2\right)^2 f\sqrt{a+b\sec(e+fx)}} + \frac{4\int \frac{\frac{3}{4}\left(a^2-b^2\right)^2 \cdots}{\cdots}}{\cdots}$$

$$= \frac{2\left(7a^2bc-3b^3c-4a^3d\right)\cot(e+fx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(\cdots}{a-\cdots}}\cdots}{3a^2(a-b)b(a+b)^{3/2}f}$$

$$= \frac{2\left(7a^2bc-3b^3c-4a^3d\right)\cot(e+fx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(\cdots}{a-\cdots}}\cdots}{3a^2(a-b)b(a+b)^{3/2}f}$$

Mathematica [C] time = 17.1037, size = 2083, normalized size = 4.21

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sec[e + f*x])/(a + b*Sec[e + f*x])^(5/2),x]
```

```
[Out] ((b + a*Cos[e + f*x])^3*Sec[e + f*x]^2*(c + d*Sec[e + f*x])*((2*(-7*a^2*b*c
 + 3*b^3*c + 4*a^3*d)*Sin[e + f*x])/(3*a^2*(a^2 - b^2)^2) - (2*(b^3*c*Sin[e
 + f*x] - a*b^2*d*Sin[e + f*x]))/(3*a^2*(a^2 - b^2)*(b + a*Cos[e + f*x])^2)
 - (2*(-8*a^2*b^2*c*Sin[e + f*x] + 4*b^4*c*Sin[e + f*x] + 5*a^3*b*d*Sin[e +
 f*x] - a*b^3*d*Sin[e + f*x]))/(3*a^2*(a^2 - b^2)^2*(b + a*Cos[e + f*x]))))
/(f*(d + c*Cos[e + f*x])*(a + b*Sec[e + f*x])^(5/2)) + (2*(b + a*Cos[e + f*
x])^(5/2)*Sec[e + f*x]^(3/2)*(c + d*Sec[e + f*x])*Sqrt[(a + b - a*Tan[(e +
f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(1 + Tan[(e + f*x)/2]^2)]*(7*a^3*b*Sqrt[(
-a + b)/(a + b)]*c*Tan[(e + f*x)/2] + 7*a^2*b^2*Sqrt[(-a + b)/(a + b)]*c*Ta
n[(e + f*x)/2] - 3*a*b^3*Sqrt[(-a + b)/(a + b)]*c*Tan[(e + f*x)/2] - 3*b^4*
Sqrt[(-a + b)/(a + b)]*c*Tan[(e + f*x)/2] - 4*a^4*Sqrt[(-a + b)/(a + b)]*d*
Tan[(e + f*x)/2] - 4*a^3*b*Sqrt[(-a + b)/(a + b)]*d*Tan[(e + f*x)/2] - 14*a
^3*b*Sqrt[(-a + b)/(a + b)]*c*Tan[(e + f*x)/2]^3 + 6*a*b^3*Sqrt[(-a + b)/(a
 + b)]*c*Tan[(e + f*x)/2]^3 + 8*a^4*Sqrt[(-a + b)/(a + b)]*d*Tan[(e + f*x)/
2]^3 + 7*a^3*b*Sqrt[(-a + b)/(a + b)]*c*Tan[(e + f*x)/2]^5 - 7*a^2*b^2*Sqrt
[(-a + b)/(a + b)]*c*Tan[(e + f*x)/2]^5 - 3*a*b^3*Sqrt[(-a + b)/(a + b)]*c*
Tan[(e + f*x)/2]^5 + 3*b^4*Sqrt[(-a + b)/(a + b)]*c*Tan[(e + f*x)/2]^5 - 4*
a^4*Sqrt[(-a + b)/(a + b)]*d*Tan[(e + f*x)/2]^5 + 4*a^3*b*Sqrt[(-a + b)/(a
+ b)]*d*Tan[(e + f*x)/2]^5 - (6*I)*a^4*c*EllipticPi[-((a + b)/(a - b)), I*A
rcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2]], (a + b)/(a - b)]*Sqrt[1 -
Tan[(e + f*x)/2]^2]*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]
^2)/(a + b)] + (12*I)*a^2*b^2*c*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sq
rt[(-a + b)/(a + b)]*Tan[(e + f*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(e +
f*x)/2]^2]*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a +
b)] - (6*I)*b^4*c*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a
 + b)]*Tan[(e + f*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqr
t[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] - (6*I)*a^
4*c*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(e
+ f*x)/2]], (a + b)/(a - b)]*Tan[(e + f*x)/2]^2*Sqrt[1 - Tan[(e + f*x)/2]^2
]*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] + (12
*I)*a^2*b^2*c*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b
)]*Tan[(e + f*x)/2]], (a + b)/(a - b)]*Tan[(e + f*x)/2]^2*Sqrt[1 - Tan[(e +
 f*x)/2]^2]*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a +
 b)] - (6*I)*b^4*c*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(
a + b)]*Tan[(e + f*x)/2]], (a + b)/(a - b)]*Tan[(e + f*x)/2]^2*Sqrt[1 - Tan
[(e + f*x)/2]^2]*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)
```

```
/(a + b)] + I*(a - b)*(-7*a^2*b*c + 3*b^3*c + 4*a^3*d)*EllipticE[I*ArcSinh[
Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(e
+ f*x)/2]^2]*(1 + Tan[(e + f*x)/2]^2)*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 +
b*Tan[(e + f*x)/2]^2)/(a + b)] + I*(a - b)*(-4*a*b^2*c - 6*b^3*c + 3*a^3*(c
 - d) + a^2*b*(9*c + d))*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(e
+ f*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*(1 + Tan[(e + f*x
)/2]^2)*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)]
))/(3*a^2*Sqrt[(-a + b)/(a + b)]*(a^2 - b^2)^2*f*(d + c*Cos[e + f*x])*(a +
b*Sec[e + f*x])^(5/2)*(-1 + Tan[(e + f*x)/2]^2)*Sqrt[(1 + Tan[(e + f*x)/2]^
2)/(1 - Tan[(e + f*x)/2]^2)]*(a*(-1 + Tan[(e + f*x)/2]^2) - b*(1 + Tan[(e +
 f*x)/2]^2)))
```

Maple [B] time = 0.368, size = 5712, normalized size = 11.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d\sec\left(fx+e\right)+c}{\left(b\sec\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e) + c)/(b*sec(f*x + e) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)+a}\left(d\sec\left(fx+e\right)+c\right)}{b^3\sec\left(fx+e\right)^3+3\,ab^2\sec\left(fx+e\right)^2+3\,a^2b\sec\left(fx+e\right)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c)/(b^3*sec(f*x + e)^3
+ 3*a*b^2*sec(f*x + e)^2 + 3*a^2*b*sec(f*x + e) + a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{c+d\sec\left(e+fx\right)}{\left(a+b\sec\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))**(5/2),x)
```

```
[Out] Integral((c + d*sec(e + f*x))/(a + b*sec(e + f*x))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d\sec\left(fx+e\right)+c}{\left(b\sec\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e) + c)/(b*sec(f*x + e) + a)^(5/2), x)
```

3.207 $\int \sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx$

Optimal. Leaf size=389

$$\frac{2\cot(e+fx)(a+b\sec(e+fx))\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b\sec(e+fx))}}\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{\frac{a+b}{c+d}}\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)}{f\sqrt{\frac{a+b}{c+d}}}-$$

```
[Out] (-2*Sqrt[c + d]*Cot[e + f*x]*EllipticPi[(a*(c + d))/((a + b)*c), ArcSin[(Sq
rt[a + b]*Sqrt[c + d*Sec[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sec[e + f*x]])]
, ((a - b)*(c + d))/((a + b)*(c - d))]*Sqrt[-(((b*c - a*d)*(1 - Sec[e + f*x
]))/((c + d)*(a + b*Sec[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sec[e + f*x]))/
((c - d)*(a + b*Sec[e + f*x]))]*(a + b*Sec[e + f*x]))/(Sqrt[a + b]*f) + (2*
Cot[e + f*x]*EllipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[(a + b)/(c +
d)]*Sqrt[c + d*Sec[e + f*x]])/Sqrt[a + b*Sec[e + f*x]]], ((a - b)*(c + d))/
((a + b)*(c - d))]*Sqrt[-(((b*c - a*d)*(1 - Sec[e + f*x]))/((c + d)*(a + b*
Sec[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sec[e + f*x]))/((c - d)*(a + b*Sec[
e + f*x]))]*(a + b*Sec[e + f*x]))/(Sqrt[(a + b)/(c + d)]*f)
```

Rubi [A] time = 0.446606, antiderivative size = 389, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {3932, 3936, 3982}

$$\frac{2\cot(e+fx)(a+b\sec(e+fx))\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b\sec(e+fx))}}\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{\frac{a+b}{c+d}}\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)}{f\sqrt{\frac{a+b}{c+d}}}-$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]],x]
```

```
[Out] (-2*Sqrt[c + d]*Cot[e + f*x]*EllipticPi[(a*(c + d))/((a + b)*c), ArcSin[(Sq
rt[a + b]*Sqrt[c + d*Sec[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sec[e + f*x]])]
, ((a - b)*(c + d))/((a + b)*(c - d))]*Sqrt[-(((b*c - a*d)*(1 - Sec[e + f*x
]))/((c + d)*(a + b*Sec[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sec[e + f*x]))/
((c - d)*(a + b*Sec[e + f*x]))]*(a + b*Sec[e + f*x]))/(Sqrt[a + b]*f) + (2*
Cot[e + f*x]*EllipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[(a + b)/(c +
d)]*Sqrt[c + d*Sec[e + f*x]])/Sqrt[a + b*Sec[e + f*x]]], ((a - b)*(c + d))/
((a + b)*(c - d))]*Sqrt[-(((b*c - a*d)*(1 - Sec[e + f*x]))/((c + d)*(a + b*
Sec[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sec[e + f*x]))/((c - d)*(a + b*Sec[
```

```
e + f*x]))]*(a + b*Sec[e + f*x]))/(Sqrt[(a + b)/(c + d)]*f)
```

Rule 3932

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Dist[c, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[c +
d*Csc[e + f*x]], x], x] + Dist[d, Int[(Csc[e + f*x]*Sqrt[a + b*Csc[e + f*x]
])/Sqrt[c + d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
b*c - a*d, 0]
```

Rule 3936

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Simp[(2*(a + b*Csc[e + f*x])*Sqrt[((b*c - a*d)
*(1 + Csc[e + f*x]))/((c - d)*(a + b*Csc[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1
 - Csc[e + f*x]))/((c + d)*(a + b*Csc[e + f*x])))]*EllipticPi[(a*(c + d))/(
c*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Csc[e + f*x]])/Sqrt[a
 + b*Csc[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(c*f*Rt[(a + b)/
(c + d), 2]*Cot[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3982

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(-2*(a + b*Csc[
e + f*x])*Sqrt[-(((b*c - a*d)*(1 - Csc[e + f*x]))/((c + d)*(a + b*Csc[e + f
*x])))]*Sqrt[((b*c - a*d)*(1 + Csc[e + f*x]))/((c - d)*(a + b*Csc[e + f*x])
)]*EllipticPi[(b*(c + d))/(d*(a + b)), ArcSin[(Sqrt[(a + b)/(c + d)]*Sqrt[c
 + d*Csc[e + f*x]])/Sqrt[a + b*Csc[e + f*x]]], ((a - b)*(c + d))/((a + b)*(
c - d))])/(d*f*Sqrt[(a + b)/(c + d)]*Cot[e + f*x]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx = c\int \frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx + d\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx$$

$$= -\frac{2\sqrt{c+d}\cot(e+fx)\Pi\left(\frac{a(c+d)}{(a+b)c};\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sqrt{-\cdots}}{\sqrt{a+b}f}$$

Mathematica [C] time = 32.5168, size = 39925, normalized size = 102.63

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]],x]
```

```
[Out] Result too large to show
```

Maple [A] time = 0.434, size = 543, normalized size = 1.4

$$2\,{\frac {\cos \left(fx+e \right) \left(\sin \left(fx+e \right) \right) ^{2}}{f \left(-1+\cos \left(fx+e \right) \right) \left(d+c\cos \left(fx+e \right) \right) \left(a\cos \left(fx+e \right) +b \right) }\left(2\,{\it EllipticPi} \left({\frac {-1+\cos \left(fx+e \right) }{\sin \left(fx+e \right) }\sqrt {{\frac {a-b}{a+b}}}},-{\frac {a+b}{a-b}},\sqrt {{\frac {c-}{c+}}}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2),x)
```

```
[Out] 2/f/((a-b)/(a+b))^(1/2)*(2*EllipticPi((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/s
in(f*x+e),-(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*a*c+2*Ellip
ticPi((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),(a+b)/(a-b),((c-d)/(c+
d))^(1/2)/((a-b)/(a+b))^(1/2))*b*d-EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))^
(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*a*c+EllipticF((-1+cos(f*x
+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*a*d+El
lipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(
c+d))^(1/2))*b*c-EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),(
(a+b)*(c-d)/(a-b)/(c+d))^(1/2))*b*d)*cos(f*x+e)*sin(f*x+e)^2*(1/cos(f*x+e)*
(a*cos(f*x+e)+b))^(1/2)*((d+c*cos(f*x+e))/cos(f*x+e))^(1/2)*(1/(c+d)*(d+c*c
os(f*x+e))/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^
(1/2)/(-1+cos(f*x+e))/(d+c*cos(f*x+e))/(a*cos(f*x+e)+b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx+e\right)+a}\sqrt{d\sec\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\sec\left(e+fx\right)}\sqrt{c+d\sec\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(1/2)*(c+d*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x))*sqrt(c + d*sec(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx+e\right)+a}\sqrt{d\sec\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c), x)
```

3.208 $\int \frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx$

Optimal. Leaf size=198

$$-\frac{2\sqrt{c+d}\cot(e+fx)(a+b\sec(e+fx))\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b\sec(e+fx))}}\Pi\left(\frac{a(c+d)}{(a+b)c};\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}\right)|\frac{(a-b}{(a+b}\right.}{cf\sqrt{a+b}}$$

```
[Out] (-2*Sqrt[c + d]*Cot[e + f*x]*EllipticPi[(a*(c + d))/((a + b)*c), ArcSin[(Sq
rt[a + b]*Sqrt[c + d*Sec[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sec[e + f*x]])]
, ((a - b)*(c + d))/((a + b)*(c - d))]*Sqrt[-(((b*c - a*d)*(1 - Sec[e + f*x
]))/((c + d)*(a + b*Sec[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sec[e + f*x]))/
((c - d)*(a + b*Sec[e + f*x]))]*(a + b*Sec[e + f*x]))/(Sqrt[a + b]*c*f)
```

Rubi [A] time = 0.106149, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {3936}

$$-\frac{2\sqrt{c+d}\cot(e+fx)(a+b\sec(e+fx))\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b\sec(e+fx))}}\Pi\left(\frac{a(c+d)}{(a+b)c};\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}\right)|\frac{(a-b}{(a+b}\right.}{cf\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]]/Sqrt[c + d*Sec[e + f*x]],x]
```

```
[Out] (-2*Sqrt[c + d]*Cot[e + f*x]*EllipticPi[(a*(c + d))/((a + b)*c), ArcSin[(Sq
rt[a + b]*Sqrt[c + d*Sec[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sec[e + f*x]])]
, ((a - b)*(c + d))/((a + b)*(c - d))]*Sqrt[-(((b*c - a*d)*(1 - Sec[e + f*x
]))/((c + d)*(a + b*Sec[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sec[e + f*x]))/
((c - d)*(a + b*Sec[e + f*x]))]*(a + b*Sec[e + f*x]))/(Sqrt[a + b]*c*f)
```

Rule 3936

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Simp[(2*(a + b*Csc[e + f*x])*Sqrt[((b*c - a*d)
*(1 + Csc[e + f*x]))/((c - d)*(a + b*Csc[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1
 - Csc[e + f*x]))/((c + d)*(a + b*Csc[e + f*x])))]*EllipticPi[(a*(c + d))/(
c*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Csc[e + f*x]])/Sqrt[a
 + b*Csc[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(c*f*Rt[(a + b)/
```

```
(c + d), 2]*Cot[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx = -\frac{2\sqrt{c+d}\cot(e+fx)\Pi\left(\frac{a(c+d)}{(a+b)c};\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{}}{\sqrt{a+b}cf}$$

Mathematica [A] time = 5.1912, size = 336, normalized size = 1.7

$$\frac{4\sin^2\left(\frac{1}{2}(e+fx)\right)\csc(e+fx)\sqrt{a+b\sec(e+fx)}\sqrt{\frac{(c+d)\cot^2\left(\frac{1}{2}(e+fx)\right)}{c-d}}\sqrt{\frac{(a+b)\csc^2\left(\frac{1}{2}(e+fx)\right)(c\cos(e+fx)+d)}{ad-bc}}\left(c(a+b)\text{EllipticF}\left(\right.\right.}{cf(a+b)\sqrt{c+d\sec(e+fx)}\sqrt{\frac{(c+}{}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]]/Sqrt[c + d*Sec[e + f*x]],x]
```

```
[Out] (4*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((a + b)*(d + c*Cos[e +
f*x])*Csc[(e + f*x)/2]^2)/(-(b*c) + a*d)]*Csc[e + f*x]*((a + b)*c*EllipticF
[ArcSin[Sqrt[((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(-(b*c) + a*
d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))] - a*(c + d)*EllipticPi[(b*
c - a*d)/(a*c + b*c), ArcSin[Sqrt[((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*
x)/2]^2)/(-(b*c) + a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))])*Sqrt
[a + b*Sec[e + f*x]]*Sin[(e + f*x)/2]^2)/((a + b)*c*f*Sqrt[((c + d)*(b + a*
Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[c + d*Sec[e + f*x]])
```

Maple [A] time = 0.376, size = 352, normalized size = 1.8

$$-2\,\frac{\cos\left(fx+e\right)\left(\sin\left(fx+e\right)\right)^2}{f\left(-1+\cos\left(fx+e\right)\right)\left(d+c\cos\left(fx+e\right)\right)\left(a\cos\left(fx+e\right)+b\right)}\left(\text{EllipticF}\left(\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\sqrt{\frac{a-b}{a+b}},\sqrt{\frac{(c-d)(a+}{(a-b)(c+}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x)
```

```
[Out] -2/f/((a-b)/(a+b))^(1/2)*(EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin
(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*a-EllipticF((-1+cos(f*x+e))*((a-b)
/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*b-2*EllipticPi((-
1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),-(a+b)/(a-b),((c-d)/(c+d))^(1/
2)/((a-b)/(a+b))^(1/2))*a)*cos(f*x+e)*(1/cos(f*x+e)*(a*cos(f*x+e)+b))^(1/2)
*sin(f*x+e)^2*((d+c*cos(f*x+e))/cos(f*x+e))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))
/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)/(-1+
cos(f*x+e))/(d+c*cos(f*x+e))/(a*cos(f*x+e)+b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sec\left(fx+e\right)+a}}{\sqrt{d\sec\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)/sqrt(d*sec(f*x + e) + c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+b\sec\left(e+fx\right)}}{\sqrt{c+d\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x))/sqrt(c + d*sec(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sec\left(fx+e\right)+a}}{\sqrt{d\sec\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)/sqrt(d*sec(f*x + e) + c), x)
```

3.209 $\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=598

$$-\frac{2d(a-b)\sqrt{a+b}\cot(e+fx)(c+d\sec(e+fx))\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{-\frac{(bc-ad)(\sec(e+fx)+1)}{(a-b)(c+d\sec(e+fx))}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sec(e+}}{\sqrt{a+b}\sqrt{c+d\sec(e+}}\right.\right.}{cf(c-d)\sqrt{c+d}(bc-ad)}$$

```
[Out] (-2*Sqrt[c + d]*Cot[e + f*x]*EllipticPi[(a*(c + d))/((a + b)*c), ArcSin[(Sq
rt[a + b]*Sqrt[c + d*Sec[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sec[e + f*x]])]
, ((a - b)*(c + d))/((a + b)*(c - d))]*Sqrt[-(((b*c - a*d)*(1 - Sec[e + f*x
]))/((c + d)*(a + b*Sec[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sec[e + f*x]))/
((c - d)*(a + b*Sec[e + f*x]))]*(a + b*Sec[e + f*x]))/(Sqrt[a + b]*c^2*f) -
 (2*Sqrt[a + b]*d*Cot[e + f*x]*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sec
[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sec[e + f*x]])], ((a + b)*(c - d))/((a
- b)*(c + d))]*(1 + Sec[e + f*x])*Sqrt[((b*c - a*d)*(1 - Sec[e + f*x]))/((a
 + b)*(c + d*Sec[e + f*x]))])/(c*(c - d)*Sqrt[c + d]*f*Sqrt[-(((b*c - a*d)*
(1 + Sec[e + f*x]))/((a - b)*(c + d*Sec[e + f*x])))]) - (2*(a - b)*Sqrt[a +
 b]*d*Cot[e + f*x]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sec[e + f*x]])/
(Sqrt[a + b]*Sqrt[c + d*Sec[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d)
)]*Sqrt[((b*c - a*d)*(1 - Sec[e + f*x]))/((a + b)*(c + d*Sec[e + f*x]))]*Sq
rt[-(((b*c - a*d)*(1 + Sec[e + f*x]))/((a - b)*(c + d*Sec[e + f*x])))]*(c +
 d*Sec[e + f*x]))/(c*(c - d)*Sqrt[c + d]*(b*c - a*d)*f)
```

Rubi [A] time = 0.90203, antiderivative size = 598, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {3939, 3936, 3986, 3984, 3994}

$$-\frac{2\sqrt{c+d}\cot(e+fx)(a+b\sec(e+fx))\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b\sec(e+fx))}}\,\Pi\left(\frac{a(c+d)}{(a+b)c};\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}\right)|\frac{(a-b}{(a+b}\right.}{c^2f\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]]/(c + d*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-2*Sqrt[c + d]*Cot[e + f*x]*EllipticPi[(a*(c + d))/((a + b)*c), ArcSin[(Sq
rt[a + b]*Sqrt[c + d*Sec[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sec[e + f*x]])]
```

```
, ((a - b)*(c + d))/((a + b)*(c - d))]*Sqrt[-(((b*c - a*d)*(1 - Sec[e + f*x
]))/((c + d)*(a + b*Sec[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sec[e + f*x]))/
((c - d)*(a + b*Sec[e + f*x]))]*(a + b*Sec[e + f*x]))/(Sqrt[a + b]*c^2*f) -
 (2*Sqrt[a + b]*d*Cot[e + f*x]*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sec
[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sec[e + f*x]])], ((a + b)*(c - d))/((a
- b)*(c + d))]*(1 + Sec[e + f*x])*Sqrt[((b*c - a*d)*(1 - Sec[e + f*x]))/((a
 + b)*(c + d*Sec[e + f*x]))]])/(c*(c - d)*Sqrt[c + d]*f*Sqrt[-(((b*c - a*d)*
(1 + Sec[e + f*x]))/((a - b)*(c + d*Sec[e + f*x])))]) - (2*(a - b)*Sqrt[a +
 b]*d*Cot[e + f*x]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sec[e + f*x]])/
(Sqrt[a + b]*Sqrt[c + d*Sec[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d)
)]*Sqrt[((b*c - a*d)*(1 - Sec[e + f*x]))/((a + b)*(c + d*Sec[e + f*x]))]*Sq
rt[-(((b*c - a*d)*(1 + Sec[e + f*x]))/((a - b)*(c + d*Sec[e + f*x])))]*(c +
 d*Sec[e + f*x]))/(c*(c - d)*Sqrt[c + d]*(b*c - a*d)*f)
```

Rule 3939

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(3/2), x_Symbol] :> Dist[1/c, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[
c + d*Csc[e + f*x]], x], x] - Dist[d/c, Int[(Csc[e + f*x]*Sqrt[a + b*Csc[e
+ f*x]])/(c + d*Csc[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3936

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Simp[(2*(a + b*Csc[e + f*x])*Sqrt[((b*c - a*d)
*(1 + Csc[e + f*x]))/((c - d)*(a + b*Csc[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1
 - Csc[e + f*x]))/((c + d)*(a + b*Csc[e + f*x])))]*EllipticPi[(a*(c + d))/(
c*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Csc[e + f*x]])/Sqrt[a
 + b*Csc[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(c*f*Rt[(a + b)/
(c + d), 2]*Cot[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3986

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(3/2), x_Symbol] :> Dist[(a - b)/(c -
d), Int[Csc[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), x
], x] + Dist[(b*c - a*d)/(c - d), Int[(Csc[e + f*x]*(1 + Csc[e + f*x]))/(Sq
rt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(3/2)), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
 0]
```

Rule 3984

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqr
t[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Simp[(-2*(c + d*Csc[
e + f*x])*Sqrt[((b*c - a*d)*(1 - Csc[e + f*x]))/((a + b)*(c + d*Csc[e + f*x
]))]*Sqrt[-(((b*c - a*d)*(1 + Csc[e + f*x]))/((a - b)*(c + d*Csc[e + f*x]))
)]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(Sqrt[a + b*Csc[e + f*x]]/Sqrt[c
 + d*Csc[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*
Rt[(c + d)/(a + b), 2]*Cot[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3994

```
Int[(sec[(e_.) + (f_.)*(x_)]*((A_) + (B_.)*sec[(e_.) + (f_.)*(x_)]))/(Sqrt[
(a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sec[(e_.) + (f_.)*(x_)]
)^(3/2)), x_Symbol] :> Simp[(2*A*(1 + Sec[e + f*x])*Sqrt[((b*c - a*d)*(1 -
Sec[e + f*x]))/((a + b)*(c + d*Sec[e + f*x]))]*EllipticE[ArcSin[(Rt[(c + d)
/(a + b), 2]*Sqrt[a + b*Sec[e + f*x]])/Sqrt[c + d*Sec[e + f*x]]], ((a + b)*
(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*Tan[e +
f*x]*Sqrt[-(((b*c - a*d)*(1 + Sec[e + f*x]))/((a - b)*(c + d*Sec[e + f*x]))
)]), x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx = \frac{\int \frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx}{c} - \frac{d\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx}{c}$$

$$= -\frac{2\sqrt{c+d}\cot(e+fx)\Pi\left(\frac{a(c+d)}{(a+b)c};\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}\right)\middle|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{}}{\sqrt{a+b}c^2f}$$

$$= -\frac{2\sqrt{c+d}\cot(e+fx)\Pi\left(\frac{a(c+d)}{(a+b)c};\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}\right)\middle|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{}}{\sqrt{a+b}c^2f}$$

Mathematica [B] time = 9.25764, size = 1678, normalized size = 2.81

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]]/(c + d*Sec[e + f*x])^(3/2),x]
```

```
[Out] ((d + c*Cos[e + f*x])^(3/2)*Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]]*((4*b*c*(
b*c - a*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*
Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[-(((a + b)*(d + c*Cos[e
 + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d))]*Csc[e + f*x]*EllipticF[ArcSin[Sq
rt[-(((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d))]/Sqrt[2
]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*(c + d)
*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) + 4*(b*c - a*d)*(a*c +
b*d)*((Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[
e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[-(((a + b)*(d + c*Cos[e + f
*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d))]*Csc[e + f*x]*EllipticF[ArcSin[Sqrt[-
(((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d))]/Sqrt[2]],
(2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*(c + d)*Sqr
t[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) - (Sqrt[((c + d)*Cot[(e + f
*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(
b*c - a*d)]*Sqrt[-(((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c -
 a*d))]*Csc[e + f*x]*EllipticPi[(b*c - a*d)/((a + b)*c), ArcSin[Sqrt[-(((a
+ b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d))]/Sqrt[2]], (2*(b
*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*c*Sqrt[b + a*Cos
[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])) + 2*a*d*((Sqrt[(-a + b)/(a + b)]*(a +
 b)*Cos[(e + f*x)/2]*Sqrt[d + c*Cos[e + f*x]]*EllipticE[ArcSin[(Sqrt[(-a +
b)/(a + b)]*Sin[(e + f*x)/2])/Sqrt[(b + a*Cos[e + f*x])/(a + b)]], (2*(b*c
- a*d))/((-a + b)*(c + d))])/(a*c*Sqrt[((a + b)*Cos[(e + f*x)/2]^2)/(b + a*
Cos[e + f*x])]*Sqrt[b + a*Cos[e + f*x]]*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*
Sqrt[((a + b)*(d + c*Cos[e + f*x]))/((c + d)*(b + a*Cos[e + f*x]))]) - (2*(
b*c - a*d)*(((b*c + (a + b)*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*S
qrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[-((
(a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d))]*Csc[e + f*x]
*EllipticF[ArcSin[Sqrt[-(((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/
(b*c - a*d))]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]
^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) - (
(b*c + a*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a
*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[-(((a + b)*(d + c*Cos[
e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d))]*Csc[e + f*x]*EllipticPi[(b*c -
a*d)/((a + b)*c), ArcSin[Sqrt[-(((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)
/2]^2)/(b*c - a*d))]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e +
f*x)/2]^4)/((a + b)*c*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])))/
(a*c) + (Sqrt[d + c*Cos[e + f*x]]*Sin[e + f*x])/(c*Sqrt[b + a*Cos[e + f*x]]
))))/((c - d)*(c + d)*f*Sqrt[b + a*Cos[e + f*x]]*(c + d*Sec[e + f*x])^(3/2)
) + (2*d*(d + c*Cos[e + f*x])*Sqrt[a + b*Sec[e + f*x]]*Tan[e + f*x])/((-c^2
 + d^2)*f*(c + d*Sec[e + f*x])^(3/2))
```

Maple [B] time = 0.484, size = 2847, normalized size = 4.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(3/2),x)
```

```
[Out] 2/f/c/(c+d)/(c-d)/((a-b)/(a+b))^(1/2)*(sin(f*x+e)*cos(f*x+e)*EllipticF((-1+
cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))
*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/
(1+cos(f*x+e)))^(1/2)*a*c*d-sin(f*x+e)*cos(f*x+e)*EllipticF((-1+cos(f*x+e))
*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(1/(c+d)*(
d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+
e)))^(1/2)*b*c*d+sin(f*x+e)*EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/s
in(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos
(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*a*c^2-sin(f
*x+e)*cos(f*x+e)*EllipticE((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),(
(a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(
1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*a*c*d+sin(f*x+e)*cos(f
*x+e)*EllipticE((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)
/(a-b)/(c+d))^(1/2))*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*(1/(a+
b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*b*c*d-b*c*d*((a-b)/(a+b))^(1/2)-s
in(f*x+e)*EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(
c-d)/(a-b)/(c+d))^(1/2))*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*(1
/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*b*c^2+cos(f*x+e)^2*((a-b)/(a+
b))^(1/2)*a*c*d-cos(f*x+e)*((a-b)/(a+b))^(1/2)*a*c*d+cos(f*x+e)*((a-b)/(a+b
))^(1/2)*b*c*d-sin(f*x+e)*EllipticE((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin
(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f
*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*a*d^2+sin(f*x
+e)*EllipticE((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(
a-b)/(c+d))^(1/2))*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*(1/(a+b)
*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*b*d^2-2*sin(f*x+e)*EllipticPi((-1+c
os(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),-(a+b)/(a-b),((c-d)/(c+d))^(1/2)/
((a-b)/(a+b))^(1/2))*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*(1/(a+
b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*a*c^2+2*sin(f*x+e)*EllipticPi((-1
+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),-(a+b)/(a-b),((c-d)/(c+d))^(1/2
)/((a-b)/(a+b))^(1/2))*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*(1/(
a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*a*d^2-cos(f*x+e)^2*((a-b)/(a+b)
)^(1/2)*a*d^2+cos(f*x+e)*((a-b)/(a+b))^(1/2)*a*d^2-cos(f*x+e)*((a-b)/(a+b))
^(1/2)*b*d^2+b*d^2*((a-b)/(a+b))^(1/2)+sin(f*x+e)*cos(f*x+e)*EllipticF((-1+
cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))
*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/
(1+cos(f*x+e)))^(1/2)*a*c^2-sin(f*x+e)*cos(f*x+e)*EllipticF((-1+cos(f*x+e))
```

```
*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(1/(c+d)*(
d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+
e)))^(1/2)*b*c^2-sin(f*x+e)*cos(f*x+e)*EllipticE((-1+cos(f*x+e))*((a-b)/(a+
b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(1/(c+d)*(d+c*cos(f*x
+e))/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*
a*d^2+sin(f*x+e)*cos(f*x+e)*EllipticE((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/s
in(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos
(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*b*d^2-2*sin
(f*x+e)*cos(f*x+e)*EllipticPi((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e
),-(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*(1/(c+d)*(d+c*cos(f
*x+e))/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2
)*a*c^2+2*sin(f*x+e)*cos(f*x+e)*EllipticPi((-1+cos(f*x+e))*((a-b)/(a+b))^(1
/2)/sin(f*x+e),-(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*(1/(c+
d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(
f*x+e)))^(1/2)*a*d^2+EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+
e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*a*c*d*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(
f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-E
llipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/
(c+d))^(1/2))*b*c*d*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*(1/(a+b
)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-EllipticE((-1+cos(f*x+e
))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*a*c*d*(1
/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+
cos(f*x+e)))^(1/2)*sin(f*x+e)+EllipticE((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)
/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*b*c*d*(1/(c+d)*(d+c*cos(f*x+e)
)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*sin
(f*x+e))*cos(f*x+e)*(1/cos(f*x+e)*(a*cos(f*x+e)+b))^(1/2)*((d+c*cos(f*x+e))
/cos(f*x+e))^(1/2)/sin(f*x+e)/(d+c*cos(f*x+e))/(a*cos(f*x+e)+b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b \sec \left(fx+e\right)+a}}{\left(d \sec \left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)/(d*sec(f*x + e) + c)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + b \sec{\left(e + f x \right)}}}{\left(c + d \sec{\left(e + f x \right)}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x))/(c + d*sec(e + f*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b \sec\left(f x + e\right) + a}}{\left(d \sec\left(f x + e\right) + c\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)/(d*sec(f*x + e) + c)^(3/2), x)
```

3.210 $\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=899

$$\frac{2\sqrt{a+b\sec(e+fx)}\sin(e+fx)d^2}{3c\left(c^2-d^2\right)f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}}+\frac{2(a-b)\sqrt{a+b}\left(6bc^3-7adc^2-2bd^2c+3ad^3\right)\sqrt{-\frac{(bc-ad)(1-\cos(e+f}{(a+b)(d+c\cos(e+f}}}{3c^2(c-d}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*d*(6*b*c^3 - 7*a*c^2*d - 2*b*c*d^2 + 3*a*d^3)*Sqrt[-
(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-((
(b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Cos
[e + f*x])^(3/2)*Csc[e + f*x]*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[
e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a -
 b)*(c + d))]*Sqrt[a + b*Sec[e + f*x]])/(3*c^2*(c - d)^2*(c + d)^(3/2)*(b*c
 - a*d)^2*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]) + (2*Sqrt[a
+ b]*(b*c^2*(3*c^2 + 3*c*d - 2*d^2) - a*d*(9*c^3 - 2*c^2*d - 6*c*d^2 + 3*d^
3))*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))
]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*
(d + c*Cos[e + f*x])^(3/2)*Csc[e + f*x]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[
b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c -
 d))/((a - b)*(c + d))]*Sqrt[a + b*Sec[e + f*x]])/(3*c^3*(c - d)^2*(c + d)^
(3/2)*(b*c - a*d)*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]) - (2
*Sqrt[a + b]*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e
+ f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e +
f*x])))]*(d + c*Cos[e + f*x])^(3/2)*Csc[e + f*x]*EllipticPi[((a + b)*c)/(a*
(c + d)), ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d
 + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sqrt[a + b*Sec[e
 + f*x]])/(c^3*Sqrt[c + d]*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*
x]]) + (2*d^2*Sqrt[a + b*Sec[e + f*x]]*Sin[e + f*x])/(3*c*(c^2 - d^2)*f*(d
+ c*Cos[e + f*x])*Sqrt[c + d*Sec[e + f*x]])
```

Rubi [A] time = 2.25756, antiderivative size = 899, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {3942, 3048, 3053, 2811, 2998, 2818, 2996}

$$\frac{2\sqrt{a+b\sec(e+fx)}\sin(e+fx)d^2}{3c\left(c^2-d^2\right)f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}}+\frac{2(a-b)\sqrt{a+b}\left(6bc^3-7adc^2-2bd^2c+3ad^3\right)\sqrt{-\frac{(bc-ad)(1-\cos(e+f}{(a+b)(d+c\cos(e+f}}}{3c^2(c-d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]]/(c + d*Sec[e + f*x])^(5/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*d*(6*b*c^3 - 7*a*c^2*d - 2*b*c*d^2 + 3*a*d^3)*Sqrt[-
(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x]))))]*Sqrt[-((
(b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Cos
[e + f*x])^(3/2)*Csc[e + f*x]*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[
e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a -
 b)*(c + d))]*Sqrt[a + b*Sec[e + f*x]])/(3*c^2*(c - d)^2*(c + d)^(3/2)*(b*c
 - a*d)^2*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]) + (2*Sqrt[a
+ b]*(b*c^2*(3*c^2 + 3*c*d - 2*d^2) - a*d*(9*c^3 - 2*c^2*d - 6*c*d^2 + 3*d^
3))*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))
]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*
(d + c*Cos[e + f*x])^(3/2)*Csc[e + f*x]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[
b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c -
 d))/((a - b)*(c + d))]*Sqrt[a + b*Sec[e + f*x]])/(3*c^3*(c - d)^2*(c + d)^
(3/2)*(b*c - a*d)*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]) - (2
*Sqrt[a + b]*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e
+ f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e +
f*x])))]*(d + c*Cos[e + f*x])^(3/2)*Csc[e + f*x]*EllipticPi[((a + b)*c)/(a*
(c + d)), ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d
 + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sqrt[a + b*Sec[e
 + f*x]])/(c^3*Sqrt[c + d]*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*
x]]) + (2*d^2*Sqrt[a + b*Sec[e + f*x]]*Sin[e + f*x])/(3*c*(c^2 - d^2)*f*(d
+ c*Cos[e + f*x])*Sqrt[c + d*Sec[e + f*x]])
```

<u>Rule 3942</u>

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[(Sqrt[d + c*Sin[e + f*x]]*Sqrt[a + b*Cs
c[e + f*x]])/(Sqrt[b + a*Sin[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int[((b +
 a*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n)/Sin[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && IntegerQ[m + 1/2
] && IntegerQ[n + 1/2] && LeQ[-2, m + n, 0]
```

<u>Rule 3048</u>

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
```

```
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{5/2}}\,dx = \frac{\left(\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\right)\int \frac{\cos^2(e+fx)\sqrt{b+a\cos(e+fx)}}{(d+c\cos(e+fx))^{5/2}}\,dx}{\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}}$$

$$= \frac{2d^2\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{3c\left(c^2-d^2\right)f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}} + \frac{\left(2\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+\right.}{3c\left(c^2-d^2\right)}$$

$$= \frac{2d^2\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{3c\left(c^2-d^2\right)f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}} + \frac{\left(a\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+\right.}{c^3\sqrt{b+a\cos(e+fx)}\sqrt{c+}}$$

$$= -\frac{2\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)\Pi\left(\right.}{c^3\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+d\,\mathrm{se}}}$$

$$= \frac{2(a-b)\sqrt{a+b}d\left(6bc^3-7ac^2d-2bcd^2+3ad^3\right)\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}}{3c^2(c-d)^2(c+d)^{3/2}(bc-ad)^2f\sqrt{b}}$$

Mathematica [B] time = 6.71899, size = 1960, normalized size = 2.18

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]]/(c + d*Sec[e + f*x])^(5/2),x]
```

```
[Out] ((d + c*Cos[e + f*x])^3*Sec[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]]*((2*d^2*Sin
[e + f*x])/(3*c*(c^2 - d^2)*(d + c*Cos[e + f*x])^2) - (2*(6*b*c^3*d*Sin[e +
```

```
f*x] - 7*a*c^2*d^2*Sin[e + f*x] - 2*b*c*d^3*Sin[e + f*x] + 3*a*d^4*Sin[e +
 f*x]))/(3*c*(b*c - a*d)*(c^2 - d^2)^2*(d + c*Cos[e + f*x])))))/(f*(c + d*Se
c[e + f*x])^(5/2)) + ((d + c*Cos[e + f*x])^(5/2)*Sec[e + f*x]^2*Sqrt[a + b*
Sec[e + f*x]]*((4*(b*c - a*d)*(3*b^2*c^4 - 3*a*b*c^3*d - a^2*c^2*d^2 + b^2*
c^2*d^2 - a*b*c*d^3 + a^2*d^4)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*S
qrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[-((
(a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d))]*Csc[e + f*x]
*EllipticF[ArcSin[Sqrt[-(((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/
(b*c - a*d))]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]
^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) + 4
*(b*c - a*d)*(3*a*b*c^4 - 3*a^2*c^3*d + 6*b^2*c^3*d - 7*a*b*c^2*d^2 - a^2*c
*d^3 - 2*b^2*c*d^3 + 4*a*b*d^4)*((Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)
]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[
-(((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d))]*Csc[e + f
*x]*EllipticF[ArcSin[Sqrt[-(((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^
2)/(b*c - a*d))]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)
/2]^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])
 - (Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e +
f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[-(((a + b)*(d + c*Cos[e + f*x])
*Csc[(e + f*x)/2]^2)/(b*c - a*d))]*Csc[e + f*x]*EllipticPi[(b*c - a*d)/((a
+ b)*c), ArcSin[Sqrt[-(((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b
*c - a*d))]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4
)/((a + b)*c*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])) + 2*(6*a*b
*c^3*d - 7*a^2*c^2*d^2 - 2*a*b*c*d^3 + 3*a^2*d^4)*((Sqrt[(-a + b)/(a + b)]*
(a + b)*Cos[(e + f*x)/2]*Sqrt[d + c*Cos[e + f*x]]*EllipticE[ArcSin[(Sqrt[(-
a + b)/(a + b)]*Sin[(e + f*x)/2])/Sqrt[(b + a*Cos[e + f*x])/(a + b)]], (2*(
b*c - a*d))/((-a + b)*(c + d))])/(a*c*Sqrt[((a + b)*Cos[(e + f*x)/2]^2)/(b
+ a*Cos[e + f*x])]*Sqrt[b + a*Cos[e + f*x]]*Sqrt[(b + a*Cos[e + f*x])/(a +
b)]*Sqrt[((a + b)*(d + c*Cos[e + f*x]))/((c + d)*(b + a*Cos[e + f*x]))]) -
(2*(b*c - a*d)*(((b*c + (a + b)*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d
)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt
[-(((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d))]*Csc[e +
f*x]*EllipticF[ArcSin[Sqrt[-(((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]
^2)/(b*c - a*d))]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x
)/2]^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])
 - ((b*c + a*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b
 + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[-(((a + b)*(d + c*
Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d))]*Csc[e + f*x]*EllipticPi[(b*
c - a*d)/((a + b)*c), ArcSin[Sqrt[-(((a + b)*(d + c*Cos[e + f*x])*Csc[(e +
f*x)/2]^2)/(b*c - a*d))]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(
e + f*x)/2]^4)/((a + b)*c*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]
)))/(a*c) + (Sqrt[d + c*Cos[e + f*x]]*Sin[e + f*x])/(c*Sqrt[b + a*Cos[e + f
*x]]))))/(3*c*(c - d)^2*(c + d)^2*(b*c - a*d)*f*Sqrt[b + a*Cos[e + f*x]]*(c
 + d*Sec[e + f*x])^(5/2))
```

Maple [B] time = 0.621, size = 15728, normalized size = 17.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sec\left(fx+e\right)+a}}{\left(d\sec\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)/(d*sec(f*x + e) + c)^(5/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sec\left(fx+e\right)+a}}{\left(d\sec\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)/(d*sec(f*x + e) + c)^(5/2), x)
```

3.211 $\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=744

$$-\frac{2\sqrt{a+b}(bc-a(2c-d))\csc(e+fx)\sqrt{a+b\sec(e+fx)}(c\cos(e+fx)+d)^{3/2}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(c\cos(e+fx)+d)}}\sqrt{-\frac{(bc-ad)(\cos(e+fx)+1)}{(a-b)(c\cos(e+fx)+d)}}\mathrm{F}}{c^2f(c-d)\sqrt{c+d}\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d
 + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d +
 c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])^(3/2)*Csc[e + f*x]*EllipticE[ArcSi
n[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*
x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sqrt[a + b*Sec[e + f*x]])/(c*(c
 - d)*Sqrt[c + d]*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]) - (2
*Sqrt[a + b]*(b*c - a*(2*c - d))*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((
a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a
- b)*(d + c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])^(3/2)*Csc[e + f*x]*Ellipt
icF[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*C
os[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sqrt[a + b*Sec[e + f*x
]])/(c^2*(c - d)*Sqrt[c + d]*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e +
f*x]]) - (2*a*Sqrt[a + b]*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*
(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d
 + c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])^(3/2)*Csc[e + f*x]*EllipticPi[((
a + b)*c)/(a*(c + d)), ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[
a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sqr
t[a + b*Sec[e + f*x]])/(c^2*Sqrt[c + d]*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c +
 d*Sec[e + f*x]])
```

Rubi [A] time = 1.10629, antiderivative size = 744, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {3942, 2798, 2811, 2998, 2818, 2996}

$$-\frac{2\sqrt{a+b}(bc-a(2c-d))\csc(e+fx)\sqrt{a+b\sec(e+fx)}(c\cos(e+fx)+d)^{3/2}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(c\cos(e+fx)+d)}}\sqrt{-\frac{(bc-ad)(\cos(e+fx)+1)}{(a-b)(c\cos(e+fx)+d)}}F}{c^2f(c-d)\sqrt{c+d}\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d
 + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d +
 c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])^(3/2)*Csc[e + f*x]*EllipticE[ArcSi
n[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*
x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sqrt[a + b*Sec[e + f*x]])/(c*(c
 - d)*Sqrt[c + d]*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]) - (2
*Sqrt[a + b]*(b*c - a*(2*c - d))*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((
a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a
- b)*(d + c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])^(3/2)*Csc[e + f*x]*Ellipt
icF[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*C
os[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sqrt[a + b*Sec[e + f*x
]])/(c^2*(c - d)*Sqrt[c + d]*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e +
f*x]]) - (2*a*Sqrt[a + b]*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*
(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d
 + c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])^(3/2)*Csc[e + f*x]*EllipticPi[((
a + b)*c)/(a*(c + d)), ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[
a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sqr
t[a + b*Sec[e + f*x]])/(c^2*Sqrt[c + d]*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c +
 d*Sec[e + f*x]])
```

Rule 3942

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[(Sqrt[d + c*Sin[e + f*x]]*Sqrt[a + b*Cs
c[e + f*x]])/(Sqrt[b + a*Sin[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int[((b +
 a*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n)/Sin[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && IntegerQ[m + 1/2
] && IntegerQ[n + 1/2] && LeQ[-2, m + n, 0]
```

Rule 2798

```
Int[((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)/((a_.) + (b_.)*sin[(e_.) +
 (f_.)*(x_)])^(3/2), x_Symbol] :> Dist[d^2/b^2, Int[Sqrt[a + b*Sin[e + f*x]
]/Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[(b*c - a*d)/b^2, Int[Simp[b*c + a
*d + 2*b*d*Sin[e + f*x], x]/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e +
f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
```

```
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{3/2}}\,dx = \frac{\left(\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\right)\int \frac{(b+a\cos(e+fx))^{3/2}}{(d+c\cos(e+fx))^{3/2}}\,dx}{\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}}$$
$$= \frac{\left(a^2\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\right)\int \frac{\sqrt{d+c\cos(e+fx)}}{\sqrt{b+a\cos(e+fx)}}\,dx}{c^2\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}} + \frac{\left((bc-ad)\sqrt{d+c\cos(e+\right.}{c^2\sqrt{}}$$
$$= -\frac{2a\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)\Pi}{c^2\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+d}}$$
$$= -\frac{2(a-b)\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+}{c(c-d)\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+}}$$

Mathematica [B] time = 9.45425, size = 1720, normalized size = 2.31

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*(d + c*Cos[e + f*x])*(a + b*Sec[e + f*x])^(3/2)*(-(b*c*Sin[e + f*x]) + a
*d*Sin[e + f*x]))/((-c^2 + d^2)*f*(b + a*Cos[e + f*x])*(c + d*Sec[e + f*x])
^(3/2)) + ((d + c*Cos[e + f*x])^(3/2)*(a + b*Sec[e + f*x])^(3/2)*((4*(b*c -
 a*d)*(a*b*c - b^2*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c +
 d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[-(((a + b)*(
d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d))]*Csc[e + f*x]*Elliptic
F[ArcSin[Sqrt[-(((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*
d))]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a +
 b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) + 4*(a^2*c -
 b^2*c)*(b*c - a*d)*((Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c +
 d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[-(((a + b)*(
d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d))]*Csc[e + f*x]*Elliptic
F[ArcSin[Sqrt[-(((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*
d))]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a +
 b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) - (Sqrt[((c
+ d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e
 + f*x)/2]^2)/(b*c - a*d)]*Sqrt[-(((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*
x)/2]^2)/(b*c - a*d))]*Csc[e + f*x]*EllipticPi[(b*c - a*d)/((a + b)*c), Arc
Sin[Sqrt[-(((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d))]/
```

```
Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*c
*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])) + 2*(-(a*b*c) + a^2*d)
*((Sqrt[(-a + b)/(a + b)]*(a + b)*Cos[(e + f*x)/2]*Sqrt[d + c*Cos[e + f*x]]
*EllipticE[ArcSin[(Sqrt[(-a + b)/(a + b)]*Sin[(e + f*x)/2])/Sqrt[(b + a*Cos
[e + f*x])/(a + b)]], (2*(b*c - a*d))/((-a + b)*(c + d))])/(a*c*Sqrt[((a +
b)*Cos[(e + f*x)/2]^2)/(b + a*Cos[e + f*x])]*Sqrt[b + a*Cos[e + f*x]]*Sqrt[
(b + a*Cos[e + f*x])/(a + b)]*Sqrt[((a + b)*(d + c*Cos[e + f*x]))/((c + d)*
(b + a*Cos[e + f*x]))]) - (2*(b*c - a*d)*(((b*c + (a + b)*d)*Sqrt[((c + d)*
Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*
x)/2]^2)/(b*c - a*d)]*Sqrt[-(((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]
^2)/(b*c - a*d))]*Csc[e + f*x]*EllipticF[ArcSin[Sqrt[-(((a + b)*(d + c*Cos[
e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d))]/Sqrt[2]], (2*(b*c - a*d))/((a +
 b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]
*Sqrt[d + c*Cos[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)
/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d
)]*Sqrt[-(((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d))]*C
sc[e + f*x]*EllipticPi[(b*c - a*d)/((a + b)*c), ArcSin[Sqrt[-(((a + b)*(d +
 c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d))]/Sqrt[2]], (2*(b*c - a*d)
)/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*c*Sqrt[b + a*Cos[e + f*x]
]*Sqrt[d + c*Cos[e + f*x]])))/(a*c) + (Sqrt[d + c*Cos[e + f*x]]*Sin[e + f*x
])/(c*Sqrt[b + a*Cos[e + f*x]]))))/((c - d)*(c + d)*f*(b + a*Cos[e + f*x])^
(3/2)*(c + d*Sec[e + f*x])^(3/2))
```

Maple [B] time = 0.442, size = 4298, normalized size = 5.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(3/2),x)
```

```
[Out] 2/f/c/(c+d)/(c-d)/((a-b)/(a+b))^(1/2)*(sin(f*x+e)*cos(f*x+e)*EllipticF((-1+
cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))
*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/
(1+cos(f*x+e)))^(1/2)*a^2*c*d-2*sin(f*x+e)*cos(f*x+e)*EllipticF((-1+cos(f*x
+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(1/(a+
b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(
f*x+e)))^(1/2)*a*b*c*d+2*sin(f*x+e)*cos(f*x+e)*EllipticE((-1+cos(f*x+e))*((
a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(1/(a+b)*(a*c
os(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e))
)^(1/2)*a*b*c*d+EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((
a+b)*(c-d)/(a-b)/(c+d))^(1/2))*a^2*c^2*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x
```

```
+e)))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(
f*x+e)*((a-b)/(a+b))^(1/2)*a*b*c^2+EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))^
(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*b^2*c^2*(1/(a+b)*(a*cos(f
*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1
/2)*sin(f*x+e)-a*b*c*d*((a-b)/(a+b))^(1/2)+a*b*d^2*((a-b)/(a+b))^(1/2)-b^2*
c*d*((a-b)/(a+b))^(1/2)-cos(f*x+e)^2*((a-b)/(a+b))^(1/2)*a^2*d^2+cos(f*x+e)
*((a-b)/(a+b))^(1/2)*a^2*d^2-cos(f*x+e)*((a-b)/(a+b))^(1/2)*b^2*c^2-cos(f*x
+e)*((a-b)/(a+b))^(1/2)*a*b*d^2+cos(f*x+e)*((a-b)/(a+b))^(1/2)*b^2*c*d+cos(
f*x+e)^2*((a-b)/(a+b))^(1/2)*a^2*c*d-cos(f*x+e)^2*((a-b)/(a+b))^(1/2)*a*b*c
^2+b^2*c^2*((a-b)/(a+b))^(1/2)-2*sin(f*x+e)*cos(f*x+e)*EllipticPi((-1+cos(f
*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),-(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-
b)/(a+b))^(1/2))*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(1/(c+d)*(
d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*a^2*c^2-sin(f*x+e)*cos(f*x+e)*Ellipti
cE((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))
^(1/2))*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(1/(c+d)*(d+c*cos(f
*x+e))/(1+cos(f*x+e)))^(1/2)*b^2*c*d-2*sin(f*x+e)*EllipticF((-1+cos(f*x+e))
*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(1/(a+b)*(
a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+
e)))^(1/2)*a*b*c*d+2*sin(f*x+e)*EllipticE((-1+cos(f*x+e))*((a-b)/(a+b))^(1/
2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(1/(a+b)*(a*cos(f*x+e)+b)/(1
+cos(f*x+e)))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*a*b*c*d
-2*sin(f*x+e)*cos(f*x+e)*EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(
f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*
x+e)))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*a*b*c^2+sin(f*
x+e)*cos(f*x+e)*EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((
a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1
/2)*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*b^2*c*d-sin(f*x+e)*cos(
f*x+e)*EllipticE((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d
)/(a-b)/(c+d))^(1/2))*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(1/(c
+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*a^2*c*d+sin(f*x+e)*cos(f*x+e)*El
lipticE((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(
c+d))^(1/2))*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(1/(c+d)*(d+c*
cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*a*b*c^2+sin(f*x+e)*cos(f*x+e)*EllipticE((
-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/
2))*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e
))/(1+cos(f*x+e)))^(1/2)*a*b*d^2+sin(f*x+e)*EllipticF((-1+cos(f*x+e))*((a-b
)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(1/(a+b)*(a*cos(
f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(
1/2)*b^2*c*d-sin(f*x+e)*EllipticE((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f
*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x
+e)))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*a^2*c*d+sin(f*x
+e)*EllipticE((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(
a-b)/(c+d))^(1/2))*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(1/(c+d)
*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*a*b*c^2+EllipticE((-1+cos(f*x+e))*(
(a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*a*b*d^2*(1/(
```

```
a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(1+co
s(f*x+e)))^(1/2)*sin(f*x+e)+2*sin(f*x+e)*cos(f*x+e)*EllipticPi((-1+cos(f*x+
e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),-(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/
(a+b))^(1/2))*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(1/(c+d)*(d+c
*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*a^2*d^2-EllipticE((-1+cos(f*x+e))*((a-b)
/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*b^2*c*d*(1/(a+b)*
(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x
+e)))^(1/2)*sin(f*x+e)-EllipticE((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*
x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*a^2*d^2*(1/(a+b)*(a*cos(f*x+e)+b)/(1+
cos(f*x+e)))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*sin(f*x+
e)-EllipticE((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a
-b)/(c+d))^(1/2))*b^2*c^2*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(
1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-2*EllipticPi((-1+
cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),-(a+b)/(a-b),((c-d)/(c+d))^(1/2)
/((a-b)/(a+b))^(1/2))*a^2*c^2*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/
2)*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+2*sin(f*x+e)*
EllipticPi((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),-(a+b)/(a-b),((c-
d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e
)))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*a^2*d^2+cos(f*x+e
)^2*((a-b)/(a+b))^(1/2)*a*b*c*d-cos(f*x+e)*((a-b)/(a+b))^(1/2)*a^2*c*d+sin(
f*x+e)*cos(f*x+e)*EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),
((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^
(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*b^2*c^2-sin(f*x+e)*co
s(f*x+e)*EllipticE((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c
-d)/(a-b)/(c+d))^(1/2))*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(1/
(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*a^2*d^2-sin(f*x+e)*cos(f*x+e)*
EllipticE((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)
/(c+d))^(1/2))*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(1/(c+d)*(d+
c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*b^2*c^2+sin(f*x+e)*EllipticF((-1+cos(f*
x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(1/(a
+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos
(f*x+e)))^(1/2)*a^2*c*d-2*sin(f*x+e)*EllipticF((-1+cos(f*x+e))*((a-b)/(a+b)
)^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(1/(a+b)*(a*cos(f*x+e)+
b)/(1+cos(f*x+e)))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*a*
b*c^2+sin(f*x+e)*cos(f*x+e)*EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/s
in(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos
(f*x+e)))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*a^2*c^2)*co
s(f*x+e)*(1/cos(f*x+e)*(a*cos(f*x+e)+b))^(1/2)*((d+c*cos(f*x+e))/cos(f*x+e)
)^(1/2)/sin(f*x+e)/(d+c*cos(f*x+e))/(a*cos(f*x+e)+b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(d\sec\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(3/2),x, algorithm="maxim
a")

[Out] integrate((b*sec(f*x + e) + a)^(3/2)/(d*sec(f*x + e) + c)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(3/2),x, algorithm="frica
s")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e))**(3/2),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(d\sec\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*sec(f*x + e) + a)^(3/2)/(d*sec(f*x + e) + c)^(3/2), x)
```

3.212 $\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=919

$$-\frac{2(a-b)\sqrt{a+b}\left(3bc^3-7adc^2+bd^2c+3ad^3\right)\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(\cos(e+fx)+1)}{(a-b)(d+c\cos(e+fx))}}\csc(e+fx)E\left(\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{b+}}{\sqrt{a+b}\sqrt{d+}}\right.\right.}{3c^2(c-d)^2(c+d)^{3/2}(bc-ad)f\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e}}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(3*b*c^3 - 7*a*c^2*d + b*c*d^2 + 3*a*d^3)*Sqrt[-(((
b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*
c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Cos[e
+ f*x])^(3/2)*Csc[e + f*x]*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e +
 f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)
*(c + d))]*Sqrt[a + b*Sec[e + f*x]])/(3*c^2*(c - d)^2*(c + d)^(3/2)*(b*c -
a*d)*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]) - (2*Sqrt[a + b]*
(b^2*c^3*(3*c + d) - 2*a*b*c^2*(3*c^2 + 2*c*d - d^2) + a^2*d*(9*c^3 - 2*c^2
*d - 6*c*d^2 + 3*d^3))*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d
+ c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d +
c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])^(3/2)*Csc[e + f*x]*EllipticF[ArcSin
[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x
]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sqrt[a + b*Sec[e + f*x]])/(3*c^3
*(c - d)^2*(c + d)^(3/2)*(b*c - a*d)*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*
Sec[e + f*x]]) - (2*a*Sqrt[a + b]*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/(
(a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a
 - b)*(d + c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])^(3/2)*Csc[e + f*x]*Ellip
ticPi[((a + b)*c)/(a*(c + d)), ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]]
)/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c +
d))]*Sqrt[a + b*Sec[e + f*x]])/(c^3*Sqrt[c + d]*f*Sqrt[b + a*Cos[e + f*x]]*
Sqrt[c + d*Sec[e + f*x]]) - (2*d*(b*c - a*d)*Sqrt[a + b*Sec[e + f*x]]*Sin[e
 + f*x])/(3*c*(c^2 - d^2)*f*(d + c*Cos[e + f*x])*Sqrt[c + d*Sec[e + f*x]])
```

Rubi [A] time = 2.13244, antiderivative size = 919, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {3942, 2989, 3053, 2811, 2998, 2818, 2996}

$$-\frac{2(a-b)\sqrt{a+b}\left(3bc^3-7adc^2+bd^2c+3ad^3\right)\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(\cos(e+fx)+1)}{(a-b)(d+c\cos(e+fx))}}\csc(e+fx)E\left(\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{b+}}{\sqrt{a+b}\sqrt{d+}}\right.\right.}{3c^2(c-d)^2(c+d)^{3/2}(bc-ad)f\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*(3*b*c^3 - 7*a*c^2*d + b*c*d^2 + 3*a*d^3)*Sqrt[-(((
b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*
c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Cos[e
+ f*x])^(3/2)*Csc[e + f*x]*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e +
 f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)
*(c + d))]*Sqrt[a + b*Sec[e + f*x]])/(3*c^2*(c - d)^2*(c + d)^(3/2)*(b*c -
a*d)*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]) - (2*Sqrt[a + b]*
(b^2*c^3*(3*c + d) - 2*a*b*c^2*(3*c^2 + 2*c*d - d^2) + a^2*d*(9*c^3 - 2*c^2
*d - 6*c*d^2 + 3*d^3))*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d
+ c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d +
c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])^(3/2)*Csc[e + f*x]*EllipticF[ArcSin
[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x
]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sqrt[a + b*Sec[e + f*x]])/(3*c^3
*(c - d)^2*(c + d)^(3/2)*(b*c - a*d)*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*
Sec[e + f*x]]) - (2*a*Sqrt[a + b]*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/(
(a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a
 - b)*(d + c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])^(3/2)*Csc[e + f*x]*Ellip
ticPi[((a + b)*c)/(a*(c + d)), ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]]
)/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c +
d))]*Sqrt[a + b*Sec[e + f*x]])/(c^3*Sqrt[c + d]*f*Sqrt[b + a*Cos[e + f*x]]*
Sqrt[c + d*Sec[e + f*x]]) - (2*d*(b*c - a*d)*Sqrt[a + b*Sec[e + f*x]]*Sin[e
 + f*x])/(3*c*(c^2 - d^2)*f*(d + c*Cos[e + f*x])*Sqrt[c + d*Sec[e + f*x]])
```

<u>Rule 3942</u>

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[(Sqrt[d + c*Sin[e + f*x]]*Sqrt[a + b*Cs
c[e + f*x]])/(Sqrt[b + a*Sin[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int[((b +
 a*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n)/Sin[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && IntegerQ[m + 1/2
] && IntegerQ[n + 1/2] && LeQ[-2, m + n, 0]
```

<u>Rule 2989</u>

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
```

```
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
```

```
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{5/2}}\,dx = \frac{\left(\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\right)\int \frac{\cos(e+fx)(b+a\cos(e+fx))^{3/2}}{(d+c\cos(e+fx))^{5/2}}\,dx}{\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}}$$

$$= -\frac{2d(bc-ad)\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{3c\left(c^2-d^2\right)f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}} + \frac{\left(2\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e\right.}{3c\left(\right.}$$

$$= -\frac{2d(bc-ad)\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{3c\left(c^2-d^2\right)f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}} + \frac{\left(a^2\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(\right.}{c^3\sqrt{b+a\cos(e+fx)}\sqrt{c}}$$

$$= -\frac{2a\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)\Pi\Big(}{c^3\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+d\,\mathrm{s}}}$$

$$= -\frac{2(a-b)\sqrt{a+b}\left(3bc^3-7ac^2d+bcd^2+3ad^3\right)\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}\Big(}{3c^2(c-d)^2(c+d)^{3/2}(bc-ad)f\sqrt{b}}$$

Mathematica [B] time = 6.63544, size = 1930, normalized size = 2.1

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^(5/2),x]
```

[Out] ((d + c*Cos[e + f*x])^3*Sec[e + f*x]*(a + b*Sec[e + f*x])^(3/2)*((2*(-(b*c*
d*Sin[e + f*x]) + a*d^2*Sin[e + f*x]))/(3*c*(c^2 - d^2)*(d + c*Cos[e + f*x]
)^2) + (2*(3*b*c^3*Sin[e + f*x] - 7*a*c^2*d*Sin[e + f*x] + b*c*d^2*Sin[e +
f*x] + 3*a*d^3*Sin[e + f*x]))/(3*c*(c^2 - d^2)^2*(d + c*Cos[e + f*x]))))/(f
*(b + a*Cos[e + f*x])*(c + d*Sec[e + f*x])^(5/2)) + ((d + c*Cos[e + f*x])^(
5/2)*Sec[e + f*x]*(a + b*Sec[e + f*x])^(3/2)*((4*(b*c - a*d)*(3*a*b*c^3 + a
^2*c^2*d - 4*b^2*c^2*d + a*b*c*d^2 - a^2*d^3)*Sqrt[((c + d)*Cot[(e + f*x)/2
]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c -
 a*d)]*Sqrt[-(((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)
)]*Csc[e + f*x]*EllipticF[ArcSin[Sqrt[-(((a + b)*(d + c*Cos[e + f*x])*Csc[(
e + f*x)/2]^2)/(b*c - a*d))]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*S
in[(e + f*x)/2]^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos
[e + f*x]]) + 4*(b*c - a*d)*(3*a^2*c^3 - 3*b^2*c^3 + 4*a*b*c^2*d + a^2*c*d^
2 - b^2*c*d^2 - 4*a*b*d^3)*((Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqr
t[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[-(((a
 + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d))]*Csc[e + f*x]*E
llipticF[ArcSin[Sqrt[-(((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b
*c - a*d))]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4
)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) - (Sq
rt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])
*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[-(((a + b)*(d + c*Cos[e + f*x])*Csc[
(e + f*x)/2]^2)/(b*c - a*d))]*Csc[e + f*x]*EllipticPi[(b*c - a*d)/((a + b)*
c), ArcSin[Sqrt[-(((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c -
a*d))]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a
 + b)*c*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])) + 2*(-3*a*b*c^3
 + 7*a^2*c^2*d - a*b*c*d^2 - 3*a^2*d^3)*((Sqrt[(-a + b)/(a + b)]*(a + b)*Co
s[(e + f*x)/2]*Sqrt[d + c*Cos[e + f*x]]*EllipticE[ArcSin[(Sqrt[(-a + b)/(a
+ b)]*Sin[(e + f*x)/2])/Sqrt[(b + a*Cos[e + f*x])/(a + b)]], (2*(b*c - a*d)
)/((-a + b)*(c + d))]])/(a*c*Sqrt[((a + b)*Cos[(e + f*x)/2]^2)/(b + a*Cos[e
+ f*x])]*Sqrt[b + a*Cos[e + f*x]]*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*Sqrt[(
(a + b)*(d + c*Cos[e + f*x]))/((c + d)*(b + a*Cos[e + f*x]))]) - (2*(b*c -
a*d)*(((b*c + (a + b)*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((
c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[-(((a + b
)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d))]*Csc[e + f*x]*Ellip
ticF[ArcSin[Sqrt[-(((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c -
 a*d))]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((
a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) - ((b*c +
 a*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e
 + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[-(((a + b)*(d + c*Cos[e + f*
x])*Csc[(e + f*x)/2]^2)/(b*c - a*d))]*Csc[e + f*x]*EllipticPi[(b*c - a*d)/(
(a + b)*c), ArcSin[Sqrt[-(((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)
/(b*c - a*d))]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2
]^4)/((a + b)*c*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])))/(a*c)
+ (Sqrt[d + c*Cos[e + f*x]]*Sin[e + f*x])/(c*Sqrt[b + a*Cos[e + f*x]]))))/(
3*c*(c - d)^2*(c + d)^2*f*(b + a*Cos[e + f*x])^(3/2)*(c + d*Sec[e + f*x])^(

```
5/2))
```

Maple [B] time = 0.536, size = 13060, normalized size = 14.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(d\sec\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(3/2)/(d*sec(f*x + e) + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}\sqrt{d\sec\left(fx+e\right)+c}}{d^{3}\sec\left(fx+e\right)^{3}+3\,cd^{2}\sec\left(fx+e\right)^{2}+3\,c^{2}d\sec\left(fx+e\right)+c^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral((b*sec(f*x + e) + a)^(3/2)*sqrt(d*sec(f*x + e) + c)/(d^3*sec(f*x +
 e)^3 + 3*c*d^2*sec(f*x + e)^2 + 3*c^2*d*sec(f*x + e) + c^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(d\sec\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*sec(f*x + e) + a)^(3/2)/(d*sec(f*x + e) + c)^(5/2), x)
```

3.213 $\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=1122

result too large to display

```
[Out] (2*(a - b)*Sqrt[a + b]*(2*a*b*c*d*(35*c^4 - 8*c^2*d^2 + 5*d^4) - a^2*d^2*(5
8*c^4 - 41*c^2*d^2 + 15*d^4) - b^2*(15*c^6 + 19*c^4*d^2 - 2*c^2*d^4))*Sqrt[
-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(
((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Co
s[e + f*x])^(3/2)*Csc[e + f*x]*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos
[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a
- b)*(c + d))]*Sqrt[a + b*Sec[e + f*x]])/(15*c^3*(c - d)^3*(c + d)^(5/2)*(b
*c - a*d)^2*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]) - (2*Sqrt[
a + b]*(b^2*c^3*(15*c^3 + 10*c^2*d + 9*c*d^2 - 2*d^3) - 2*a*b*c^2*(15*c^4 +
 20*c^3*d - 4*c^2*d^2 - 4*c*d^3 + 5*d^4) + a^2*d*(60*c^5 - 2*c^4*d - 66*c^3
*d^2 + 25*c^2*d^3 + 30*c*d^4 - 15*d^5))*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*
x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]
))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])^(3/2)*Csc[e + f*x]
*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[
d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sqrt[a + b*Sec[
e + f*x]])/(15*c^4*(c - d)^3*(c + d)^(5/2)*(b*c - a*d)*f*Sqrt[b + a*Cos[e +
 f*x]]*Sqrt[c + d*Sec[e + f*x]]) - (2*a*Sqrt[a + b]*Sqrt[-(((b*c - a*d)*(1
- Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 +
Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])^(3/2)*
Csc[e + f*x]*EllipticPi[((a + b)*c)/(a*(c + d)), ArcSin[(Sqrt[c + d]*Sqrt[b
 + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c -
d))/((a - b)*(c + d))]*Sqrt[a + b*Sec[e + f*x]])/(c^4*Sqrt[c + d]*f*Sqrt[b
+ a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]) + (2*d^2*(b + a*Cos[e + f*x])*S
qrt[a + b*Sec[e + f*x]]*Sin[e + f*x])/(5*c*(c^2 - d^2)*f*(d + c*Cos[e + f*x
])^2*Sqrt[c + d*Sec[e + f*x]]) - (2*d*(10*b*c^3 - 13*a*c^2*d - 2*b*c*d^2 +
5*a*d^3)*Sqrt[a + b*Sec[e + f*x]]*Sin[e + f*x])/(15*c^2*(c^2 - d^2)^2*f*(d
+ c*Cos[e + f*x])*Sqrt[c + d*Sec[e + f*x]])
```

Rubi [A] time = 3.16279, antiderivative size = 1122, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {3942, 3048, 3047, 3053, 2811, 2998, 2818, 2996}

$$\frac{2(b+a\cos(e+fx))\sqrt{a+b\sec(e+fx)}\sin(e+fx)d^2}{5c\left(c^2-d^2\right)f(d+c\cos(e+fx))^2\sqrt{c+d\sec(e+fx)}}-\frac{2\left(10bc^3-13adc^2-2bd^2c+5ad^3\right)\sqrt{a+b\sec(e+fx)}\sin(e+fx}{15c^2\left(c^2-d^2\right)^2f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^(7/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(2*a*b*c*d*(35*c^4 - 8*c^2*d^2 + 5*d^4) - a^2*d^2*(5
8*c^4 - 41*c^2*d^2 + 15*d^4) - b^2*(15*c^6 + 19*c^4*d^2 - 2*c^2*d^4))*Sqrt[
-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(
((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Co
s[e + f*x])^(3/2)*Csc[e + f*x]*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos
[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a
- b)*(c + d))]*Sqrt[a + b*Sec[e + f*x]])/(15*c^3*(c - d)^3*(c + d)^(5/2)*(b
*c - a*d)^2*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]) - (2*Sqrt[
a + b]*(b^2*c^3*(15*c^3 + 10*c^2*d + 9*c*d^2 - 2*d^3) - 2*a*b*c^2*(15*c^4 +
 20*c^3*d - 4*c^2*d^2 - 4*c*d^3 + 5*d^4) + a^2*d*(60*c^5 - 2*c^4*d - 66*c^3
*d^2 + 25*c^2*d^3 + 30*c*d^4 - 15*d^5))*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*
x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]
))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])^(3/2)*Csc[e + f*x]
*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[
d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sqrt[a + b*Sec[
e + f*x]])/(15*c^4*(c - d)^3*(c + d)^(5/2)*(b*c - a*d)*f*Sqrt[b + a*Cos[e +
 f*x]]*Sqrt[c + d*Sec[e + f*x]]) - (2*a*Sqrt[a + b]*Sqrt[-(((b*c - a*d)*(1
- Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 +
Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])^(3/2)*
Csc[e + f*x]*EllipticPi[((a + b)*c)/(a*(c + d)), ArcSin[(Sqrt[c + d]*Sqrt[b
 + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c -
d))/((a - b)*(c + d))]*Sqrt[a + b*Sec[e + f*x]])/(c^4*Sqrt[c + d]*f*Sqrt[b
+ a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]) + (2*d^2*(b + a*Cos[e + f*x])*S
qrt[a + b*Sec[e + f*x]]*Sin[e + f*x])/(5*c*(c^2 - d^2)*f*(d + c*Cos[e + f*x
])^2*Sqrt[c + d*Sec[e + f*x]]) - (2*d*(10*b*c^3 - 13*a*c^2*d - 2*b*c*d^2 +
5*a*d^3)*Sqrt[a + b*Sec[e + f*x]]*Sin[e + f*x])/(15*c^2*(c^2 - d^2)^2*f*(d
+ c*Cos[e + f*x])*Sqrt[c + d*Sec[e + f*x]])
```

Rule 3942

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[(Sqrt[d + c*Sin[e + f*x]]*Sqrt[a + b*Cs
c[e + f*x]])/(Sqrt[b + a*Sin[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int[((b +
 a*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n)/Sin[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && IntegerQ[m + 1/2
] && IntegerQ[n + 1/2] && LeQ[-2, m + n, 0]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
```

```
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{7/2}}\,dx = \frac{\left(\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\right)\int \frac{\cos^2(e+fx)(b+a\cos(e+fx))^{3/2}}{(d+c\cos(e+fx))^{7/2}}\,dx}{\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}}$$

$$= \frac{2d^2(b+a\cos(e+fx))\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{5c\left(c^2-d^2\right)f(d+c\cos(e+fx))^2\sqrt{c+d\sec(e+fx)}} + \frac{\left(2\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e\right.}{5c}$$

$$= \frac{2d^2(b+a\cos(e+fx))\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{5c\left(c^2-d^2\right)f(d+c\cos(e+fx))^2\sqrt{c+d\sec(e+fx)}} - \frac{2d\left(10bc^3-13ac^2d-2bcd^2+5ad^3\right.}{15c^2\left(c^2-d^2\right)^2 f(d+c\cos(e}$$

$$= \frac{2d^2(b+a\cos(e+fx))\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{5c\left(c^2-d^2\right)f(d+c\cos(e+fx))^2\sqrt{c+d\sec(e+fx)}} - \frac{2d\left(10bc^3-13ac^2d-2bcd^2+5ad^3\right.}{15c^2\left(c^2-d^2\right)^2 f(d+c\cos(e}$$

$$= -\frac{2a\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)\Pi\left(\right.}{c^4\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+d\,\mathrm{s}}}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(2abcd\left(35c^4-8c^2d^2+5d^4\right)-a^2d^2\left(58c^4-41c^2d^2+15d^4\right)-b^2\left(15c^6+19\right.\right.}{15c^3(c}$$

Mathematica [B] time = 7.3191, size = 2355, normalized size = 2.1

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^(7/2),x]
```

```
[Out] ((d + c*Cos[e + f*x])^4*Sec[e + f*x]^2*(a + b*Sec[e + f*x])^(3/2)*((-2*(-(b
*c*d^2*Sin[e + f*x]) + a*d^3*Sin[e + f*x]))/(5*c^2*(c^2 - d^2)*(d + c*Cos[e
 + f*x])^3) - (4*(5*b*c^3*d*Sin[e + f*x] - 8*a*c^2*d^2*Sin[e + f*x] - b*c*d
^3*Sin[e + f*x] + 4*a*d^4*Sin[e + f*x]))/(15*c^2*(c^2 - d^2)^2*(d + c*Cos[e
 + f*x])^2) + (2*(15*b^2*c^6*Sin[e + f*x] - 70*a*b*c^5*d*Sin[e + f*x] + 58*
a^2*c^4*d^2*Sin[e + f*x] + 19*b^2*c^4*d^2*Sin[e + f*x] + 16*a*b*c^3*d^3*Sin
[e + f*x] - 41*a^2*c^2*d^4*Sin[e + f*x] - 2*b^2*c^2*d^4*Sin[e + f*x] - 10*a
*b*c*d^5*Sin[e + f*x] + 15*a^2*d^6*Sin[e + f*x]))/(15*c^2*(b*c - a*d)*(c^2
- d^2)^3*(d + c*Cos[e + f*x]))))/(f*(b + a*Cos[e + f*x])*(c + d*Sec[e + f*x
])^(7/2)) + ((d + c*Cos[e + f*x])^(7/2)*Sec[e + f*x]^2*(a + b*Sec[e + f*x])
```

```
^(3/2)*((4*(b*c - a*d)*(-15*a*b^2*c^6 + 5*a^2*b*c^5*d + 25*b^3*c^5*d + 13*a
^3*c^4*d^2 - 38*a*b^2*c^4*d^2 + 25*a^2*b*c^3*d^3 + 7*b^3*c^3*d^3 - 18*a^3*c
^2*d^4 - 11*a*b^2*c^2*d^4 + 2*a^2*b*c*d^5 + 5*a^3*d^6)*Sqrt[((c + d)*Cot[(e
 + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^
2)/(b*c - a*d)]*Sqrt[-(((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b
*c - a*d))]*Csc[e + f*x]*EllipticF[ArcSin[Sqrt[-(((a + b)*(d + c*Cos[e + f*
x])*Csc[(e + f*x)/2]^2)/(b*c - a*d))]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c
 - d))]*Sin[(e + f*x)/2]^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[
d + c*Cos[e + f*x]]) + 4*(b*c - a*d)*(-15*a^2*b*c^6 + 15*b^3*c^6 + 15*a^3*c
^5*d - 55*a*b^2*c^5*d + 33*a^2*b*c^4*d^2 + 19*b^3*c^4*d^2 + 13*a^3*c^3*d^3
+ 35*a*b^2*c^3*d^3 - 70*a^2*b*c^2*d^4 - 2*b^3*c^2*d^4 + 4*a^3*c*d^5 - 12*a*
b^2*c*d^5 + 20*a^2*b*d^6)*((Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt
[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[-(((a
+ b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d))]*Csc[e + f*x]*El
lipticF[ArcSin[Sqrt[-(((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*
c - a*d))]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)
/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) - (Sqr
t[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*
Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[-(((a + b)*(d + c*Cos[e + f*x])*Csc[(
e + f*x)/2]^2)/(b*c - a*d))]*Csc[e + f*x]*EllipticPi[(b*c - a*d)/((a + b)*c
), ArcSin[Sqrt[-(((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a
*d))]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a
+ b)*c*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])) + 2*(15*a*b^2*c^
6 - 70*a^2*b*c^5*d + 58*a^3*c^4*d^2 + 19*a*b^2*c^4*d^2 + 16*a^2*b*c^3*d^3 -
 41*a^3*c^2*d^4 - 2*a*b^2*c^2*d^4 - 10*a^2*b*c*d^5 + 15*a^3*d^6)*((Sqrt[(-a
 + b)/(a + b)]*(a + b)*Cos[(e + f*x)/2]*Sqrt[d + c*Cos[e + f*x]]*EllipticE[
ArcSin[(Sqrt[(-a + b)/(a + b)]*Sin[(e + f*x)/2])/Sqrt[(b + a*Cos[e + f*x])/
(a + b)]], (2*(b*c - a*d))/((-a + b)*(c + d))])/(a*c*Sqrt[((a + b)*Cos[(e +
 f*x)/2]^2)/(b + a*Cos[e + f*x])]*Sqrt[b + a*Cos[e + f*x]]*Sqrt[(b + a*Cos[
e + f*x])/(a + b)]*Sqrt[((a + b)*(d + c*Cos[e + f*x]))/((c + d)*(b + a*Cos[
e + f*x]))]) - (2*(b*c - a*d)*(((b*c + (a + b)*d)*Sqrt[((c + d)*Cot[(e + f*
x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b
*c - a*d)]*Sqrt[-(((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c -
a*d))]*Csc[e + f*x]*EllipticF[ArcSin[Sqrt[-(((a + b)*(d + c*Cos[e + f*x])*C
sc[(e + f*x)/2]^2)/(b*c - a*d))]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d)
)]*Sin[(e + f*x)/2]^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c
*Cos[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*S
qrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[-((
(a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d))]*Csc[e + f*x]
*EllipticPi[(b*c - a*d)/((a + b)*c), ArcSin[Sqrt[-(((a + b)*(d + c*Cos[e +
f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d))]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*
(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*c*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d +
c*Cos[e + f*x]])))/(a*c) + (Sqrt[d + c*Cos[e + f*x]]*Sin[e + f*x])/(c*Sqrt[
b + a*Cos[e + f*x]]))))/(15*c^2*(c - d)^3*(c + d)^3*(-(b*c) + a*d)*f*(b + a
*Cos[e + f*x])^(3/2)*(c + d*Sec[e + f*x])^(7/2))
```

Maple [B] time = 1.249, size = 39418, normalized size = 35.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(7/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(d\sec\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(7/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(3/2)/(d*sec(f*x + e) + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}\sqrt{d\sec\left(fx+e\right)+c}}{d^4\sec\left(fx+e\right)^4+4\,cd^3\sec\left(fx+e\right)^3+6\,c^2d^2\sec\left(fx+e\right)^2+4\,c^3d\sec\left(fx+e\right)+c^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(7/2),x, algorithm="frica
s")
```

```
[Out] integral((b*sec(f*x + e) + a)^(3/2)*sqrt(d*sec(f*x + e) + c)/(d^4*sec(f*x +
 e)^4 + 4*c*d^3*sec(f*x + e)^3 + 6*c^2*d^2*sec(f*x + e)^2 + 4*c^3*d*sec(f*x
```

```
+ e) + c^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(d\sec\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(7/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*sec(f*x + e) + a)^(3/2)/(d*sec(f*x + e) + c)^(7/2), x)
```

3.214 $\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=891

$$-\frac{2\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(\cos(e+fx)+1)}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)\Pi\left(\frac{(a+b)c}{a(c+d)};\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{b+a\cos(e+fx)}}{\sqrt{a+b}\sqrt{d+c\cos(e+fx)}}\right)\right)}{c^3\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(7*a*c^2 - 4*b*c*d - 3*a*d^2)*Sqrt[-(((b*c - a*d)*(
1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1
+ Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])^(3/2
)*Csc[e + f*x]*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqr
t[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*S
qrt[a + b*Sec[e + f*x]])/(3*c^2*(c - d)^2*(c + d)^(3/2)*f*Sqrt[b + a*Cos[e
+ f*x]]*Sqrt[c + d*Sec[e + f*x]]) + (2*Sqrt[a + b]*(b^2*c^2*(c + 3*d) - a*b
*c*(7*c^2 + 4*c*d - 3*d^2) + a^2*(9*c^3 - 2*c^2*d - 6*c*d^2 + 3*d^3))*Sqrt[
-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(
((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Co
s[e + f*x])^(3/2)*Csc[e + f*x]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos
[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a
- b)*(c + d))]*Sqrt[a + b*Sec[e + f*x]])/(3*c^3*(c - d)^2*(c + d)^(3/2)*f*S
qrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]) - (2*a^2*Sqrt[a + b]*Sqrt
[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-
(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*C
os[e + f*x])^(3/2)*Csc[e + f*x]*EllipticPi[((a + b)*c)/(a*(c + d)), ArcSin[
(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]
])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sqrt[a + b*Sec[e + f*x]])/(c^3*Sq
rt[c + d]*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]) + (2*(b*c -
a*d)^2*Sqrt[a + b*Sec[e + f*x]]*Sin[e + f*x])/(3*c*(c^2 - d^2)*f*(d + c*Cos
[e + f*x])*Sqrt[c + d*Sec[e + f*x]])
```

Rubi [A] time = 2.00892, antiderivative size = 891, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {3942, 2792, 3053, 2811, 2998, 2818, 2996}

$$-\frac{2\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(\cos(e+fx)+1)}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)\Pi\left(\frac{(a+b)c}{a(c+d)};\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{b+a\cos(e+fx)}}{\sqrt{a+b}\sqrt{d+c\cos(e+fx)}}\right)\right)}{c^3\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*(7*a*c^2 - 4*b*c*d - 3*a*d^2)*Sqrt[-(((b*c - a*d)*(
1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1
+ Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])^(3/2
)*Csc[e + f*x]*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqr
t[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*S
qrt[a + b*Sec[e + f*x]])/(3*c^2*(c - d)^2*(c + d)^(3/2)*f*Sqrt[b + a*Cos[e
+ f*x]]*Sqrt[c + d*Sec[e + f*x]]) + (2*Sqrt[a + b]*(b^2*c^2*(c + 3*d) - a*b
*c*(7*c^2 + 4*c*d - 3*d^2) + a^2*(9*c^3 - 2*c^2*d - 6*c*d^2 + 3*d^3))*Sqrt[
-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(
((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Co
s[e + f*x])^(3/2)*Csc[e + f*x]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos
[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a
- b)*(c + d))]*Sqrt[a + b*Sec[e + f*x]])/(3*c^3*(c - d)^2*(c + d)^(3/2)*f*S
qrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]) - (2*a^2*Sqrt[a + b]*Sqrt
[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-
(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*C
os[e + f*x])^(3/2)*Csc[e + f*x]*EllipticPi[((a + b)*c)/(a*(c + d)), ArcSin[
(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]
])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sqrt[a + b*Sec[e + f*x]])/(c^3*Sq
rt[c + d]*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]) + (2*(b*c -
a*d)^2*Sqrt[a + b*Sec[e + f*x]]*Sin[e + f*x])/(3*c*(c^2 - d^2)*f*(d + c*Cos
[e + f*x])*Sqrt[c + d*Sec[e + f*x]])
```

<u>Rule 3942</u>

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[(Sqrt[d + c*Sin[e + f*x]]*Sqrt[a + b*Cs
c[e + f*x]])/(Sqrt[b + a*Sin[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int[((b +
 a*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n)/Sin[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && IntegerQ[m + 1/2
] && IntegerQ[n + 1/2] && LeQ[-2, m + n, 0]
```

<u>Rule 2792</u>

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
```

```
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
```

```
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{5/2}}\,dx = \frac{\left(\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\right)\int \frac{(b+a\cos(e+fx))^{5/2}}{(d+c\cos(e+fx))^{5/2}}\,dx}{\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}}$$

$$= \frac{2(bc-ad)^2\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{3c\left(c^2-d^2\right)f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}} + \frac{\left(2\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e\right.}{}$$

$$= \frac{2(bc-ad)^2\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{3c\left(c^2-d^2\right)f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}} + \frac{\left(a^3\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(\right.}{c^3\sqrt{b+a\cos(e+fx)}\sqrt{c}}$$

$$= -\frac{2a^2\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)\Gamma}{c^3\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+c}}$$

$$= -\frac{2(a-b)\sqrt{a+b}\left(7ac^2-4bcd-3ad^2\right)\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\,c}{3c^2(c-d)^2(c+d)^{3/2}f\sqrt{b+a\,co}}$$

Mathematica [B] time = 6.69505, size = 1996, normalized size = 2.24

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^(5/2),x]
```

```
[Out] ((d + c*Cos[e + f*x])^3*(a + b*Sec[e + f*x])^(5/2)*((2*(b^2*c^2*Sin[e + f*x
] - 2*a*b*c*d*Sin[e + f*x] + a^2*d^2*Sin[e + f*x]))/(3*c*(c^2 - d^2)*(d + c
*Cos[e + f*x])^2) + (2*(7*a*b*c^3*Sin[e + f*x] - 7*a^2*c^2*d*Sin[e + f*x] -
 4*b^2*c^2*d*Sin[e + f*x] + a*b*c*d^2*Sin[e + f*x] + 3*a^2*d^3*Sin[e + f*x]
))/(3*c*(c^2 - d^2)^2*(d + c*Cos[e + f*x]))))/(f*(b + a*Cos[e + f*x])^2*(c
+ d*Sec[e + f*x])^(5/2)) + ((d + c*Cos[e + f*x])^(5/2)*(a + b*Sec[e + f*x])
^(5/2)*((4*(b*c - a*d)*(2*a^2*b*c^3 + b^3*c^3 + a^3*c^2*d - 8*a*b^2*c^2*d +
 2*a^2*b*c*d^2 + 3*b^3*c*d^2 - a^3*d^3)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(
c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]
*Sqrt[-(((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d))]*Csc
[e + f*x]*EllipticF[ArcSin[Sqrt[-(((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*
x)/2]^2)/(b*c - a*d))]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e
+ f*x)/2]^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f
*x]]) + 4*(b*c - a*d)*(3*a^3*c^3 - 7*a*b^2*c^3 + 4*b^3*c^2*d + a^3*c*d^2 +
3*a*b^2*c*d^2 - 4*a^2*b*d^3)*((Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*S
qrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[-((
(a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d))]*Csc[e + f*x]
*EllipticF[ArcSin[Sqrt[-(((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/
(b*c - a*d))]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]
^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) - (
Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x
])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[-(((a + b)*(d + c*Cos[e + f*x])*Cs
c[(e + f*x)/2]^2)/(b*c - a*d))]*Csc[e + f*x]*EllipticPi[(b*c - a*d)/((a + b
)*c), ArcSin[Sqrt[-(((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c
- a*d))]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/(
(a + b)*c*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])) + 2*(-7*a^2*b
*c^3 + 7*a^3*c^2*d + 4*a*b^2*c^2*d - a^2*b*c*d^2 - 3*a^3*d^3)*((Sqrt[(-a +
b)/(a + b)]*(a + b)*Cos[(e + f*x)/2]*Sqrt[d + c*Cos[e + f*x]]*EllipticE[Arc
Sin[(Sqrt[(-a + b)/(a + b)]*Sin[(e + f*x)/2])/Sqrt[(b + a*Cos[e + f*x])/(a
+ b)]], (2*(b*c - a*d))/((-a + b)*(c + d))])/(a*c*Sqrt[((a + b)*Cos[(e + f*
x)/2]^2)/(b + a*Cos[e + f*x])]*Sqrt[b + a*Cos[e + f*x]]*Sqrt[(b + a*Cos[e +
 f*x])/(a + b)]*Sqrt[((a + b)*(d + c*Cos[e + f*x]))/((c + d)*(b + a*Cos[e +
 f*x]))]) - (2*(b*c - a*d)*(((b*c + (a + b)*d)*Sqrt[((c + d)*Cot[(e + f*x)/
2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c
- a*d)]*Sqrt[-(((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d
))]*Csc[e + f*x]*EllipticF[ArcSin[Sqrt[-(((a + b)*(d + c*Cos[e + f*x])*Csc[
(e + f*x)/2]^2)/(b*c - a*d))]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*
Sin[(e + f*x)/2]^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Co
s[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt
[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[-(((a
+ b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d))]*Csc[e + f*x]*El
lipticPi[(b*c - a*d)/((a + b)*c), ArcSin[Sqrt[-(((a + b)*(d + c*Cos[e + f*x
])*Csc[(e + f*x)/2]^2)/(b*c - a*d))]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c
- d))]*Sin[(e + f*x)/2]^4)/((a + b)*c*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*C
os[e + f*x]])))/(a*c) + (Sqrt[d + c*Cos[e + f*x]]*Sin[e + f*x])/(c*Sqrt[b +
```

```
a*Cos[e + f*x]])))/(3*c*(c - d)^2*(c + d)^2*f*(b + a*Cos[e + f*x])^(5/2)*
(c + d*Sec[e + f*x])^(5/2))
```

Maple [B] time = 0.607, size = 15912, normalized size = 17.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(5/2),x)

[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \sec\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(d \sec\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(5/2),x, algorithm="maxim
a")

[Out] integrate((b*sec(f*x + e) + a)^(5/2)/(d*sec(f*x + e) + c)^(5/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(5/2),x, algorithm="frica
s")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sec\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(d\sec\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*sec(f*x + e) + a)^(5/2)/(d*sec(f*x + e) + c)^(5/2), x)
```

3.215 $\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=1150

result too large to display

```
[Out] (2*(a - b)*Sqrt[a + b]*(b^2*c^2*d*(29*c^2 + 3*d^2) - a*b*c*(35*c^4 + 34*c^2
*d^2 - 5*d^4) + a^2*(58*c^4*d - 41*c^2*d^3 + 15*d^5))*Sqrt[-(((b*c - a*d)*(
1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1
+ Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])^(3/2
)*Csc[e + f*x]*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqr
t[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*S
qrt[a + b*Sec[e + f*x]])/(15*c^3*(c - d)^3*(c + d)^(5/2)*(b*c - a*d)*f*Sqrt
[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]) + (2*Sqrt[a + b]*(b^3*c^4*(5
*c^2 + 24*c*d + 3*d^2) - a*b^2*c^3*(35*c^3 + 42*c^2*d + 21*c*d^2 - 2*d^3) +
 a^2*b*c^2*(45*c^4 + 48*c^3*d + c^2*d^2 - 8*c*d^3 + 10*d^4) - a^3*d*(60*c^5
 - 2*c^4*d - 66*c^3*d^2 + 25*c^2*d^3 + 30*c*d^4 - 15*d^5))*Sqrt[-(((b*c - a
*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d
)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])
^(3/2)*Csc[e + f*x]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])
/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d
))]*Sqrt[a + b*Sec[e + f*x]])/(15*c^4*(c - d)^3*(c + d)^(5/2)*(b*c - a*d)*f
*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]) - (2*a^2*Sqrt[a + b]*Sq
rt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt
[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c
*Cos[e + f*x])^(3/2)*Csc[e + f*x]*EllipticPi[((a + b)*c)/(a*(c + d)), ArcSi
n[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*
x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sqrt[a + b*Sec[e + f*x]])/(c^4*
Sqrt[c + d]*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]) - (2*d*(b*
c - a*d)*(b + a*Cos[e + f*x])*Sqrt[a + b*Sec[e + f*x]]*Sin[e + f*x])/(5*c*(
c^2 - d^2)*f*(d + c*Cos[e + f*x])^2*Sqrt[c + d*Sec[e + f*x]]) + (2*(b*c - a
*d)*(5*b*c^3 - 13*a*c^2*d + 3*b*c*d^2 + 5*a*d^3)*Sqrt[a + b*Sec[e + f*x]]*S
in[e + f*x])/(15*c^2*(c^2 - d^2)^2*f*(d + c*Cos[e + f*x])*Sqrt[c + d*Sec[e
+ f*x]])
```

Rubi [A] time = 3.43514, antiderivative size = 1150, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {3942, 2989, 3047, 3053, 2811, 2998, 2818, 2996}

$$-\frac{2\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(\cos(e+fx)+1)}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)\Pi\left(\frac{(a+b)c}{a(c+d)};\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{b+a\cos(e+fx)}}{\sqrt{a+b}\sqrt{d+c\cos(e+fx)}}\right.\right.}{c^4\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^(7/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(b^2*c^2*d*(29*c^2 + 3*d^2) - a*b*c*(35*c^4 + 34*c^2
*d^2 - 5*d^4) + a^2*(58*c^4*d - 41*c^2*d^3 + 15*d^5))*Sqrt[-(((b*c - a*d)*(
1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1
+ Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])^(3/2
)*Csc[e + f*x]*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqr
t[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*S
qrt[a + b*Sec[e + f*x]])/(15*c^3*(c - d)^3*(c + d)^(5/2)*(b*c - a*d)*f*Sqrt
[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]) + (2*Sqrt[a + b]*(b^3*c^4*(5
*c^2 + 24*c*d + 3*d^2) - a*b^2*c^3*(35*c^3 + 42*c^2*d + 21*c*d^2 - 2*d^3) +
 a^2*b*c^2*(45*c^4 + 48*c^3*d + c^2*d^2 - 8*c*d^3 + 10*d^4) - a^3*d*(60*c^5
 - 2*c^4*d - 66*c^3*d^2 + 25*c^2*d^3 + 30*c*d^4 - 15*d^5))*Sqrt[-(((b*c - a
*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d
)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])
^(3/2)*Csc[e + f*x]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])
/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d
))]*Sqrt[a + b*Sec[e + f*x]])/(15*c^4*(c - d)^3*(c + d)^(5/2)*(b*c - a*d)*f
*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]) - (2*a^2*Sqrt[a + b]*Sq
rt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt
[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c
*Cos[e + f*x])^(3/2)*Csc[e + f*x]*EllipticPi[((a + b)*c)/(a*(c + d)), ArcSi
n[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*
x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sqrt[a + b*Sec[e + f*x]])/(c^4*
Sqrt[c + d]*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]) - (2*d*(b*
c - a*d)*(b + a*Cos[e + f*x])*Sqrt[a + b*Sec[e + f*x]]*Sin[e + f*x])/(5*c*(
c^2 - d^2)*f*(d + c*Cos[e + f*x])^2*Sqrt[c + d*Sec[e + f*x]]) + (2*(b*c - a
*d)*(5*b*c^3 - 13*a*c^2*d + 3*b*c*d^2 + 5*a*d^3)*Sqrt[a + b*Sec[e + f*x]]*S
in[e + f*x])/(15*c^2*(c^2 - d^2)^2*f*(d + c*Cos[e + f*x])*Sqrt[c + d*Sec[e
+ f*x]])
```

Rule 3942

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[(Sqrt[d + c*Sin[e + f*x]]*Sqrt[a + b*Cs
c[e + f*x]])/(Sqrt[b + a*Sin[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int[((b +
 a*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n)/Sin[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && IntegerQ[m + 1/2
] && IntegerQ[n + 1/2] && LeQ[-2, m + n, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
```

```
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{7/2}}\,dx = \frac{\left(\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\right)\int \frac{\cos(e+fx)(b+a\cos(e+fx))^{5/2}}{(d+c\cos(e+fx))^{7/2}}\,dx}{\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}}$$

$$= -\frac{2d(bc-ad)(b+a\cos(e+fx))\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{5c\left(c^2-d^2\right)f(d+c\cos(e+fx))^2\sqrt{c+d\sec(e+fx)}} + \frac{\left(2\sqrt{d+c\cos(e+fx)}\sqrt{\right.}{}$$

$$= -\frac{2d(bc-ad)(b+a\cos(e+fx))\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{5c\left(c^2-d^2\right)f(d+c\cos(e+fx))^2\sqrt{c+d\sec(e+fx)}} + \frac{2(bc-ad)\left(5bc^3-13ac\right.}{15c^2\left(c^2-d^2\right.}$$

$$= -\frac{2d(bc-ad)(b+a\cos(e+fx))\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{5c\left(c^2-d^2\right)f(d+c\cos(e+fx))^2\sqrt{c+d\sec(e+fx)}} + \frac{2(bc-ad)\left(5bc^3-13ac\right.}{15c^2\left(c^2-d^2\right.}$$

$$= -\frac{2a^2\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)\Pi}{c^4\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+d}}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(b^2c^2d\left(29c^2+3d^2\right)-abc\left(35c^4+34c^2d^2-5d^4\right)+a^2\left(58c^4d-41c^2d^3+1\right.\right.}{15c^3(c-}$$

Mathematica [B] time = 7.16679, size = 2314, normalized size = 2.01

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^(7/2),x]
```

```
[Out] ((d + c*Cos[e + f*x])^4*Sec[e + f*x]*(a + b*Sec[e + f*x])^(5/2)*((-2*(b^2*c
^2*d*Sin[e + f*x] - 2*a*b*c*d^2*Sin[e + f*x] + a^2*d^3*Sin[e + f*x]))/(5*c^
2*(c^2 - d^2)*(d + c*Cos[e + f*x])^3) + (2*(5*b^2*c^4*Sin[e + f*x] - 21*a*b
*c^3*d*Sin[e + f*x] + 16*a^2*c^2*d^2*Sin[e + f*x] + 3*b^2*c^2*d^2*Sin[e + f
*x] + 5*a*b*c*d^3*Sin[e + f*x] - 8*a^2*d^4*Sin[e + f*x]))/(15*c^2*(c^2 - d^
2)^2*(d + c*Cos[e + f*x])^2) + (2*(35*a*b*c^5*Sin[e + f*x] - 58*a^2*c^4*d*S
in[e + f*x] - 29*b^2*c^4*d*Sin[e + f*x] + 34*a*b*c^3*d^2*Sin[e + f*x] + 41*
a^2*c^2*d^3*Sin[e + f*x] - 3*b^2*c^2*d^3*Sin[e + f*x] - 5*a*b*c*d^4*Sin[e +
 f*x] - 15*a^2*d^5*Sin[e + f*x]))/(15*c^2*(c^2 - d^2)^3*(d + c*Cos[e + f*x]
))))/(f*(b + a*Cos[e + f*x])^2*(c + d*Sec[e + f*x])^(7/2)) + ((d + c*Cos[e
```

```
+ f*x])^(7/2)*Sec[e + f*x]*(a + b*Sec[e + f*x])^(5/2)*((4*(b*c - a*d)*(10*a
^2*b*c^5 + 5*b^3*c^5 + 13*a^3*c^4*d - 48*a*b^2*c^4*d + 15*a^2*b*c^3*d^2 + 2
7*b^3*c^3*d^2 - 18*a^3*c^2*d^3 - 16*a*b^2*c^2*d^3 + 7*a^2*b*c*d^4 + 5*a^3*d
^5)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e +
 f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[-(((a + b)*(d + c*Cos[e + f*x]
)*Csc[(e + f*x)/2]^2)/(b*c - a*d))]*Csc[e + f*x]*EllipticF[ArcSin[Sqrt[-(((
a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d))]/Sqrt[2]], (2*
(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*(c + d)*Sqrt[b
 + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) + 4*(b*c - a*d)*(15*a^3*c^5 -
35*a*b^2*c^5 + 23*a^2*b*c^4*d + 29*b^3*c^4*d + 13*a^3*c^3*d^2 - 5*a*b^2*c^3
*d^2 - 75*a^2*b*c^2*d^3 + 3*b^3*c^2*d^3 + 4*a^3*c*d^4 + 8*a*b^2*c*d^4 + 20*
a^2*b*d^5)*((Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b +
a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[-(((a + b)*(d + c*Cos
[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d))]*Csc[e + f*x]*EllipticF[ArcSin[
Sqrt[-(((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d))]/Sqrt
[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*(c +
d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) - (Sqrt[((c + d)*Cot[
(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2
]^2)/(b*c - a*d)]*Sqrt[-(((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/
(b*c - a*d))]*Csc[e + f*x]*EllipticPi[(b*c - a*d)/((a + b)*c), ArcSin[Sqrt[
-(((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d))]/Sqrt[2]],
 (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*c*Sqrt[b +
 a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])) + 2*(-35*a^2*b*c^5 + 58*a^3*c^4
*d + 29*a*b^2*c^4*d - 34*a^2*b*c^3*d^2 - 41*a^3*c^2*d^3 + 3*a*b^2*c^2*d^3 +
 5*a^2*b*c*d^4 + 15*a^3*d^5)*((Sqrt[(-a + b)/(a + b)]*(a + b)*Cos[(e + f*x)
/2]*Sqrt[d + c*Cos[e + f*x]]*EllipticE[ArcSin[(Sqrt[(-a + b)/(a + b)]*Sin[(
e + f*x)/2])/Sqrt[(b + a*Cos[e + f*x])/(a + b)]], (2*(b*c - a*d))/((-a + b)
*(c + d))]])/(a*c*Sqrt[((a + b)*Cos[(e + f*x)/2]^2)/(b + a*Cos[e + f*x])]*Sq
rt[b + a*Cos[e + f*x]]*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*Sqrt[((a + b)*(d
+ c*Cos[e + f*x]))/((c + d)*(b + a*Cos[e + f*x]))]]) - (2*(b*c - a*d)*(((b*c
 + (a + b)*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b +
 a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[-(((a + b)*(d + c*Co
s[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d))]*Csc[e + f*x]*EllipticF[ArcSin
[Sqrt[-(((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d))]/Sqr
t[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*(c +
 d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) - ((b*c + a*d)*Sqrt[
((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Cs
c[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[-(((a + b)*(d + c*Cos[e + f*x])*Csc[(e
+ f*x)/2]^2)/(b*c - a*d))]*Csc[e + f*x]*EllipticPi[(b*c - a*d)/((a + b)*c),
 ArcSin[Sqrt[-(((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d
))]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a +
b)*c*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])))/(a*c) + (Sqrt[d +
 c*Cos[e + f*x]]*Sin[e + f*x])/(c*Sqrt[b + a*Cos[e + f*x]])))/(15*c^2*(c -
 d)^3*(c + d)^3*f*(b + a*Cos[e + f*x])^(5/2)*(c + d*Sec[e + f*x])^(7/2))
```

Maple [B] time = 1.121, size = 32283, normalized size = 28.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(7/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sec\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(d\sec\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(7/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(5/2)/(d*sec(f*x + e) + c)^(7/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(7/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sec\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(d\sec\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(7/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*sec(f*x + e) + a)^(5/2)/(d*sec(f*x + e) + c)^(7/2), x)
```

3.216 $\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=1428

result too large to display

```
[Out] (2*(a - b)*Sqrt[a + b]*(2*b^3*c^3*d*(133*c^4 + 62*c^2*d^2 - 3*d^4) + 2*a^2*
b*c*d*(406*c^6 + 73*c^4*d^2 + 132*c^2*d^4 - 35*d^6) - a*b^2*c^2*(245*c^6 +
852*c^4*d^2 + 41*c^2*d^4 + 14*d^6) - a^3*(582*c^6*d^2 - 485*c^4*d^4 + 392*c
^2*d^6 - 105*d^8))*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*
Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Co
s[e + f*x])))]*(d + c*Cos[e + f*x])^(3/2)*Csc[e + f*x]*EllipticE[ArcSin[(Sq
rt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])]
, ((a + b)*(c - d))/((a - b)*(c + d))]*Sqrt[a + b*Sec[e + f*x]])/(105*c^4*(
c - d)^4*(c + d)^(7/2)*(b*c - a*d)^2*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*
Sec[e + f*x]]) + (2*Sqrt[a + b]*(b^3*c^4*(35*c^4 + 231*c^3*d + 67*c^2*d^2 +
 57*c*d^3 - 6*d^4) - a*b^2*c^3*(245*c^5 + 413*c^4*d + 439*c^3*d^2 + 53*c^2*
d^3 - 12*c*d^4 + 14*d^5) + a^2*b*c^2*(315*c^6 + 497*c^5*d + 219*c^4*d^2 - 7
3*c^3*d^3 + 208*c^2*d^4 + 56*c*d^5 - 70*d^6) - a^3*d*(525*c^7 + 57*c^6*d -
699*c^5*d^2 + 214*c^4*d^3 + 672*c^3*d^4 - 280*c^2*d^5 - 210*c*d^6 + 105*d^7
))*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]
*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(
d + c*Cos[e + f*x])^(3/2)*Csc[e + f*x]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[b
 + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c -
d))/((a - b)*(c + d))]*Sqrt[a + b*Sec[e + f*x]])/(105*c^5*(c - d)^4*(c + d)
^(7/2)*(b*c - a*d)*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]) - (
2*a^2*Sqrt[a + b]*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*C
os[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos
[e + f*x])))]*(d + c*Cos[e + f*x])^(3/2)*Csc[e + f*x]*EllipticPi[((a + b)*c
)/(a*(c + d)), ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*S
qrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sqrt[a + b*
Sec[e + f*x]])/(c^5*Sqrt[c + d]*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e
 + f*x]]) + (2*d^2*(b + a*Cos[e + f*x])^2*Sqrt[a + b*Sec[e + f*x]]*Sin[e +
f*x])/(7*c*(c^2 - d^2)*f*(d + c*Cos[e + f*x])^3*Sqrt[c + d*Sec[e + f*x]]) -
 (2*d*(14*b*c^3 - 19*a*c^2*d - 2*b*c*d^2 + 7*a*d^3)*(b + a*Cos[e + f*x])*Sq
rt[a + b*Sec[e + f*x]]*Sin[e + f*x])/(35*c^2*(c^2 - d^2)^2*f*(d + c*Cos[e +
 f*x])^2*Sqrt[c + d*Sec[e + f*x]]) - (2*(2*a*b*c*d*(91*c^4 - 2*c^2*d^2 + 7*
d^4) - a^2*d^2*(162*c^4 - 101*c^2*d^2 + 35*d^4) - b^2*(35*c^6 + 67*c^4*d^2
- 6*c^2*d^4))*Sqrt[a + b*Sec[e + f*x]]*Sin[e + f*x])/(105*c^3*(c^2 - d^2)^3
*f*(d + c*Cos[e + f*x])*Sqrt[c + d*Sec[e + f*x]])
```

Rubi [A] time = 5.43789, antiderivative size = 1428, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {3942, 3048, 3047, 3053, 2811, 2998, 2818, 2996}

$$-\frac{2\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(\cos(e+fx)+1)}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)\Pi\left(\frac{(a+b)c}{a(c+d)};\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{b+a\cos(e+fx)}}{\sqrt{a+b}\sqrt{d+c\cos(e+fx)}}\right)\right.}{c^5\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^(9/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(2*b^3*c^3*d*(133*c^4 + 62*c^2*d^2 - 3*d^4) + 2*a^2*
b*c*d*(406*c^6 + 73*c^4*d^2 + 132*c^2*d^4 - 35*d^6) - a*b^2*c^2*(245*c^6 +
852*c^4*d^2 + 41*c^2*d^4 + 14*d^6) - a^3*(582*c^6*d^2 - 485*c^4*d^4 + 392*c
^2*d^6 - 105*d^8))*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*
Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Co
s[e + f*x])))]*(d + c*Cos[e + f*x])^(3/2)*Csc[e + f*x]*EllipticE[ArcSin[(Sq
rt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])]
, ((a + b)*(c - d))/((a - b)*(c + d))]*Sqrt[a + b*Sec[e + f*x]])/(105*c^4*(
c - d)^4*(c + d)^(7/2)*(b*c - a*d)^2*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*
Sec[e + f*x]]) + (2*Sqrt[a + b]*(b^3*c^4*(35*c^4 + 231*c^3*d + 67*c^2*d^2 +
 57*c*d^3 - 6*d^4) - a*b^2*c^3*(245*c^5 + 413*c^4*d + 439*c^3*d^2 + 53*c^2*
d^3 - 12*c*d^4 + 14*d^5) + a^2*b*c^2*(315*c^6 + 497*c^5*d + 219*c^4*d^2 - 7
3*c^3*d^3 + 208*c^2*d^4 + 56*c*d^5 - 70*d^6) - a^3*d*(525*c^7 + 57*c^6*d -
699*c^5*d^2 + 214*c^4*d^3 + 672*c^3*d^4 - 280*c^2*d^5 - 210*c*d^6 + 105*d^7
))*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]
*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(
d + c*Cos[e + f*x])^(3/2)*Csc[e + f*x]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[b
 + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c -
d))/((a - b)*(c + d))]*Sqrt[a + b*Sec[e + f*x]])/(105*c^5*(c - d)^4*(c + d)
^(7/2)*(b*c - a*d)*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]) - (
2*a^2*Sqrt[a + b]*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*C
os[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos
[e + f*x])))]*(d + c*Cos[e + f*x])^(3/2)*Csc[e + f*x]*EllipticPi[((a + b)*c
)/(a*(c + d)), ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*S
qrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sqrt[a + b*
Sec[e + f*x]])/(c^5*Sqrt[c + d]*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e
 + f*x]]) + (2*d^2*(b + a*Cos[e + f*x])^2*Sqrt[a + b*Sec[e + f*x]]*Sin[e +
f*x])/(7*c*(c^2 - d^2)*f*(d + c*Cos[e + f*x])^3*Sqrt[c + d*Sec[e + f*x]]) -
 (2*d*(14*b*c^3 - 19*a*c^2*d - 2*b*c*d^2 + 7*a*d^3)*(b + a*Cos[e + f*x])*Sq
rt[a + b*Sec[e + f*x]]*Sin[e + f*x])/(35*c^2*(c^2 - d^2)^2*f*(d + c*Cos[e +
 f*x])^2*Sqrt[c + d*Sec[e + f*x]]) - (2*(2*a*b*c*d*(91*c^4 - 2*c^2*d^2 + 7*
d^4) - a^2*d^2*(162*c^4 - 101*c^2*d^2 + 35*d^4) - b^2*(35*c^6 + 67*c^4*d^2
```

```
- 6*c^2*d^4))*Sqrt[a + b*Sec[e + f*x]]*Sin[e + f*x])/(105*c^3*(c^2 - d^2)^3
*f*(d + c*Cos[e + f*x])*Sqrt[c + d*Sec[e + f*x]])
```

Rule 3942

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[(Sqrt[d + c*Sin[e + f*x]]*Sqrt[a + b*Cs
c[e + f*x]])/(Sqrt[b + a*Sin[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int[((b +
 a*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n)/Sin[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && IntegerQ[m + 1/2
] && IntegerQ[n + 1/2] && LeQ[-2, m + n, 0]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
```

```
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
```

```
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{9/2}}\,dx = \frac{\left(\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\right)\int \frac{\cos^2(e+fx)(b+a\cos(e+fx))^{5/2}}{(d+c\cos(e+fx))^{9/2}}\,dx}{\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}}$$

$$= \frac{2d^2(b+a\cos(e+fx))^2\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{7c\left(c^2-d^2\right)f(d+c\cos(e+fx))^3\sqrt{c+d\sec(e+fx)}} + \frac{\left(2\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec\right.}{}$$

$$= \frac{2d^2(b+a\cos(e+fx))^2\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{7c\left(c^2-d^2\right)f(d+c\cos(e+fx))^3\sqrt{c+d\sec(e+fx)}} - \frac{2d\left(14bc^3-19ac^2d-2bcd^2+7\right.}{35c^2\left(c^2-d^2\right)^2 f}$$

$$= \frac{2d^2(b+a\cos(e+fx))^2\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{7c\left(c^2-d^2\right)f(d+c\cos(e+fx))^3\sqrt{c+d\sec(e+fx)}} - \frac{2d\left(14bc^3-19ac^2d-2bcd^2+7\right.}{35c^2\left(c^2-d^2\right)^2 f}$$

$$= \frac{2d^2(b+a\cos(e+fx))^2\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{7c\left(c^2-d^2\right)f(d+c\cos(e+fx))^3\sqrt{c+d\sec(e+fx)}} - \frac{2d\left(14bc^3-19ac^2d-2bcd^2+7\right.}{35c^2\left(c^2-d^2\right)^2 f}$$

$$= -\frac{2a^2\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)\Gamma}{c^5\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+}}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(2b^3c^3d\left(133c^4+62c^2d^2-3d^4\right)+2a^2bcd\left(406c^6+73c^4d^2+132c^2d^4-35\right.\right.}{}$$

Mathematica [B] time = 8.17393, size = 2949, normalized size = 2.07

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^(9/2),x]
```

```
[Out] ((d + c*Cos[e + f*x])^5*Sec[e + f*x]^2*(a + b*Sec[e + f*x])^(5/2)*((2*(b^2*
c^2*d^2*Sin[e + f*x] - 2*a*b*c*d^3*Sin[e + f*x] + a^2*d^4*Sin[e + f*x]))/(7
```

c^3(c^2 - d^2)*(d + c*Cos[e + f*x])^4) + (2*(-14*b^2*c^4*d*Sin[e + f*x] + 43*a*b*c^3*d^2*Sin[e + f*x] - 29*a^2*c^2*d^3*Sin[e + f*x] + 2*b^2*c^2*d^3*Sin[e + f*x] - 19*a*b*c*d^4*Sin[e + f*x] + 17*a^2*d^5*Sin[e + f*x]))/(35*c^3*(c^2 - d^2)^2*(d + c*Cos[e + f*x])^3) + (2*(35*b^2*c^6*Sin[e + f*x] - 224*a*b*c^5*d*Sin[e + f*x] + 234*a^2*c^4*d^2*Sin[e + f*x] + 67*b^2*c^4*d^2*Sin[e + f*x] + 52*a*b*c^3*d^3*Sin[e + f*x] - 209*a^2*c^2*d^4*Sin[e + f*x] - 6*b^2*c^2*d^4*Sin[e + f*x] - 20*a*b*c*d^5*Sin[e + f*x] + 71*a^2*d^6*Sin[e + f*x]))/(105*c^3*(c^2 - d^2)^3*(d + c*Cos[e + f*x])^2) + (2*(245*a*b^2*c^8*Sin[e + f*x] - 812*a^2*b*c^7*d*Sin[e + f*x] - 266*b^3*c^7*d*Sin[e + f*x] + 582*a^3*c^6*d^2*Sin[e + f*x] + 852*a*b^2*c^6*d^2*Sin[e + f*x] - 146*a^2*b*c^5*d^3*Sin[e + f*x] - 124*b^3*c^5*d^3*Sin[e + f*x] - 485*a^3*c^4*d^4*Sin[e + f*x] + 41*a*b^2*c^4*d^4*Sin[e + f*x] - 264*a^2*b*c^3*d^5*Sin[e + f*x] + 6*b^3*c^3*d^5*Sin[e + f*x] + 392*a^3*c^2*d^6*Sin[e + f*x] + 14*a*b^2*c^2*d^6*Sin[e + f*x] + 70*a^2*b*c*d^7*Sin[e + f*x] - 105*a^3*d^8*Sin[e + f*x]))/(105*c^3*(b*c - a*d)*(c^2 - d^2)^4*(d + c*Cos[e + f*x]))))/(f*(b + a*Cos[e + f*x])^2*(c + d*Sec[e + f*x])^(9/2)) + ((d + c*Cos[e + f*x])^(9/2)*Sec[e + f*x]^2*(a + b*Sec[e + f*x])^(5/2)*((4*(b*c - a*d)*(-70*a^2*b^2*c^8 - 35*b^4*c^8 - 77*a^3*b*c^7*d + 427*a*b^3*c^7*d + 162*a^4*c^6*d^2 - 522*a^2*b^2*c^6*d^2 - 298*b^4*c^6*d^2 + 348*a^3*b*c^5*d^3 + 666*a*b^3*c^5*d^3 - 263*a^4*c^4*d^4 - 586*a^2*b^2*c^4*d^4 - 51*b^4*c^4*d^4 + 127*a^3*b*c^3*d^5 + 59*a*b^3*c^3*d^5 + 136*a^4*c^2*d^6 + 26*a^2*b^2*c^2*d^6 - 14*a^3*b*c*d^7 - 35*a^4*d^8)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[-(((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d))]*Csc[e + f*x]*EllipticF[ArcSin[Sqrt[-(((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d))]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) + 4*(b*c - a*d)*(-105*a^3*b*c^8 + 245*a*b^3*c^8 + 105*a^4*c^7*d - 567*a^2*b^2*c^7*d - 266*b^4*c^7*d + 190*a^3*b*c^6*d^2 + 586*a*b^3*c^6*d^2 + 162*a^4*c^5*d^3 + 706*a^2*b^2*c^5*d^3 - 124*b^4*c^5*d^3 - 1261*a^3*b*c^4*d^4 - 83*a*b^3*c^4*d^4 + 145*a^4*c^3*d^5 - 223*a^2*b^2*c^3*d^5 + 6*b^4*c^3*d^5 + 548*a^3*b*c^2*d^6 + 20*a*b^3*c^2*d^6 - 28*a^4*c*d^7 + 84*a^2*b^2*c*d^7 - 140*a^3*b*d^8)*((Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[-(((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d))]*Csc[e + f*x]*EllipticF[ArcSin[Sqrt[-(((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d))]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) - (Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[-(((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d))]*Csc[e + f*x]*EllipticPi[(b*c - a*d)/((a + b)*c), ArcSin[Sqrt[-(((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d))]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*c*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])) + 2*(245*a^2*b^2*c^8 - 812*a^3*b*c^7*d - 266*a*b^3*c^7*d + 582*a^4*c^6*d^2 + 852*a^2*b^2*c^6*d^2 - 146*a^3*b*c^5*d^3 - 124*a*b^3*c^5*d^3 - 48

```
5*a^4*c^4*d^4 + 41*a^2*b^2*c^4*d^4 - 264*a^3*b*c^3*d^5 + 6*a*b^3*c^3*d^5 +
392*a^4*c^2*d^6 + 14*a^2*b^2*c^2*d^6 + 70*a^3*b*c*d^7 - 105*a^4*d^8)*((Sqrt
[(-a + b)/(a + b)]*(a + b)*Cos[(e + f*x)/2]*Sqrt[d + c*Cos[e + f*x]]*Ellipt
icE[ArcSin[(Sqrt[(-a + b)/(a + b)]*Sin[(e + f*x)/2])/Sqrt[(b + a*Cos[e + f*
x])/(a + b)]], (2*(b*c - a*d))/((-a + b)*(c + d))])/(a*c*Sqrt[((a + b)*Cos[
(e + f*x)/2]^2)/(b + a*Cos[e + f*x])]*Sqrt[b + a*Cos[e + f*x]]*Sqrt[(b + a*
Cos[e + f*x])/(a + b)]*Sqrt[((a + b)*(d + c*Cos[e + f*x]))/((c + d)*(b + a*
Cos[e + f*x]))]) - (2*(b*c - a*d)*(((b*c + (a + b)*d)*Sqrt[((c + d)*Cot[(e
+ f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2
)/(b*c - a*d)]*Sqrt[-(((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*
c - a*d))]*Csc[e + f*x]*EllipticF[ArcSin[Sqrt[-(((a + b)*(d + c*Cos[e + f*x
])*Csc[(e + f*x)/2]^2)/(b*c - a*d))]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c
- d))]*Sin[(e + f*x)/2]^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d
 + c*Cos[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d
)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt
[-(((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d))]*Csc[e +
f*x]*EllipticPi[(b*c - a*d)/((a + b)*c), ArcSin[Sqrt[-(((a + b)*(d + c*Cos[
e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d))]/Sqrt[2]], (2*(b*c - a*d))/((a +
 b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*c*Sqrt[b + a*Cos[e + f*x]]*Sqrt[
d + c*Cos[e + f*x]])))/(a*c) + (Sqrt[d + c*Cos[e + f*x]]*Sin[e + f*x])/(c*S
qrt[b + a*Cos[e + f*x]])))))/(105*c^3*(c - d)^4*(c + d)^4*(-(b*c) + a*d)*f*(
b + a*Cos[e + f*x])^(5/2)*(c + d*Sec[e + f*x])^(9/2))
```

Maple [B] time = 2.485, size = 75468, normalized size = 52.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(9/2),x)

[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sec\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(d\sec\left(fx+e\right)+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(9/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(5/2)/(d*sec(f*x + e) + c)^(9/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(9/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sec\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(d\sec\left(fx+e\right)+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(9/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*sec(f*x + e) + a)^(5/2)/(d*sec(f*x + e) + c)^(9/2), x)
```

3.217 $\int \frac{(c+d\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}}\,dx$

Optimal. Leaf size=652

$$\frac{2(bc-ad)\cot(e+fx)\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\sqrt{\frac{(bc-ad)(\sec(e+fx)-1)}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b\sec(e+fx))}}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{(}{(}}\right.\right.}{abf\sqrt{\frac{(a+b)(c+d\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}}$$

```
[Out] (-2*c*(c + d)*Cot[e + f*x]*EllipticPi[(a*(c + d))/((a + b)*c), ArcSin[Sqrt[
((a + b)*(c + d*Sec[e + f*x]))/((c + d)*(a + b*Sec[e + f*x]))]], ((a - b)*(
c + d))/((a + b)*(c - d))]*Sqrt[((b*c - a*d)*(1 + Sec[e + f*x]))/((c - d)*(
a + b*Sec[e + f*x]))]*(a + b*Sec[e + f*x])^(3/2)*Sqrt[((a + b)*(b*c - a*d)*
(-1 + Sec[e + f*x])*(c + d*Sec[e + f*x]))/((c + d)^2*(a + b*Sec[e + f*x])^2
)])/(a*(a + b)*f*Sqrt[c + d*Sec[e + f*x]]) + (2*d*(c + d)*Cot[e + f*x]*Elli
pticPi[(b*(c + d))/((a + b)*d), ArcSin[Sqrt[((a + b)*(c + d*Sec[e + f*x]))/
((c + d)*(a + b*Sec[e + f*x]))]], ((a - b)*(c + d))/((a + b)*(c - d))]*Sqrt
[((b*c - a*d)*(1 + Sec[e + f*x]))/((c - d)*(a + b*Sec[e + f*x]))]*(a + b*Se
c[e + f*x])^(3/2)*Sqrt[-(((a + b)*(-(b*c) + a*d)*(-1 + Sec[e + f*x])*(c + d
*Sec[e + f*x]))/((c + d)^2*(a + b*Sec[e + f*x])^2))])/(b*(a + b)*f*Sqrt[c +
 d*Sec[e + f*x]]) + (2*(b*c - a*d)*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[((a +
 b)*(c + d*Sec[e + f*x]))/((c + d)*(a + b*Sec[e + f*x]))]], ((a - b)*(c + d
))/((a + b)*(c - d))]*Sqrt[((b*c - a*d)*(-1 + Sec[e + f*x]))/((c + d)*(a +
b*Sec[e + f*x]))]*Sqrt[((b*c - a*d)*(1 + Sec[e + f*x]))/((c - d)*(a + b*Sec
[e + f*x]))]*Sqrt[a + b*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])/(a*b*f*Sqrt
[((a + b)*(c + d*Sec[e + f*x]))/((c + d)*(a + b*Sec[e + f*x]))])
```

Rubi [F] time = 0.092558, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{(c+d\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*Sec[e + f*x])^(3/2)/Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] Defer[Int][(c + d*Sec[e + f*x])^(3/2)/Sqrt[a + b*Sec[e + f*x]], x]
```

Rubi steps

$$\int \frac{(c+d\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}}\,dx = \int \frac{(c+d\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}}\,dx$$

Mathematica [C] time = 32.6576, size = 49385, normalized size = 75.74

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sec[e + f*x])^(3/2)/Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] Result too large to show
```

Maple [A] time = 0.386, size = 491, normalized size = 0.8

$$2\,\frac{\cos\left(fx+e\right)\left(\sin\left(fx+e\right)\right)^{2}}{f\left(-1+\cos\left(fx+e\right)\right)\left(d+c\cos\left(fx+e\right)\right)\left(a\cos\left(fx+e\right)+b\right)}\left(2\,\mathrm{EllipticPi}\left(\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\sqrt{\frac{a-b}{a+b}},-\frac{a+b}{a-b},\sqrt{\frac{c}{c}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))^(1/2),x)
```

```
[Out] 2/f/((a-b)/(a+b))^(1/2)*(2*EllipticPi((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/s
in(f*x+e),-(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*c^2+2*Ellip
ticPi((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),(a+b)/(a-b),((c-d)/(c+
d))^(1/2)/((a-b)/(a+b))^(1/2))*d^2-EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))^
(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*c^2+2*EllipticF((-1+cos(f
*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*c*d-
EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)
/(c+d))^(1/2))*d^2)*cos(f*x+e)*sin(f*x+e)^2*((d+c*cos(f*x+e))/cos(f*x+e))^(
1/2)*(1/cos(f*x+e)*(a*cos(f*x+e)+b))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos
(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)/(-1+cos(f*x
+e))/(d+c*cos(f*x+e))/(a*cos(f*x+e)+b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\sqrt{b\sec\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((d*sec(f*x + e) + c)^(3/2)/sqrt(b*sec(f*x + e) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))**(3/2)/(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\sqrt{b\sec\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((d*sec(f*x + e) + c)^(3/2)/sqrt(b*sec(f*x + e) + a), x)
```

3.218 $\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\,dx$

Optimal. Leaf size=198

$$-\frac{2\sqrt{a+b}\cot(e+fx)(c+d\sec(e+fx))\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{-\frac{(bc-ad)(\sec(e+fx)+1)}{(a-b)(c+d\sec(e+fx))}}\,\Pi\left(\frac{(a+b)c}{a(c+d)};\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{af\sqrt{c+d}}$$

```
[Out] (-2*Sqrt[a + b]*Cot[e + f*x]*EllipticPi[((a + b)*c)/(a*(c + d)), ArcSin[(Sq
rt[c + d]*Sqrt[a + b*Sec[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sec[e + f*x]])]
, ((a + b)*(c - d))/((a - b)*(c + d))]*Sqrt[((b*c - a*d)*(1 - Sec[e + f*x])
)/((a + b)*(c + d*Sec[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sec[e + f*x]))/(
(a - b)*(c + d*Sec[e + f*x])))]*(c + d*Sec[e + f*x]))/(a*Sqrt[c + d]*f)
```

Rubi [A] time = 0.110988, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {3936}

$$-\frac{2\sqrt{a+b}\cot(e+fx)(c+d\sec(e+fx))\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{-\frac{(bc-ad)(\sec(e+fx)+1)}{(a-b)(c+d\sec(e+fx))}}\,\Pi\left(\frac{(a+b)c}{a(c+d)};\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{af\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Sec[e + f*x]]/Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] (-2*Sqrt[a + b]*Cot[e + f*x]*EllipticPi[((a + b)*c)/(a*(c + d)), ArcSin[(Sq
rt[c + d]*Sqrt[a + b*Sec[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sec[e + f*x]])]
, ((a + b)*(c - d))/((a - b)*(c + d))]*Sqrt[((b*c - a*d)*(1 - Sec[e + f*x])
)/((a + b)*(c + d*Sec[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sec[e + f*x]))/(
(a - b)*(c + d*Sec[e + f*x])))]*(c + d*Sec[e + f*x]))/(a*Sqrt[c + d]*f)
```

Rule 3936

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Simp[(2*(a + b*Csc[e + f*x])*Sqrt[((b*c - a*d)
*(1 + Csc[e + f*x]))/((c - d)*(a + b*Csc[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1
 - Csc[e + f*x]))/((c + d)*(a + b*Csc[e + f*x])))]*EllipticPi[(a*(c + d))/(
c*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Csc[e + f*x]])/Sqrt[a
 + b*Csc[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(c*f*Rt[(a + b)/
```

```
(c + d), 2]*Cot[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\,dx = -\frac{2\sqrt{a+b}\cot(e+fx)\Pi\left(\frac{(a+b)c}{a(c+d)};\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{-}}{a\sqrt{c+d}f}$$

Mathematica [A] time = 5.24961, size = 325, normalized size = 1.64

$$\frac{4\sin^2\left(\frac{1}{2}(e+fx)\right)\csc(e+fx)\sqrt{\frac{(a+b)\cot^2\left(\frac{1}{2}(e+fx)\right)}{a-b}}\sqrt{c+d\sec(e+fx)}\sqrt{\frac{(c+d)\csc^2\left(\frac{1}{2}(e+fx)\right)(a\cos(e+fx)+b)}{bc-ad}}\left(a(c+d)\text{EllipticF}\left(\right.\right.}{af(c+d)\sqrt{a+b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*Sec[e + f*x]]/Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] (4*Sqrt[((a + b)*Cot[(e + f*x)/2]^2)/(a - b)]*Sqrt[((c + d)*(b + a*Cos[e +
f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*(a*(c + d)*EllipticF[Ar
cSin[Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(2*b*c - 2*a*d)
]], (2*(-(b*c) + a*d))/((a - b)*(c + d))] - (a + b)*c*EllipticPi[(-(b*c) +
a*d)/(a*(c + d)), ArcSin[Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2
]^2)/(2*b*c - 2*a*d)]], (2*(-(b*c) + a*d))/((a - b)*(c + d))])*Sqrt[c + d*S
ec[e + f*x]]*Sin[(e + f*x)/2]^2)/(a*(c + d)*f*Sqrt[((a + b)*(d + c*Cos[e +
f*x])*Csc[(e + f*x)/2]^2)/(-(b*c) + a*d)]*Sqrt[a + b*Sec[e + f*x]])
```

Maple [A] time = 0.372, size = 352, normalized size = 1.8

$$2\,\frac{\cos(fx+e)(\sin(fx+e))^2}{f(-1+\cos(fx+e))(d+c\cos(fx+e))(a\cos(fx+e)+b)}\left(2\,\text{EllipticPi}\left(\frac{-1+\cos(fx+e)}{\sin(fx+e)}\sqrt{\frac{a-b}{a+b}},-\frac{a+b}{a-b},\sqrt{\frac{c}{c}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e))^(1/2),x)
```

```
[Out] 2/f/((a-b)/(a+b))^(1/2)*(2*EllipticPi((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/s
in(f*x+e),-(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*c-EllipticF
((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(
1/2))*c+EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-
d)/(a-b)/(c+d))^(1/2))*d)*cos(f*x+e)*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e
)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*((d+c*cos(f*x+e))
/cos(f*x+e))^(1/2)*(1/cos(f*x+e)*(a*cos(f*x+e)+b))^(1/2)*sin(f*x+e)^2/(-1+c
os(f*x+e))/(d+c*cos(f*x+e))/(a*cos(f*x+e)+b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d \sec\left(fx+e\right)+c}}{\sqrt{b \sec\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(d*sec(f*x + e) + c)/sqrt(b*sec(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d \sec\left(fx+e\right)+c}}{\sqrt{b \sec\left(fx+e\right)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(d*sec(f*x + e) + c)/sqrt(b*sec(f*x + e) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+d\sec\left(e+fx\right)}}{\sqrt{a+b\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))**(1/2)/(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*sec(e + f*x))/sqrt(a + b*sec(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\sec\left(fx+e\right)+c}}{\sqrt{b\sec\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(d*sec(f*x + e) + c)/sqrt(b*sec(f*x + e) + a), x)
```

3.219 $\int \frac{1}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$

Optimal. Leaf size=398

$$-\frac{2b\sqrt{a+b}\cot(e+fx)(c+d\sec(e+fx))\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{-\frac{(bc-ad)(\sec(e+fx)+1)}{(a-b)(c+d\sec(e+fx))}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}\right),\frac{(}{(}\right.}{af\sqrt{c+d}(bc-ad)}$$

```
[Out] (-2*Sqrt[c + d]*Cot[e + f*x]*EllipticPi[(a*(c + d))/((a + b)*c), ArcSin[(Sq
rt[a + b]*Sqrt[c + d*Sec[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sec[e + f*x]])]
, ((a - b)*(c + d))/((a + b)*(c - d))]*Sqrt[-(((b*c - a*d)*(1 - Sec[e + f*x
]))/((c + d)*(a + b*Sec[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sec[e + f*x]))/
((c - d)*(a + b*Sec[e + f*x]))]*(a + b*Sec[e + f*x]))/(a*Sqrt[a + b]*c*f) -
 (2*b*Sqrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sec
[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sec[e + f*x]])], ((a + b)*(c - d))/((a
- b)*(c + d))]*Sqrt[((b*c - a*d)*(1 - Sec[e + f*x]))/((a + b)*(c + d*Sec[e
+ f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sec[e + f*x]))/((a - b)*(c + d*Sec[e + f
*x])))]*(c + d*Sec[e + f*x]))/(a*Sqrt[c + d]*(b*c - a*d)*f)
```

Rubi [A] time = 0.436726, antiderivative size = 398, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {3938, 3936, 3984}

$$-\frac{2b\sqrt{a+b}\cot(e+fx)(c+d\sec(e+fx))\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{-\frac{(bc-ad)(\sec(e+fx)+1)}{(a-b)(c+d\sec(e+fx))}}F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{af\sqrt{c+d}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]),x]
```

```
[Out] (-2*Sqrt[c + d]*Cot[e + f*x]*EllipticPi[(a*(c + d))/((a + b)*c), ArcSin[(Sq
rt[a + b]*Sqrt[c + d*Sec[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sec[e + f*x]])]
, ((a - b)*(c + d))/((a + b)*(c - d))]*Sqrt[-(((b*c - a*d)*(1 - Sec[e + f*x
]))/((c + d)*(a + b*Sec[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sec[e + f*x]))/
((c - d)*(a + b*Sec[e + f*x]))]*(a + b*Sec[e + f*x]))/(a*Sqrt[a + b]*c*f) -
 (2*b*Sqrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sec
[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sec[e + f*x]])], ((a + b)*(c - d))/((a
- b)*(c + d))]*Sqrt[((b*c - a*d)*(1 - Sec[e + f*x]))/((a + b)*(c + d*Sec[e
+ f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sec[e + f*x]))/((a - b)*(c + d*Sec[e + f
```

```
*x]))))]*(c + d*Sec[e + f*x]))/(a*Sqrt[c + d]*(b*c - a*d)*f)
```

Rule 3938

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqrt[csc[(e_.) + (f_.)*(x
_)]*(d_.) + (c_)]), x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqr
t[c + d*Csc[e + f*x]], x], x] - Dist[b/a, Int[Csc[e + f*x]/(Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0]
```

Rule 3936

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Simp[(2*(a + b*Csc[e + f*x])*Sqrt[((b*c - a*d)
*(1 + Csc[e + f*x]))/((c - d)*(a + b*Csc[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1
 - Csc[e + f*x]))/((c + d)*(a + b*Csc[e + f*x])))]*EllipticPi[(a*(c + d))/(
c*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Csc[e + f*x]])/Sqrt[a
 + b*Csc[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(c*f*Rt[(a + b)/
(c + d), 2]*Cot[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3984

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqr
t[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Simp[(-2*(c + d*Csc[
e + f*x])*Sqrt[((b*c - a*d)*(1 - Csc[e + f*x]))/((a + b)*(c + d*Csc[e + f*x
]))]*Sqrt[-(((b*c - a*d)*(1 + Csc[e + f*x]))/((a - b)*(c + d*Csc[e + f*x]))
)]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(Sqrt[a + b*Csc[e + f*x]]/Sqrt[c
 + d*Csc[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*
Rt[(c + d)/(a + b), 2]*Cot[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \frac{\int \frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx}{a} - \frac{b\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx}{a}$$

$$= -\frac{2\sqrt{c+d}\cot(e+fx)\Pi\left(\frac{a(c+d)}{(a+b)c};\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sqrt{-}}{a\sqrt{a+b}cf}$$

Mathematica [C] time = 1.98328, size = 249, normalized size = 0.63

$$\frac{4i\cos^2\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\sqrt{\frac{a\cos(e+fx)+b}{(a+b)(\cos(e+fx)+1)}}\sqrt{\frac{c\cos(e+fx)+d}{(c+d)(\cos(e+fx)+1)}}\left(\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{b-a}{a+b}}\tan\left(\frac{1}{2}(e+fx)\right)\right),\frac{(a+b)(c-\dots}{(a-b)(c+\dots}\right.\right.}{f\sqrt{\frac{b-a}{a+b}}\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]),x]
```

```
[Out] ((4*I)*Cos[(e + f*x)/2]^2*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e + f
*x]))]*Sqrt[(d + c*Cos[e + f*x])/((c + d)*(1 + Cos[e + f*x]))]*(EllipticF[I
*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2]], ((a + b)*(c - d))/((a -
b)*(c + d))] - 2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a
+ b)]*Tan[(e + f*x)/2]], ((a + b)*(c - d))/((a - b)*(c + d))])*Sec[e + f*x]
)/(Sqrt[(-a + b)/(a + b)]*f*Sqrt[a + b*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x
]])
```

Maple [A] time = 0.372, size = 292, normalized size = 0.7

$$-2\,\frac{\cos\left(fx+e\right)\left(\sin\left(fx+e\right)\right)^{2}}{f\left(-1+\cos\left(fx+e\right)\right)\left(d+c\cos\left(fx+e\right)\right)\left(a\cos\left(fx+e\right)+b\right)}\left(\text{EllipticF}\left(\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\sqrt{\frac{a-b}{a+b}},\sqrt{\frac{(c-d)\,(a+\dots}{(a-b)\,(c+\dots}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e))^(1/2),x)
```

```
[Out] -2/f/((a-b)/(a+b))^(1/2)*(EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin
(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))-2*EllipticPi((-1+cos(f*x+e))*((a-b
)/(a+b))^(1/2)/sin(f*x+e),-(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1
/2)))*cos(f*x+e)*sin(f*x+e)^2*((d+c*cos(f*x+e))/cos(f*x+e))^(1/2)*(1/cos(f*
x+e)*(a*cos(f*x+e)+b))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2
)*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)/(-1+cos(f*x+e))/(d+c*cos(
f*x+e))/(a*cos(f*x+e)+b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sec\left(fx+e\right)+a}\sqrt{d\sec\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)+a}\sqrt{d\sec\left(fx+e\right)+c}}{bd\sec\left(fx+e\right)^{2}+ac+(bc+ad)\sec\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)/(b*d*sec(f*x + e
)^2 + a*c + (b*c + a*d)*sec(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+b\sec\left(e+fx\right)}\sqrt{c+d\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*sec(f*x+e))**(1/2)/(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*sec(e + f*x))*sqrt(c + d*sec(e + f*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sec\left(fx+e\right)+a}\sqrt{d\sec\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)), x)
```

3.220 $\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=622

$$-\frac{2d\sqrt{a+b}(2c-d)\cot(e+fx)(c+d\sec(e+fx))\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{-\frac{(bc-ad)(\sec(e+fx)+1)}{(a-b)(c+d\sec(e+fx))}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sec}}{\sqrt{a+b}\sqrt{c+d\sec}}\right.\right.}{c^2f(c-d)\sqrt{c+d}(bc-ad)}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*d^2*Cot[e + f*x]*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt
[a + b*Sec[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sec[e + f*x]])], ((a + b)*(c
- d))/((a - b)*(c + d))]*Sqrt[((b*c - a*d)*(1 - Sec[e + f*x]))/((a + b)*(c
+ d*Sec[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sec[e + f*x]))/((a - b)*(c + d
*Sec[e + f*x])))]*(c + d*Sec[e + f*x]))/(c*(c - d)*Sqrt[c + d]*(b*c - a*d)^
2*f) - (2*Sqrt[a + b]*(2*c - d)*d*Cot[e + f*x]*EllipticF[ArcSin[(Sqrt[c + d
]*Sqrt[a + b*Sec[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sec[e + f*x]])], ((a +
b)*(c - d))/((a - b)*(c + d))]*Sqrt[((b*c - a*d)*(1 - Sec[e + f*x]))/((a +
b)*(c + d*Sec[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sec[e + f*x]))/((a - b)*
(c + d*Sec[e + f*x])))]*(c + d*Sec[e + f*x]))/(c^2*(c - d)*Sqrt[c + d]*(b*c
 - a*d)*f) - (2*Sqrt[a + b]*Cot[e + f*x]*EllipticPi[((a + b)*c)/(a*(c + d))
, ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sec[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sec
[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sqrt[((b*c - a*d)*(1 - S
ec[e + f*x]))/((a + b)*(c + d*Sec[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sec[
e + f*x]))/((a - b)*(c + d*Sec[e + f*x])))]*(c + d*Sec[e + f*x]))/(a*c^2*Sq
rt[c + d]*f)
```

Rubi [A] time = 1.31597, antiderivative size = 763, normalized size of antiderivative = 1.23, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {3942, 3054, 2811, 2998, 2818, 2996}

$$-\frac{2d\sqrt{a+b}(2c-d)\csc(e+fx)\sqrt{a+b\sec(e+fx)}(c\cos(e+fx)+d)^{3/2}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(c\cos(e+fx)+d)}}\sqrt{-\frac{(bc-ad)(\cos(e+fx)+1)}{(a-b)(c\cos(e+fx)+d)}}F\left(\sin\right.}{c^2f(c-d)\sqrt{c+d}(bc-ad)\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])^(3/2)),x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*d^2*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b
)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*
(d + c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])^(3/2)*Csc[e + f*x]*EllipticE[A
```

```
rcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e
+ f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sqrt[a + b*Sec[e + f*x]])/(
c*(c - d)*Sqrt[c + d]*(b*c - a*d)^2*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*S
ec[e + f*x]]) - (2*Sqrt[a + b]*(2*c - d)*d*Sqrt[-(((b*c - a*d)*(1 - Cos[e +
 f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f
*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])^(3/2)*Csc[e + f
*x]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sq
rt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sqrt[a + b*S
ec[e + f*x]])/(c^2*(c - d)*Sqrt[c + d]*(b*c - a*d)*f*Sqrt[b + a*Cos[e + f*x
]]*Sqrt[c + d*Sec[e + f*x]]) - (2*Sqrt[a + b]*Sqrt[-(((b*c - a*d)*(1 - Cos[
e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e
+ f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])^(3/2)*Csc[e
+ f*x]*EllipticPi[((a + b)*c)/(a*(c + d)), ArcSin[(Sqrt[c + d]*Sqrt[b + a*C
os[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((
a - b)*(c + d))]*Sqrt[a + b*Sec[e + f*x]])/(a*c^2*Sqrt[c + d]*f*Sqrt[b + a*
Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])
```

<u>Rule 3942</u>

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[(Sqrt[d + c*Sin[e + f*x]]*Sqrt[a + b*Cs
c[e + f*x]])/(Sqrt[b + a*Sin[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int[((b +
 a*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n)/Sin[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && IntegerQ[m + 1/2
] && IntegerQ[n + 1/2] && LeQ[-2, m + n, 0]
```

<u>Rule 3054</u>

```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(((a_.) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :
> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[1/b^2, Int[(A*b^2 - a^2*C - 2*a*b*C*Sin[e + f*x])/((a + b*Sin[e + f
*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

<u>Rule 2811</u>

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))^{3/2}}\,dx = \frac{\left(\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\right)\int \frac{\cos^2(e+fx)}{\sqrt{b+a\cos(e+fx)}(d+c\cos(e+fx))^{3/2}}\,d}{\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}}$$

$$= \frac{\left(\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\right)\int \frac{\sqrt{d+c\cos(e+fx)}}{\sqrt{b+a\cos(e+fx)}}\,dx}{c^2\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}} + \frac{\left(\sqrt{d+c\,c\right.}{}}{}$$

$$= -\frac{2\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3}}{ac^2\sqrt{c+d}f\sqrt{b+a\,c}}$$

$$= -\frac{2(a-b)\sqrt{a+b}d^2\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(}{c(c-d)\sqrt{c+d}(bc-ad)^2f}$$

Mathematica [B] time = 9.53524, size = 1731, normalized size = 2.78

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])^(3/2)),x]
```

```
[Out] (Sqrt[b + a*Cos[e + f*x]]*(d + c*Cos[e + f*x])^(3/2)*Sec[e + f*x]^2*((-4*b*
c*d*(b*c - a*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b
 + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[-(((a + b)*(d + c*
Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d))]*Csc[e + f*x]*EllipticF[ArcS
in[Sqrt[-(((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d))]/S
qrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*(c
 + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) + 4*(b*c - a*d)*(b
*c^2 - a*c*d - 2*b*d^2)*((Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[(
(c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[-(((a +
b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d))]*Csc[e + f*x]*Elli
pticF[ArcSin[Sqrt[-(((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c
- a*d))]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/(
(a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) - (Sqrt[
((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Cs
c[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[-(((a + b)*(d + c*Cos[e + f*x])*Csc[(e
+ f*x)/2]^2)/(b*c - a*d))]*Csc[e + f*x]*EllipticPi[(b*c - a*d)/((a + b)*c),
 ArcSin[Sqrt[-(((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d
))]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a +
b)*c*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])) - 2*a*d^2*((Sqrt[(
```

```
-a + b)/(a + b)]*(a + b)*Cos[(e + f*x)/2]*Sqrt[d + c*Cos[e + f*x]]*Elliptic
E[ArcSin[(Sqrt[(-a + b)/(a + b)]*Sin[(e + f*x)/2])/Sqrt[(b + a*Cos[e + f*x]
)/(a + b)]], (2*(b*c - a*d))/((-a + b)*(c + d))])/(a*c*Sqrt[((a + b)*Cos[(e
 + f*x)/2]^2)/(b + a*Cos[e + f*x])]*Sqrt[b + a*Cos[e + f*x]]*Sqrt[(b + a*Co
s[e + f*x])/(a + b)]*Sqrt[((a + b)*(d + c*Cos[e + f*x]))/((c + d)*(b + a*Co
s[e + f*x]))]) - (2*(b*c - a*d)*(((b*c + (a + b)*d)*Sqrt[((c + d)*Cot[(e +
f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/
(b*c - a*d)]*Sqrt[-(((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c
- a*d))]*Csc[e + f*x]*EllipticF[ArcSin[Sqrt[-(((a + b)*(d + c*Cos[e + f*x])
*Csc[(e + f*x)/2]^2)/(b*c - a*d))]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c -
d))]*Sin[(e + f*x)/2]^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d +
 c*Cos[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]
*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[-
(((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d))]*Csc[e + f*
x]*EllipticPi[(b*c - a*d)/((a + b)*c), ArcSin[Sqrt[-(((a + b)*(d + c*Cos[e
+ f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d))]/Sqrt[2]], (2*(b*c - a*d))/((a + b
)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*c*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d
+ c*Cos[e + f*x]])))/(a*c) + (Sqrt[d + c*Cos[e + f*x]]*Sin[e + f*x])/(c*Sqr
t[b + a*Cos[e + f*x]])))))/((c - d)*(c + d)*(b*c - a*d)*f*Sqrt[a + b*Sec[e +
 f*x]]*(c + d*Sec[e + f*x])^(3/2)) + (2*d^2*(b + a*Cos[e + f*x])*(d + c*Cos
[e + f*x])*Sec[e + f*x]*Tan[e + f*x])/((-(b*c) + a*d)*(-c^2 + d^2)*f*Sqrt[a
 + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])^(3/2))
```

Maple [B] time = 0.463, size = 3451, normalized size = 5.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+d*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))^(1/2),x)
```

```
[Out] 2/f/c/(c+d)/(c-d)/(a*d-b*c)/((a-b)/(a+b))^(1/2)*(-b*c*d^2*((a-b)/(a+b))^(1/
2)-EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a
-b)/(c+d))^(1/2))*b*c^3*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(1/
(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+sin(f*x+e)*cos(f*x+
e)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e)
)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f
*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*a*c^2*d-EllipticE((-1+cos(f*x+e))*((
a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*a*d^3*(1/(a+b
)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f
*x+e)))^(1/2)*sin(f*x+e)+sin(f*x+e)*cos(f*x+e)*(1/(a+b)*(a*cos(f*x+e)+b)/(1
+cos(f*x+e)))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*Ellipti
```

```
cF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))
^(1/2))*a*c*d^2-sin(f*x+e)*cos(f*x+e)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+
e)))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+co
s(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*b
*c^2*d-sin(f*x+e)*cos(f*x+e)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2
)*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*EllipticE((-1+cos(f*x+e))
*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*a*c*d^2+si
n(f*x+e)*cos(f*x+e)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(1/(c+d
)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*EllipticE((-1+cos(f*x+e))*((a-b)/(
a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*b*c*d^2-2*sin(f*x+e
)*cos(f*x+e)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(1/(c+d)*(d+c*
cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((a-b)/(a+b))^
(1/2)/sin(f*x+e),-(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*a*c^
2*d-2*sin(f*x+e)*cos(f*x+e)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)
*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))
*((a-b)/(a+b))^(1/2)/sin(f*x+e),-(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+
b))^(1/2))*b*c*d^2+cos(f*x+e)^2*((a-b)/(a+b))^(1/2)*a*c*d^2-cos(f*x+e)*((a-
b)/(a+b))^(1/2)*a*c*d^2+cos(f*x+e)*((a-b)/(a+b))^(1/2)*b*c*d^2+b*d^3*((a-b)
/(a+b))^(1/2)-cos(f*x+e)^2*((a-b)/(a+b))^(1/2)*a*d^3+cos(f*x+e)*((a-b)/(a+b
))^(1/2)*a*d^3-cos(f*x+e)*((a-b)/(a+b))^(1/2)*b*d^3-sin(f*x+e)*cos(f*x+e)*(
1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(1
+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e
),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*b*c^3-sin(f*x+e)*cos(f*x+e)*(1/(a+b)*(a*
cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)
))^(1/2)*EllipticE((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c
-d)/(a-b)/(c+d))^(1/2))*a*d^3+sin(f*x+e)*cos(f*x+e)*(1/(a+b)*(a*cos(f*x+e)+
b)/(1+cos(f*x+e)))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*El
lipticE((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(
c+d))^(1/2))*b*d^3+2*sin(f*x+e)*cos(f*x+e)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos
(f*x+e)))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*EllipticPi(
(-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),-(a+b)/(a-b),((c-d)/(c+d))^(
1/2)/((a-b)/(a+b))^(1/2))*a*d^3+2*sin(f*x+e)*cos(f*x+e)*(1/(a+b)*(a*cos(f*x
+e)+b)/(1+cos(f*x+e)))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2
)*EllipticPi((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),-(a+b)/(a-b),((
c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*b*c^3+sin(f*x+e)*(1/(a+b)*(a*cos(f*x
+e)+b)/(1+cos(f*x+e)))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2
)*EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-
b)/(c+d))^(1/2))*a*c^2*d+EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(
f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*a*c*d^2*(1/(a+b)*(a*cos(f*x+e)+b)/(
1+cos(f*x+e)))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*sin(f*
x+e)-sin(f*x+e)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(1/(c+d)*(d
+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((a-b)/(a+b)
)^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*b*c^2*d-EllipticE((-1+c
os(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*
a*c*d^2*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(1/(c+d)*(d+c*cos(f
```

```
*x+e))/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+EllipticE((-1+cos(f*x+e))*((a-b)/(a
+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*b*c*d^2*(1/(a+b)*(a*
cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)
))^(1/2)*sin(f*x+e)-2*EllipticPi((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*
x+e),-(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*a*c^2*d*(1/(a+b)
*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*
x+e)))^(1/2)*sin(f*x+e)-2*EllipticPi((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/si
n(f*x+e),-(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*b*c*d^2*(1/(
a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(1+co
s(f*x+e)))^(1/2)*sin(f*x+e)+EllipticE((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/s
in(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*b*d^3*(1/(a+b)*(a*cos(f*x+e)+b)/
(1+cos(f*x+e)))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*sin(f
*x+e)+2*EllipticPi((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),-(a+b)/(a
-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*a*d^3*(1/(a+b)*(a*cos(f*x+e)+b
)/(1+cos(f*x+e)))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*sin
(f*x+e)+2*EllipticPi((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),-(a+b)/
(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*b*c^3*(1/(a+b)*(a*cos(f*x+e)
+b)/(1+cos(f*x+e)))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*s
in(f*x+e))*cos(f*x+e)*((d+c*cos(f*x+e))/cos(f*x+e))^(1/2)*(1/cos(f*x+e)*(a*
cos(f*x+e)+b))^(1/2)/sin(f*x+e)/(d+c*cos(f*x+e))/(a*cos(f*x+e)+b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sec\left(fx+e\right)+a}\left(d\sec\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+b\sec\left(e+fx\right)}\left(c+d\sec\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*sec(f*x+e))**(3/2)/(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*sec(e + f*x))*(c + d*sec(e + f*x))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sec\left(fx+e\right)+a}\left(d\sec\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c)^(3/2)), x)
```

3.221 $\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{\sqrt[3]{c+d\sec(e+fx)}}\,dx$

Optimal. Leaf size=88

$$\frac{\sqrt[3]{a+b\sec(e+fx)}\sqrt[3]{c\cos(e+fx)+d}\text{Unintegrable}\left(\frac{\sqrt[3]{a\cos(e+fx)+b}}{\sqrt[3]{c\cos(e+fx)+d}},x\right)}{\sqrt[3]{a\cos(e+fx)+b}\sqrt[3]{c+d\sec(e+fx)}}$$

```
[Out] ((d + c*Cos[e + f*x])^(1/3)*(a + b*Sec[e + f*x])^(1/3)*Unintegrable[(b + a*
Cos[e + f*x])^(1/3)/(d + c*Cos[e + f*x])^(1/3), x])/((b + a*Cos[e + f*x])^(
1/3)*(c + d*Sec[e + f*x])^(1/3))
```

Rubi [A] time = 0.190879, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{\sqrt[3]{c+d\sec(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(1/3),x]
```

```
[Out] ((d + c*Cos[e + f*x])^(1/3)*(a + b*Sec[e + f*x])^(1/3)*Defer[Int][(b + a*Co
s[e + f*x])^(1/3)/(d + c*Cos[e + f*x])^(1/3), x])/((b + a*Cos[e + f*x])^(1/
3)*(c + d*Sec[e + f*x])^(1/3))
```

Rubi steps

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{\sqrt[3]{c+d\sec(e+fx)}}\,dx = \frac{\left(\sqrt[3]{d+c\cos(e+fx)}\sqrt[3]{a+b\sec(e+fx)}\right)\int \frac{\sqrt[3]{b+a\cos(e+fx)}}{\sqrt[3]{d+c\cos(e+fx)}}\,dx}{\sqrt[3]{b+a\cos(e+fx)}\sqrt[3]{c+d\sec(e+fx)}}$$

Mathematica [A] time = 2.15962, size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{\sqrt[3]{c+d\sec(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(1/3),x]
```

```
[Out] Integrate[(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(1/3), x]
```

Maple [A] time = 0.257, size = 0, normalized size = 0.

$$\int \sqrt[3]{a+b\sec\left(fx+e\right)}\frac{1}{\sqrt[3]{c+d\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(1/3),x)
```

```
[Out] int((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(1/3),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sec\left(fx+e\right)+a\right)^{\frac{1}{3}}}{\left(d\sec\left(fx+e\right)+c\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(1/3),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(1/3)/(d*sec(f*x + e) + c)^(1/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(1/3),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{a + b \sec\left(e + fx\right)}}{\sqrt[3]{c + d \sec\left(e + fx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(1/3)/(c+d*sec(f*x+e))**(1/3),x)
```

```
[Out] Integral((a + b*sec(e + f*x))**(1/3)/(c + d*sec(e + f*x))**(1/3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \sec\left(fx + e\right) + a\right)^{\frac{1}{3}}}{\left(d \sec\left(fx + e\right) + c\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(1/3),x, algorithm="giac"
)
```

```
[Out] integrate((b*sec(f*x + e) + a)^(1/3)/(d*sec(f*x + e) + c)^(1/3), x)
```

3.222 $\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{4/3}}\,dx$

Optimal. Leaf size=31

$$\text{Unintegrable}\left(\frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{4/3}},x\right)$$

```
[Out] Unintegrable[(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(4/3), x]
```

Rubi [A] time = 0.0920152, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{4/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(4/3),x]
```

```
[Out] Defer[Int][(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(4/3), x]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{4/3}}\,dx = \int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{4/3}}\,dx$$

Mathematica [A] time = 46.9511, size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{4/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(4/3),x]
```

```
[Out] Integrate[(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(4/3), x]
```

Maple [A] time = 0.143, size = 0, normalized size = 0.

$$\int \sqrt[3]{a+b\sec\left(fx+e\right)}\left(c+d\sec\left(fx+e\right)\right)^{-\frac{4}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(4/3),x)
```

```
[Out] int((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(4/3),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sec\left(fx+e\right)+a\right)^{\frac{1}{3}}}{\left(d\sec\left(fx+e\right)+c\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(4/3),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(1/3)/(d*sec(f*x + e) + c)^(4/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(4/3),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(1/3)/(c+d*sec(f*x+e))**(4/3),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sec\left(fx+e\right)+a\right)^{\frac{1}{3}}}{\left(d\sec\left(fx+e\right)+c\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(4/3),x, algorithm="giac"
)
```

```
[Out] integrate((b*sec(f*x + e) + a)^(1/3)/(d*sec(f*x + e) + c)^(4/3), x)
```

3.223 $\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{7/3}}\,dx$

Optimal. Leaf size=31

$$\text{Unintegrable}\left(\frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{7/3}},x\right)$$

```
[Out] Unintegrable[(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(7/3), x]
```

Rubi [A] time = 0.091233, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{7/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(7/3),x]
```

```
[Out] Defer[Int][(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(7/3), x]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{7/3}}\,dx=\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{7/3}}\,dx$$

Mathematica [A] time = 77.6866, size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{7/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(7/3),x]
```

```
[Out] Integrate[(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(7/3), x]
```

Maple [A] time = 0.18, size = 0, normalized size = 0.

$$\int \sqrt[3]{a+b\sec\left(fx+e\right)}\left(c+d\sec\left(fx+e\right)\right)^{-\frac{7}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(7/3),x)
```

```
[Out] int((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(7/3),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sec\left(fx+e\right)+a\right)^{\frac{1}{3}}}{\left(d\sec\left(fx+e\right)+c\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(7/3),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(1/3)/(d*sec(f*x + e) + c)^(7/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(7/3),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(1/3)/(c+d*sec(f*x+e))**(7/3),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sec\left(fx+e\right)+a\right)^{\frac{1}{3}}}{\left(d\sec\left(fx+e\right)+c\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(7/3),x, algorithm="giac"
)
```

```
[Out] integrate((b*sec(f*x + e) + a)^(1/3)/(d*sec(f*x + e) + c)^(7/3), x)
```

3.224 $\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{2/3}}\,dx$

Optimal. Leaf size=88

$$\frac{(a+b\sec(e+fx))^{2/3}(c\cos(e+fx)+d)^{2/3}\text{Unintegrable}\left(\frac{(a\cos(e+fx)+b)^{2/3}}{(c\cos(e+fx)+d)^{2/3}},x\right)}{(a\cos(e+fx)+b)^{2/3}(c+d\sec(e+fx))^{2/3}}$$

```
[Out] ((d + c*Cos[e + f*x])^(2/3)*(a + b*Sec[e + f*x])^(2/3)*Unintegrable[(b + a*
Cos[e + f*x])^(2/3)/(d + c*Cos[e + f*x])^(2/3), x])/((b + a*Cos[e + f*x])^(
2/3)*(c + d*Sec[e + f*x])^(2/3))
```

Rubi [A] time = 0.220201, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{2/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(2/3),x]
```

```
[Out] ((d + c*Cos[e + f*x])^(2/3)*(a + b*Sec[e + f*x])^(2/3)*Defer[Int][(b + a*Co
s[e + f*x])^(2/3)/(d + c*Cos[e + f*x])^(2/3), x])/((b + a*Cos[e + f*x])^(2/
3)*(c + d*Sec[e + f*x])^(2/3))
```

Rubi steps

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{2/3}}\,dx = \frac{\left((d+c\cos(e+fx))^{2/3}(a+b\sec(e+fx))^{2/3}\right)\int \frac{(b+a\cos(e+fx))^{2/3}}{(d+c\cos(e+fx))^{2/3}}\,dx}{(b+a\cos(e+fx))^{2/3}(c+d\sec(e+fx))^{2/3}}$$

Mathematica [A] time = 2.17062, size = 0, normalized size = 0.

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{2/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(2/3),x]
```

```
[Out] Integrate[(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(2/3), x]
```

Maple [A] time = 0.227, size = 0, normalized size = 0.

$$\int \left(a+b\sec\left(fx+e\right)\right)^{\frac{2}{3}}\left(c+d\sec\left(fx+e\right)\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(2/3),x)
```

```
[Out] int((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(2/3),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sec\left(fx+e\right)+a\right)^{\frac{2}{3}}}{\left(d\sec\left(fx+e\right)+c\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(2/3),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(2/3)/(d*sec(f*x + e) + c)^(2/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(2/3),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + b\sec\left(e + fx\right)\right)^{\frac{2}{3}}}{\left(c + d\sec\left(e + fx\right)\right)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(2/3)/(c+d*sec(f*x+e))**(2/3),x)
```

```
[Out] Integral((a + b*sec(e + f*x))**(2/3)/(c + d*sec(e + f*x))**(2/3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sec\left(fx + e\right) + a\right)^{\frac{2}{3}}}{\left(d\sec\left(fx + e\right) + c\right)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(2/3),x, algorithm="giac"
)
```

```
[Out] integrate((b*sec(f*x + e) + a)^(2/3)/(d*sec(f*x + e) + c)^(2/3), x)
```

3.225 $\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{5/3}}\,dx$

Optimal. Leaf size=31

$$\text{Unintegrable}\left(\frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{5/3}},x\right)$$

```
[Out] Unintegrable[(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(5/3), x]
```

Rubi [A] time = 0.0967324, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{5/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(5/3),x]
```

```
[Out] Defer[Int][(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(5/3), x]
```

Rubi steps

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{5/3}}\,dx = \int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{5/3}}\,dx$$

Mathematica [A] time = 47.8521, size = 0, normalized size = 0.

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{5/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(5/3),x]
```

```
[Out] Integrate[(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(5/3), x]
```

Maple [A] time = 0.16, size = 0, normalized size = 0.

$$\int \left(a+b\sec\left(fx+e\right)\right)^{\frac{2}{3}} \left(c+d\sec\left(fx+e\right)\right)^{-\frac{5}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(5/3),x)
```

```
[Out] int((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(5/3),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sec\left(fx+e\right)+a\right)^{\frac{2}{3}}}{\left(d\sec\left(fx+e\right)+c\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(5/3),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(2/3)/(d*sec(f*x + e) + c)^(5/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(5/3),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(2/3)/(c+d*sec(f*x+e))**(5/3),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sec\left(fx+e\right)+a\right)^{\frac{2}{3}}}{\left(d\sec\left(fx+e\right)+c\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(5/3),x, algorithm="giac"
)
```

```
[Out] integrate((b*sec(f*x + e) + a)^(2/3)/(d*sec(f*x + e) + c)^(5/3), x)
```

3.226 $\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{8/3}}\,dx$

Optimal. Leaf size=31

$$\text{Unintegrable}\left(\frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{8/3}},x\right)$$

```
[Out] Unintegrable[(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(8/3), x]
```

Rubi [A] time = 0.0962087, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{8/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(8/3),x]
```

```
[Out] Defer[Int][(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(8/3), x]
```

Rubi steps

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{8/3}}\,dx = \int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{8/3}}\,dx$$

Mathematica [A] time = 82.0668, size = 0, normalized size = 0.

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{8/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(8/3),x]
```

```
[Out] Integrate[(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(8/3), x]
```

Maple [A] time = 0.172, size = 0, normalized size = 0.

$$\int \left(a+b\sec\left(fx+e\right)\right)^{\frac{2}{3}} \left(c+d\sec\left(fx+e\right)\right)^{-\frac{8}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(8/3),x)
```

```
[Out] int((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(8/3),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sec\left(fx+e\right)+a\right)^{\frac{2}{3}}}{\left(d\sec\left(fx+e\right)+c\right)^{\frac{8}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(8/3),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(2/3)/(d*sec(f*x + e) + c)^(8/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(8/3),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(2/3)/(c+d*sec(f*x+e))**(8/3),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sec\left(fx+e\right)+a\right)^{\frac{2}{3}}}{\left(d\sec\left(fx+e\right)+c\right)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(8/3),x, algorithm="giac"
)
```

```
[Out] integrate((b*sec(f*x + e) + a)^(2/3)/(d*sec(f*x + e) + c)^(8/3), x)
```

3.227 $\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{4/3}}\,dx$

Optimal. Leaf size=88

$$\frac{(a+b\sec(e+fx))^{4/3}(c\cos(e+fx)+d)^{4/3}\text{Unintegrable}\left(\frac{(a\cos(e+fx)+b)^{4/3}}{(c\cos(e+fx)+d)^{4/3}},x\right)}{(a\cos(e+fx)+b)^{4/3}(c+d\sec(e+fx))^{4/3}}$$

```
[Out] ((d + c*Cos[e + f*x])^(4/3)*(a + b*Sec[e + f*x])^(4/3)*Unintegrable[(b + a*
Cos[e + f*x])^(4/3)/(d + c*Cos[e + f*x])^(4/3), x])/((b + a*Cos[e + f*x])^(
4/3)*(c + d*Sec[e + f*x])^(4/3))
```

Rubi [A] time = 0.218174, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{4/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(4/3),x]
```

```
[Out] ((d + c*Cos[e + f*x])^(4/3)*(a + b*Sec[e + f*x])^(4/3)*Defer[Int][(b + a*Co
s[e + f*x])^(4/3)/(d + c*Cos[e + f*x])^(4/3), x])/((b + a*Cos[e + f*x])^(4/
3)*(c + d*Sec[e + f*x])^(4/3))
```

Rubi steps

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{4/3}}\,dx=\frac{\left((d+c\cos(e+fx))^{4/3}(a+b\sec(e+fx))^{4/3}\right)\int \frac{(b+a\cos(e+fx))^{4/3}}{(d+c\cos(e+fx))^{4/3}}\,dx}{(b+a\cos(e+fx))^{4/3}(c+d\sec(e+fx))^{4/3}}$$

Mathematica [A] time = 57.3319, size = 0, normalized size = 0.

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{4/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(4/3),x]
```

```
[Out] Integrate[(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(4/3), x]
```

Maple [A] time = 0.143, size = 0, normalized size = 0.

$$\int \left(a+b\sec\left(fx+e\right)\right)^{\frac{4}{3}}\left(c+d\sec\left(fx+e\right)\right)^{-\frac{4}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(4/3),x)
```

```
[Out] int((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(4/3),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sec\left(fx+e\right)+a\right)^{\frac{4}{3}}}{\left(d\sec\left(fx+e\right)+c\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(4/3),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(4/3)/(d*sec(f*x + e) + c)^(4/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(4/3),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(4/3)/(c+d*sec(f*x+e))**(4/3),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sec\left(fx+e\right)+a\right)^{\frac{4}{3}}}{\left(d\sec\left(fx+e\right)+c\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(4/3),x, algorithm="giac"
)
```

```
[Out] integrate((b*sec(f*x + e) + a)^(4/3)/(d*sec(f*x + e) + c)^(4/3), x)
```

3.228 $\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{7/3}}\,dx$

Optimal. Leaf size=31

$$\text{Unintegrable}\left(\frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{7/3}},x\right)$$

```
[Out] Unintegrable[(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(7/3), x]
```

Rubi [A] time = 0.0974406, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{7/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(7/3),x]
```

```
[Out] Defer[Int][(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(7/3), x]
```

Rubi steps

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{7/3}}\,dx = \int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{7/3}}\,dx$$

Mathematica [A] time = 92.5433, size = 0, normalized size = 0.

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{7/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(7/3),x]
```

```
[Out] Integrate[(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(7/3), x]
```

Maple [A] time = 0.158, size = 0, normalized size = 0.

$$\int \left(a+b\sec\left(fx+e\right)\right)^{\frac{4}{3}} \left(c+d\sec\left(fx+e\right)\right)^{-\frac{7}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(7/3),x)
```

```
[Out] int((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(7/3),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sec\left(fx+e\right)+a\right)^{\frac{4}{3}}}{\left(d\sec\left(fx+e\right)+c\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(7/3),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(4/3)/(d*sec(f*x + e) + c)^(7/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(7/3),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(4/3)/(c+d*sec(f*x+e))**(7/3),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sec\left(fx+e\right)+a\right)^{\frac{4}{3}}}{\left(d\sec\left(fx+e\right)+c\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(7/3),x, algorithm="giac"
)
```

```
[Out] integrate((b*sec(f*x + e) + a)^(4/3)/(d*sec(f*x + e) + c)^(7/3), x)
```

3.229 $\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{10/3}}\,dx$

Optimal. Leaf size=31

$$\text{Unintegrable}\left(\frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{10/3}},x\right)$$

```
[Out] Unintegrable[(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(10/3), x]
```

Rubi [A] time = 0.096693, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{10/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(10/3),x]
```

```
[Out] Defer[Int][(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(10/3), x]
```

Rubi steps

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{10/3}}\,dx=\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{10/3}}\,dx$$

Mathematica [A] time = 132.591, size = 0, normalized size = 0.

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{10/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(10/3),x]
```

```
[Out] Integrate[(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(10/3), x]
```

Maple [A] time = 0.168, size = 0, normalized size = 0.

$$\int \left(a+b\sec\left(fx+e\right)\right)^{\frac{4}{3}} \left(c+d\sec\left(fx+e\right)\right)^{-\frac{10}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(10/3),x)
```

```
[Out] int((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(10/3),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sec\left(fx+e\right)+a\right)^{\frac{4}{3}}}{\left(d\sec\left(fx+e\right)+c\right)^{\frac{10}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(10/3),x, algorithm="maxi
ma")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(4/3)/(d*sec(f*x + e) + c)^(10/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(10/3),x, algorithm="fric
as")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(4/3)/(c+d*sec(f*x+e))**(10/3),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sec\left(fx+e\right)+a\right)^{\frac{4}{3}}}{\left(d\sec\left(fx+e\right)+c\right)^{\frac{10}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(10/3),x, algorithm="giac
")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(4/3)/(d*sec(f*x + e) + c)^(10/3), x)
```

3.230 $\int \left(c(d\sec(e+fx))^p\right)^n (a+a\sec(e+fx))^m\,dx$

Optimal. Leaf size=106

$$-\frac{\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m F_1\left(np;\frac{1}{2},\frac{1}{2}-m;np+1;\sec(e+fx),-\sec(e+fx)\right)\left(c(d\sec(e+\right.}{fnp\sqrt{1-\sec(e+fx)}}$$

```
[Out] -((AppellF1[n*p, 1/2, 1/2 - m, 1 + n*p, Sec[e + f*x], -Sec[e + f*x]]*(c*(d*
Sec[e + f*x])^p)^n*(1 + Sec[e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan
[e + f*x])/(f*n*p*Sqrt[1 - Sec[e + f*x]]))
```

Rubi [A] time = 0.189358, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3948, 3828, 3827, 133}

$$-\frac{\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m F_1\left(np;\frac{1}{2},\frac{1}{2}-m;np+1;\sec(e+fx),-\sec(e+fx)\right)\left(c(d\sec(e+\right.}{fnp\sqrt{1-\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sec[e + f*x])^p)^n*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] -((AppellF1[n*p, 1/2, 1/2 - m, 1 + n*p, Sec[e + f*x], -Sec[e + f*x]]*(c*(d*
Sec[e + f*x])^p)^n*(1 + Sec[e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan
[e + f*x])/(f*n*p*Sqrt[1 - Sec[e + f*x]]))
```

Rule 3948

```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c^IntPart[n]*(c*(d*Sec[e + f*x
])^p)^FracPart[n])/(d*Sec[e + f*x])^(p*FracPart[n]), Int[(a + b*Sec[e + f*x
])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] &&  !IntegerQ[n]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
```

```
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \left(c(d\sec(e+fx))^p\right)^n (a+a\sec(e+fx))^m\,dx = \left((d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\right)\int (d\sec(e+fx))^{np}(a+a\sec$$

$$= \left((d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n (1+\sec(e+fx))^{-m}(a+a\sec($$

$$= -\frac{\left(d(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n (1+\sec(e+fx))^{-\frac{1}{2}-m}(a+\right.}{f\sqrt{1-}}$$

$$= -\frac{F_1\left(np;\frac{1}{2},\frac{1}{2}-m;1+np;\sec(e+fx),-\sec(e+fx)\right)\left(c(d\sec(e+fx)\right.}{fnp\sqrt{1-\sec(e}}$$

Mathematica [B] time = 14.4402, size = 2425, normalized size = 22.88

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*(d*Sec[e + f*x])^p)^n*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] (3*2^(1 + m)*AppellF1[1/2, m + n*p, 1 - n*p, 3/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n*p)*(Cos[(e + f*x)/2]^2*Sec[e +
 f*x])^(m + n*p)*(c*(d*Sec[e + f*x])^p)^n*(a*(1 + Sec[e + f*x]))^m*Tan[(e +
```

```
f*x)/2])/(f*(3*AppellF1[1/2, m + n*p, 1 - n*p, 3/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2] + 2*((-1 + n*p)*AppellF1[3/2, m + n*p, 2 - n*p, 5/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n*p)*AppellF1[3/2, 1 + m + n*p
, 1 - n*p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^
2)*((3*2^m*AppellF1[1/2, m + n*p, 1 - n*p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(n*p)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^
(m + n*p))/(3*AppellF1[1/2, m + n*p, 1 - n*p, 3/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2] + 2*((-1 + n*p)*AppellF1[3/2, m + n*p, 2 - n*p, 5/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n*p)*AppellF1[3/2, 1 + m + n*p,
1 - n*p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)
 + (3*2^(1 + m)*(-1 + n*p)*AppellF1[1/2, m + n*p, 1 - n*p, 3/2, Tan[(e + f*
x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n*p)*(Cos[(e + f*x
)/2]^2*Sec[e + f*x])^(m + n*p)*Tan[(e + f*x)/2]^2)/(3*AppellF1[1/2, m + n*p
, 1 - n*p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n*p)*Ap
pellF1[3/2, m + n*p, 2 - n*p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
 + (m + n*p)*AppellF1[3/2, 1 + m + n*p, 1 - n*p, 5/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*2^(1 + m)*(Sec[(e + f*x)/2]^2
)^(-1 + n*p)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n*p)*Tan[(e + f*x)/2]*(
-((1 - n*p)*AppellF1[3/2, m + n*p, 2 - n*p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3 + ((m + n*p)*AppellF1
[3/2, 1 + m + n*p, 1 - n*p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*S
ec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3))/(3*AppellF1[1/2, m + n*p, 1 - n*p,
3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n*p)*AppellF1[3/2,
 m + n*p, 2 - n*p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n*p
)*AppellF1[3/2, 1 + m + n*p, 1 - n*p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2])*Tan[(e + f*x)/2]^2) - (3*2^(1 + m)*AppellF1[1/2, m + n*p, 1 - n*p
, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 +
n*p)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n*p)*Tan[(e + f*x)/2]*(2*((-1 +
 n*p)*AppellF1[3/2, m + n*p, 2 - n*p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2] + (m + n*p)*AppellF1[3/2, 1 + m + n*p, 1 - n*p, 5/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2] + 3*(-((1
- n*p)*AppellF1[3/2, m + n*p, 2 - n*p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3 + ((m + n*p)*AppellF1[3/2,
 1 + m + n*p, 1 - n*p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e
 + f*x)/2]^2*Tan[(e + f*x)/2])/3) + 2*Tan[(e + f*x)/2]^2*((-1 + n*p)*((-3*(
2 - n*p)*AppellF1[5/2, m + n*p, 3 - n*p, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(m + n*p)*AppellF1[
5/2, 1 + m + n*p, 2 - n*p, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Se
c[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5) + (m + n*p)*((-3*(1 - n*p)*AppellF1[5
/2, 1 + m + n*p, 2 - n*p, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec
[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(1 + m + n*p)*AppellF1[5/2, 2 + m
+ n*p, 1 - n*p, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)
/2]^2*Tan[(e + f*x)/2])/5))))/(3*AppellF1[1/2, m + n*p, 1 - n*p, 3/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n*p)*AppellF1[3/2, m + n*p,
2 - n*p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n*p)*AppellF1
```

```
[3/2, 1 + m + n*p, 1 - n*p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*
Tan[(e + f*x)/2]^2)^2 + (3*2^(1 + m)*(m + n*p)*AppellF1[1/2, m + n*p, 1 - n
*p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1
+ n*p)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-1 + m + n*p)*Tan[(e + f*x)/2]*(-
(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]) + Cos[(e + f*x)/2]^2*Sec[e
 + f*x]*Tan[e + f*x]))/(3*AppellF1[1/2, m + n*p, 1 - n*p, 3/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n*p)*AppellF1[3/2, m + n*p, 2 - n*p
, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n*p)*AppellF1[3/2, 1
 + m + n*p, 1 - n*p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e
+ f*x)/2]^2)))
```

Maple [F] time = 0.349, size = 0, normalized size = 0.

$$\int \left(c \left(d \sec \left(fx+e\right)\right)^{p}\right)^{n} \left(a+a \sec \left(fx+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^m,x)

[Out] int((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(d \sec \left(fx+e\right)\right)^{p} c\right)^{n} \left(a \sec \left(fx+e\right)+a\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^m,x, algorithm="maxima")

[Out] integrate(((d*sec(f*x + e))^p*c)^n*(a*sec(f*x + e) + a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(\left(d \sec \left(fx+e\right)\right)^{p} c\right)^{n} \left(a \sec \left(fx+e\right)+a\right)^{m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral(((d*sec(f*x + e))^p*c)^n*(a*sec(f*x + e) + a)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\left(\sec\left(e+fx\right)+1\right)\right)^{m}\left(c\left(d\sec\left(e+fx\right)\right)^{p}\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))**p)**n*(a+a*sec(f*x+e))**m,x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**m*(c*(d*sec(e + f*x))**p)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(d\sec\left(fx+e\right)\right)^{p}c\right)^{n}\left(a\sec\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate(((d*sec(f*x + e))^p*c)^n*(a*sec(f*x + e) + a)^m, x)
```

3.231 $\int \left(c(d\sec(e+fx))^p\right)^n (a+a\sec(e+fx))^3\,dx$

Optimal. Leaf size=275

$$-\frac{a^3(4np+1)\sin(e+fx)\cos(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1-np),\frac{1}{2}(3-np),\cos^2(e+fx)\right)\left(c(d\sec(e+fx))^p\right)^n}{f\left(1-n^2p^2\right)\sqrt{\sin^2(e+fx)}}$$

```
[Out] (a^3*(7 + 4*n*p)*Hypergeometric2F1[1/2, -(n*p)/2, (2 - n*p)/2, Cos[e + f*x]
^2]*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x])/(f*n*p*(2 + n*p)*Sqrt[Sin[e + f*
x]^2]) - (a^3*(1 + 4*n*p)*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 - n*p)/2,
(3 - n*p)/2, Cos[e + f*x]^2]*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x])/(f*(1 -
 n^2*p^2)*Sqrt[Sin[e + f*x]^2]) + (a^3*(5 + 2*n*p)*(c*(d*Sec[e + f*x])^p)^n
*Tan[e + f*x])/(f*(1 + n*p)*(2 + n*p)) + ((c*(d*Sec[e + f*x])^p)^n*(a^3 + a
^3*Sec[e + f*x])*Tan[e + f*x])/(f*(2 + n*p))
```

Rubi [A] time = 0.436603, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3948, 3814, 3997, 3787, 3772, 2643}

$$-\frac{a^3(4np+1)\sin(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1-np);\frac{1}{2}(3-np);\cos^2(e+fx)\right)\left(c(d\sec(e+fx))^p\right)^n}{f\left(1-n^2p^2\right)\sqrt{\sin^2(e+fx)}}+\frac{a^3(4np+7)\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sec[e + f*x])^p)^n*(a + a*Sec[e + f*x])^3,x]
```


```
[Out] (a^3*(7 + 4*n*p)*Hypergeometric2F1[1/2, -(n*p)/2, (2 - n*p)/2, Cos[e + f*x]
^2]*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x])/(f*n*p*(2 + n*p)*Sqrt[Sin[e + f*
x]^2]) - (a^3*(1 + 4*n*p)*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 - n*p)/2,
(3 - n*p)/2, Cos[e + f*x]^2]*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x])/(f*(1 -
 n^2*p^2)*Sqrt[Sin[e + f*x]^2]) + (a^3*(5 + 2*n*p)*(c*(d*Sec[e + f*x])^p)^n
*Tan[e + f*x])/(f*(1 + n*p)*(2 + n*p)) + ((c*(d*Sec[e + f*x])^p)^n*(a^3 + a
^3*Sec[e + f*x])*Tan[e + f*x])/(f*(2 + n*p))
```


Rule 3948

```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c^IntPart[n]*(c*(d*Sec[e + f*x
])^p)^FracPart[n])/(d*Sec[e + f*x])^(p*FracPart[n]), Int[(a + b*Sec[e + f*x
```

```
])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] &&  !IntegerQ[n]
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \left(c(d\sec(e+fx))^p\right)^n (a+a\sec(e+fx))^3\,dx = \left((d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\right)\int (d\sec(e+fx))^{np}(a+a\sec($$

$$= \frac{\left(c(d\sec(e+fx))^p\right)^n\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{f(2+np)} + \frac{\left(a(d\sec(e+\right.}{}$$

$$= \frac{a^3(5+2np)\left(c(d\sec(e+fx))^p\right)^n\tan(e+fx)}{f(1+np)(2+np)} + \frac{\left(c(d\sec(e+fx))^p\right)^n\left(a\right.}{}$$

$$= \frac{a^3(5+2np)\left(c(d\sec(e+fx))^p\right)^n\tan(e+fx)}{f(1+np)(2+np)} + \frac{\left(c(d\sec(e+fx))^p\right)^n\left(a\right.}{}$$

$$= \frac{a^3(5+2np)\left(c(d\sec(e+fx))^p\right)^n\tan(e+fx)}{f(1+np)(2+np)} + \frac{\left(c(d\sec(e+fx))^p\right)^n\left(a\right.}{}$$

$$= \frac{a^3(7+4np)\,{}_2F_1\left(\frac{1}{2},-\frac{np}{2};\frac{1}{2}(2-np);\cos^2(e+fx)\right)\left(c(d\sec(e+fx))^p\right)^n \mathrm{s}}{fnp(2+np)\sqrt{\sin^2(e+fx)}}$$

Mathematica [F] time = 2.39777, size = 0, normalized size = 0.

$$\int \left(c(d\sec(e+fx))^p\right)^n (a+a\sec(e+fx))^3\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c*(d*Sec[e + f*x])^p)^n*(a + a*Sec[e + f*x])^3,x]
```

```
[Out] Integrate[(c*(d*Sec[e + f*x])^p)^n*(a + a*Sec[e + f*x])^3, x]
```

Maple [F] time = 0.173, size = 0, normalized size = 0.

$$\int \left(c\left(d\sec\left(fx+e\right)\right)^p\right)^n \left(a+a\sec\left(fx+e\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^3,x)
```

```
[Out] int((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^3,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^3 \sec\left(fx+e\right)^3 + 3\,a^3 \sec\left(fx+e\right)^2 + 3\,a^3 \sec\left(fx+e\right) + a^3\right)\left(\left(d \sec\left(fx+e\right)\right)^p c\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral((a^3*sec(f*x + e)^3 + 3*a^3*sec(f*x + e)^2 + 3*a^3*sec(f*x + e) +
a^3)*((d*sec(f*x + e))^p*c)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3\left(\int \left(c\left(d\sec\left(e+fx\right)\right)^p\right)^n dx + \int 3\left(c\left(d\sec\left(e+fx\right)\right)^p\right)^n \sec\left(e+fx\right)dx + \int 3\left(c\left(d\sec\left(e+fx\right)\right)^p\right)^n \sec^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))**p)**n*(a+a*sec(f*x+e))**3,x)
```

```
[Out] a**3*(Integral((c*(d*sec(e + f*x))**p)**n, x) + Integral(3*(c*(d*sec(e + f*
x))**p)**n*sec(e + f*x), x) + Integral(3*(c*(d*sec(e + f*x))**p)**n*sec(e +
 f*x)**2, x) + Integral((c*(d*sec(e + f*x))**p)**n*sec(e + f*x)**3, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec\left(fx+e\right)+a\right)^3\left(\left(d\sec\left(fx+e\right)\right)^p c\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^3*((d*sec(f*x + e))^p*c)^n, x)
```

3.232 $\int \left(c(d\sec(e+fx))^p\right)^n (a+a\sec(e+fx))^2\,dx$

Optimal. Leaf size=205

$$-\frac{a^2(2np+1)\sin(e+fx)\cos(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1-np),\frac{1}{2}(3-np),\cos^2(e+fx)\right)\left(c(d\sec(e+fx))^p\right)^n}{f\left(1-n^2p^2\right)\sqrt{\sin^2(e+fx)}}+$$

```
[Out] (2*a^2*Hypergeometric2F1[1/2, -(n*p)/2, (2 - n*p)/2, Cos[e + f*x]^2]*(c*(d*
Sec[e + f*x])^p)^n*Sin[e + f*x])/(f*n*p*Sqrt[Sin[e + f*x]^2]) - (a^2*(1 + 2
*n*p)*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 - n*p)/2, (3 - n*p)/2, Cos[e +
 f*x]^2]*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x])/(f*(1 - n^2*p^2)*Sqrt[Sin[e
 + f*x]^2]) + (a^2*(c*(d*Sec[e + f*x])^p)^n*Tan[e + f*x])/(f*(1 + n*p))
```

Rubi [A] time = 0.248729, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {3948, 3788, 3772, 2643, 4046}

$$-\frac{a^2(2np+1)\sin(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1-np);\frac{1}{2}(3-np);\cos^2(e+fx)\right)\left(c(d\sec(e+fx))^p\right)^n}{f\left(1-n^2p^2\right)\sqrt{\sin^2(e+fx)}}+\frac{2a^2\sin(e+fx)\,{}_2F}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sec[e + f*x])^p)^n*(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (2*a^2*Hypergeometric2F1[1/2, -(n*p)/2, (2 - n*p)/2, Cos[e + f*x]^2]*(c*(d*
Sec[e + f*x])^p)^n*Sin[e + f*x])/(f*n*p*Sqrt[Sin[e + f*x]^2]) - (a^2*(1 + 2
*n*p)*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 - n*p)/2, (3 - n*p)/2, Cos[e +
 f*x]^2]*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x])/(f*(1 - n^2*p^2)*Sqrt[Sin[e
 + f*x]^2]) + (a^2*(c*(d*Sec[e + f*x])^p)^n*Tan[e + f*x])/(f*(1 + n*p))
```

Rule 3948

```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c^IntPart[n]*(c*(d*Sec[e + f*x
])^p)^FracPart[n])/(d*Sec[e + f*x])^(p*FracPart[n]), Int[(a + b*Sec[e + f*x
])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && !IntegerQ[n]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \left(c(d\sec(e+fx))^p\right)^n (a+a\sec(e+fx))^2\,dx = \left((d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\right)\int (d\sec(e+fx))^{np}(a+a\sec($$

$$= \left((d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\right)\int (d\sec(e+fx))^{np}\left(a^2+a^2\,\mathrm{s}\right.$$

$$= \frac{a^2\left(c(d\sec(e+fx))^p\right)^n\tan(e+fx)}{f(1+np)} + \frac{\left(2a^2\left(\frac{\cos(e+fx)}{d}\right)^{np}\left(c(d\sec(e+f\right.\right.}{d}$$

$$= \frac{2a^2\,{}_2F_1\left(\frac{1}{2},-\frac{np}{2};\frac{1}{2}(2-np);\cos^2(e+fx)\right)\left(c(d\sec(e+fx))^p\right)^n\sin(e+f}{fnp\sqrt{\sin^2(e+fx)}}$$

$$= \frac{2a^2\,{}_2F_1\left(\frac{1}{2},-\frac{np}{2};\frac{1}{2}(2-np);\cos^2(e+fx)\right)\left(c(d\sec(e+fx))^p\right)^n\sin(e+f}{fnp\sqrt{\sin^2(e+fx)}}$$

Mathematica [F] time = 1.00003, size = 0, normalized size = 0.

$$\int \left(c(d\sec(e+fx))^p\right)^n (a+a\sec(e+fx))^2\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c*(d*Sec[e + f*x])^p)^n*(a + a*Sec[e + f*x])^2,x]
```

```
[Out] Integrate[(c*(d*Sec[e + f*x])^p)^n*(a + a*Sec[e + f*x])^2, x]
```

Maple [F] time = 0.161, size = 0, normalized size = 0.

$$\int \left(c\left(d\sec\left(fx+e\right)\right)^{p}\right)^{n}\left(a+a\sec\left(fx+e\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^2,x)
```

```
[Out] int((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec\left(fx+e\right)+a\right)^{2}\left(\left(d\sec\left(fx+e\right)\right)^{p}c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^2*((d*sec(f*x + e))^p*c)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^{2}\sec\left(fx+e\right)^{2}+2\,a^{2}\sec\left(fx+e\right)+a^{2}\right)\left(\left(d\sec\left(fx+e\right)\right)^{p}c\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*((d*sec(f*x + e))^
p*c)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int\left(c\left(d\sec\left(e+fx\right)\right)^p\right)^n dx + \int 2\left(c\left(d\sec\left(e+fx\right)\right)^p\right)^n \sec\left(e+fx\right)dx + \int\left(c\left(d\sec\left(e+fx\right)\right)^p\right)^n \sec^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))**p)**n*(a+a*sec(f*x+e))**2,x)
```

```
[Out] a**2*(Integral((c*(d*sec(e + f*x))**p)**n, x) + Integral(2*(c*(d*sec(e + f*
x))**p)**n*sec(e + f*x), x) + Integral((c*(d*sec(e + f*x))**p)**n*sec(e + f
*x)**2, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sec\left(fx+e\right)+a\right)^2\left(\left(d\sec\left(fx+e\right)\right)^p c\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^2*((d*sec(f*x + e))^p*c)^n, x)
```

3.233 $\int \left(c(d\sec(e+fx))^p\right)^n (a+a\sec(e+fx))\,dx$

Optimal. Leaf size=156

$$\frac{a\sin(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{np}{2},\frac{1}{2}(2-np),\cos^2(e+fx)\right)\left(c(d\sec(e+fx))^p\right)^n}{fnp\sqrt{\sin^2(e+fx)}}-\frac{a\sin(e+fx)\cos(e+fx)\text{Hy}}{}$$

```
[Out] (a*Hypergeometric2F1[1/2, -(n*p)/2, (2 - n*p)/2, Cos[e + f*x]^2]*(c*(d*Sec[
e + f*x])^p)^n*Sin[e + f*x])/(f*n*p*Sqrt[Sin[e + f*x]^2]) - (a*Cos[e + f*x]
*Hypergeometric2F1[1/2, (1 - n*p)/2, (3 - n*p)/2, Cos[e + f*x]^2]*(c*(d*Sec
[e + f*x])^p)^n*Sin[e + f*x])/(f*(1 - n*p)*Sqrt[Sin[e + f*x]^2])
```

Rubi [A] time = 0.146012, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3948, 3787, 3772, 2643}

$$\frac{a\sin(e+fx)\,{}_2F_1\left(\frac{1}{2},-\frac{np}{2};\frac{1}{2}(2-np);\cos^2(e+fx)\right)\left(c(d\sec(e+fx))^p\right)^n}{fnp\sqrt{\sin^2(e+fx)}}-\frac{a\sin(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1-np);\frac{1}{2}(3}{f(1-np)\sqrt{\sin^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sec[e + f*x])^p)^n*(a + a*Sec[e + f*x]),x]
```

```
[Out] (a*Hypergeometric2F1[1/2, -(n*p)/2, (2 - n*p)/2, Cos[e + f*x]^2]*(c*(d*Sec[
e + f*x])^p)^n*Sin[e + f*x])/(f*n*p*Sqrt[Sin[e + f*x]^2]) - (a*Cos[e + f*x]
*Hypergeometric2F1[1/2, (1 - n*p)/2, (3 - n*p)/2, Cos[e + f*x]^2]*(c*(d*Sec
[e + f*x])^p)^n*Sin[e + f*x])/(f*(1 - n*p)*Sqrt[Sin[e + f*x]^2])
```

Rule 3948

```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c^IntPart[n]*(c*(d*Sec[e + f*x
])^p)^FracPart[n])/(d*Sec[e + f*x])^(p*FracPart[n]), Int[(a + b*Sec[e + f*x
])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && !IntegerQ[n]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \left(c(d\sec(e+fx))^p\right)^n (a+a\sec(e+fx))\,dx = \left((d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\right)\int (d\sec(e+fx))^{np}(a+a\sec(e$$

$$= \left(a(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\right)\int (d\sec(e+fx))^{np}\,dx + \frac{\left(a(d\right.}{}$$

$$= \left(a\left(\frac{\cos(e+fx)}{d}\right)^{np}\left(c(d\sec(e+fx))^p\right)^n\right)\int \left(\frac{\cos(e+fx)}{d}\right)^{-np}dx + \frac{\left(a\left(\right.\right.}{}$$

$$= \frac{a\,{}_2F_1\left(\frac{1}{2},-\frac{np}{2};\frac{1}{2}(2-np);\cos^2(e+fx)\right)\left(c(d\sec(e+fx))^p\right)^n\sin(e+fx)}{fnp\sqrt{\sin^2(e+fx)}}$$

Mathematica [A] time = 0.201515, size = 124, normalized size = 0.79

$$\frac{a\sqrt{-\tan^2(e+fx)}\csc(e+fx)\left(c(d\sec(e+fx))^p\right)^n\left(np\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(np+1),\frac{1}{2}(np+3),\sec^2(e+fx)\right)+\right.}{fnp(np+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(d*Sec[e + f*x])^p)^n*(a + a*Sec[e + f*x]),x]
```

```
[Out] (a*Csc[e + f*x]*((1 + n*p)*Cos[e + f*x]*Hypergeometric2F1[1/2, (n*p)/2, 1 +
 (n*p)/2, Sec[e + f*x]^2] + n*p*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*
p)/2, Sec[e + f*x]^2])*(c*(d*Sec[e + f*x])^p)^n*Sqrt[-Tan[e + f*x]^2])/(f*n
*p*(1 + n*p))
```

Maple [F] time = 0.148, size = 0, normalized size = 0.

$$\int \left(c \left(d \sec \left(f x+e\right)\right)^{p}\right)^{n} \left(a+a \sec \left(f x+e\right)\right) d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e)),x)
```

```
[Out] int((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \sec \left(f x+e\right)+a\right) \left(\left(d \sec \left(f x+e\right)\right)^{p} c\right)^{n} d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)*((d*sec(f*x + e))^p*c)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a \sec \left(f x+e\right)+a\right) \left(\left(d \sec \left(f x+e\right)\right)^{p} c\right)^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((a*sec(f*x + e) + a)*((d*sec(f*x + e))^p*c)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int \left(c\left(d\sec\left(e+fx\right)\right)^{p}\right)^{n}\,dx+\int \left(c\left(d\sec\left(e+fx\right)\right)^{p}\right)^{n}\sec\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))**p)**n*(a+a*sec(f*x+e)),x)
```

```
[Out] a*(Integral((c*(d*sec(e + f*x))**p)**n, x) + Integral((c*(d*sec(e + f*x))**
p)**n*sec(e + f*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec\left(fx+e\right)+a\right)\left(\left(d\sec\left(fx+e\right)\right)^{p}c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)*((d*sec(f*x + e))^p*c)^n, x)
```

3.234 $\int \frac{(c(d\sec(e+fx))^p)^n}{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=208

$$-\frac{\sin(e+fx)\cos(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1-np),\frac{1}{2}(3-np),\cos^2(e+fx)\right)\left(c(d\sec(e+fx))^p\right)^n}{af\sqrt{\sin^2(e+fx)}}+\frac{(1-np)\sin}{}$$

```
[Out] ((c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x])/(f*(a + a*Sec[e + f*x])) - (Cos[e +
 f*x]*Hypergeometric2F1[1/2, (1 - n*p)/2, (3 - n*p)/2, Cos[e + f*x]^2]*(c*(
d*Sec[e + f*x])^p)^n*Sin[e + f*x])/(a*f*Sqrt[Sin[e + f*x]^2]) + ((1 - n*p)*
Cos[e + f*x]^2*Hypergeometric2F1[1/2, (2 - n*p)/2, (4 - n*p)/2, Cos[e + f*x
]^2]*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x])/(a*f*(2 - n*p)*Sqrt[Sin[e + f*x
]^2])
```

Rubi [A] time = 0.271023, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {3948, 3820, 3787, 3772, 2643}

$$-\frac{\sin(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1-np);\frac{1}{2}(3-np);\cos^2(e+fx)\right)\left(c(d\sec(e+fx))^p\right)^n}{af\sqrt{\sin^2(e+fx)}}+\frac{(1-np)\sin(e+fx)\cos^2(e+}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sec[e + f*x])^p)^n/(a + a*Sec[e + f*x]),x]
```


```
[Out] ((c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x])/(f*(a + a*Sec[e + f*x])) - (Cos[e +
 f*x]*Hypergeometric2F1[1/2, (1 - n*p)/2, (3 - n*p)/2, Cos[e + f*x]^2]*(c*(
d*Sec[e + f*x])^p)^n*Sin[e + f*x])/(a*f*Sqrt[Sin[e + f*x]^2]) + ((1 - n*p)*
Cos[e + f*x]^2*Hypergeometric2F1[1/2, (2 - n*p)/2, (4 - n*p)/2, Cos[e + f*x
]^2]*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x])/(a*f*(2 - n*p)*Sqrt[Sin[e + f*x
]^2])
```


Rule 3948

```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c^IntPart[n]*(c*(d*Sec[e + f*x
])^p)^FracPart[n])/(d*Sec[e + f*x])^(p*FracPart[n]), Int[(a + b*Sec[e + f*x
])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
```

```
] &&  !IntegerQ[n]
```

Rule 3820

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/(a*f*(
a + b*Csc[e + f*x])), x] + Dist[(d*(n - 1))/(a*b), Int[(d*Csc[e + f*x])^(n
- 1)*(a - b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^
2 - b^2, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\left(c(d\sec(e+fx))^p\right)^n}{a+a\sec(e+fx)}\,dx = \left((d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\right)\int \frac{(d\sec(e+fx))^{np}}{a+a\sec(e+fx)}\,dx$$

$$= \frac{\left(c(d\sec(e+fx))^p\right)^n\sin(e+fx)}{f(a+a\sec(e+fx))} - \frac{\left(d(1-np)(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\right)\int(d\,\mathrm{se}}{a^2}$$

$$= \frac{\left(c(d\sec(e+fx))^p\right)^n\sin(e+fx)}{f(a+a\sec(e+fx))} + \frac{\left((1-np)(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\right)\int(d\sec}{a}$$

$$= \frac{\left(c(d\sec(e+fx))^p\right)^n\sin(e+fx)}{f(a+a\sec(e+fx))} + \frac{\left((1-np)\left(\frac{\cos(e+fx)}{d}\right)^{np}\left(c(d\sec(e+fx))^p\right)^n\right)\int\left(\frac{\cos(e+fx)}{d}\right)}{a}$$

$$= \frac{\left(c(d\sec(e+fx))^p\right)^n\sin(e+fx)}{f(a+a\sec(e+fx))} - \frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1-np);\frac{1}{2}(3-np);\cos^2(e+fx)\right)\left(c\right.}{af\sqrt{\sin^2(e+fx)}}$$

Mathematica [F] time = 1.20468, size = 0, normalized size = 0.

$$\int \frac{\left(c(d\sec(e+fx))^p\right)^n}{a+a\sec(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c*(d*Sec[e + f*x])^p)^n/(a + a*Sec[e + f*x]),x]
```

```
[Out] Integrate[(c*(d*Sec[e + f*x])^p)^n/(a + a*Sec[e + f*x]), x]
```

Maple [F] time = 0.15, size = 0, normalized size = 0.

$$\int \frac{\left(c\left(d\sec\left(fx+e\right)\right)^p\right)^n}{a+a\sec\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e)),x)
```

```
[Out] int((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(d\sec\left(fx+e\right)\right)^{p}c\right)^{n}}{a\sec\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(((d*sec(f*x + e))^p*c)^n/(a*sec(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\left(d\sec\left(fx+e\right)\right)^{p}c\right)^{n}}{a\sec\left(fx+e\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(((d*sec(f*x + e))^p*c)^n/(a*sec(f*x + e) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\left(c\left(d\sec\left(e+fx\right)\right)^{p}\right)^{n}}{\sec\left(e+fx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))**p)**n/(a+a*sec(f*x+e)),x)
```

```
[Out] Integral((c*(d*sec(e + f*x))**p)**n/(sec(e + f*x) + 1), x)/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(d \sec\left(fx+e\right)\right)^{p} c\right)^{n}}{a \sec\left(fx+e\right)+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(((d*sec(f*x + e))^p*c)^n/(a*sec(f*x + e) + a), x)
```

3.235 $\int \frac{\left(c(d\sec(e+fx))^p\right)^n}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=248

$$\frac{2(2-np)\sin(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{np}{2},\frac{1}{2}(2-np),\cos^2(e+fx)\right)\left(c(d\sec(e+fx))^p\right)^n}{3a^2f\sqrt{\sin^2(e+fx)}}-\frac{(3-2np)\sin(e+f}{}$$

```
[Out] (2*(2 - n*p)*Hypergeometric2F1[1/2, -(n*p)/2, (2 - n*p)/2, Cos[e + f*x]^2]*
(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x])/(3*a^2*f*Sqrt[Sin[e + f*x]^2]) - ((3
 - 2*n*p)*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 - n*p)/2, (3 - n*p)/2, Cos
[e + f*x]^2]*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x])/(3*a^2*f*Sqrt[Sin[e + f
*x]^2]) - (2*(2 - n*p)*(c*(d*Sec[e + f*x])^p)^n*Tan[e + f*x])/(3*a^2*f*(1 +
 Sec[e + f*x])) - ((c*(d*Sec[e + f*x])^p)^n*Tan[e + f*x])/(3*f*(a + a*Sec[e
 + f*x])^2)
```

Rubi [A] time = 0.451474, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3948, 3817, 4020, 3787, 3772, 2643}

$$\frac{2(2-np)\sin(e+fx)\,{}_2F_1\left(\frac{1}{2},-\frac{np}{2};\frac{1}{2}(2-np);\cos^2(e+fx)\right)\left(c(d\sec(e+fx))^p\right)^n}{3a^2f\sqrt{\sin^2(e+fx)}}-\frac{(3-2np)\sin(e+fx)\cos(e+fx)\,{}_2F_1}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sec[e + f*x])^p)^n/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (2*(2 - n*p)*Hypergeometric2F1[1/2, -(n*p)/2, (2 - n*p)/2, Cos[e + f*x]^2]*
(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x])/(3*a^2*f*Sqrt[Sin[e + f*x]^2]) - ((3
 - 2*n*p)*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 - n*p)/2, (3 - n*p)/2, Cos
[e + f*x]^2]*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x])/(3*a^2*f*Sqrt[Sin[e + f
*x]^2]) - (2*(2 - n*p)*(c*(d*Sec[e + f*x])^p)^n*Tan[e + f*x])/(3*a^2*f*(1 +
 Sec[e + f*x])) - ((c*(d*Sec[e + f*x])^p)^n*Tan[e + f*x])/(3*f*(a + a*Sec[e
 + f*x])^2)
```

Rule 3948

```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c^IntPart[n]*(c*(d*Sec[e + f*x
```

```
])^p)^FracPart[n])/(d*Sec[e + f*x])^(p*FracPart[n]), Int[(a + b*Sec[e + f*x
])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] &&  !IntegerQ[n]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a+a\sec(e+fx))^2}\,dx = \left((d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\right)\int \frac{(d\sec(e+fx))^{np}}{(a+a\sec(e+fx))^2}\,dx$$

$$= -\frac{\left(c(d\sec(e+fx))^p\right)^n\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{\left((d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\right)\int \frac{(d\sec(e+fx))}{}}{3a^2}$$

$$= -\frac{2(2-np)\left(c(d\sec(e+fx))^p\right)^n\tan(e+fx)}{3a^2f(1+\sec(e+fx))} - \frac{\left(c(d\sec(e+fx))^p\right)^n\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{\left((d\sec(\right.}{}$$

$$= -\frac{2(2-np)\left(c(d\sec(e+fx))^p\right)^n\tan(e+fx)}{3a^2f(1+\sec(e+fx))} - \frac{\left(c(d\sec(e+fx))^p\right)^n\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{\left((3-2\right.}{}$$

$$= -\frac{2(2-np)\left(c(d\sec(e+fx))^p\right)^n\tan(e+fx)}{3a^2f(1+\sec(e+fx))} - \frac{\left(c(d\sec(e+fx))^p\right)^n\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{\left((3-2\right.}{}$$

$$= \frac{2(2-np)\,{}_2F_1\left(\frac{1}{2},-\frac{np}{2};\frac{1}{2}(2-np);\cos^2(e+fx)\right)\left(c(d\sec(e+fx))^p\right)^n\sin(e+fx)}{3a^2f\sqrt{\sin^2(e+fx)}} - \frac{(3-2np}{}$$

Mathematica [F] time = 1.79699, size = 0, normalized size = 0.

$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a+a\sec(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c*(d*Sec[e + f*x])^p)^n/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] Integrate[(c*(d*Sec[e + f*x])^p)^n/(a + a*Sec[e + f*x])^2, x]
```

Maple [F] time = 0.161, size = 0, normalized size = 0.

$$\int \frac{\left(c\left(d\sec\left(fx+e\right)\right)^p\right)^n}{\left(a+a\sec\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e))^2,x)
```

```
[Out] int((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e))^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\left(d\sec\left(fx+e\right)\right)^{p}c\right)^{n}}{a^{2}\sec\left(fx+e\right)^{2}+2\,a^{2}\sec\left(fx+e\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(((d*sec(f*x + e))^p*c)^n/(a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e)
+ a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\left(c\left(d\sec\left(e+fx\right)\right)^{p}\right)^{n}}{\sec^{2}\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))**p)**n/(a+a*sec(f*x+e))**2,x)
```

```
[Out] Integral((c*(d*sec(e + f*x))**p)**n/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1),
 x)/a**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(d\sec\left(fx+e\right)\right)^{p}c\right)^{n}}{\left(a\sec\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(((d*sec(f*x + e))^p*c)^n/(a*sec(f*x + e) + a)^2, x)
```

3.236 $\int \left(c(d\sec(e+fx))^p\right)^n (a+b\sec(e+fx))^m\,dx$

Optimal. Leaf size=55

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n \text{Unintegrable}\left((a+b\sec(e+fx))^m(d\sec(e+fx))^{np},x\right)$$

```
[Out] ((c*(d*Sec[e + f*x])^p)^n*Unintegrable[(d*Sec[e + f*x])^(n*p)*(a + b*Sec[e
+ f*x])^m, x])/(d*Sec[e + f*x])^(n*p)
```

Rubi [A] time = 0.116231, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(c(d\sec(e+fx))^p\right)^n (a+b\sec(e+fx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x])^m,x]
```

```
[Out] ((c*(d*Sec[e + f*x])^p)^n*Defer[Int][(d*Sec[e + f*x])^(n*p)*(a + b*Sec[e +
f*x])^m, x])/(d*Sec[e + f*x])^(n*p)
```

Rubi steps

$$\int \left(c(d\sec(e+fx))^p\right)^n (a+b\sec(e+fx))^m\,dx = \left((d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\right)\int (d\sec(e+fx))^{np}(a+b\sec(e$$

Mathematica [A] time = 2.421, size = 0, normalized size = 0.

$$\int \left(c(d\sec(e+fx))^p\right)^n (a+b\sec(e+fx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x])^m,x]
```

```
[Out] Integrate[(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x])^m, x]
```

Maple [A] time = 0.251, size = 0, normalized size = 0.

$$\int \left(c \left(d \sec \left(f x + e\right)\right)^{p}\right)^{n} \left(a + b \sec \left(f x + e\right)\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^m,x)
```

```
[Out] int((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^m,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(d \sec \left(f x + e\right)\right)^{p} c\right)^{n} \left(b \sec \left(f x + e\right) + a\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate(((d*sec(f*x + e))^p*c)^n*(b*sec(f*x + e) + a)^m, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(\left(d \sec \left(f x + e\right)\right)^{p} c\right)^{n} \left(b \sec \left(f x + e\right) + a\right)^{m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral(((d*sec(f*x + e))^p*c)^n*(b*sec(f*x + e) + a)^m, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \left(c \left(d \sec \left(e + f x\right)\right)^{p}\right)^{n} \left(a + b \sec \left(e + f x\right)\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))**p)**n*(a+b*sec(f*x+e))**m,x)
```

```
[Out] Integral((c*(d*sec(e + f*x))**p)**n*(a + b*sec(e + f*x))**m, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(d \sec \left(fx+e\right)\right)^{p} c\right)^{n} \left(b \sec \left(fx+e\right)+a\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate(((d*sec(f*x + e))^p*c)^n*(b*sec(f*x + e) + a)^m, x)
```

3.237 $\int \left(c(d\sec(e+fx))^p\right)^n (a+b\sec(e+fx))^3\,dx$

Optimal. Leaf size=296

$$-\frac{a\left(a^2(np+1)+3b^2np\right)\sin(e+fx)\cos(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1-np),\frac{1}{2}(3-np),\cos^2(e+fx)\right)\left(c(d\sec(e\right.}{f\left(1-n^2p^2\right)\sqrt{\sin^2(e+fx)}}$$

```
[Out] (b*(b^2*(1 + n*p) + 3*a^2*(2 + n*p))*Hypergeometric2F1[1/2, -(n*p)/2, (2 -
n*p)/2, Cos[e + f*x]^2]*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x])/(f*n*p*(2 +
n*p)*Sqrt[Sin[e + f*x]^2]) - (a*(3*b^2*n*p + a^2*(1 + n*p))*Cos[e + f*x]*Hy
pergeometric2F1[1/2, (1 - n*p)/2, (3 - n*p)/2, Cos[e + f*x]^2]*(c*(d*Sec[e
+ f*x])^p)^n*Sin[e + f*x])/(f*(1 - n^2*p^2)*Sqrt[Sin[e + f*x]^2]) + (a*b^2*
(5 + 2*n*p)*(c*(d*Sec[e + f*x])^p)^n*Tan[e + f*x])/(f*(1 + n*p)*(2 + n*p))
+ (b^2*(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x])*Tan[e + f*x])/(f*(2 +
n*p))
```

Rubi [A] time = 0.514726, antiderivative size = 296, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3948, 3842, 4047, 3772, 2643, 4046}

$$-\frac{a\left(a^2(np+1)+3b^2np\right)\sin(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1-np);\frac{1}{2}(3-np);\cos^2(e+fx)\right)\left(c(d\sec(e+fx))^p\right)^n}{f\left(1-n^2p^2\right)\sqrt{\sin^2(e+fx)}}+\frac{b\left(3a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x])^3,x]
```


```
[Out] (b*(b^2*(1 + n*p) + 3*a^2*(2 + n*p))*Hypergeometric2F1[1/2, -(n*p)/2, (2 -
n*p)/2, Cos[e + f*x]^2]*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x])/(f*n*p*(2 +
n*p)*Sqrt[Sin[e + f*x]^2]) - (a*(3*b^2*n*p + a^2*(1 + n*p))*Cos[e + f*x]*Hy
pergeometric2F1[1/2, (1 - n*p)/2, (3 - n*p)/2, Cos[e + f*x]^2]*(c*(d*Sec[e
+ f*x])^p)^n*Sin[e + f*x])/(f*(1 - n^2*p^2)*Sqrt[Sin[e + f*x]^2]) + (a*b^2*
(5 + 2*n*p)*(c*(d*Sec[e + f*x])^p)^n*Tan[e + f*x])/(f*(1 + n*p)*(2 + n*p))
+ (b^2*(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x])*Tan[e + f*x])/(f*(2 +
n*p))
```


Rule 3948

```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c^IntPart[n]*(c*(d*Sec[e + f*x
```

```
])^p)^FracPart[n])/(d*Sec[e + f*x])^(p*FracPart[n]), Int[(a + b*Sec[e + f*x
])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && !IntegerQ[n]
```

Rule 3842

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)), Int[(a
+ b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b^2
*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d*(
3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x] &
& NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&
 !(IGtQ[n, 2] && !IntegerQ[m])
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rubi steps

$$\int \left(c(d\sec(e+fx))^p\right)^n (a+b\sec(e+fx))^3\,dx = \left((d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\right)\int (d\sec(e+fx))^{np}(a+b\sec($$

$$= \frac{b^2\left(c(d\sec(e+fx))^p\right)^n(a+b\sec(e+fx))\tan(e+fx)}{f(2+np)} + \frac{\left((d\sec(e+f\right.}{}$$

$$= \frac{b^2\left(c(d\sec(e+fx))^p\right)^n(a+b\sec(e+fx))\tan(e+fx)}{f(2+np)} + \frac{\left((d\sec(e+f\right.}{}$$

$$= \frac{ab^2(5+2np)\left(c(d\sec(e+fx))^p\right)^n\tan(e+fx)}{f(1+np)(2+np)} + \frac{b^2\left(c(d\sec(e+fx))^p\right)}{}$$

$$= \frac{b\left(b^2(1+np)+3a^2(2+np)\right)\,{}_2F_1\left(\frac{1}{2},-\frac{np}{2};\frac{1}{2}(2-np);\cos^2(e+fx)\right)\left(c(d\right.}{fnp(2+np)\sqrt{\sin^2(e+fx)}}$$

$$= \frac{b\left(b^2(1+np)+3a^2(2+np)\right)\,{}_2F_1\left(\frac{1}{2},-\frac{np}{2};\frac{1}{2}(2-np);\cos^2(e+fx)\right)\left(c(d\right.}{fnp(2+np)\sqrt{\sin^2(e+fx)}}$$

Mathematica [A] time = 1.05196, size = 278, normalized size = 0.94

$$-\frac{\left(-\tan^2(e+fx)\right)^{3/2}\csc^3(e+fx)\left(c(d\sec(e+fx))^p\right)^n\left(bnp\left((np+2)\left(3a^2(np+3)\cos^2(e+fx)\text{Hypergeometric2F1}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x])^3,x]
```

```
[Out] -((Csc[e + f*x]^3*(a^3*(6 + 11*n*p + 6*n^2*p^2 + n^3*p^3)*Cos[e + f*x]^3*Hy
pergeometric2F1[1/2, (n*p)/2, 1 + (n*p)/2, Sec[e + f*x]^2] + b*n*p*(3*a*b*(
3 + 4*n*p + n^2*p^2)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1 + (n*p)/2, 2 + (
n*p)/2, Sec[e + f*x]^2] + (2 + n*p)*(3*a^2*(3 + n*p)*Cos[e + f*x]^2*Hyperge
ometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sec[e + f*x]^2] + b^2*(1 + n*p)*H
ypergeometric2F1[1/2, (3 + n*p)/2, (5 + n*p)/2, Sec[e + f*x]^2])))*(c*(d*Se
c[e + f*x])^p)^n*(-Tan[e + f*x]^2)^(3/2))/(f*n*p*(1 + n*p)*(2 + n*p)*(3 + n
*p)))
```

Maple [F] time = 0.163, size = 0, normalized size = 0.

$$\int \left(c \left(d \sec \left(fx+e\right)\right)^{p}\right)^{n} \left(a+b \sec \left(fx+e\right)\right)^{3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^3,x)
```

```
[Out] int((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^3,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b^{3} \sec \left(fx+e\right)^{3}+3\,ab^{2} \sec \left(fx+e\right)^{2}+3\,a^{2}b \sec \left(fx+e\right)+a^{3}\right) \left(\left(d \sec \left(fx+e\right)\right)^{p} c\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*sec(f*x + e)^3 + 3*a*b^2*sec(f*x + e)^2 + 3*a^2*b*sec(f*x + e
) + a^3)*((d*sec(f*x + e))^p*c)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c \left(d \sec \left(e+fx\right)\right)^{p}\right)^{n} \left(a+b \sec \left(e+fx\right)\right)^{3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))**p)**n*(a+b*sec(f*x+e))**3,x)
```

```
[Out] Integral((c*(d*sec(e + f*x))**p)**n*(a + b*sec(e + f*x))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sec\left(fx + e\right) + a\right)^{3} \left(\left(d \sec\left(fx + e\right)\right)^{p} c\right)^{n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e) + a)^3*((d*sec(f*x + e))^p*c)^n, x)
```

3.238 $\int \left(c(d\sec(e+fx))^p\right)^n (a+b\sec(e+fx))^2\,dx$

Optimal. Leaf size=211

$$-\frac{\left(a^2(np+1)+b^2np\right)\sin(e+fx)\cos(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1-np),\frac{1}{2}(3-np),\cos^2(e+fx)\right)\left(c(d\sec(e+f\right.}{f\left(1-n^2p^2\right)\sqrt{\sin^2(e+fx)}}$$

```
[Out] (2*a*b*Hypergeometric2F1[1/2, -(n*p)/2, (2 - n*p)/2, Cos[e + f*x]^2]*(c*(d*
Sec[e + f*x])^p)^n*Sin[e + f*x])/(f*n*p*Sqrt[Sin[e + f*x]^2]) - ((b^2*n*p +
 a^2*(1 + n*p))*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 - n*p)/2, (3 - n*p)/
2, Cos[e + f*x]^2]*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x])/(f*(1 - n^2*p^2)*
Sqrt[Sin[e + f*x]^2]) + (b^2*(c*(d*Sec[e + f*x])^p)^n*Tan[e + f*x])/(f*(1 +
 n*p))
```

Rubi [A] time = 0.242389, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {3948, 3788, 3772, 2643, 4046}

$$-\frac{\left(a^2(np+1)+b^2np\right)\sin(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1-np);\frac{1}{2}(3-np);\cos^2(e+fx)\right)\left(c(d\sec(e+fx))^p\right)^n}{f\left(1-n^2p^2\right)\sqrt{\sin^2(e+fx)}}+\frac{2ab\sin(e}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x])^2,x]
```


```
[Out] (2*a*b*Hypergeometric2F1[1/2, -(n*p)/2, (2 - n*p)/2, Cos[e + f*x]^2]*(c*(d*
Sec[e + f*x])^p)^n*Sin[e + f*x])/(f*n*p*Sqrt[Sin[e + f*x]^2]) - ((b^2*n*p +
 a^2*(1 + n*p))*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 - n*p)/2, (3 - n*p)/
2, Cos[e + f*x]^2]*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x])/(f*(1 - n^2*p^2)*
Sqrt[Sin[e + f*x]^2]) + (b^2*(c*(d*Sec[e + f*x])^p)^n*Tan[e + f*x])/(f*(1 +
 n*p))
```


Rule 3948

```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c^IntPart[n]*(c*(d*Sec[e + f*x
])^p)^FracPart[n])/(d*Sec[e + f*x])^(p*FracPart[n]), Int[(a + b*Sec[e + f*x
])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
```

```
] &&  !IntegerQ[n]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \left(c(d\sec(e+fx))^p\right)^n (a+b\sec(e+fx))^2\,dx = \left((d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\right)\int (d\sec(e+fx))^{np}(a+b\sec(e$$

$$= \left((d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\right)\int (d\sec(e+fx))^{np}\left(a^2+b^2\sec\right.$$

$$= \frac{b^2\left(c(d\sec(e+fx))^p\right)^n\tan(e+fx)}{f(1+np)} + \frac{\left(2ab\left(\frac{\cos(e+fx)}{d}\right)^{np}\left(c(d\sec(e+fx)\right.\right.}{d}$$

$$= \frac{2ab\,{}_2F_1\left(\frac{1}{2},-\frac{np}{2};\frac{1}{2}(2-np);\cos^2(e+fx)\right)\left(c(d\sec(e+fx))^p\right)^n\sin(e+fx}{fnp\sqrt{\sin^2(e+fx)}}$$

$$= \frac{2ab\,{}_2F_1\left(\frac{1}{2},-\frac{np}{2};\frac{1}{2}(2-np);\cos^2(e+fx)\right)\left(c(d\sec(e+fx))^p\right)^n\sin(e+fx}{fnp\sqrt{\sin^2(e+fx)}}$$

Mathematica [A] time = 0.492597, size = 200, normalized size = 0.95

$$\frac{\sqrt{-\tan^2(e+fx)}\csc(e+fx)\sec(e+fx)\left(c(d\sec(e+fx))^p\right)^n\left(a^2\left(n^2p^2+3np+2\right)\cos^2(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x])^2,x]
```

```
[Out] (Csc[e + f*x]*(a^2*(2 + 3*n*p + n^2*p^2)*Cos[e + f*x]^2*Hypergeometric2F1[1
/2, (n*p)/2, 1 + (n*p)/2, Sec[e + f*x]^2] + b*n*p*(b*(1 + n*p)*Hypergeometr
ic2F1[1/2, 1 + (n*p)/2, 2 + (n*p)/2, Sec[e + f*x]^2] + 2*a*(2 + n*p)*Cos[e
+ f*x]*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sec[e + f*x]^2]))*S
ec[e + f*x]*(c*(d*Sec[e + f*x])^p)^n*Sqrt[-Tan[e + f*x]^2])/(f*n*p*(1 + n*p
)*(2 + n*p))
```

Maple [F] time = 0.15, size = 0, normalized size = 0.

$$\int \left(c\left(d\sec\left(fx+e\right)\right)^p\right)^n\left(a+b\sec\left(fx+e\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^2,x)
```

```
[Out] int((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)+a\right)^2\left(\left(d\sec\left(fx+e\right)\right)^p c\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e) + a)^2*((d*sec(f*x + e))^p*c)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b^2\sec\left(fx+e\right)^2+2\,ab\sec\left(fx+e\right)+a^2\right)\left(\left(d\sec\left(fx+e\right)\right)^p c\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(f*x + e)^2 + 2*a*b*sec(f*x + e) + a^2)*((d*sec(f*x + e))^
p*c)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c\left(d\sec\left(e+fx\right)\right)^p\right)^n\left(a+b\sec\left(e+fx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))**p)**n*(a+b*sec(f*x+e))**2,x)
```

```
[Out] Integral((c*(d*sec(e + f*x))**p)**n*(a + b*sec(e + f*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)+a\right)^{2}\left(\left(d\sec\left(fx+e\right)\right)^{p}c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e) + a)^2*((d*sec(f*x + e))^p*c)^n, x)
```

3.239 $\int \left(c(d\sec(e+fx))^p\right)^n (a+b\sec(e+fx))\,dx$

Optimal. Leaf size=156

$$\frac{b\sin(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{np}{2},\frac{1}{2}(2-np),\cos^2(e+fx)\right)\left(c(d\sec(e+fx))^p\right)^n}{fnp\sqrt{\sin^2(e+fx)}} - \frac{a\sin(e+fx)\cos(e+fx)\text{H}}{}$$

```
[Out] (b*Hypergeometric2F1[1/2, -(n*p)/2, (2 - n*p)/2, Cos[e + f*x]^2]*(c*(d*Sec[
e + f*x])^p)^n*Sin[e + f*x])/(f*n*p*Sqrt[Sin[e + f*x]^2]) - (a*Cos[e + f*x]
*Hypergeometric2F1[1/2, (1 - n*p)/2, (3 - n*p)/2, Cos[e + f*x]^2]*(c*(d*Sec
[e + f*x])^p)^n*Sin[e + f*x])/(f*(1 - n*p)*Sqrt[Sin[e + f*x]^2])
```

Rubi [A] time = 0.143564, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3948, 3787, 3772, 2643}

$$\frac{b\sin(e+fx)\,{}_2F_1\left(\frac{1}{2},-\frac{np}{2};\frac{1}{2}(2-np);\cos^2(e+fx)\right)\left(c(d\sec(e+fx))^p\right)^n}{fnp\sqrt{\sin^2(e+fx)}} - \frac{a\sin(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1-np);\frac{1}{2}(\right.}{f(1-np)\sqrt{\sin}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x]),x]
```

```
[Out] (b*Hypergeometric2F1[1/2, -(n*p)/2, (2 - n*p)/2, Cos[e + f*x]^2]*(c*(d*Sec[
e + f*x])^p)^n*Sin[e + f*x])/(f*n*p*Sqrt[Sin[e + f*x]^2]) - (a*Cos[e + f*x]
*Hypergeometric2F1[1/2, (1 - n*p)/2, (3 - n*p)/2, Cos[e + f*x]^2]*(c*(d*Sec
[e + f*x])^p)^n*Sin[e + f*x])/(f*(1 - n*p)*Sqrt[Sin[e + f*x]^2])
```

Rule 3948

```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c^IntPart[n]*(c*(d*Sec[e + f*x
])^p)^FracPart[n])/(d*Sec[e + f*x])^(p*FracPart[n]), Int[(a + b*Sec[e + f*x
])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] &&  !IntegerQ[n]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \left(c(d\sec(e+fx))^p\right)^n (a+b\sec(e+fx))\,dx = \left((d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\right)\int (d\sec(e+fx))^{np}(a+b\sec(e+$$

$$= \left(a(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\right)\int (d\sec(e+fx))^{np}\,dx + \frac{\left(b(d\,\mathrm{s}\right.}{}$$

$$= \left(a\left(\frac{\cos(e+fx)}{d}\right)^{np}\left(c(d\sec(e+fx))^p\right)^n\right)\int \left(\frac{\cos(e+fx)}{d}\right)^{-np}\,dx + \frac{\left(b\left(\frac{\mathrm{co}}{}\right.\right.}{}$$

$$= \frac{b\,{}_2F_1\left(\frac{1}{2},-\frac{np}{2};\frac{1}{2}(2-np);\cos^2(e+fx)\right)\left(c(d\sec(e+fx))^p\right)^n\sin(e+fx)}{fnp\sqrt{\sin^2(e+fx)}} -$$

Mathematica [A] time = 0.222007, size = 125, normalized size = 0.8

$$\frac{\sqrt{-\tan^2(e+fx)}\csc(e+fx)\left(c(d\sec(e+fx))^p\right)^n\left(a(np+1)\cos(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{np}{2},\frac{np}{2}+1,\sec^2(e+fx\right.\right.}{fnp(np+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x]),x]
```

```
[Out] (Csc[e + f*x]*(a*(1 + n*p)*Cos[e + f*x]*Hypergeometric2F1[1/2, (n*p)/2, 1 +
 (n*p)/2, Sec[e + f*x]^2] + b*n*p*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 +
n*p)/2, Sec[e + f*x]^2])*(c*(d*Sec[e + f*x])^p)^n*Sqrt[-Tan[e + f*x]^2])/(f
*n*p*(1 + n*p))
```

Maple [F] time = 0.133, size = 0, normalized size = 0.

$$\int \left(c\left(d\sec\left(fx+e\right)\right)^{p}\right)^{n}\left(a+b\sec\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e)),x)
```

```
[Out] int((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)+a\right)\left(\left(d\sec\left(fx+e\right)\right)^{p}c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e) + a)*((d*sec(f*x + e))^p*c)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sec\left(fx+e\right)+a\right)\left(\left(d\sec\left(fx+e\right)\right)^{p}c\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e) + a)*((d*sec(f*x + e))^p*c)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c\left(d\sec\left(e+fx\right)\right)^{p}\right)^{n}\left(a+b\sec\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))**p)**n*(a+b*sec(f*x+e)),x)
```

```
[Out] Integral((c*(d*sec(e + f*x))**p)**n*(a + b*sec(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)+a\right)\left(\left(d\sec\left(fx+e\right)\right)^{p}c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e) + a)*((d*sec(f*x + e))^p*c)^n, x)
```

3.240 $\int \frac{(c(d\sec(e+fx))^p)^n}{a+b\sec(e+fx)}\,dx$

Optimal. Leaf size=206

$$\frac{a\sin(e+fx)\cos(e+fx)\cos^2(e+fx)^{\frac{1}{2}(np-1)}\left(c(d\sec(e+fx))^p\right)^n F_1\left(\frac{1}{2};\frac{1}{2}(np-1),1;\frac{3}{2};\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)}-\frac{b\sin}{}$$

```
[Out] -((b*AppellF1[1/2, (n*p)/2, 1, 3/2, Sin[e + f*x]^2, (a^2*Sin[e + f*x]^2)/(a
^2 - b^2)]*(Cos[e + f*x]^2)^((n*p)/2)*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x]
)/((a^2 - b^2)*f)) + (a*AppellF1[1/2, (-1 + n*p)/2, 1, 3/2, Sin[e + f*x]^2,
 (a^2*Sin[e + f*x]^2)/(a^2 - b^2)]*Cos[e + f*x]*(Cos[e + f*x]^2)^((-1 + n*p
)/2)*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x])/((a^2 - b^2)*f)
```

Rubi [A] time = 0.402304, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {3948, 3869, 2823, 3189, 429}

$$\frac{a\sin(e+fx)\cos(e+fx)\cos^2(e+fx)^{\frac{1}{2}(np-1)}\left(c(d\sec(e+fx))^p\right)^n F_1\left(\frac{1}{2};\frac{1}{2}(np-1),1;\frac{3}{2};\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)}-\frac{b\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sec[e + f*x])^p)^n/(a + b*Sec[e + f*x]),x]
```


```
[Out] -((b*AppellF1[1/2, (n*p)/2, 1, 3/2, Sin[e + f*x]^2, (a^2*Sin[e + f*x]^2)/(a
^2 - b^2)]*(Cos[e + f*x]^2)^((n*p)/2)*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x]
)/((a^2 - b^2)*f)) + (a*AppellF1[1/2, (-1 + n*p)/2, 1, 3/2, Sin[e + f*x]^2,
 (a^2*Sin[e + f*x]^2)/(a^2 - b^2)]*Cos[e + f*x]*(Cos[e + f*x]^2)^((-1 + n*p
)/2)*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x])/((a^2 - b^2)*f)
```


Rule 3948

```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c^IntPart[n]*(c*(d*Sec[e + f*x
])^p)^FracPart[n])/(d*Sec[e + f*x])^(p*FracPart[n]), Int[(a + b*Sec[e + f*x
])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] &&  !IntegerQ[n]
```

Rule 3869

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Dist[Sin[e + f*x]^n*(d*Csc[e + f*x])^n, Int[(b +
a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rule 2823

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[a, Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]^
2), x], x] - Dist[b/d, Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] &&  !IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(c(d\sec(e+fx))^p\right)^n}{a+b\sec(e+fx)}\,dx = \left((d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\right)\int \frac{(d\sec(e+fx))^{np}}{a+b\sec(e+fx)}\,dx$$

$$= \left(\cos^{np}(e+fx)\left(c(d\sec(e+fx))^p\right)^n\right)\int \frac{\cos^{1-np}(e+fx)}{b+a\cos(e+fx)}\,dx$$

$$= -\left(\left(a\cos^{np}(e+fx)\left(c(d\sec(e+fx))^p\right)^n\right)\int \frac{\cos^{2-np}(e+fx)}{b^2-a^2\cos^2(e+fx)}\,dx\right) + \left(b\cos^{np}(e+fx)\left(c\right.\right.$$

$$= \frac{\left(b\cos^2(e+fx)^{\frac{np}{2}}\left(c(d\sec(e+fx))^p\right)^n\right)\text{Subst}\left(\int \frac{\left(1-x^2\right)^{-\frac{np}{2}}}{-a^2+b^2+a^2x^2}\,dx, x, \sin(e+fx)\right)}{f} - \frac{\left(a\cos^{np}\right.}{}$$

$$= -\frac{bF_1\left(\frac{1}{2};\frac{np}{2},1;\frac{3}{2};\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos^2(e+fx)^{\frac{np}{2}}\left(c(d\sec(e+fx))^p\right)^n\sin(e+fx)}{\left(a^2-b^2\right)f}$$

Mathematica [B] time = 25.5924, size = 5411, normalized size = 26.27

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*(d*Sec[e + f*x])^p)^n/(a + b*Sec[e + f*x]),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.147, size = 0, normalized size = 0.

$$\int \frac{\left(c\left(d\sec\left(fx+e\right)\right)^p\right)^n}{a+b\sec\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e)),x)
```

```
[Out] int((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(d \sec\left(fx+e\right)\right)^{p} c\right)^{n}}{b \sec\left(fx+e\right)+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(((d*sec(f*x + e))^p*c)^n/(b*sec(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(\left(d \sec\left(fx+e\right)\right)^{p} c\right)^{n}}{b \sec\left(fx+e\right)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(((d*sec(f*x + e))^p*c)^n/(b*sec(f*x + e) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c \left(d \sec\left(e+fx\right)\right)^{p}\right)^{n}}{a+b \sec\left(e+fx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))**p)**n/(a+b*sec(f*x+e)),x)
```

```
[Out] Integral((c*(d*sec(e + f*x))**p)**n/(a + b*sec(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(d\sec\left(fx+e\right)\right)^{p}c\right)^{n}}{b\sec\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(((d*sec(f*x + e))^p*c)^n/(b*sec(f*x + e) + a), x)
```

3.241 $\int \frac{\left(c(d\sec(e+fx))^p\right)^n}{(a+b\sec(e+fx))^2}\,dx$

Optimal. Leaf size=322

$$-\frac{2ab\sin(e+fx)\cos^2(e+fx)^{\frac{np}{2}}\left(c(d\sec(e+fx))^p\right)^n F_1\left(\frac{1}{2};\frac{1}{2}(np-2),2;\frac{3}{2};\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)^2}+\frac{a^2\sin(e+fx)\cos}{}$$

```
[Out] (-2*a*b*AppellF1[1/2, (-2 + n*p)/2, 2, 3/2, Sin[e + f*x]^2, (a^2*Sin[e + f*
x]^2)/(a^2 - b^2)]*(Cos[e + f*x]^2)^((n*p)/2)*(c*(d*Sec[e + f*x])^p)^n*Sin[
e + f*x])/((a^2 - b^2)^2*f) + (a^2*AppellF1[1/2, (-3 + n*p)/2, 2, 3/2, Sin[
e + f*x]^2, (a^2*Sin[e + f*x]^2)/(a^2 - b^2)]*Cos[e + f*x]*(Cos[e + f*x]^2)
^((-1 + n*p)/2)*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x])/((a^2 - b^2)^2*f) +
(b^2*AppellF1[1/2, (-1 + n*p)/2, 2, 3/2, Sin[e + f*x]^2, (a^2*Sin[e + f*x]^
2)/(a^2 - b^2)]*Cos[e + f*x]*(Cos[e + f*x]^2)^((-1 + n*p)/2)*(c*(d*Sec[e +
f*x])^p)^n*Sin[e + f*x])/((a^2 - b^2)^2*f)
```

Rubi [A] time = 0.559336, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {3948, 3869, 2824, 3189, 429}

$$-\frac{2ab\sin(e+fx)\cos^2(e+fx)^{\frac{np}{2}}\left(c(d\sec(e+fx))^p\right)^n F_1\left(\frac{1}{2};\frac{1}{2}(np-2),2;\frac{3}{2};\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)^2}+\frac{a^2\sin(e+fx)\cos}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sec[e + f*x])^p)^n/(a + b*Sec[e + f*x])^2,x]
```

```
[Out] (-2*a*b*AppellF1[1/2, (-2 + n*p)/2, 2, 3/2, Sin[e + f*x]^2, (a^2*Sin[e + f*
x]^2)/(a^2 - b^2)]*(Cos[e + f*x]^2)^((n*p)/2)*(c*(d*Sec[e + f*x])^p)^n*Sin[
e + f*x])/((a^2 - b^2)^2*f) + (a^2*AppellF1[1/2, (-3 + n*p)/2, 2, 3/2, Sin[
e + f*x]^2, (a^2*Sin[e + f*x]^2)/(a^2 - b^2)]*Cos[e + f*x]*(Cos[e + f*x]^2)
^((-1 + n*p)/2)*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x])/((a^2 - b^2)^2*f) +
(b^2*AppellF1[1/2, (-1 + n*p)/2, 2, 3/2, Sin[e + f*x]^2, (a^2*Sin[e + f*x]^
2)/(a^2 - b^2)]*Cos[e + f*x]*(Cos[e + f*x]^2)^((-1 + n*p)/2)*(c*(d*Sec[e +
f*x])^p)^n*Sin[e + f*x])/((a^2 - b^2)^2*f)
```

Rule 3948

```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c^IntPart[n]*(c*(d*Sec[e + f*x
])^p)^FracPart[n])/(d*Sec[e + f*x])^(p*FracPart[n]), Int[(a + b*Sec[e + f*x
])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] &&  !IntegerQ[n]
```

Rule 3869

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Dist[Sin[e + f*x]^n*(d*Csc[e + f*x])^n, Int[(b +
a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rule 2824

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^n/((a - b*sin[e +
 f*x])^m/(a^2 - b^2*sin[e + f*x]^2)^m), x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && NeQ[a^2 - b^2, 0] && ILtQ[m, -1]
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] &&  !IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(c(d\sec(e+fx))^p\right)^n}{(a+b\sec(e+fx))^2}\,dx = \left((d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\right)\int \frac{(d\sec(e+fx))^{np}}{(a+b\sec(e+fx))^2}\,dx$$

$$= \left(\cos^{np}(e+fx)\left(c(d\sec(e+fx))^p\right)^n\right)\int \frac{\cos^{2-np}(e+fx)}{(b+a\cos(e+fx))^2}\,dx$$

$$= \left(\cos^{np}(e+fx)\left(c(d\sec(e+fx))^p\right)^n\right)\int\left(\frac{b^2\cos^{2-np}(e+fx)}{\left(b^2-a^2\cos^2(e+fx)\right)^2}-\frac{2ab\cos^{3-np}(e+fx)}{\left(b^2-a^2\cos^2(e+fx)\right)^2}+$$

$$= \left(a^2\cos^{np}(e+fx)\left(c(d\sec(e+fx))^p\right)^n\right)\int \frac{\cos^{4-np}(e+fx)}{\left(-b^2+a^2\cos^2(e+fx)\right)^2}\,dx-\left(2ab\cos^{np}(e+fx)\right.$$

$$= -\frac{\left(2ab\cos^2(e+fx)^{\frac{np}{2}}\left(c(d\sec(e+fx))^p\right)^n\right)\operatorname{Subst}\left(\int\frac{\left(1-x^2\right)^{\frac{1}{2}(2-np)}}{\left(-a^2+b^2+a^2x^2\right)^2}\,dx,x,\sin(e+fx)\right)}{f}+\frac{\left(a^2\right.}{}$$

$$= -\frac{2abF_1\left(\frac{1}{2};\frac{1}{2}(-2+np),2;\frac{3}{2};\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos^2(e+fx)^{\frac{np}{2}}\left(c(d\sec(e+fx))^p\right)^n\mathrm{si}}{\left(a^2-b^2\right)^2f}$$

Mathematica [B] time = 32.8527, size = 10678, normalized size = 33.16

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*(d*Sec[e + f*x])^p)^n/(a + b*Sec[e + f*x])^2,x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.154, size = 0, normalized size = 0.

$$\int \frac{\left(c\left(d\sec\left(fx+e\right)\right)^p\right)^n}{\left(a+b\sec\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e))^2,x)
```

```
[Out] int((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e))^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\left(d\sec\left(fx+e\right)\right)^{p}c\right)^{n}}{b^{2}\sec\left(fx+e\right)^{2}+2\,ab\sec\left(fx+e\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(((d*sec(f*x + e))^p*c)^n/(b^2*sec(f*x + e)^2 + 2*a*b*sec(f*x + e)
+ a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(c\left(d\sec\left(e+fx\right)\right)^{p}\right)^{n}}{\left(a+b\sec\left(e+fx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))**p)**n/(a+b*sec(f*x+e))**2,x)
```

```
[Out] Integral((c*(d*sec(e + f*x))**p)**n/(a + b*sec(e + f*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(d\sec\left(fx+e\right)\right)^{p}c\right)^{n}}{\left(b\sec\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(((d*sec(f*x + e))^p*c)^n/(b*sec(f*x + e) + a)^2, x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
```

```

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
```

```
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
```

```
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
```

```
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
```

```
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
```

```
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
```

```
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(
    flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```
def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
```

```
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0], Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1], Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0], Rational)
                return 1
```

```
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)
```

```
    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
   expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```